COSMOLOGICAL CONSTANT RUNNING - DECOUPLING EFFECTS AND THE HIGGS MASS

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COSMOLOGICAL CONSTANT PROBLEM

Hilbert-Einstein action (neglecting higher order R- and derivative terms):

$$S_{HE} = \int d^4x \sqrt{-g} \left\{ \mathcal{L}_{SM} + \xi\, \varphi^\dagger \varphi\, R - \frac{1}{16\pi G}(R + 2\Lambda_{vac}) \right\}$$

Vacuum energy density

$$\rho_\Lambda^{vac}(\mu) \equiv \frac{\Lambda(\mu)}{8\pi G(\mu)}$$

Induced vacuum energy density

$$V_0 = -\frac{1}{2}m^2\phi^2 + \frac{1}{8}\lambda\phi^4$$

$$\rho_{ind}(\mu) \equiv V_0(\langle\phi\rangle) = -\frac{m^4(\mu)}{2\lambda(\mu)}$$

Higgs condensate contribution

$$\rho_{ind}(\mu = 246\ GeV) \sim\ 10^8\ GeV^4$$

$$\rho_{phys} = \rho_\Lambda^{\text{vac}}(\mu_c) + \rho_{ind}(\mu_c) + \ldots = 10^{-47}\ \text{GeV}^4$$

Today $\mu_c = \mathcal{O}(10^{-3})\, eV$

ONE NEEDS TO ACCOUNT FOR RUNNING AND FINE TUNNING (10^{55})

DECOUPLING EFFECTS

At low energies & for $m_{light} \ll \mu \ll m$ ➡ expectations from the dimensional analysis and the decoupling theorem (Appelquist-Carazzone):

$$\beta\left(m_{light}, \frac{\mu}{m}\right) = a\, m_{light}^4 + b \left(\frac{\mu}{m}\right)^2 m^4 + c \left(\frac{\mu}{m}\right)^4 m^4 + d\, \mu^2 R + \ldots$$

$$= \mu^2 m^2$$

[it could be potentially dangerous for the CC running for large masses m]

To account properly for the decoupling effects one neeed to go to the mass-dependent renormalization schemes

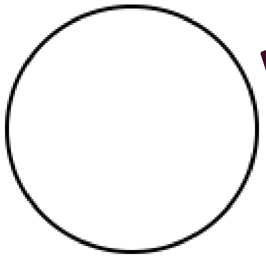
(decoupling does not hold in the mass-independent schemes like MS)

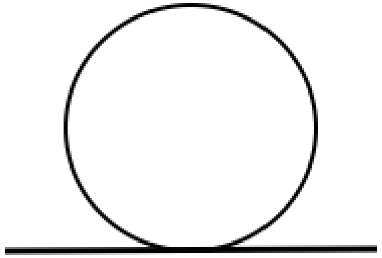
TOY MODEL – REAL SCALAR FIELD

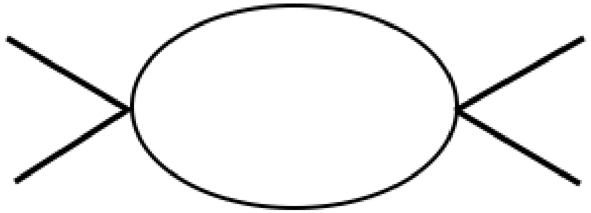
$$L = \frac{1}{2}m^2\phi^2 + \frac{1}{8}\lambda\phi^4$$

One-loop effective potential:

$$V = V_{vac}(\rho_\Lambda^{\text{vac}}, m^2, \lambda, \mu) + V_{scalar}(\phi, m^2, \lambda, \mu)$$





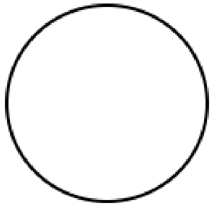


Separate pieces satisfy separate RGE equations: $$\frac{dV(\langle\phi\rangle)}{d\mu} = 0$$

TOY MODEL – MASS INDEPENDENT SCHEME (MS)

$$\left(\mu\frac{\partial}{\partial\mu}+\beta_\lambda\frac{\partial}{\partial\lambda}+\gamma_m m^2\frac{\partial}{\partial m^2}+\beta_{\rho_\Lambda^{\rm vac}}\frac{\partial}{\partial\rho_\Lambda^{\rm vac}}\right)V_{vac}(m^2,\lambda,\rho_\Lambda^{\rm vac},\mu)\;=\;0$$

vacuum part:

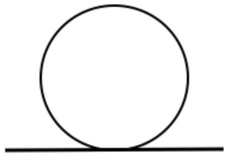


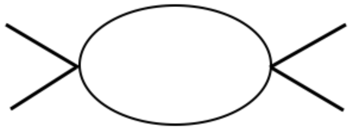
$$\mu\frac{\partial\rho_\Lambda^{\rm vac}}{\partial\mu}\equiv\beta_{\rho_\Lambda^{\rm vac}}=\frac{m^4}{32\pi^2}$$

$\overline{MS}$

$$\left(\mu\frac{\partial}{\partial\mu}+\beta_\lambda\frac{\partial}{\partial\lambda}+\gamma_m m^2\frac{\partial}{\partial m^2}\right)V_{scal}(\langle\phi\rangle,m^2,\lambda,\mu)=0$$

induced part:





$\overline{MS}$

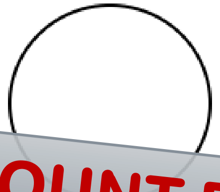
$$\mu\frac{\partial\rho_{ind}(\mu)}{\partial\mu}=\mu\frac{\partial}{\partial\mu}\left(-\frac{m^4(\mu)}{2\lambda(\mu)}\right)=\rho_{ind}(\mu)\;\left(2\frac{\beta_{m^2}}{m^2}-\frac{\beta_\lambda}{\lambda}\right)$$

valid ONLY in the UV regime ! $\mu\gg m$

TOY MODEL – MASS INDEPENDENT SCHEME (MS)

$$\left(\mu\frac{\partial}{\partial\mu}+\beta_\lambda\frac{\partial}{\partial\lambda}+\gamma_m m^2\frac{\partial}{\partial m^2}+\beta_{\rho_\Lambda^{\rm vac}}\frac{\partial}{\partial\rho_\Lambda^{\rm vac}}\right)V_{vac}(m^2,\lambda,\rho_\Lambda^{\rm vac},\mu)\;=\;0$$

vacuum part:



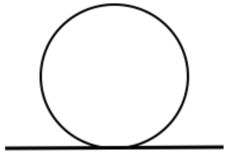
$$\mu\frac{\partial\rho_\Lambda^{\rm vac}}{\partial\mu}\equiv\beta_{\rho_\Lambda^{\rm vac}}=\frac{m^4}{32\pi^2}$$

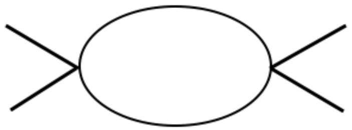
$\overline{MS}$

HOW TO ACCOUNT FOR DECOUPLING OF MASSIVE PARTICLES FOR m $\ll \mu$??

$$\left(\mu\frac{\partial}{\partial\mu}+\beta_\lambda\frac{\partial}{\partial\lambda}+\gamma_m m^2\frac{\partial}{\partial m^2}\right)V_{scal}(\langle\phi\rangle,m^2,\lambda,\mu)=0$$

induced part:





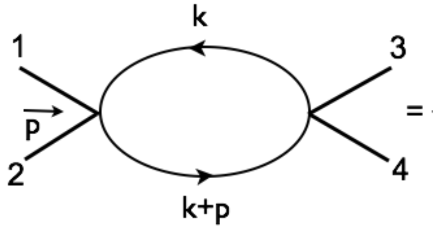
$\overline{MS}$

$$\mu\frac{\partial\rho_{ind}(\mu)}{\partial\mu}=\mu\frac{\partial}{\partial\mu}\left(-\frac{m^4(\mu)}{2\lambda(\mu)}\right)=\rho_{ind}(\mu)\left(2\frac{\beta_{m^2}}{m^2}-\frac{\beta_\lambda}{\lambda}\right)$$

valid ONLY in the UV regime ! $\mu\gg m$

TOY MODEL – MASS DEPENDENT SCHEME (MOM)

Example:



$$= \quad A(p^2)_{\overline{MS}} = A(p^2) + c.t. = \frac{1}{32\pi^2}\int_0^1 dx \ \log\left(\frac{m^2 - x(1-x)p^2}{\mu^2_{\overline{MS}}}\right)$$

$$A(p^2)_{\mathrm{MOM}} = A(p^2) + c.t. = \frac{1}{32\pi^2}\int_0^1 dx \ \log\left(\frac{m^2 - x(1-x)p^2}{m^2 + x(1-x)\mu^2}\right)$$

$$\mu\frac{\partial \rho_{ind}(\mu)}{\partial \mu} = \rho_{ind}(\mu)\ \left(2\frac{\beta_{m^2}}{m^2} - \frac{\beta_\lambda}{\lambda}\right) \int_0^1 \frac{x(1-x)\mu^2 dx}{m^2 + x(1-x)\mu^2}$$

MOM

Similarly one can derive the vacuum part in the MOM scheme:

$$\mu\frac{\partial \rho_\Lambda^{\mathrm{vac}}}{\partial \mu}_{|\mathrm{MOM}} = \frac{m^4}{32\pi^2}\int_0^1 \frac{x(1-x)\mu^2 dx}{m^2 + x(1-x)\mu^2}$$

MOM

valid in the UV and IR regime !

STANDARD MODEL – MOM SCHEME

Contributions of SM particles to the effective potential:

Φ	i	n_i	κ_i	κ_i^m	c_i
$W^\pm$	1	6	$g^2/4$	0	5/6
Z^0	2	3	$(g^2+g'^2)/4$	0	5/6
t	3	−12	$y_t^2/2$	0	3/2
ϕ	4	1	$3\lambda/2$	1	3/2
χ_i	5	3	$\lambda/2$	1	3/2

By using the condition

$$\mu\frac{dV}{d\mu} \sim \left(\ \phi^4[...] + m^2\phi^2[....] + m^4[...]\ \right) = 0$$

calculated RGE give:

$$\frac{1}{8}\beta_\lambda - \frac{1}{2}\gamma_\phi\lambda = \sum_i \frac{n_i\kappa_i^2}{32\pi^2}$$

$$\frac{1}{2}\gamma_m - \gamma_\phi = \sum_i \frac{n_i\kappa_i\kappa_i^m}{16\pi^2}$$

$$\mu\frac{\partial\rho_\Lambda^{vac}}{\partial\mu} = m^4\sum_i \frac{n_i(\kappa_i^m)^2}{32\pi^2}$$

from which follow then CC running ⇨

STANDARD MODEL – MOM SCHEME

Contributions of SM particles to the CC running for $\phi = \langle\phi\rangle \Rightarrow \gamma_\phi = 0$

$$\mu \frac{\partial \rho_\Lambda^{\rm vac}(\mu)}{\partial \mu}_{|\rm MOM} = \frac{\langle\phi\rangle^4}{32\pi^2} \sum_i n_i \left[\frac{\kappa_i^m \lambda}{2}\right]^2 \int_0^1 \frac{x(1-x)\mu^2 dx}{(M^2_{phys})_i + x(1-x)\mu^2}$$

$$\mu \frac{\partial \rho_{ind}(\mu)}{\partial \mu}_{|\rm MOM} = \frac{\langle\phi\rangle^4}{32\pi^2} \sum_i n_i \kappa_i \left[\kappa_i - \lambda \kappa_i^m\right] \int_0^1 \frac{x(1-x)\mu^2 dx}{(M^2_{phys})_i + x(1-x)\mu^2}$$

$M_i^2(\phi) = \kappa_i \phi^2 - m^2 \kappa_i^m$ background-dependent mass matrix $\langle\phi\rangle^2 = \frac{2m^2}{\lambda}$

$$\mu \frac{\partial (\rho_{ind} + \rho_\Lambda^{\rm vac})}{\partial \mu}_{|\rm MOM} = \sum_i \frac{n_i}{32\pi^2} (M^4_{phys})_i \int_0^1 \frac{x(1-x)\mu^2 dx}{(M^2_{phys})_i + x(1-x)\mu^2}$$

STANDARD MODEL – DECOUPLING

$$\mu \frac{\partial(\rho_{ind} + \rho_\Lambda^{\rm vac})}{\partial \mu}_{|\rm MOM} = \sum_i \frac{n_i}{32\pi^2} (M^4_{phys})_i \int_0^1 \frac{x(1-x)\mu^2 dx}{(M^2_{phys})_i + x(1-x)\mu^2}$$

$$\int_0^1 \frac{x(1-x)\mu^2 dx}{(M^2_{phys})_i + x(1-x)\mu^2} = \begin{cases} 1 & \mu^2 \gg (M^2_{phys})_i \quad \text{as before} \\ \frac{\mu^2}{6(M^2_{phys})_i} - \frac{\mu^4}{30(M^4_{phys})_i} & \mu^2 \ll (M^2_{phys})_i \quad \textbf{DECOUPLING} \end{cases}$$

➡ Appelquist-Carazzone theorem for CC running:

$$\mu \frac{\partial(\rho_\Lambda^{\rm vac} + \rho_{ind})}{\partial \mu} \approx \sum_j \frac{n_j (m^4_{light})_j}{32\pi^2} - \frac{\mu^2}{12(4\pi)^2} \left[12M_t^2 - 6M_W^2 - 3M_Z^2 - M_H^2 \right] + \frac{\mu^4}{30(4\pi)^2}$$

(Goldstons cancel in the sum – only physical particles contribute)

(similar to the Veltman condition)

THIS PART IS RESPONSIBLE FOR THE CC RUNNING !

$$12M_t^2 - 6M_W^2 - 3M_Z^2 - M_H^2 \simeq 0?$$ ➡ FAILS IN THE SM !

STANDARD MODEL IN THE CONSTANT CURVATURE SPACE

$$S_{HE} = \int d^4x\sqrt{-g}\left\{ \mathcal{L}_{SM} + \xi\, \varphi^\dagger \varphi\, R - \frac{1}{16\pi G}(R + 2\Lambda_{vac}) \right\}$$

Φ	i	n_i	κ_i	κ_i^m	κ_i^R
$W^\pm$(ghost)	1	−2	$g^2/4$	0	1/2
$W^\pm$	2	8	$g^2/4$	0	1/2
Z^0(ghost)	3	−1	$(g^2+g'^2)/4$	0	1/2
Z^0	4	4	$(g^2+g'^2)/4$	0	1/2
t	5	−12	$y_t^2/2$	0	1/4
ϕ	6	1	$3\lambda/2$	1	1/2
χ_i	7	3	$\lambda/2$	1	1/2

$$\mathcal{M}_i^2(\langle\phi\rangle) = \kappa_i\langle\phi\rangle^2 - m^2\kappa_i^m + \left(\kappa_i^R - \frac{1}{6}\right)R$$

$$\langle\phi\rangle^2 = \frac{2(m^2 - \xi R)}{\lambda}$$ (vacuum is R dependent)

$$\tilde{M}_i^2 = M_i^2(\langle\phi\rangle) - 2\kappa_i\frac{\xi R}{\lambda}$$ (masses are R dependent)

→ Appelquist-Carazzone th. for CC running in the curved space:

$$\mu\frac{\partial(\rho_{ind} + \rho_\Lambda^{\rm vac} + \kappa R)}{\partial\mu} \simeq \sum_j \frac{n_j(\mathcal{M}^4_{light})_j}{32\pi^2} + \frac{\mu^2}{12(4\pi)^2}\left(-12\tilde{M}_t^2 + 6\tilde{M}_W^2 + 3\tilde{M}_Z^2 + \tilde{M}_H^2 + \frac{7}{3}R\right)$$

MASSLESS THEORY

$$\mu \frac{\partial(\rho_{ind} + \rho_\Lambda^{\text{vac}})}{\partial \mu}_{|\text{MOM}} = \sum_i \frac{n_i}{32\pi^2} (M^4_{phys})_i \int_0^1 \frac{x(1-x)\mu^2 dx}{(M^2_{phys})_i + x(1-x)\mu^2}$$

$$\mu \frac{\partial \rho_\Lambda^{\text{vac}}}{\partial \mu} \equiv \beta_{\rho_\Lambda^{\text{vac}}} = \frac{m^4}{32\pi^2}$$ = 0 (m = 0) → $\rho_\Lambda^{\text{vac}}$ IS CONSTANT

$$\mu \frac{\partial \rho_{ind}}{\partial \mu} = \frac{\langle\phi\rangle^4}{32\pi^2} \sum_i n_i \kappa_i^2 \int_0^1 \frac{x(1-x)\mu^2 dx}{(M^2_{phys})_i + x(1-x)\mu^2} = \frac{\mu^2 \langle\phi\rangle^2}{12(4\pi)^2} \sum_i \kappa_i n_i + \frac{\mu^4}{20(4\pi)^2}$$

$$M_i^2(\phi) = \kappa_i \phi^2$$

[simple structure of CC running]

in massless models fine-tuning of Higgs mass and CC are linked →

MASSLESS THEORY: SM WITH A REAL SCALAR

[O.Antipin, M. Mojaza, F. Sannino, arXiv: 1310.0957]

$$V_0 = V_0^{SM} + \lambda_{HS} H^\dagger H S^2 + \frac{\lambda_S}{4} S^4$$

[new real scalar particle S]

Generalized Veltman condition –from the CC running (M_H = 0 in the massless theory):

$$12M_t^2 - 6M_W^2 - 3M_Z^2 - M_S^2 = 0 \quad \Rightarrow \quad M_S^2(\phi) \approx (550\ GeV)^2$$

$$M_S^2(\phi) = \lambda_{HS}\phi^2 \quad \rightarrow \quad \lambda_{HS}(\mu) = 6y_t^2(\mu) - \frac{9}{4}g^2(\mu) - \frac{3}{4}g'^2(\mu) \approx 4.8$$

M_H is generated by the one-loop Coleman-Weinberg mechanism:

$$M_H^2 = \frac{3}{8\pi^2}\left[\frac{1}{16}(3g^4 + 2g^2g'^2 + g'^4) - y_t^4 + \frac{1}{3}\lambda_{HS}^2\right] v_{EW}^2 \approx (125\ GeV)^2$$

prediction!

connection between the CC running and BSM physics !

CONCLUSIONS

- we have considered the RG running of the Cosmological Constant

- we have showed that only RG running of the total (induced + vacuum) CC exhibits behavior consistent with the decoupling theorem

- we have provided generalization to the constant curvature space

- we have provided a simple extension of the SM with addition of one massless real scalar where condition of absence of leading RG effect allowed us to predict the (radiative) Higgs mass correctly