Observable Gravitational Waves From Higgs Inflation in SUGRA

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Based on:

- C.P., *Phys. Rev. D* **92**, no. 12, 121305(R) (2015) [arXiv:1511.01456].
- C.P., *J. Cosmol. Astropart. Phys.* **16**, no. 10, 037 (2016) [arXiv:1606.09607].

Outline

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SUGRA (i.e. Supergravity) Potential

- The General Einstein Frame Action For The Scalar Fields z^α Plus Gravity In Four Dimensional, $\mathcal{N}=1$ SUGRA is:

$$\mathcal{S}=\int d^4x\sqrt{-\widehat{g}}\left(-\frac{1}{2}\widehat{\mathcal{R}}+K_{\alpha\bar\beta}\widehat{g}^{\mu\nu}D_\mu z^\alpha D_\nu z^{*\bar\beta}-\widehat{V}\right)$$ Where We Use Units With $m_{\rm P}$=1.

Also K is The **Kähler Potential** With $K_{\alpha\bar\beta}=\dfrac{\partial^2K}{\partial z^\alpha\partial z^{*\bar\beta}}>0$ and $K^{\bar\beta\alpha}K_{\alpha\bar\gamma}=\delta^{\bar\beta}_{\bar\gamma}$; $D_\mu z^\alpha=\partial_\mu z^\alpha+igA^a_\mu T^a_{\alpha\beta}z^\beta$, Where A^a_μ is The Vector Gauge Fields and T_a are the Generators of the Gauge Transformations Of z^α; Finally, $\widehat{V}=\widehat{V}_{\rm F}+\widehat{V}_{\rm D}$ With $\widehat{V}_{\rm F}=e^K\left(K^{\alpha\bar\beta}{\rm F}_\alpha{\rm F}^*_{\bar\beta}-3|W|^2\right)$ With W The **Superpotential** and ${\rm F}_\alpha=W_{,z^\alpha}+K_{,z^\alpha}W$; $\widehat{V}_{\rm D}=\dfrac{1}{2}g^2{\rm D}_a^2$ with ${\rm D}_a=z_\alpha\,(T_a)^\alpha_\beta\,K_{,z^\beta}$.

- We Concentrate on **Higgs Inflation** (**HI**) Driven by $\widehat{V}_{\rm F}$ Since We Can Easily Assure $\widehat{V}_{\rm D}=0$ During HI.

Therefore, HI Within SUGRA Requires The Appropriate Selection of the Functions W and K

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- **Difficulties And Possible Ways Out**
  - **The η Problem.** Coefficients of Order Unity in K May Spoil The Flatness of $V_{\rm F}$ Due To The Factor e^K. This Can Be Evaded If We Impose A **Shift Symmetry** so That $K=K(\Phi-\Phi^*)=K({\rm Im}(\Phi))$ and the Inflaton be $\phi=\sqrt{2}{\rm Re}(\Phi)$.

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- Complementarily, From Models of **non-Minimal Chaotic Inflation** (**nMI**) in SUGRA We know that $\widehat{V}_{\rm F}$ is Sufficiently Flat, If We Adopt $K=-{\bf N}\ln\left(1+c_{\mathcal{R}}(\Phi^{\bf n}+\Phi^{*{\bf n}})\right)+\cdots$ and Tune $N>0$ and n With The Exponent m of Φ in $W=\lambda S\Phi^{\bf m}$. E.g.,

If we Select $W=\lambda S\Phi^2$ and $K=-2\ln\left(1+2c_{\mathcal{R}}(\Phi^2+\Phi^{*2})\right)-(\Phi-\Phi^*)^2/2+|S|^2$

We Obtain $\widehat{V}_{\rm F}=e^KK^{SS^*}\left|W_{,S}\right|^2=\lambda\phi^4/4(1+c_{\mathcal{R}}\phi^2)^2\sim{\rm const}$ for $c_{\mathcal{R}}\gg1$.

How We can Apply These General Ideas to HI?

Selecting Conveniently the Superpotential and Kähler Potentials

- We Use 3 Superfields $z^1 = \Phi$, $z^2 = \bar{\Phi}$, **Charged** Under a Local Symmetry, e.g. $U(1)_{B-L}$, and $z^3 = S$ (**"Stabilizer" Field**).
  - **Superpotential** $W = \lambda S\left(\bar{\Phi}\Phi - M^2/4\right)$
  - W Is Uniquely Determined Using $U(1)_{B-L}$ and an R Symmetry and Leads to a **Grand Unified Theory (GUT)** Phase Transition

Charge Assignments

Superfields:	S	Φ	$\bar{\Phi}$
$U(1)_R$	1	0	0
$U(1)_{B-L}$	0	1	−1

At The SUSY Vacuum $\langle S\rangle = 0, |\langle\Phi\rangle| = |\langle\bar{\Phi}\rangle| = M/2$,

Since in The SUSY Limit, After HI, We Get $V_{\rm HI} \sim \lambda^2\left|\Phi\bar{\Phi} - M^2/4\right|^2 + \lambda^2|S|^2(|\Phi|^2 + |\bar{\Phi}|^2) + {\rm D-terms}$.

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- **Possible Kähler Potentials – Softly Broken Shift Symmetry For Higgs Fields**
- The Shift Symmetry Can Be Formulated By **The Functions** $F_\pm = \left|\Phi \pm \bar{\Phi}^*\right|^2$ With Coefficients c_+ and c_-, $c_+ \leq c_-$.
- HI can be Obtained Selecting the Following K's Which Are **Quadratic and Invariant** Under $U(1)_{B-L}$ and R Symmetries:

$$K_1 = -N\ln\left(1 + c_+F_+ + F_{1S}(|S|^2)\right) + c_-F_-,\quad K_2 = -N\ln(1 + c_+F_+) + c_-F_- + F_{2S}(|S|^2),$$

$$K_3 = -N\ln(1 + c_+F_+) + F_{3S}(F_-, |S|^2)$$ Where We Choose The Functions[1]

$$F_{1S} = \begin{cases} -\ln(1 + |S|^2/N) \\ \exp\left(-|S|^2/N\right) - 1 \end{cases},\quad F_{2S} = \begin{cases} N_S\ln(1 + |S|^2/N_S) \\ -N_S\left(e^{-|S|^2/N_S} - 1\right) \end{cases}\ \text{And}\ F_{3S} = \begin{cases} N_S\ln(1 + c_-F_-/N_S + |S|^2/N_S) \\ -N_S\left(e^{-(c_-F_-/N_S + |S|^2/N_S)} - 1\right) \end{cases}\ \text{With}\ N, N_S > 0$$

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- The **Free Parameters**, For Fixed n, are $r_{\pm} = c_+/c_-$ and λ/c_- (not c_+, c_- and λ) Since If We Perform the Rescalings

$$\Phi \to \Phi/\sqrt{c_-},\ \bar{\Phi} \to \bar{\Phi}/\sqrt{c_-},\ \text{ and }\ S \to S,$$ we see That W Depends on λ/c_- and K on $r_{\pm}$.

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Inflationary Potential

- If We Use The Parametrization $\Phi = \frac{1}{\sqrt{2}}\phi e^{i\theta}\cos\theta_\Phi$ and $\bar{\Phi} = \frac{1}{\sqrt{2}}\phi e^{i\bar{\theta}}\sin\theta_\Phi$ With $0 \leq \theta_\Phi \leq \frac{\pi}{2}$ and $S = \frac{1}{\sqrt{2}}(s + i\bar{s})$

We Can Show That **A D-Flat Direction** Is $\theta = \bar{\theta} = s = \bar{s} = 0$ and $\theta_\Phi = \pi/4$ **(: I)**

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- The Only **Surviving Term** of $\widehat{V}_{\rm F}$ Along the Path in Eq. (I) is

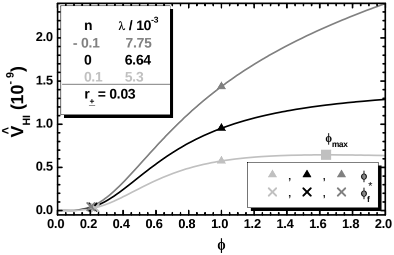
$$\widehat{V}_{\rm HI} = e^K K^{SS^*}\,|W_{,S}|^2 = \frac{\lambda^2(\phi^2 - M^2)^2}{16 f_{\mathcal{R}}^{2(1+n)}} \quad \text{With} \quad f_{\mathcal{R}} = 1 + c_+\phi^2$$

Playing The Role Of A **Non-Minimal Coupling to Gravity**. Also,

$$n = \begin{cases}(N-3)/2 \\ N/2 - 1\end{cases} \quad \text{and} \quad K^{SS^*} = \begin{cases} f_{\mathcal{R}} \\ 1 \end{cases} \quad \text{for} \quad \begin{cases} K = K_1 \\ K = K_{2,3}\end{cases}$$

- For $n > 0$, $\widehat{V}_{\rm HI}$ Develops A Local **Maximum**

$$\widehat{V}_{\rm HI}(\phi_{\rm max}) = \frac{\lambda^2 n^{2n}}{16 c_+^2 (1+n)^{2(1+n)}} \quad \text{at} \quad \phi_{\rm max} = \frac{1}{\sqrt{c_+ n}}$$



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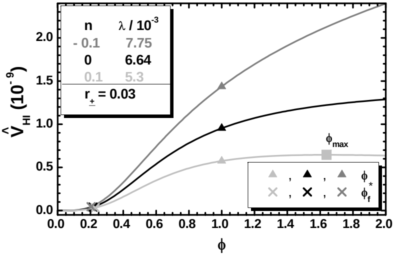

- The EF **Canonically Normalized Fields**, Which Are Denoted By Hat, Can Be Obtained As Follows:

$$\frac{d\widehat\phi}{d\phi} = J = \sqrt{\kappa_+},\ \widehat\theta_+ = \frac{J\phi\theta_+}{\sqrt2},\ \widehat\theta_- = \sqrt{\frac{\kappa_-}{2}}\phi\theta_-,\ \widehat\theta_\Phi = \phi\sqrt{\kappa_-}\left(\theta_\Phi - \frac{\pi}{4}\right) \quad \text{and} \quad (\widehat s, \widehat{\bar s}) = \sqrt{K_{SS^*}}(s, \bar s)\cdot$$

Where $\theta_\pm = (\theta \pm \bar\theta)/\sqrt2$, $\kappa_+ = c_-\left(1 + N r_\pm(c_+\phi^2 - 1)/f_{\mathcal R}^2\right) \simeq c_-$ and $\kappa_- = c_-\left(1 - N r_\pm/f_{\mathcal R}\right) > 0 \quad \Rightarrow \quad$ $\mathbf{r_\pm < 1/N}$.

- We Can Check the **Stability** of the Inflationary Trajectory in Eq. (I) w.r.t the Fluctuations Of The Various Fields, i.e.

$$\left.\frac{\partial V}{\partial\widehat\chi^\alpha}\right|_{\rm Eq.\ (I)} = 0 \quad \text{and} \quad \widehat m^2_{\chi^\alpha} > 0 \quad \text{Where} \quad \widehat m^2_{\chi^\alpha} = \text{Egv}\left[\widehat M^2_{\alpha\beta}\right] \quad \text{With} \quad \widehat M^2_{\alpha\beta} = \left.\frac{\partial^2 V}{\partial\widehat\chi^\alpha\partial\widehat\chi^\beta}\right|_{\rm Eq.\ (I)} \quad \text{and} \quad \chi^\alpha = \theta_-, \theta_+, \theta_\Phi, s, \bar s.$$

Stability and Radiative Corrections

The Mass Spectrum Along The Inflationary Trajectory

Fields	Eingestates	Masses Squared			
			$K=K_1$	$K=K_2$	$K=K_3$
2 Real Scalars	$\widehat{\theta}_+$	$\widehat{m}^2_{\theta+}$	$6\widehat{H}^2_{\rm HI}$		$6(1-1/N_S)\widehat{H}^2_{\rm HI}$
	$\widehat{\theta}_\Phi$	$\widehat{m}^2_{\theta_\Phi}$	$M^2_{BL}+6\widehat{H}^2_{\rm HI}$		$M^2_{BL}+6(1-1/N_S)\widehat{H}^2_{\rm HI}$
1 Complex Scalar	$\widehat{s},\widehat{\bar{s}}$	$\widehat{m}^2_s$	$6c_+\phi^2\widehat{H}^2_{\rm HI}/N$	$6\widehat{H}^2_{\rm HI}/N_S$	
1 Gauge boson	A_{BL}	M^2_{BL}	$g^2c_-\left(1-Nr_\pm/f_{\mathcal{R}}\right)\phi^2$		
4 Weyl Spinors	$\widehat{\psi}_\pm$	$\widehat{m}^2_{\psi\pm}$	$6(c_+(N-3)\phi^2-2)^2\widehat{H}^2_{\rm HI}/c_-\phi^2f^2_{\mathcal{R}}$	$6(c_+(N-2)\phi^2-2)^2\widehat{H}^2_{\rm HI}/c_-\phi^2f^2_{\mathcal{R}}$	
	$\lambda_{BL},\widehat{\psi}_{\Phi-}$	M^2_{BL}	$g^2c_-\left(1-Nr_\pm/f_{\mathcal{R}}\right)\phi^2$		

- $\forall\,\alpha,\ \widehat{m}^2_{\chi^\alpha}>0$. Especially $\widehat{m}^2_s>\widehat{H}^2_{\rm HI}\ \Leftrightarrow\ 0<N<6$ for $K=K_1$ and $0<N_S<6$ for $K=K_{2,3}$;
- $\forall\,\alpha,\ \widehat{m}^2_{\chi^\alpha}>\widehat{H}^2_{\rm HI}$ and So Any Inflationary **Perturbations** Of The Fields Other Than The Inflaton Are Safely **Eliminated**;
- $M_{BL}\neq 0$ Signals The Fact That $U(1)_{B-L}$ **Is Broken** During non-MHI;
- The One-Loop **Radiative Corrections** à la Coleman-Weinberg to $\widehat{V}_{\rm HI}$ Have The Form:

$$\Delta\widehat{V}_{\rm HI}=\frac{1}{64\pi^2}\left(\widehat{m}^4_{\theta+}\ln\frac{\widehat{m}^2_{\theta+}}{\Lambda^2}+2\widehat{m}^4_s\ln\frac{\widehat{m}^2_s}{\Lambda^2}-4\widehat{m}^4_{\psi_\pm}\ln\frac{\widehat{m}^2_{\psi\pm}}{\Lambda^2}\right)$$ Where

- $M^2_{BL}>m^2_{\rm P}$ and $\widehat{m}^2_{\theta_\Phi}>m^2_{\rm P}$ Are not Taken Into Account;
- $\Lambda\simeq(1-5)\cdot10^{14}$ is A Renormalization Group Mass Scale Determined By Requiring $\Delta\widehat{V}_{\rm HI}(\phi_\star)=0$ or $\Delta\widehat{V}_{\rm HI}(\phi_{\rm f})=0$. As a Consequence, $\Delta\widehat{V}_{\rm HI}$ Has **No Significant Effect** On The Results.

Approximating the Inflationary Dynamics

- The **Slow-Roll Parameters** Are Determined Using the Standard Formulae Employing The Canonically Normalized $\widehat{\phi}$:

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$$\text{For}\;\; n=0,\quad \phi_\star\leq 1 \quad\Rightarrow\quad c_-\geq (f_{n\star}-1)/r_\pm \simeq 100,\;\; \text{with}\;\; f_{n\star} = \left(1+16r_\pm\widehat{N}_\star\right)^{1/2} \;\;\text{and}\;\; \widehat{N}_\star\simeq 58.$$

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- A Clear **Dependence of The Observables** (Spectral Index $n_{\rm s}$ and Tensor-To-Scalar Ratio, r) On $r_\pm$ and n Arises, I.e.,

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With Negligible $n_{\rm s}$ Running, $\alpha_{\rm s}$. The Variables With Subscript $\star$ Are Evaluated at $\widehat{\phi}=\widehat{\phi}_\star$.

Ultraviolet (UV) Cut-off Scale ($\Lambda_{\rm UV}$)

- The Implementation Of Our Inflationary Model With $\phi \leq 1$ Requires **Relatively Large c_-'s**. Therefore, We Have To Check If the Resulting Effective Theory Respects Perturbative unitarity up to $m_{\rm P} = 1$, Analyzing The Small-Field Behavior[2] Of the Theory. I.e., We Expand About $\langle\phi\rangle = 0$ the Action $\mathcal{S}$ Along The Inflationary Path

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• This Has to Be Contrasted With the Situation in **Standard non-Minimal HI** Which is Defined for

$$f_{\rm K} = 1 \;\text{ and }\; f_{\mathcal{R}} = 1 + c_{\mathcal{R}}\phi^2 \;\text{ Leading to }\; \langle J\rangle = 1\,.$$

This Results to $\underline{\Lambda_{\rm UV} = m_{\rm P}/c_{\mathcal{R}} \ll m_{\rm P}}$ For $c_{\mathcal{R}} > 1$ Since The Expansions About $\langle\phi\rangle \simeq 0$ Are $c_{\mathcal{R}}$ Dependent, I.E.,

$$J^2\dot{\phi}^2 = \left(1 - c_{\mathcal{R}}\widehat{\phi}^2 + 6c_{\mathcal{R}}^2\widehat{\phi}^2 + c_{\mathcal{R}}^2\widehat{\phi}^4 + \cdots\right)\dot{\widehat{\phi}}^2 \;\text{ and }\; \widehat{V}_{\rm HI} = \frac{\lambda^2\widehat{\phi}^4}{2}\left(1 - 2c_{\mathcal{R}}\widehat{\phi}^2 + 3c_{\mathcal{R}}^2\widehat{\phi}^4 - 4c_{\mathcal{R}}^3\widehat{\phi}^6 + \cdots\right).$$

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Testing Against Observations

• The **Combined** Bicep2/*Keck Array* **and** *Planck* **Results**[3] Although Do Not Exclude Inflationary Models With Negligible r's, They **Seem to Favor** Those With r's of Order 0.01 Since

$$r = 0.028^{+0.026}_{-0.025} \;\; \Rightarrow \;\; 0.003 \lesssim r \lesssim 0.054 \;\; \text{at } 68\% \text{ c.l. And} \;\; r \leq 0.07 \;\; \text{at } 95\% \text{ c.l.}$$

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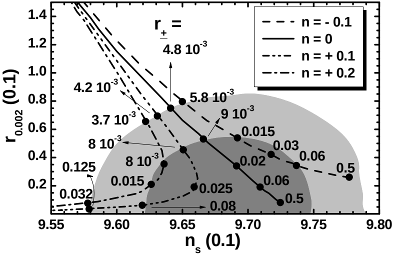

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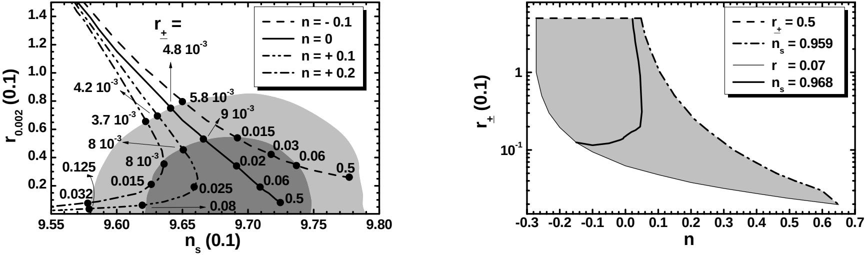

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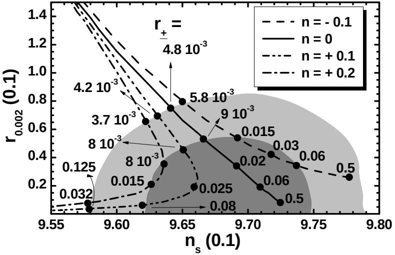

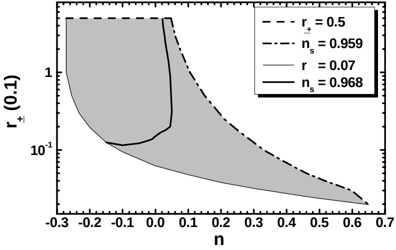

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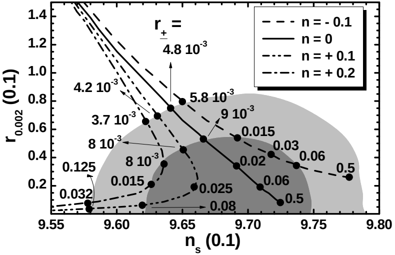

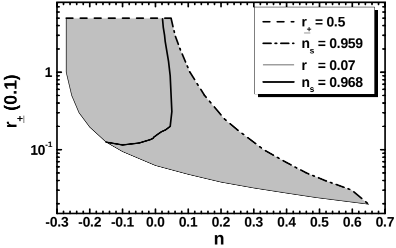

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• **Special Cases:** $(n, r_\pm) = (\mathbf{0}, 0.015) \Rightarrow (n_s, r) = (\mathbf{0.968}, 0.043)$ and $(n, r_\pm) = (0.042, 0.025) \Rightarrow (n_s, r) = (\mathbf{0.968}, \mathbf{0.028})$.

[3] *Planck Collaboration (2015);* Bicep2/*Keck Array and Planck Collaborations (2015)*

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Thank You!

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