Detecting quantum gravity in the sky

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Motivations

- Dimensional flow: Changing behaviour of correlation functions, spacetime with scale-dependent dimension ($d_{\rm H}$, $d_{\rm S}$, ...). $d < 4$ in the UV. Universal feature in QG ['t Hooft 1993; Carlip 2009; G.C. PRL 2010] (perturbative QG, asymptotic safety, CDT, HL gravity, noncommutative spacetimes, nonlocal gravity, LQG, spin foams, GFT,...). All QGs are multiscale.

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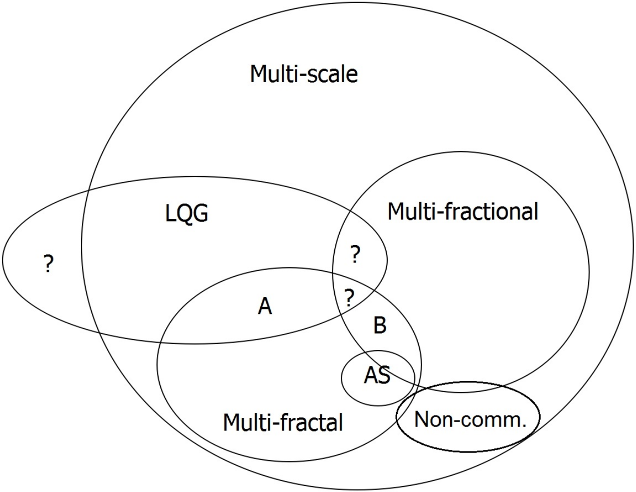
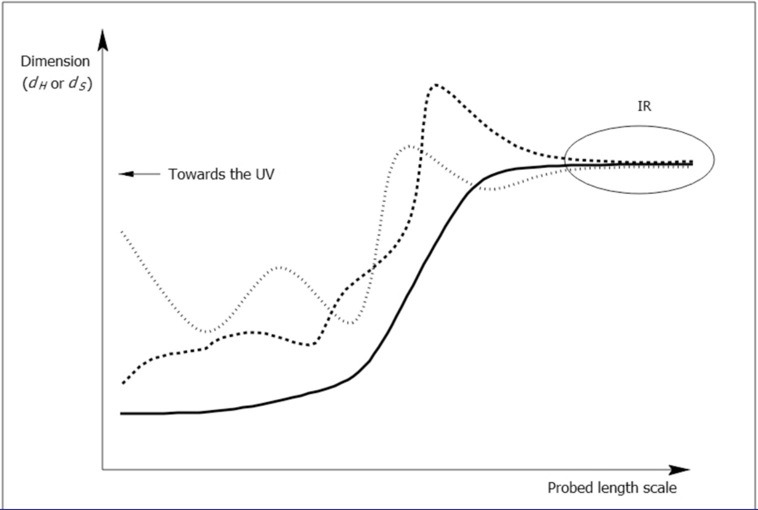
- Dim. flow and UV finiteness? Renormalizabile gravity?
- Is it observable?
- Theory and phenomenology (from particle physics to cosmology) of fractal spacetimes? Very preliminary results in the 1980s [Svozil 1986; Eynk 1989a,b; Müller, Schäfer 1986a,b].

Landscape of multiscale theories

G.C. EPJC **76** (2016) 181 [arXiv:1602.01470]



Universal property: dimensional flow



Dimensions near the IR

$$d(\ell) \simeq D + b\left(\frac{\ell_*}{\ell}\right)^c + \text{(log oscillations)}, \qquad d = d_{\rm H}, d_{\rm S}$$

		D	$b_{\rm H}$	$c_{\rm H}$	$b_{\rm S}$	$c_{\rm S}$
Asymptotic safety		4	0	—	<0	>0
CDT		4	0	—	<0	2
Near black holes		D	0	—	$\frac{D+1}{2}$	2
Nonlocal gravity and string field theory		D	0	—	<0	2
Fuzzy spacetimes		D	0	—	$-D$	2
Gravity with quantum particles		3	0	—	$-\frac{21}{16}$	2
κ-Minkowski	bicovariant ∇^2, AN(3)	4	0	—	-2	2
κ-Minkowski	bicovariant ∇^2, AN(2)	3	0	—	$-\frac{3}{2}$	2
κ-Minkowski	bicrossproduct ∇^2	4	0	—	1	2
κ-Minkowski	cyclic invariance (o.s.)	D	<0	1	?	?
Hořava–Lifshitz gravity		D	0	—	<0	>0
GFT, spin foams, LQG (o.s.)		$D(=4)$	<0	2	>0	2

Flow-equation theorem

G.C., PRD **95** (2017) 064057 [arXiv:1609.02776]

Varying $d_{\rm H}$

If the Hausdorff dimension of spacetime changes with the probed scale, and if it does so slowly at large scales, then the D-volume is uniquely determined as

$$\mathcal{V}(\ell) \simeq \ell^D + \left|\frac{\ell}{\ell_*}\right|^{D\alpha} F_\omega(\ell), \qquad F_\omega(\ell) = 1 + \sum_{n>0} F_n(\ell)$$

$$F_n(\ell) = A_n \cos\left(n\omega \ln \frac{\ell}{\ell_\infty}\right) + \sum_n B_n \sin\left(n\omega \ln \frac{\ell}{\ell_\infty}\right)$$

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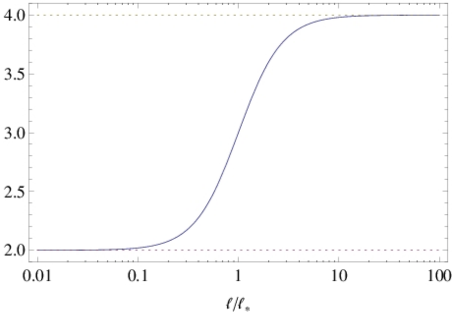
Similar statement for the spectral dimension $d_{\rm S}$.

Multiscaling

Scaling property: $\mathcal{V}(\lambda\ell) \sim \lambda^{D\alpha}\mathcal{V}(\ell) \qquad \Rightarrow \qquad d_{\rm H} = D\alpha$



Discreteness

G.C., JHEP **01** (2012) 065 [arXiv:1107.5041]; G.C., arXiv:1705.01619

Exactly the same measure found in fractal geometry [Ren et al. 1996–2003; Nigmatullin & Le Méhauté 2003].

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Oscillatory part of $\mathcal{V}(\ell) = \ell^D + \ell^{D\alpha} F_\omega(\ell)$ invariant under a DSI:

$$F_\omega(\lambda_\omega^n \ell) = F_\omega(\ell)\,, \qquad \lambda_\omega = \exp(-2\pi/\omega)\,, \qquad n = 0, 1, 2, \ldots$$

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DSIs appear in fractals and in complex and critical systems (earthquakes, financial crashes,. . .) [Sornette 1998].

$\omega = d_{\mathrm{H}\mathbb{C}}$ complex dimension!

Can we observe complex dimensions?

Multifractional theories

G.C., JHEP **03** (2017) 138 [arXiv:1612.05632]

$$S[\phi^i] = \int \mathrm{d}\varrho(x)\, \mathcal{L}[\mathcal{D}_x, \phi^i]\,, \qquad \mathrm{d}\varrho(x) = \prod_\mu \mathrm{d}q^\mu(x^\mu)$$

Same measure, different choices of Lagrangian symmetries ($v = \partial_x q$):

1. Weighted derivatives: $\mathcal{D}_x = v^{-1/2}\partial_x(v^{1/2}\,\cdot\,)$.
2. q-derivatives (multifractal): $\partial_q = v^{-1}\partial_x$.
3. Fractional derivatives (multifractal): $\mathbb{D}_x \sim \partial_x^\alpha$.

Status: 2012–2017

	$\mathcal{D}^2$	$\Box_q$	$\mathbb{D}^2$
Foundations	✓	✓	?
Relativistic mechanics	✓	✓	?
QFT and Standard Model	✓	✓	✓?
Perturbative renormalizability	✗	✗	✓?
Gravity and cosmology	✓	✓	?
Phenomenology: particles	✓	✓	?
Phenomenology: astrophysics	✓	✓	?
Phenomenology: inflation	?	✓	?
Phenomenology: dark energy	?	?	?

Observational constraints

WEIGHTED DER.	t_* (s)	ℓ_* (m)	E_* (eV)	source
$\alpha_{\rm QED}$ quasars	—	—	—	G.C., Magueijo, Rodríguez, PRD 2014
CMB black body	—	—	—	G.C., Kuroyanagi, Tsujikawa, JCAP 2016
Lamb shift	$< \mathbf{10^{-23}}$	$< 10^{-14}$	$> 10^{7}$	G.C., Nardelli, Rodríguez, PRD 2016(b)
$\alpha_{\rm QED}$ measurements	$< \mathbf{10^{-26}}$	$< 10^{-17}$	$> 10^{10}$	G.C., Nardelli, Rodríguez, PRD 2016(b)
GWs and GRBs	—	—	—	G.C., EPJC 2017

q-DER.	t_* (s)	ℓ_* (m)	E_* (eV)	source
primordial CMB (!)	weak	weak	weak	G.C., Kuroyanagi, Tsujikawa, JCAP 2016
CMB black body	—	—	—	G.C., Kuroyanagi, Tsujikawa, JCAP 2016
Muon lifetime	$< \mathbf{10^{-13}}$	$< 10^{-5}$	$> 10^{-3}$	G.C., Nardelli, Rodríguez, PRD 2016(a)
Lamb shift	$< 10^{-23}$	$< 10^{-15}$	$> \mathbf{10^{7}}$	G.C., Nardelli, Rodríguez, PRD 2016(a)
$\alpha_{\rm QED}$ measurements	—	—	—	G.C., Nardelli, Rodríguez, PRD 2016(b)
GWs	$< 10^{-22}$	$< 10^{-14}$	$> \mathbf{10^{7}}$	G.C., EPJC 2017
GRBs $\sim$	$< 10^{-32}$	$< 10^{-24}$	$> \mathbf{10^{26}}$	G.C., EPJC 2017

q-derivatives: gravity

G.C., JCAP **12** (2013) 041 [arXiv:1307.6382]

$$ {}^q\Gamma^\rho_{\mu\nu} := \tfrac{1}{2} g^{\rho\sigma}\left(\frac{1}{v_\mu}\partial_\mu g_{\nu\sigma} + \frac{1}{v_\nu}\partial_\nu g_{\mu\sigma} - \frac{1}{v_\sigma}\partial_\sigma g_{\mu\nu}\right) , $$

$$ {}^q R^\rho{}_{\mu\sigma\nu} := \frac{1}{v_\sigma}\partial_\sigma {}^q\Gamma^\rho_{\mu\nu} - \frac{1}{v_\nu}\partial_\nu {}^q\Gamma^\rho_{\mu\sigma} + {}^q\Gamma^\tau_{\mu\nu}\, {}^q\Gamma^\rho_{\sigma\tau} - {}^q\Gamma^\tau_{\mu\sigma}\, {}^q\Gamma^\rho_{\nu\tau} . $$

Action:

$$ S = \frac{1}{2\kappa^2}\int \mathrm{d}^D x\, v\, \sqrt{-g}\,({}^q R - 2\Lambda) + S_{\rm m} . $$

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G.C., JCAP **12** (2013) 041 [arXiv:1307.6382]

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Action:

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Einstein equations:

$$ {}^q R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}({}^q R - 2\Lambda) = \kappa^2\, {}^q T_{\mu\nu}\,. $$

FRW cosmology

$$\frac{H^2}{v^2} = \frac{\kappa^2}{3}\rho + \frac{\Lambda}{3} - \frac{\mathsf{K}}{a^2}\,, \qquad \dot\rho + 3H(\rho + P) = 0$$

$$\rho = \frac{\dot\phi^2}{2v^2} + V(\phi)$$

Ordinary slow-roll approximation unnecessary.

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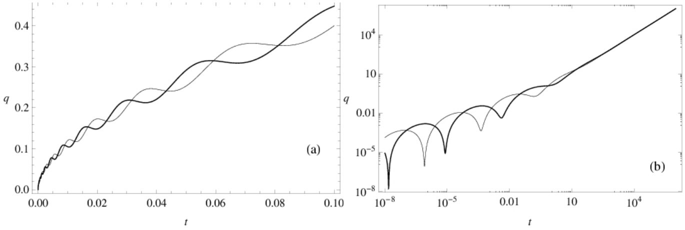
Ordinary slow-roll approximation unnecessary. **Cyclic** universe



Inflationary spectra

General behaviour (from $p(k) = 1/q(1/k)$):

$$P_{\rm s} = \mathcal{A}_{\rm s}\tilde{k}^{n_{\rm s}-1} \sim \mathcal{A}_{\rm s}\left(\frac{k}{k_*}\right)^{\alpha(n_{\rm s}-1)}[F_\omega(\ln k)]^{1-n_{\rm s}}\,.$$

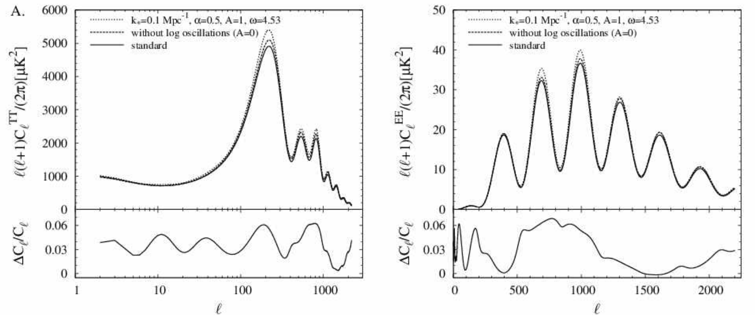
Scale invariance without strong slow-roll approximation and a log-oscillating pattern.

$\to$ Spacetime discrete at scales $\sim \ell_\infty$ (totally disconnected?).

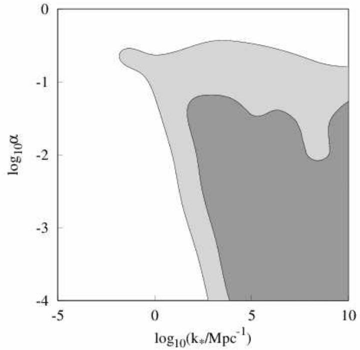
Visible effect of this geometry: not "holes" in the fabric of spacetime but a logarithmic modulation of the power spectrum of primordial fluctuations!

CMB spectra

G.C., Kuroyanagi, Tsujikawa, JCAP **08** (2016) 039 [arXiv:1606.08449]



2D contours



E.g. $N = 4$: **Upper bound** $\alpha < 0.1$. In general, $\alpha \lesssim 0.1 - 0.6$.

Consequences for spacetime dimension

The Hausdorff dimension of space $d_{\rm H}^{\rm space} = 3\alpha$ in the UV cannot exceed

$$N = 2: \qquad d_{\rm H}^{\rm space} \lesssim 0.3 \qquad \text{(UV)}$$
$$N = 3: \qquad d_{\rm H}^{\rm space} \lesssim 1.9 \qquad \text{(UV)}$$
$$N = 4: \qquad d_{\rm H}^{\rm space} \lesssim 1.7 \qquad \text{(UV)}$$

Counter-intuitive!

Upper bounds on A_1, B_1

Amplitudes of log oscillations of geometry cannot exceed

$$N=2: \qquad A_1 < 0.3,\ B_1 < 0.4$$
$$N=3: \qquad A_1 < 0.3,\ B_1 < 0.2$$
$$N=4: \qquad A_1 < 0.4,\ B_1 < 1.0$$

First constraints of this kind.

Beyond first harmonic

G.C., arXiv:1705.01619; G.C., Ronco, arXiv:1706.02159

Parametrization inspired by critical systems [Gluzman, Sornette 2002]:

$$A_n = 2\xi \frac{e^{-\gamma n}}{n^u} \cos(\psi_n + \beta_n), \quad B_n = -2\xi \frac{e^{-\gamma n}}{n^u} \sin(\psi_n + \beta_n), \quad \beta_n := n\omega \frac{\ell_\infty}{\ell_*}.$$

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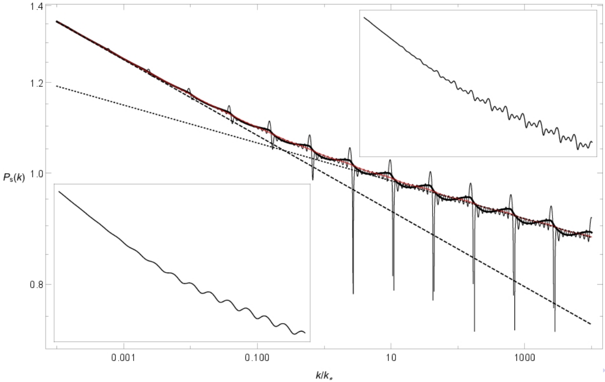
Nowhere differentiable measure (stochastic spacetime):
fast-varying (ergodic mixing) phases

$$\psi_n = \Omega n \ln(\Omega n)\,, \qquad \psi_n = \Omega n^2\,, \qquad \psi_n = \Omega \mathrm{e}^{n/\Omega}$$

Beyond first harmonic

G.C., arXiv:1705.01619; G.C., Shafieloo, *in progress*

Discrete Scale Invariance:
imprint of complex dimensions in the CMB



どうもありがとうございました！

Thank you!

¡Muchas gracias!

Grazie!

Muito obrigado!

Danke schön!