

Axion Inflation with Dissipation and Thermalization

Alessio Notari ¹

Universitat de Barcelona

¹ In collaboration with Ricardo Z.Ferreira, and Konrad Tywoniuk

Inflation

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- **Inflation** is the best candidate for initial conditions of our Universe
- Can explain:
 - Observed **small spatial curvature** Ω_k ("flatness problem").
 - Why Universe **causally connected** on large scales ("horizon problem").
 - In addition: **provides the seeds** for observed cosmological density fluctuations.

Inflation

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- In FLRW metric, expansion described by $a(t)$

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2$$

- $a(t) \simeq e^{Ht}$
- Need $N = Ht \gtrsim 60$
- Need fluid with $p \sim -\rho$ dominating energy density

Standard slow-roll Inflation

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- Simple approach: a new scalar field

$$\rho = V(\phi) + \frac{\dot{\phi}^2}{2} \quad p = -V(\phi) + \frac{\dot{\phi}^2}{2} \quad (1)$$

$$(\dot{\phi}^2/2 \ll V)$$

Standard slow-roll Inflation

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- Simple approach: a new scalar field

$$\rho = V(\phi) + \frac{\dot{\phi}^2}{2} \quad p = -V(\phi) + \frac{\dot{\phi}^2}{2} \quad (1)$$

$$(\dot{\phi}^2/2 \ll V)$$

Hubble friction dominates:

$$\ddot{\phi} + 3H\dot{\phi} + V_\phi(\phi) = 0 \quad (2)$$

$$(\ddot{\phi} \ll 3H\dot{\phi})$$

Standard slow-roll Inflation

- Simple approach: a new scalar field

$$\rho = V(\phi) + \frac{\dot{\phi}^2}{2} \quad p = -V(\phi) + \frac{\dot{\phi}^2}{2} \quad (1)$$

$$(\dot{\phi}^2/2 \ll V)$$

Hubble friction dominates:

$$\ddot{\phi} + 3H\dot{\phi} + V_\phi(\phi) = 0 \quad (2)$$

$$(\ddot{\phi} \ll 3H\dot{\phi})$$

- \Rightarrow Slowly rolls for $\left(\frac{a_I}{a_F}\right) = e^N \gtrsim e^{60}$

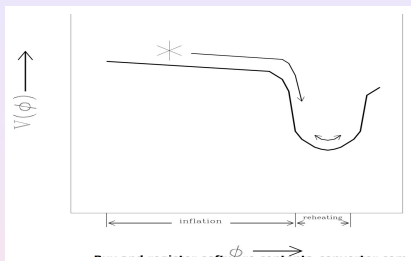
Slow-roll

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

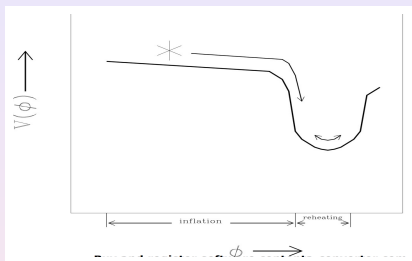
Inflation with
Dissipation

Thermalized Axion
Inflation



- **Slow-roll:** “Vacuum” state
- Quantum fluctuations \Rightarrow **Density fluctuations**

Slow-roll



- **Slow-roll**: "Vacuum" state
- Quantum fluctuations \Rightarrow **Density fluctuations**
- Then fast roll and create particles, thermalization ("**Reheating**")

Slow-roll Inflation: simple but...

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- Which V ?
- Why unusually flat V ?
- Unknown couplings and Reheating process
- Not many observables, beyond cosmological density perturbations (A , n_s , r , perhaps f_{NL} ...)

Slow-roll Inflation: simple but...

- Which V ?
- Why unusually flat V ?
- Unknown couplings and Reheating process
- Not many observables, beyond cosmological density perturbations (A , n_s , r , perhaps f_{NL} ...)
- Interesting to explore non-standard features of inflation:
 - New observables?
 - New dynamics? (not based on flat potentials)

Slow-roll Inflation: simple but...

- Which V ?
- Why unusually flat V ?
- Unknown couplings and Reheating process
- Not many observables, beyond cosmological density perturbations (A , n_s , r , perhaps f_{NL} ...)
- Interesting to explore non-standard features of inflation:
 - New observables?
 - New dynamics? (not based on flat potentials)
 - Temperature already during inflation
 - Strong Dissipation during inflation (friction?)

Inflation with Axial coupling

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- ϕ coupled to **U(1) gauge fields**, “axion-like”:

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + V(\phi) + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\phi}{4f_\gamma} F_{\mu\nu} \tilde{F}^{\mu\nu} \right)$$

Inflation with Axial coupling

- ϕ coupled to **U(1) gauge fields**, “axion-like”:

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + V(\phi) + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\phi}{4f_\gamma} F_{\mu\nu} \tilde{F}^{\mu\nu} \right)$$

- $F_{\mu\nu} \tilde{F}^{\mu\nu}$ odd under CP (and so T)
 \implies **Instability** $\propto \dot{\phi}$

Inflation with Axial coupling

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- ϕ coupled to **U(1) gauge fields**, “axion-like”:

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + V(\phi) + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\phi}{4f_\gamma} F_{\mu\nu} \tilde{F}^{\mu\nu} \right)$$

- $F_{\mu\nu} \tilde{F}^{\mu\nu}$ odd under CP (and so T)
 \implies **Instability** $\propto \dot{\phi}$
- Photons are **massless** at constant ϕ
 \implies **Efficient production**

Inflation with Axial coupling

- ϕ coupled to **U(1) gauge fields**, “axion-like”:

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + V(\phi) + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\phi}{4f_\gamma} F_{\mu\nu} \tilde{F}^{\mu\nu} \right)$$

- $F_{\mu\nu} \tilde{F}^{\mu\nu}$ odd under CP (and so T)
 \implies **Instability** $\propto \dot{\phi}$
- Photons are **massless** at constant ϕ
 \implies **Efficient production**
- Derivative coupling \implies **No corrections to $V(\phi)$**

Inflation with Axial coupling

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- In a time dependent ϕ and in FLRW
(conformal time $ad\tau = dt$, \pm positive (negative) helicity) :

$$A''_{\pm} + \left(k^2 \mp \frac{k\phi'}{f} \right) A_{\pm} = 0,$$

(e.g. I. Tkachev, Pisma Astron.Zh. 12 (1986))

- $\phi' = a\dot{\phi} \neq 0$

Inflation with Axial coupling

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- In a time dependent ϕ and in FLRW
(conformal time $ad\tau = dt$, \pm positive (negative) helicity) :

$$A''_{\pm} + \left(k^2 \mp \frac{k\phi'}{f} \right) A_{\pm} = 0,$$

(e.g. I. Tkachev, Pisma Astron.Zh. 12 (1986))

- $\phi' = a\dot{\phi} \neq 0$
- **One helicity is unstable**: gauge fields become quickly large

Constant $\dot{\phi}$ and de Sitter

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- (Sorbo & Anber '09) assumed: $\dot{\phi} = \text{const}$ in de Sitter

$$a(t) = -\frac{1}{H\tau}$$

$$A''_{\pm} + \left(k^2 \mp \frac{2k\xi}{\tau} \right) A_{\pm} = 0, \quad \xi \equiv \frac{\dot{\phi}}{2fH}$$

Constant $\dot{\phi}$ and de Sitter

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- (Sorbo & Anber '09) assumed: $\dot{\phi} = \text{const}$ in de Sitter
 $a(t) = -\frac{1}{H\tau}$

$$A''_{\pm} + \left(k^2 \mp \frac{2k\xi}{\tau} \right) A_{\pm} = 0, \quad \xi \equiv \frac{\dot{\phi}}{2fH}$$

- Impose **vacuum fluctuations** $A_k = \frac{e^{ik\tau}}{\sqrt{2k}}$ at $\tau \rightarrow -\infty$
(past)
(*Almost, up to a $\ln(\tau)$ phase.)

Constant $\dot{\phi}$ and de Sitter

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- (Sorbo & Anber '09) assumed: $\dot{\phi} = \text{const}$ in de Sitter
 $a(t) = -\frac{1}{H\tau}$

$$A''_{\pm} + \left(k^2 \mp \frac{2k\xi}{\tau} \right) A_{\pm} = 0, \quad \xi \equiv \frac{\dot{\phi}}{2fH}$$

- Impose **vacuum fluctuations** $A_k = \frac{e^{ik\tau}}{\sqrt{2k}}$ at $\tau \rightarrow -\infty$
(past)
(*Almost, up to a $\ln(\tau)$ phase.)
- Solution at $\tau \rightarrow 0^-$ (future):

$$A_+ \approx \frac{1}{\sqrt{2k}} \left(\frac{k|\tau|}{2\xi} \right)^{1/4} e^{\pi\xi - 2\sqrt{2\xi k|\tau|}}$$

Consequences

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- (Sorbo & Anber '09) estimated:

$$\frac{\langle F\tilde{F} \rangle}{4} = \frac{1}{2a^4} \int \frac{d^3k}{(2\pi)^3} k \frac{d[|A_+|^2 - |A_-|^2]}{d\tau} \approx \left(\frac{H}{\xi}\right)^4 e^{2\pi\xi}$$

²Barnaby & Peloso PRL 106 (2011), Barnaby et al. PRD85 (2012), Namba et al. JCAP 1601 (2016).
Ferreira & Sloth, JHEP 1412 (2014) 139. Anber & Sorbo PRD85 (2012) 123537. Lin & Ng (Taiwan, Inst.
Phys.), Phys.Lett. B718 (2013),....

Consequences

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- (Sorbo & Anber '09) estimated:

$$\frac{\langle F\tilde{F} \rangle}{4} = \frac{1}{2a^4} \int \frac{d^3k}{(2\pi)^3} k \frac{d[|A_+|^2 - |A_-|^2]}{d\tau} \approx \left(\frac{H}{\xi}\right)^4 e^{2\pi\xi}$$

$$\rho_\gamma = \frac{1}{2a^4} \int \frac{d^3k}{(2\pi)^3} k \frac{d[|A_+|^2 - |A_-|^2]}{d\tau} \approx \left(\frac{H}{\xi}\right)^4 e^{2\pi\xi}$$

²Barnaby & Peloso PRL 106 (2011), Barnaby et al. PRD85 (2012), Namba et al. JCAP 1601 (2016).
Ferreira & Sloth, JHEP 1412 (2014) 139. Anber & Sorbo PRD85 (2012) 123537. Lin & Ng (Taiwan, Inst.
Phys.), Phys.Lett. B718 (2013),....

Consequences

- (Sorbo & Anber '09) estimated:

$$\frac{\langle F\tilde{F} \rangle}{4} = \frac{1}{2a^4} \int \frac{d^3k}{(2\pi)^3} k \frac{d[|A_+|^2 - |A_-|^2]}{d\tau} \approx \left(\frac{H}{\xi}\right)^4 e^{2\pi\xi}$$

$$\rho_\gamma = \frac{1}{2a^4} \int \frac{d^3k}{(2\pi)^3} k \frac{d[|A_+|^2 - |A_-|^2]}{d\tau} \approx \left(\frac{H}{\xi}\right)^4 e^{2\pi\xi}$$

- **New features:** ²
 - Fields are **not** in the vacuum:
 - Contribution to **2-point** function $\langle \delta\phi\delta\phi \rangle_{loop}$
 - Contribution to **3-point** function $\langle \delta\phi\delta\phi\delta\phi \rangle_{loop}$
 - Contribution to tensors (**gravitational waves**)

²Barnaby & Peloso PRL 106 (2011), Barnaby et al. PRD85 (2012), Namba et al. JCAP 1601 (2016).
Ferreira & Sloth, JHEP 1412 (2014) 139. Anber & Sorbo PRD85 (2012) 123537. Lin & Ng (Taiwan, Inst.
Phys.), Phys.Lett. B718 (2013),....

Consequences

- (Sorbo & Anber '09) estimated:

$$\frac{\langle F\tilde{F} \rangle}{4} = \frac{1}{2a^4} \int \frac{d^3k}{(2\pi)^3} k \frac{d[|A_+|^2 - |A_-|^2]}{d\tau} \approx \left(\frac{H}{\xi}\right)^4 e^{2\pi\xi}$$

$$\rho_\gamma = \frac{1}{2a^4} \int \frac{d^3k}{(2\pi)^3} k \frac{d[|A_+|^2 - |A_-|^2]}{d\tau} \approx \left(\frac{H}{\xi}\right)^4 e^{2\pi\xi}$$

- **New features:** ²
 - Fields are **not** in the vacuum:
 - Contribution to **2-point** function $\langle \delta\phi\delta\phi \rangle_{loop}$
 - Contribution to **3-point** function $\langle \delta\phi\delta\phi\delta\phi \rangle_{loop}$
 - Contribution to tensors (**gravitational waves**)
 - At **large ξ** : **Backreaction** on ϕ dynamics

²Barnaby & Peloso PRL 106 (2011), Barnaby et al. PRD85 (2012), Namba et al. JCAP 1601 (2016).
Ferreira & Sloth, JHEP 1412 (2014) 139. Anber & Sorbo PRD85 (2012) 123537. Lin & Ng (Taiwan, Inst.
Phys.), Phys.Lett. B718 (2013),....

Outline

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

1 Introduction

2 Inflation with Dissipation

- Thermalized Axion Inflation

Thermalized Axion Inflation

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- Include **scattering** of gauge bosons ³:
 - Very large occupation number $N_\gamma \rightarrow$ **scatterings** enhanced $\gamma\gamma \leftrightarrow \gamma\gamma, \gamma\gamma \leftrightarrow \phi\phi, \gamma\phi \leftrightarrow \gamma\phi$

³“Thermalized Axion Inflation” Ricardo Z. Ferreira, A.N.1706.00373

Thermalized Axion Inflation

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- Include **scattering** of gauge bosons ³:
 - Very large occupation number $N_\gamma \rightarrow$ **scatterings** enhanced $\gamma\gamma \leftrightarrow \gamma\gamma, \gamma\gamma \leftrightarrow \phi\phi, \gamma\phi \leftrightarrow \gamma\phi$
 - If **Standard Model gauge fields**: $\gamma\gamma \leftrightarrow \ell^+\ell^-$ (very efficient!)

³“Thermalized Axion Inflation” Ricardo Z. Ferreira, A.N.1706.00373

Thermalized Axion Inflation

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- Include **scattering** of gauge bosons ³:
 - Very large occupation number $N_\gamma \rightarrow$ **scatterings** enhanced $\gamma\gamma \leftrightarrow \gamma\gamma, \gamma\gamma \leftrightarrow \phi\phi, \gamma\phi \leftrightarrow \gamma\phi$
 - If **Standard Model gauge fields**: $\gamma\gamma \leftrightarrow \ell^+\ell^-$ (very efficient!)
 - \implies **Thermalization** during Inflation, with $T > H$

³“Thermalized Axion Inflation” Ricardo Z. Ferreira, A.N.1706.00373

Thermalized Axion Inflation

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- New phenomenology:
 - **Cut-off** of the exponential instability
 - Quasi-**Equilibration between** A_+ and A_-

Thermalized Axion Inflation

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- New phenomenology:
 - **Cut-off** of the exponential instability
 - Quasi-**Equilibration between** A_+ and A_-

$\implies \langle F\tilde{F} \rangle$ reduced!

Thermalized Axion Inflation

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- New phenomenology:
 - **Cut-off** of the exponential instability
 - Quasi-**Equilibration between** A_+ and A_-

 $\implies \langle F\tilde{F} \rangle$ reduced!
 - Moves power of gauge fields IR \rightarrow UV
 $k/a \sim T$, more inside the horizon

Thermalized Axion Inflation

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- New phenomenology:
 - **Cut-off** of the exponential instability
 - Quasi-**Equilibration between** A_+ and A_-
 $\implies \langle F\tilde{F} \rangle$ reduced!
 - Moves power of gauge fields IR \rightarrow UV
 $k/a \sim T$, more inside the horizon
 - Expect: **suppression** of fluctuations in $\langle \delta\phi\delta\phi \rangle_{loop}$,
 $\langle \delta\phi\delta\phi\delta\phi \rangle_{loop}$ at Hubble scale (IR)

Particle production and thermalization

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- **Instability** \Rightarrow particle production of modes: $\frac{k}{a} \lesssim 2\xi H$.
- Instability starts **subhorizon** (if $\xi > 1$) where particle interpretation meaningful.

Particle production and thermalization

- **Instability** \Rightarrow particle production of modes: $\frac{k}{a} \lesssim 2\xi H$.
- Instability starts **subhorizon** (if $\xi > 1$) where particle interpretation meaningful.
- Define **Particle number** per mode k as

$$\frac{\rho_\gamma(k)}{2k} = \frac{A'^2 + k^2 A^2}{2k} \equiv \frac{1}{2} + N_\gamma(k) \quad \Rightarrow$$

Particle production and thermalization

- **Instability** \Rightarrow particle production of modes: $\frac{k}{a} \lesssim 2\xi H$.
- Instability starts **subhorizon** (if $\xi > 1$) where particle interpretation meaningful.
- Define **Particle number** per mode k as

$$\frac{\rho_\gamma(k)}{2k} = \frac{A'^2 + k^2 A^2}{2k} \equiv \frac{1}{2} + N_\gamma(k) \quad \Rightarrow$$

$$\begin{cases} N_\gamma(k) \simeq 0, & k/a \gg H \\ N_\gamma(k) \simeq \frac{e^{2\pi\xi}}{8\pi\xi}, & k/a \ll H \end{cases}$$

Scatterings

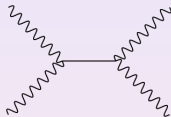
Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

Scatterings are enhanced by powers of N_γ



$$\frac{dN_\gamma(k)}{d\tau} = S(k)$$

$$S = \frac{1}{\omega(k)} \int \prod_{i=2}^4 \left(\frac{d^3 p_i}{(2\pi)^3 (2E_i)} \right) |M_n|^2 (2\pi)^4 \delta^{(4)}(k^\mu + p_2^\mu - p_3^\mu - p_4^\mu) \cdot N_\gamma(k) N_\gamma(p_2) [1 + N_\gamma(p_3)] [1 + N_\gamma(p_4)]$$

Scatterings

Axion Inflation
with
Dissipation
and Thermal-
ization

- Scattering rates $\propto N_\gamma^3 \implies$ For large N_γ :

$$t_{\text{scatterings}} \ll H^{-1} \implies \text{thermalization}$$

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

Scatterings

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- Scattering rates $\propto N_\gamma^3 \implies$ For large N_γ :

$$t_{\text{scatterings}} \ll H^{-1} \implies \text{thermalization}$$

- $S \approx 10^{-4} \frac{\omega^5}{f^4} N_+^3$.

- Compare: $N_+ H \ll S$

Scatterings

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- Scattering rates $\propto N_\gamma^3 \implies$ For large N_γ :

$$t_{\text{scatterings}} \ll H^{-1} \implies \text{thermalization}$$

- $S \approx 10^{-4} \frac{\omega^5}{f^4} N_+^3$.

- Compare: $N_+ H \ll S$

$$N_+ \gg 10^2 \sqrt{\frac{Hf^4}{\omega^5}} \xrightarrow{\omega \approx H} \xi \gtrsim 0.45 \ln \left(\frac{f}{H} \right) + 2.7,$$

(Using $N_+ \approx 10^{-4} e^{4.5\xi}$)

Scatterings

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- Scattering rates $\propto N_\gamma^3 \implies$ For large N_γ :

$$t_{\text{scatterings}} \ll H^{-1} \implies \text{thermalization}$$

- $S \approx 10^{-4} \frac{\omega^5}{f^4} N_+^3$.

- Compare: $N_+ H \ll S$

$$N_+ \gg 10^2 \sqrt{\frac{Hf^4}{\omega^5}} \xrightarrow{\omega \approx H} \xi \gtrsim 0.45 \ln \left(\frac{f}{H} \right) + 2.7,$$

(Using $N_+ \approx 10^{-4} e^{4.5\xi}$)

- Expectation: **thermal bath of photons** with temperature

$$T \approx \rho_{\gamma, \text{initial}}^{1/4} \approx H e^{\pi\xi/2}$$

Boltzmann-like equations

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- We rewrite the eom as a Boltzmann-like eq.
($g \equiv A'_+(k, \tau)/A_+(k, \tau)$) :

$$\frac{dN_+(k)}{d\tau} = -\frac{4k\xi}{\tau} \frac{\text{Re}[g]}{|g|^2 + k^2} \left(N_+(k) + \frac{1}{2} \right)$$

Boltzmann-like equations

- We rewrite the eom as a Boltzmann-like eq.
($g \equiv A'_+(k, \tau)/A_+(k, \tau)$) :

$$\frac{dN_+(k)}{d\tau} = -\frac{4k\xi}{\tau} \frac{\text{Re}[g]}{|g|^2 + k^2} \left(N_+(k) + \frac{1}{2} \right) + S$$

(approximation: g computed without S)

Boltzmann-like equations

- We rewrite the eom as a Boltzmann-like eq.
($g \equiv A'_+(k, \tau)/A_+(k, \tau)$) :

$$\frac{dN_+(k)}{d\tau} = -\frac{4k\xi}{\tau} \frac{\text{Re}[g]}{|g|^2 + k^2} \left(N_+(k) + \frac{1}{2} \right) + S$$

(approximation: g computed without S)

- Full system (γ_+, γ_-, ϕ)

$$\begin{cases} N'_+ = -\frac{4k\xi}{\tau} \frac{\text{Re}[g]}{|g|^2 + k^2} \left(N_+ + \frac{1}{2} \right) + S^{++} + S^{+\phi} + D^{+\phi} + S^{+-}, \\ N'_\phi = -S^{+\phi} - D^{+\phi}, \\ N'_- = -S^{+-}, \end{cases}$$

Numerical results

- Discretize: $\mathcal{O}(10)$ modes of comoving momentum:
 $k \in [1, \mathcal{O}(10)]H$.
- Duration of simulation: $\mathcal{O}(1)$ e-fold, $\{\tau_0 = -2, \tau_f = -1\}$

Numerical results

- Discretize: $\mathcal{O}(10)$ modes of comoving momentum: $k \in [1, \mathcal{O}(10)]H$.
- Duration of simulation: $\mathcal{O}(1)$ e-fold, $\{\tau_0 = -2, \tau_f = -1\}$
- Distribution of particles **approaches Bose-Einstein distribution** at ξ, f in agreement with estimations.

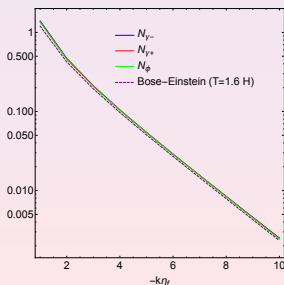


Figure: $\xi = 2, f/H = 0.1$

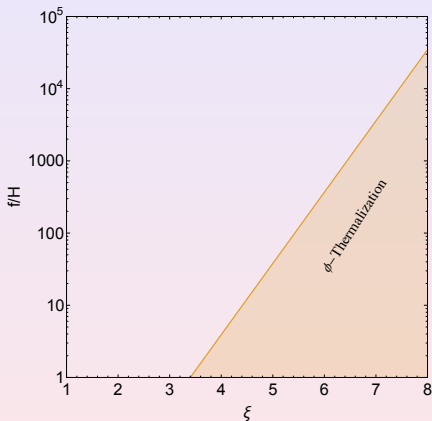
Thermalization

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation



$$\Gamma_s \gg H \quad \Rightarrow \quad \xi \gtrsim 0.45 \ln \left(\frac{f}{H} \right) + 2.7,$$

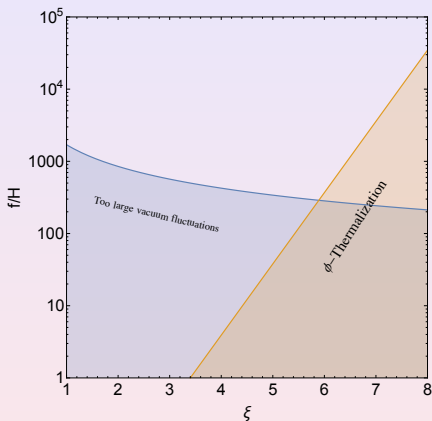
Thermalization

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation



Vacuum fluctuations:

$$P_{\zeta}^{\text{vac}} = \frac{H_*^4}{4\pi^2 \dot{\phi}_*^2} = \frac{H^2}{16\pi^2 f^2 \xi^2} \leq 2.2 \times 10^{-9} \quad \Rightarrow \quad \frac{f}{H} \gtrsim \frac{2 \times 10^3}{\xi}.$$

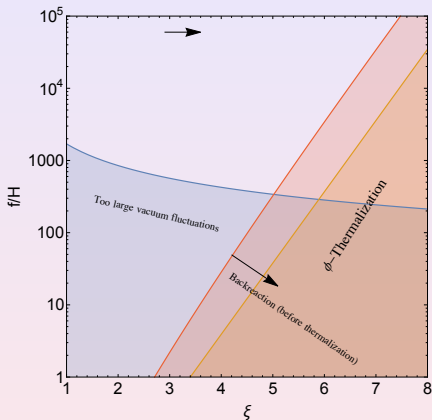
Thermalization

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation



Backreaction on ϕ (before reaching thermalization):

$$\frac{\langle F\tilde{F} \rangle}{f} \gtrsim V'(\phi) \simeq 3H\dot{\phi} \quad \Rightarrow \quad f/H \lesssim 4 \times 10^{-3} e^{\pi\xi} / \xi^{5/2}.$$

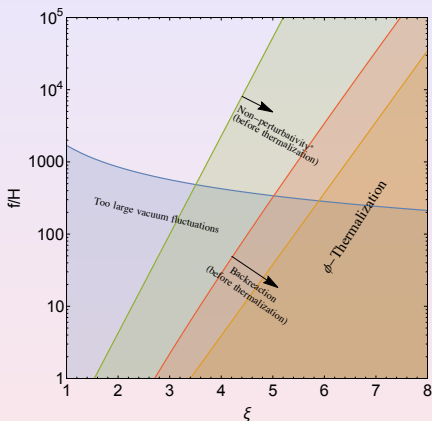
Thermalization

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation



* Requiring perturbativity on loop expansion for cosmological correlators, in absence of thermalization (Ferreira *et al.*, JCAP 1604 (2016)):

$$\frac{H^2}{f^2} \frac{e^{2\pi\xi}}{16\pi^2 l} < 1,$$

Standard Model couplings

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- If gauge fields belong to SM: many other interactions with known couplings ($\gamma\gamma \leftrightarrow e^+e^-, \dots$)

Standard Model couplings

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- If gauge fields belong to SM: many other interactions with known couplings ($\gamma\gamma \leftrightarrow e^+e^-, \dots$)
- **More predictive**, only depends on ξ . Interactions **not suppressed** by powers of $1/f$.

Standard Model couplings

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- If gauge fields belong to SM: many other interactions with known couplings ($\gamma\gamma \leftrightarrow e^+e^-, \dots$)
- **More predictive**, only depends on ξ . Interactions **not suppressed** by powers of $1/f$.
- **More realistic**, the inflaton has anyway to couple efficiently to the SM to reheat the universe.

Standard Model couplings

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- If gauge fields belong to SM: many other interactions with known couplings ($\gamma\gamma \leftrightarrow e^+e^-, \dots$)
- **More predictive**, only depends on ξ . Interactions **not suppressed** by powers of $1/f$.
- **More realistic**, the inflaton has anyway to couple efficiently to the SM to reheat the universe.
- Using $\sigma_{\gamma\gamma \leftrightarrow l^+l^-} \approx \frac{\alpha_{EM}^2}{\omega^2}$

Standard Model couplings

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- If gauge fields belong to SM: many other interactions with known couplings ($\gamma\gamma \leftrightarrow e^+e^-, \dots$)
- **More predictive**, only depends on ξ . Interactions **not suppressed** by powers of $1/f$.
- **More realistic**, the inflaton has anyway to couple efficiently to the SM to reheat the universe.
- Using $\sigma_{\gamma\gamma \leftrightarrow l+l^-} \approx \frac{\alpha_{EM}^2}{\omega^2} \implies$

Requirement for thermalization ($\Gamma_s \gg H$) :

$$N_\gamma H \ll \frac{\alpha_{EM}^2}{\omega^2} \cdot H^3 N_\gamma^2 \implies \xi \gtrsim 2.9$$

SM Thermalization

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

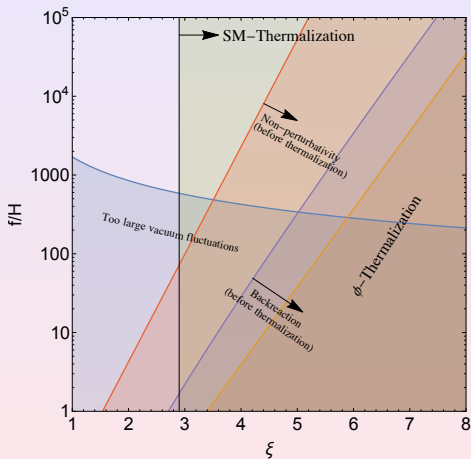


Figure: Summing over all $U_Y(1)$ charged particles in SM.

Enter a new regime...

- **Thermal** gauge field **masses** appear: $m_T \propto gT$

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

Enter a new regime...

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- **Thermal** gauge field **masses** appear: $m_T \propto gT$

$$A''_{\pm} + \omega_T^2(k) A_{\pm} = 0, \quad \omega_T^2(k) = \left(k^2 \pm \frac{2k\xi}{\tau} + \frac{m_T^2}{H^2\tau^2} \right).$$

Enter a new regime...

- **Thermal** gauge field **masses** appear: $m_T \propto gT$

$$A''_{\pm} + \omega_T^2(k) A_{\pm} = 0, \quad \omega_T^2(k) = \left(k^2 \pm \frac{2k\xi}{\tau} + \frac{m_T^2}{H^2\tau^2} \right).$$

- When $m_T \geq \xi H$ completely **shields** the instability band ($\omega^2 > 0$)

Enter a new regime...

- **Thermal** gauge field **masses** appear: $m_T \propto gT$

$$A''_{\pm} + \omega_T^2(k) A_{\pm} = 0, \quad \omega_T^2(k) = \left(k^2 \pm \frac{2k\xi}{\tau} + \frac{m_T^2}{H^2\tau^2} \right).$$

- When $m_T \geq \xi H$ completely **shields** the instability band ($\omega^2 > 0$)
- Expect $T_{eq} \approx \frac{\xi H}{g}$ (or maybe oscillations?)

Enter a new regime...

- **Thermal** gauge field **masses** appear: $m_T \propto gT$

$$A''_{\pm} + \omega_T^2(k) A_{\pm} = 0, \quad \omega_T^2(k) = \left(k^2 \pm \frac{2k\xi}{\tau} + \frac{m_T^2}{H^2\tau^2} \right).$$

- When $m_T \geq \xi H$ completely **shields** the instability band ($\omega^2 > 0$)
- Expect $T_{eq} \approx \frac{\xi H}{g}$ (or maybe oscillations?)
- New regime: linear in ξ not exponential: $T_{eq} \ll e^{\frac{\pi\xi}{2}} H !$

At equilibrium temperature

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

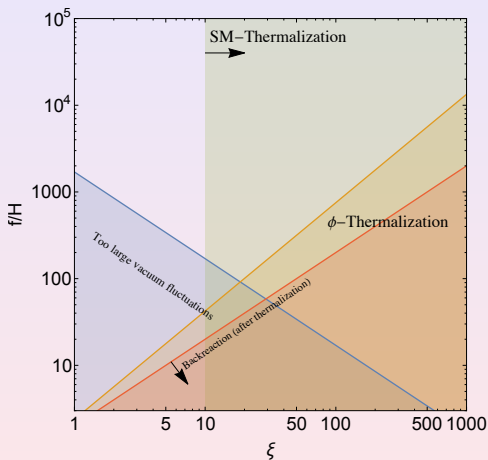


Figure: Using $T_{eq} \approx \frac{\xi H}{g}$, with $g = 0.5$.

Thermal Spectrum of ζ

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- $\zeta \equiv \frac{H}{\dot{\phi}} \delta\phi$ curvature perturbation: assume **conservation superhorizon**
- Its value fixed by vacuum fluctuations at Horizon Crossing

Thermal Spectrum of ζ

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- $\zeta \equiv \frac{H}{\dot{\phi}} \delta\phi$ curvature perturbation: assume **conservation superhorizon**
- Its value fixed by vacuum fluctuations at Horizon Crossing
- If ζ thermalizes

Thermal Spectrum of ζ

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- $\zeta \equiv \frac{H}{\dot{\phi}} \delta\phi$ curvature perturbation: assume **conservation superhorizon**
- Its value fixed by vacuum fluctuations at Horizon Crossing
- If ζ thermalizes
 \implies **not vacuum, but thermal** at Horizon Crossing

Thermal Spectrum of ζ

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- $\zeta \equiv \frac{H}{\dot{\phi}} \delta\phi$ curvature perturbation: assume **conservation superhorizon**
- Its value fixed by vacuum fluctuations at Horizon Crossing
- If ζ thermalizes
 \implies **not vacuum, but thermal** at Horizon Crossing
- $|u_k|^2 = \left| \frac{1}{\sqrt{2k}} \right|^2$

Thermal Spectrum of ζ

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- $\zeta \equiv \frac{H}{\dot{\phi}} \delta\phi$ curvature perturbation: assume **conservation superhorizon**
- Its value fixed by vacuum fluctuations at Horizon Crossing
- If ζ thermalizes
 \implies **not vacuum, but thermal** at Horizon Crossing
- $|u_k|^2 = \left| \frac{1}{\sqrt{2k}} \right|^2 = \frac{1}{k} \cdot \frac{1}{2}$

Thermal Spectrum of ζ

- $\zeta \equiv \frac{H}{\dot{\phi}} \delta\phi$ curvature perturbation: assume **conservation superhorizon**
- Its value fixed by vacuum fluctuations at Horizon Crossing
- If ζ thermalizes
 \implies **not vacuum, but thermal** at Horizon Crossing
- $|u_k|^2 = \left| \frac{1}{\sqrt{2k}} \right|^2 = \frac{1}{k} \cdot \frac{1}{2} \implies \frac{1}{k} \left(\frac{1}{2} + N(k_*) \right)$

Thermal Spectrum of ζ

- $\zeta \equiv \frac{H}{\dot{\phi}} \delta\phi$ curvature perturbation: assume **conservation superhorizon**

- Its value fixed by vacuum fluctuations at Horizon Crossing

- If ζ thermalizes
 \implies **not vacuum, but thermal** at Horizon Crossing

- $|u_k|^2 = \left| \frac{1}{\sqrt{2k}} \right|^2 = \frac{1}{k} \cdot \frac{1}{2} \quad \implies \quad \frac{1}{k} \left(\frac{1}{2} + N(k_*) \right)$

- $N_k = \frac{1}{e^{\frac{k/a}{T}} - 1}$

Thermal Spectrum of ζ

- $\zeta \equiv \frac{H}{\dot{\phi}} \delta\phi$ curvature perturbation: assume **conservation superhorizon**

- Its value fixed by vacuum fluctuations at Horizon Crossing

- If ζ thermalizes
 \implies **not vacuum, but thermal** at Horizon Crossing

- $|u_k|^2 = \left| \frac{1}{\sqrt{2k}} \right|^2 = \frac{1}{k} \cdot \frac{1}{2} \implies \frac{1}{k} \left(\frac{1}{2} + N(k_*) \right)$

- $N_k = \frac{1}{e^{\frac{k/a}{T}} - 1}$. At $\frac{k}{a} = H \implies N_k \approx \frac{T_*}{H_*}$

Thermal Spectrum of ζ

- $\zeta \equiv \frac{H}{\dot{\phi}} \delta\phi$ curvature perturbation: assume **conservation superhorizon**

- Its value fixed by vacuum fluctuations at Horizon Crossing

- If ζ thermalizes
 \implies **not vacuum, but thermal** at Horizon Crossing

- $|u_k|^2 = \left| \frac{1}{\sqrt{2k}} \right|^2 = \frac{1}{k} \cdot \frac{1}{2} \implies \frac{1}{k} \left(\frac{1}{2} + N(k_*) \right)$

- $N_k = \frac{1}{e^{\frac{k/a}{T}} - 1}$. At $\frac{k}{a} = H \implies N_k \approx \frac{T_*}{H_*}$

- $P_\zeta = P_\zeta^{vac} \cdot \frac{2T_*}{H_*}$

Thermal Spectrum of ζ

Axion Inflation
with
Dissipation
and Thermal-
ization

$$\bullet P_{\zeta} = P_{\zeta}^{vac} \cdot \frac{2T_*}{H_*} \implies$$

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

Thermal Spectrum of ζ

Axion Inflation
with
Dissipation
and Thermal-
ization

$$\bullet P_{\zeta} = P_{\zeta}^{vac} \cdot \frac{2T_*}{H_*} \implies$$

$$n_s - 1 \equiv \frac{d \ln P_{\zeta}^{therm}}{d \ln k} = -6\epsilon_H + 2\eta + \frac{d \ln(T_*/H_*)}{d \ln k},$$

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

Thermal Spectrum of ζ

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- $P_{\zeta} = P_{\zeta}^{\text{vac}} \cdot \frac{2T_*}{H_*} \implies$

$$n_s - 1 \equiv \frac{d \ln P_{\zeta}^{\text{therm}}}{d \ln k} = -6\epsilon_H + 2\eta + \frac{d \ln(T_*/H_*)}{d \ln k},$$

- If $T = T_{\text{eq}} = \frac{\xi}{g} H$, \implies

$$\begin{cases} P_{\zeta}^{\text{therm}} = \frac{\xi}{g} \frac{H_*^4}{2\pi^2 \dot{\phi}_*^2} = \frac{H^2}{8\pi^2 f^2 \bar{g} \xi}, \\ n_s - 1 \equiv -6\epsilon_H + 2\eta + \frac{\dot{\xi}}{H\xi} = -4\epsilon_H + \eta. \end{cases}$$

New regime at T_{eq}

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

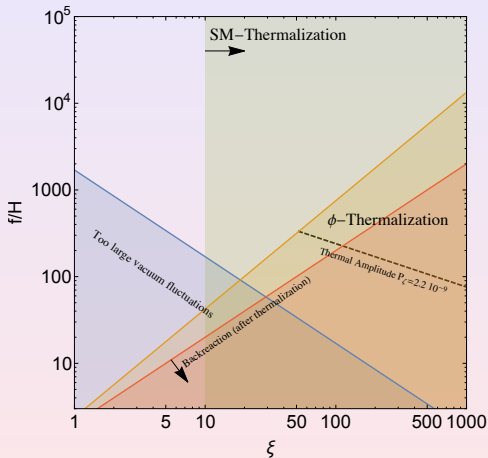


Figure: $P_\zeta = P_\zeta^{vac} \cdot \frac{2T_*}{H_*} = P_\zeta^{vac} \cdot \frac{2\xi}{g}$, ($g = 0.5$.)

Phenomenology in the thermal regime

- Loop effects on ζ **drastically modified!**
- Thermalization shifts gauge fields from horizon size to UV. At horizon crossing N_γ is reduced.
- We expect (work in progress) **much smaller** $\langle \zeta \zeta \zeta \rangle_{\text{loop}}$

- $f_{NL} \simeq \frac{\langle \zeta^3 \rangle}{\langle \zeta^2 \rangle^2}$

Phenomenology in the thermal regime

- Loop effects on ζ **drastically modified!**
- Thermalization shifts gauge fields from horizon size to UV. At horizon crossing N_γ is reduced.
- We expect (work in progress) **much smaller** $\langle \zeta \zeta \zeta \rangle_{\text{loop}}$

$$\bullet f_{NL} \simeq \frac{\langle \zeta^3 \rangle}{\langle \zeta^2 \rangle^2} \propto P_\zeta^{\text{vac}} \mathcal{O} \left(\frac{T^4}{H^4} \right)$$

Phenomenology in the thermal regime

- Loop effects on ζ **drastically modified!**
- Thermalization shifts gauge fields from horizon size to UV. At horizon crossing N_γ is reduced.
- We expect (work in progress) **much smaller** $\langle \zeta \zeta \zeta \rangle_{\text{loop}}$

$$\bullet f_{NL} \simeq \frac{\langle \zeta^3 \rangle}{\langle \zeta^2 \rangle^2} \propto P_\zeta^{\text{vac}} \mathcal{O} \left(\frac{T^4}{H^4} \right) \propto c \xi^4 \text{ (c small number)}$$

Phenomenology in the thermal regime

- Loop effects on ζ **drastically modified!**
- Thermalization shifts gauge fields from horizon size to UV. At horizon crossing N_γ is reduced.
- We expect (work in progress) **much smaller** $\langle \zeta \zeta \zeta \rangle_{\text{loop}}$

- $f_{NL} \simeq \frac{\langle \zeta^3 \rangle}{\langle \zeta^2 \rangle^2} \propto P_\zeta^{\text{vac}} \mathcal{O} \left(\frac{T^4}{H^4} \right) \propto c \xi^4$ (c small number)

- Instead of non-thermal case: $f_{NL} \propto e^{4\pi\xi}$!

Phenomenology in the thermal regime

- Loop effects on ζ **drastically modified!**
- Thermalization shifts gauge fields from horizon size to UV. At horizon crossing N_γ is reduced.
- We expect (work in progress) **much smaller** $\langle \zeta \zeta \zeta \rangle_{\text{loop}}$

- $f_{NL} \simeq \frac{\langle \zeta^3 \rangle}{\langle \zeta^2 \rangle^2} \propto P_\zeta^{\text{vac}} \mathcal{O} \left(\frac{T^4}{H^4} \right) \propto c \xi^4$ (c small number)

- Instead of non-thermal case: $f_{NL} \propto e^{4\pi\xi}$!

- Constraints on ξ become weaker and **(maybe) allow for the backreacting regime?** (Work in progress)

Phenomenology of tensor modes (gravitational waves)

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- Assuming **tensor** modes to be in the **vacuum**:

Phenomenology of tensor modes (gravitational waves)

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- Assuming **tensor** modes to be in the **vacuum**:

$$r \equiv \frac{P_T}{P_{\zeta}^{\text{therm}}} = 16 \epsilon \frac{H_*}{2T_*}.$$

- **Suppressed by $\frac{H_*}{2T_*} = \frac{g}{2\xi}$**
- At least $\mathcal{O}(10^{-2})$ suppression.
- **$r \lesssim 10^{-4}$**

Summary: Thermalized Axion Inflation

Axion Inflation
with
Dissipation
and Thermal-
ization

Introduction

Inflation with
Dissipation

Thermalized Axion
Inflation

- **Thermalization** can be achieved **during inflation**
- More easily in SM case
- **Reheating** is automatic and fixed (when $\rho_\gamma > V(\phi)$)

Summary: Thermalized Axion Inflation

- **Thermalization** can be achieved **during inflation**
- More easily in SM case
- **Reheating** is automatic and fixed (when $\rho_\gamma > V(\phi)$)
- P_ζ can be **thermal**

Summary: Thermalized Axion Inflation

- **Thermalization** can be achieved **during inflation**
- More easily in SM case
- **Reheating** is automatic and fixed (when $\rho_\gamma > V(\phi)$)
- P_ζ can be **thermal**
- *Work in progress:*
 - Inclusion of **Thermal masses** (expect T_{eq})
 - **Non-gaussianity** (expect reduction)
 - **Backreaction** (Large friction without flat potential: maybe phenomenologically possible?)