COSMOLOGICAL CONSTANT RUNNING - DECOUPLING EFFECTS AND THE HIGGS MASS

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COSMOLOGICAL CONSTANT PROBLEM

Hilbert-Einstein action (neglecting higher order R- and derivative terms):

\[ S_{HE} = \int d^4x \sqrt{-g} \left\{ \mathcal{L}_{SM} + \xi \phi^{\dagger} \phi R - \frac{1}{16\pi G} (R + 2\Lambda_{vac}) \right\} \]

Vacuum energy density

\[ \rho^\text{vac}_\Lambda (\mu) = \frac{\Lambda(\mu)}{8\pi G(\mu)} \]

Induced vacuum energy density

\[ V_0 = -\frac{1}{2}m^2\phi^2 + \frac{1}{8}\lambda\phi^4 \]

\[ \rho_{\text{ind}}(\mu) \equiv V_0(\langle \phi \rangle) = -\frac{m^4(\mu)}{2\lambda(\mu)} \]

\[ \rho_{\text{ind}}(\mu = 246 \text{ GeV}) \sim 10^8 \text{ GeV}^4 \]

\[ \rho_{\text{phys}} = \rho^\text{vac}_\Lambda (\mu_c) + \rho_{\text{ind}}(\mu_c) + \ldots = 10^{-47} \text{ GeV}^4 \]

Today \[ \mu_c = \mathcal{O}(10^{-3}) \text{ eV} \]

ONE NEEDS TO ACCOUNT FOR RUNNING AND FINE TUNNING \(10^{55}\)
At low energies & for $m_{\text{light}} \ll \mu \ll m$ expectations from the dimensional analysis and the decoupling theorem (Appelquist-Carazzone):

$$
\beta(m_{\text{light}}, \frac{\mu}{m}) = a m_{\text{light}}^4 + b \left( \frac{\mu}{m} \right)^2 m^4 + c \left( \frac{\mu}{m} \right)^4 m^4 + d \mu^2 R + \ldots
$$

$$
= \mu^2 m^2
$$

[ it could be potentially dangerous for the CC running for large masses $m$ ]

To account properly for the decoupling effects one need to go to the mass-dependent renormalization schemes

(decoupling does not hold in the mass-independent schemes like MS)
TOY MODEL – REAL SCALAR FIELD

\[ L = \frac{1}{2} m^2 \phi^2 + \frac{1}{8} \lambda \phi^4 \]

One-loop effective potential:

\[ V = V_{vac}(\rho_{\Lambda}^{\text{vac}}, m^2, \lambda, \mu) + V_{\text{scalar}}(\phi, m^2, \lambda, \mu) \]

Separate pieces satisfy separate RGE equations:

\[ \frac{dV(\langle \phi \rangle)}{d\mu} = 0 \]
TOY MODEL – MASS INDEPENDENT SCHEME (MS)

\[
\left( \mu \frac{\partial}{\partial \mu} + \beta \lambda \frac{\partial}{\partial \lambda} + \gamma m m^2 \frac{\partial}{\partial m^2} + \beta \rho_{\Lambda}^{\text{vac}} \frac{\partial}{\partial \rho_{\Lambda}^{\text{vac}}} \right) V_{\text{vac}}(m^2, \lambda, \rho_{\Lambda}^{\text{vac}}, \mu) = 0
\]

vacuum part:

\[
\mu \frac{\partial \rho_{\Lambda}^{\text{vac}}}{\partial \mu} \equiv \beta \rho_{\Lambda}^{\text{vac}} = \frac{m^4}{32\pi^2}
\]

induced part:

\[
\left( \mu \frac{\partial}{\partial \mu} + \beta \lambda \frac{\partial}{\partial \lambda} + \gamma m m^2 \frac{\partial}{\partial m^2} \right) V_{\text{scal}}(\langle \phi \rangle, m^2, \lambda, \mu) = 0
\]

valid ONLY in the UV regime! \( \mu \gg m \)
TOY MODEL – MASS INDEPENDENT SCHEME (MS)

\[
\left( \mu \frac{\partial}{\partial \mu} + \beta \frac{\partial}{\partial \lambda} + \gamma m^2 \frac{\partial}{\partial m^2} + \beta \rho_{\lambda}^{\text{vac}} \frac{\partial}{\partial \rho_{\lambda}^{\text{vac}}} \right) V_{\text{vac}}(m^2, \lambda, \rho_{\lambda}^{\text{vac}}, \mu) = 0
\]

vacuum part:

\[
\mu \frac{\partial \rho_{\lambda}^{\text{vac}}}{\partial \mu} \equiv \beta \rho_{\lambda}^{\text{vac}} = \frac{m^4}{32\pi^2}
\]

\[\text{MS}\]

induced part:

\[
\left( \mu \frac{\partial}{\partial \mu} + \beta \frac{\partial}{\partial \lambda} + \gamma m^2 \frac{\partial}{\partial m^2} \right) V_{\text{scal}}(\langle \phi \rangle, m^2, \lambda, \mu) = 0
\]

\[
\mu \frac{\partial \rho_{\text{ind}}(\mu)}{\partial \mu} = \mu \frac{\partial}{\partial \mu} \left( -\frac{m^4(\mu)}{2\lambda(\mu)} \right) = \rho_{\text{ind}}(\mu) \left( 2\frac{\beta m^2}{m^2} - \frac{\beta}{\lambda} \right)
\]

valid ONLY in the UV regime! \[\mu \gg m\]
TOY MODEL – MASS DEPENDENT SCHEME (MOM)

Example:

\[ A(p^2)_{\text{MS}} = A(p^2) + \text{c.t.} = \frac{1}{32\pi^2} \int_0^1 dx \log \left( \frac{m^2 - x(1-x)p^2}{\mu_{\text{MS}}^2} \right) \]

\[ A(p^2)_{\text{MOM}} = A(p^2) + \text{c.t.} = \frac{1}{32\pi^2} \int_0^1 dx \log \left( \frac{m^2 - x(1-x)p^2}{m^2 + x(1-x)\mu^2} \right) \]

Similarly one can derive the vacuum part in the MOM scheme:

\[ \mu \frac{\partial \rho_{\text{ind}}(\mu)}{\partial \mu} = \rho_{\text{ind}}(\mu) \left( 2\frac{\beta m^2}{m^2} - \frac{\beta \lambda}{\lambda} \right) \int_0^1 \frac{x(1-x)\mu^2 dx}{m^2 + x(1-x)\mu^2} \]

\[ \mu \frac{\partial \rho_{\text{vac}}(\mu)}{\partial \mu} \bigg|_{\text{MOM}} = \frac{m^4}{32\pi^2} \int_0^1 \frac{x(1-x)\mu^2 dx}{m^2 + x(1-x)\mu^2} \]

valid in the UV and IR regime!
Contributions of SM particles to the effective potential:

\[
\mu \frac{dV}{d\mu} \sim (\phi^4 [...] + m^2 \phi^2 [...] + m^4 [...] ) = 0
\]

By using the condition

\[
\frac{1}{8} \beta_\lambda - \frac{1}{2} \gamma_\phi \lambda = \sum_i \frac{n_i k_i^2}{32\pi^2}
\]
\[
\frac{1}{2} \gamma_m - \gamma_\phi = \sum_i \frac{n_i k_i k_i^m}{16\pi^2}
\]

calculated RGE give:

\[
\mu \frac{\partial \rho^\text{vac}_\lambda}{\partial \mu} = m^4 \sum_i \frac{n_i (k_i^m)^2}{32\pi^2}
\]

from which follow then CC running
STANDARD MODEL – MOM SCHEME

Contributions of SM particles to the CC running for \( \phi = \langle \phi \rangle \rightarrow \gamma_\phi = 0 \)

\[
\mu \frac{\partial \rho_{\Lambda}^{\text{vac}}(\mu)}{\partial \mu} \bigg|_{\text{MOM}} = \frac{\langle \phi \rangle^4}{32\pi^2} \sum_i n_i \left[ \frac{\kappa_i^m \lambda}{2} \right]^2 \int_0^1 \frac{x(1-x)\mu^2 dx}{(M_{\text{phys}}^2)_i + x(1-x)\mu^2}
\]

\[
\mu \frac{\partial \rho_{\text{ind}}(\mu)}{\partial \mu} \bigg|_{\text{MOM}} = \frac{\langle \phi \rangle^4}{32\pi^2} \sum_i n_i \kappa_i \left[ \kappa_i - \lambda \kappa_i^m \right] \int_0^1 \frac{x(1-x)\mu^2 dx}{(M_{\text{phys}}^2)_i + x(1-x)\mu^2}
\]

\[
M_i^2(\phi) = \kappa_i \phi^2 - m^2 \kappa_i^m \quad \text{background-dependent mass matrix} \quad \langle \phi \rangle^2 = \frac{2m^2}{\lambda}
\]

\[
\mu \frac{\partial (\rho_{\text{ind}} + \rho_{\Lambda}^{\text{vac}})}{\partial \mu} \bigg|_{\text{MOM}} = \sum_i \frac{n_i}{32\pi^2} (M_{\text{phys}}^4)_i \int_0^1 \frac{x(1-x)\mu^2 dx}{(M_{\text{phys}}^2)_i + x(1-x)\mu^2}
\]
\[
\mu \frac{\partial (\rho_{\text{ind}} + \rho_{\Lambda}^{\text{vac}})}{\partial \mu} \bigg|_{\text{MOM}} = \sum_i \frac{n_i}{32\pi^2} (M_{\text{phys}}^4)_i \int_0^1 \frac{x(1-x)\mu^2 dx}{(M_{\text{phys}}^2)_i + x(1-x)\mu^2} 
\]

\[
\int_0^1 \frac{x(1-x)\mu^2 dx}{(M_{\text{phys}}^2)_i + x(1-x)\mu^2} = \left\{ \begin{array}{l}
1 \\
\frac{\mu^2}{6(M_{\text{phys}}^2)_i} - \frac{\mu^4}{30(M_{\text{phys}}^4)_i}
\end{array} \right. 
\]

\[\mu^2 \gg (M_{\text{phys}}^2)_i \quad \text{as before}
\]

\[\mu^2 \ll (M_{\text{phys}}^2)_i \quad \text{DECOUPLING}
\]

\[\rightarrow \quad \text{Appelquist-Carazzone theorem for CC running:}
\]

\[
\mu \frac{\partial (\rho_{\Lambda}^{\text{vac}} + \rho_{\text{ind}})}{\partial \mu} \approx \sum_j \frac{n_j (m_{\text{light}}^4)_j}{32\pi^2} - \frac{\mu^2}{12(4\pi)^2} \left[ 12M_t^2 - 6M_W^2 - 3M_Z^2 - M_H^2 \right] + \frac{\mu^4}{30(4\pi)^2}
\]

(Goldstones cancel in the sum – only physical particles contribute)

(similar to the Veltman condition)

\[\text{THIS PART IS RESPONSIBLE FOR THE CC RUNNING!}
\]

\[12M_t^2 - 6M_W^2 - 3M_Z^2 - M_H^2 \simeq 0? \quad \rightarrow \quad \text{FAILS IN THE SM!}
\]
STANDARD MODEL IN THE CONSTANT CURVATURE SPACE

\[ S_{HE} = \int d^4x \sqrt{-g} \left\{ \mathcal{L}_{SM} + \xi \varphi^\dagger \varphi R - \frac{1}{16\pi G} (R + 2\Lambda_{\text{vac}}) \right\} \]

\[ \mathcal{M}_{i}^2(\langle \phi \rangle) = \kappa_i \langle \phi \rangle^2 - m^2 \kappa_i^m + \left( \kappa_i^R - \frac{1}{6} \right) R \]

\[ \langle \phi \rangle^2 = \frac{2(m^2 - \xi R)}{\lambda} \quad \text{(vacuum is R dependent)} \]

\[ \tilde{M}_{i}^2 = M_{i}^2(\langle \phi \rangle) - 2\kappa_i \frac{\xi R}{\lambda} \quad \text{(masses are R dependent)} \]

Appelquist-Carazzone th. for CC running in the curved space:

\[ \mu \frac{\partial (\rho_{\text{ind}} + \rho_A^{\text{vac}} + \kappa R)}{\partial \mu} \approx \sum_j n_j (\mathcal{M}_{\text{light}}^4)^j \frac{\mu^2}{32\pi^2} + \frac{\mu^2}{12(4\pi)^2} \left( -12\tilde{M}_t^2 + 6\tilde{M}_W^2 + 3\tilde{M}_Z^2 + \tilde{M}_H^2 + \frac{7}{3} R \right) \]
in massless models fine-tuning of Higgs mass and CC are linked
MASSLESS THEORY: SM WITH A REAL SCALAR

\[ V_0 = V_0^{SM} + \lambda_{HS} H^\dagger H S^2 + \frac{\lambda_S}{4} S^4 \]

[ new real scalar particle S ]

Generalized Veltman condition –from the CC running \((M_H = 0 \text{ in the massless theory})\):

\[ 12M_t^2 - 6M_W^2 - 3M_Z^2 - \left( \frac{M_S^2}{2} \right) = 0 \quad \Rightarrow \quad M_S^2(\phi) \approx (550 \text{ GeV})^2 \]

\[ M_S^2(\phi) = \lambda_{HS} \phi^2 \quad \Rightarrow \quad \lambda_{HS}(\mu) = 6y_t^2(\mu) - \frac{9}{4} g^2(\mu) - \frac{3}{4} g'(\mu) \approx 4.8 \]

\( M_H \) is generated by the one-loop Coleman-Weinberg mechanism:

\[ M_H^2 = \frac{3}{8\pi^2} \left[ \frac{1}{16} (3g^4 + 2g^2 g'^2 + g'^4) - y_t^4 + \frac{1}{3} \lambda_{HS}^2 \right] v_{EW}^2 \approx (125 \text{ GeV})^2 \]


connection between the CC running and BSM physics!
we have considered the RG running of the Cosmological Constant

we have showed that only RG running of the total (induced + vacuum) CC exhibits behavior consistent with the decoupling theorem

we have provided generalization to the constant curvature space

we have provided a simple extension of the SM with addition of one massless real scalar where condition of absence of leading RG effect allowed us to predict the (radiative) Higgs mass correctly