CONSTRAINING THE FLAVOR STRUCTURE OF LORENTZ VIOLATION HAMILTONIAN WITH THE MEASUREMENT OF ASTROPHYSICAL NEUTRINO FLAVOR COMPOSITIONS

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K.-C. Lai, W.-H. Lai and G.-L. Lin, arXiv: 1704.04027

OUTLINE

1. Introduction
2. Lorentz violation effects to neutrino flavor transitions
3. Lorentz violation and the current IceCube results on astrophysical neutrino flavor compositions
4. IceCube Gen2 and its potential of constraining Lorentz violation Hamiltonian
5. Summary and conclusions

1. INTRODUCTION

- Violations of Lorentz symmetry could arise in Planck scale physics

  V. A. Kostelecky and S. Samuel, Phys. Rev. D 39, 683 (1989)

  V. A. Kostelecky and R. Potting, Nucl. Phys. B 359, 545 (1991)

- The effects of Lorentz violations (LV) to neutrino oscillations have been studied before

  V. A. Kostelecky and M. Mewes, Phys. Rev. D 69, 016005 (2004)

  V. A. Kostelecky and M. Mewes, Phys. Rev. D 70, 031902 (2004)

  V. A. Kostelecky and M. Mewes, Phys. Rev. D 70, 076002 (2004)

- The standard model neutrino Hamiltonian in vacuum

U : PMNS matrix

$$H_{\rm SM} = U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2_{21} & 0 \\ 0 & 0 & \Delta m^2_{31} \end{pmatrix} U^\dagger / 2E$$

$H_{\rm SM}$ behaves as $1/E$

- With Lorentz violation

$$H = H_{\rm SM} + H_{\rm LV}$$

$H_{\rm LV}$ contains E^0 and E^1 terms

- Standard neutrino oscillation probability depends on *L/E*
- Lorentz violation introduces *L* and *LE* terms. It also generates directional dependence due to the breaking of rotation symmetry
- Experiments probing LV focus on (i) spectral anomalies of the oscillated neutrino flux, (ii) sidereal variations of neutrino oscillation probabilities.

Short baseline beams (1-4), long baseline beams (5-6), reactor neutrino (7-8), atmospheric neutrinos in IceCube (9) and Super-Kamiokande (10)

(1) L. B. Auerbach *et al*. [LSND Collaboration], Phys. Rev. D 72, 076004 (2005).
(2) P. Adamson *et al*. [MINOS Collaboration], Phys. Rev. Lett. 101, 151601 (2008).
(3)A. A. Aguilar-Arevalo *et al*. [MiniBooNE Collaboration], Phys. Lett. B 718, 1303 (2013).
(4) P. Adamson *et al*. [MINOS Collaboration], Phys. Rev. D 85, 031101 (2012).
(5) P. Adamson *et al*. [MINOS Collaboration], Phys. Rev. Lett. 105, 151601 (2010).
(6) B. Rebel and S. Mufson, Astropart. Phys. 48, 78 (2013).
(7) Y. Abe et al. [Double Chooz Collaboration], Phys. Rev. D 86, 112009 (2012).
(8) J. S. Diaz, T. Katori, J. Spitz and J. M. Conrad, Phys. Lett. B 727, 412 (2013).
(9) R. Abbasi *et al*. [IceCube Collaboration], Phys. Rev. D 82, 112003 (2010).
[10] K. Abe *et al*. [Super-Kamiokande Collaboration], Phys. Rev. D 91, no. 5, 052003 (2015).

- We shall study the LV effects with high energy astrophysical neutrino source. The neutrino flavor transition probability this case is

$$P(\nu_\alpha \to \nu_\beta) = |V_{\alpha i}|^2 |V_{\beta i}|^2,$$

where V is the matrix that diagonalizes the full Hamiltonian

$$H = H_{\rm SM} + H_{\rm LV}$$

Diagonalized by V

Diagonalized by U

$1/E$

E^0 and E^1

- V approaches to PMNS matrix U for $H_{\rm LV}$=0.
- When the neutrino energy is sufficiently high, the structure of V is dictated by $H_{\rm LV}$.

$$H = H_{\rm SM} + H_{\rm LV}$$

$1/E$ E^0 and E^1

The structure of $H_{\rm SM}$ in the limit: $\theta_{23} = 45^\circ,\ \theta_{13} = 0$

$$H_{\rm SM} = \begin{pmatrix} s_{12}^2 \frac{\Delta m_{21}^2}{2E} & \frac{\sqrt{2}}{2} s_{12} c_{12} \frac{\Delta m_{21}^2}{2E} & -\frac{\sqrt{2}}{2} s_{12} c_{12} \frac{\Delta m_{21}^2}{2E} \\ \frac{\sqrt{2}}{2} s_{12} c_{12} \frac{\Delta m_{21}^2}{2E} & \frac{1}{2}(c_{12}^2 \frac{\Delta m_{21}^2}{2E} + \frac{\Delta m_{31}^2}{2E}) & \frac{1}{2}(-c_{12}^2 \frac{\Delta m_{21}^2}{2E} + \frac{\Delta m_{31}^2}{2E}) \\ -\frac{\sqrt{2}}{2} s_{12} c_{12} \frac{\Delta m_{21}^2}{2E} & \frac{1}{2}(-c_{12}^2 \frac{\Delta m_{21}^2}{2E} + \frac{\Delta m_{31}^2}{2E}) & \frac{1}{2}(c_{12}^2 \frac{\Delta m_{21}^2}{2E} + \frac{\Delta m_{31}^2}{2E}) \end{pmatrix}$$

Large values at 2nd and 3rd rows and columns—special flavor structure ($\mu\tau$ symmetry)

Flavor structure of $H_{\rm LV}$ may differ significantly from $H_{\rm SM}$
Detectable from the flavor composition of astrophysical neutrinos reaching to the Earth

Common astrophysical neutrino sources

(1) *pp* collisions: roughly the same number of π^+ and π^- are produced. Neutrinos and anti-neutrinos are produced equally

$$\pi^+(\pi^-) \to \mu^+(\mu^-)\nu_\mu(\bar{\nu}_\mu)$$
$$\mu^+(\mu^-) \to \nu_\mu(\bar{\nu}_\mu)e^+(e^-)\bar{\nu}_e(\nu_e)$$

(a) secondary muons decay immediately

Pion source

$$\nu_e : \nu_\mu : \nu_\tau = 1 : 2 : 0 \qquad \bar{\nu}_e : \bar{\nu}_\mu : \bar{\nu}_\tau = 1 : 2 : 0$$

(b) secondary muons lose significant energies before decay

$$\nu_e : \nu_\mu : \nu_\tau = 0 : 1 : 0 \qquad \bar{\nu}_e : \bar{\nu}_\mu : \bar{\nu}_\tau = 0 : 1 : 0$$

Muon-damped source

(2) $p\gamma$ collisions: leading contributions

$$p + \gamma \to \Delta^+ \to n + \pi^+$$
$$\pi^+ \to \mu^+ \nu_\mu$$
$$\mu^+ \to \bar{\nu}_\mu e^+ \nu_e$$

(a) secondary muons decay immediately

Pion source

$$\nu_e : \nu_\mu : \nu_\tau = 1 : 1 : 0 \qquad \bar{\nu}_e : \bar{\nu}_\mu : \bar{\nu}_\tau = 0 : 1 : 0$$

(b) secondary muons lose significant energies before decay

$$\nu_e : \nu_\mu : \nu_\tau = 0 : 1 : 0$$

Muon-damped source

(2) $p\gamma$ collisions: sub-leading contributions

$$p\gamma \to p\pi^+\pi^-$$

Neutrinos and anti-neutrinos are produced equally
Non-negligible if gamma spectrum is hard enough

(a) secondary muons decay immediately Pion source

$$\nu_e : \nu_\mu : \nu_\tau = 1/3 : 2/3 : 0 \quad \bar{\nu}_e : \bar{\nu}_\mu : \bar{\nu}_\tau = 1/3 : 2/3 : 0$$

(b) secondary muons lose significant energies before decay

$$\nu_e : \nu_\mu : \nu_\tau = 0 : 1 : 0 \qquad \bar{\nu}_e : \bar{\nu}_\mu : \bar{\nu}_\tau = 0 : 1 : 0$$

Muon-damped source

We shall focus on pion source from *pp* collisions, i.e.,

$$\nu_e : \nu_\mu : \nu_\tau = 1 : 2 : 0 \quad \bar{\nu}_e : \bar{\nu}_\mu : \bar{\nu}_\tau = 1 : 2 : 0$$

Defining neutrino flavor fraction:

$$f_\alpha^0 \equiv \Phi^0(\nu_\alpha)/(\Phi^0(\nu_e) + \Phi^0(\nu_\mu) + \Phi^0(\nu_\tau))$$

total flux of neutrinos and anti-neutrinos of flavor α at the source

Hence

$$(f_e^0, f_\mu^0, f_\tau^0) = (1/3,\ 2/3,\ 0)$$

$$f_\alpha \equiv \Phi(\nu_\alpha)/(\Phi(\nu_e) + \Phi(\nu_\mu) + \Phi(\nu_\tau))$$

total flux of neutrinos and anti-neutrinos
of flavor at the terrestrial detector

$$f_\alpha = P_{\alpha\beta} f^0_\beta \qquad P_{\alpha\beta} \equiv P(\nu_\beta \to \nu_\alpha)$$

With $(f^0_e, f^0_\mu, f^0_\tau) = (1/3,\ 2/3,\ 0)$

$$f_e = 1/3 + (P_{e\mu} - P_{e\tau})/3$$

$$f_\mu = 1/3 + (P_{\mu\mu} - P_{\mu\tau})/3$$

$$f_\tau = 1/3 + (P_{\mu\tau} - P_{\tau\tau})/3$$

A test of μτ symmetry breaking

μτ symmetry breaking effects are small in the standard model neutrino Hamiltonian

$$(P_{e\mu} - P_{e\tau}) = 2\epsilon \quad (P_{\mu\mu} - P_{\mu\tau}) = (P_{\mu\tau} - P_{\tau\tau}) = -\epsilon$$

Here $\epsilon = 2\cos 2\theta_{23}/9 + \sqrt{2}\sin\theta_{13}\cos\delta/9$

(1) Taking $\sin^2\theta_{12} = 1/3$
and keeping only the leading-order symmetry breaking terms
(2) Lorentz violating Hamiltonian may contain large μτ symmetry breaking effects

2. LV EFFECTS TO NEUTRINO FLAVOR TRANSITIONS

For neutrinos, the general form of LV Hamiltonian

$$H^{\nu}_{\rm LV} = \frac{p_\lambda}{E}\begin{pmatrix} a^{\lambda}_{ee} & a^{\lambda}_{e\mu} & a^{\lambda}_{e\tau} \\ a^{\lambda *}_{e\mu} & a^{\lambda}_{\mu\mu} & a^{\lambda}_{\mu\tau} \\ a^{\lambda *}_{e\tau} & a^{\lambda *}_{\mu\tau} & a^{\lambda}_{\tau\tau} \end{pmatrix} - \frac{p^\rho p^\lambda}{E}\begin{pmatrix} c^{\rho\lambda}_{ee} & c^{\rho\lambda}_{e\mu} & c^{\rho\lambda}_{e\tau} \\ c^{\rho\lambda *}_{e\mu} & c^{\rho\lambda}_{\mu\mu} & c^{\rho\lambda}_{\mu\tau} \\ c^{\rho\lambda *}_{e\tau} & c^{\rho\lambda *}_{\mu\tau} & c^{\rho\lambda}_{\tau\tau} \end{pmatrix}$$

For rotationally invariant LV effects

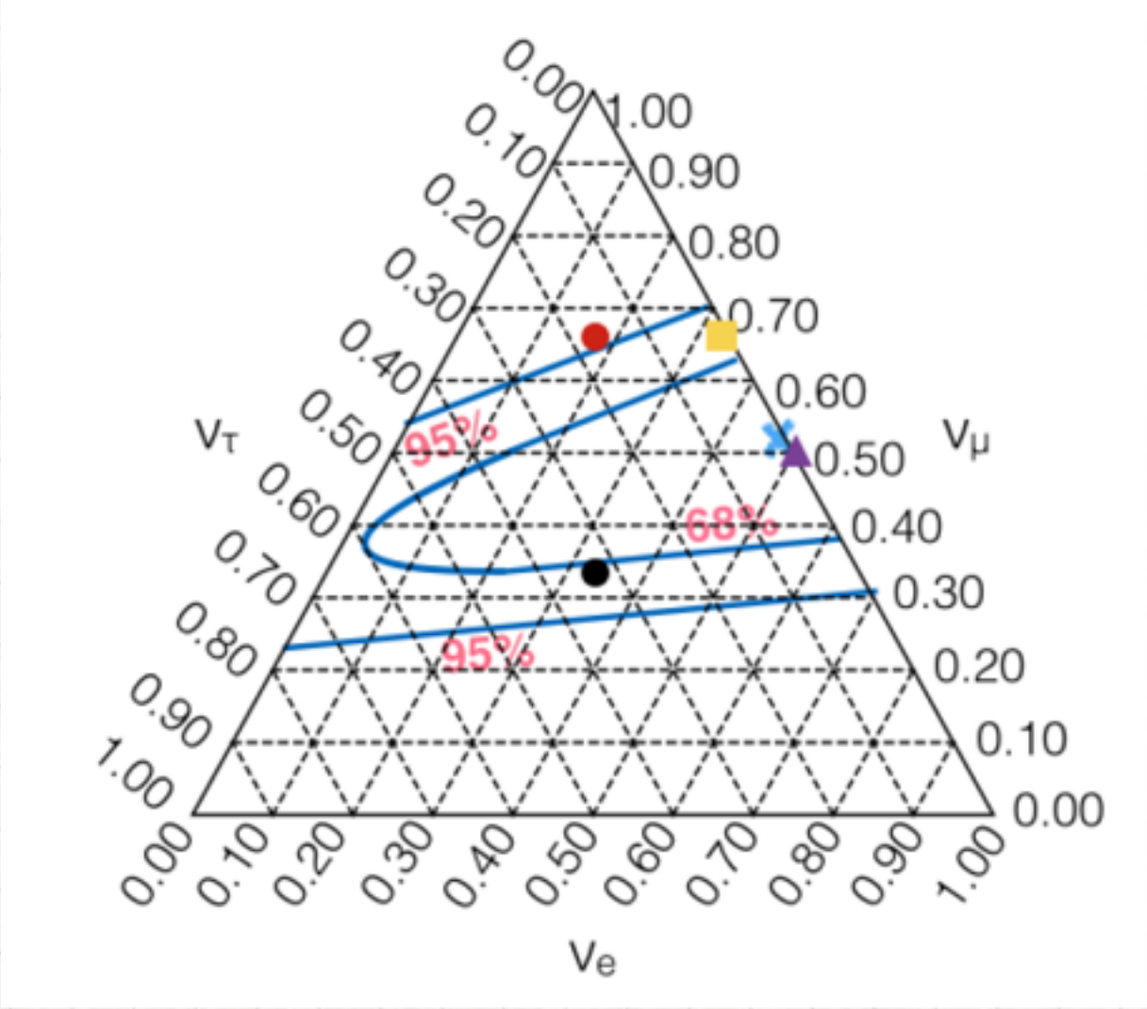
$$H^{\nu}_{\rm LV} = \begin{pmatrix} a^{T}_{ee} & a^{T}_{e\mu} & a^{T}_{e\tau} \\ a^{T *}_{e\mu} & a^{T}_{\mu\mu} & a^{T}_{\mu\tau} \\ a^{T *}_{e\tau} & a^{T *}_{\mu\tau} & a^{T}_{\tau\tau} \end{pmatrix} - \frac{4E}{3}\begin{pmatrix} c^{TT}_{ee} & c^{TT}_{e\mu} & c^{TT}_{e\tau} \\ c^{TT *}_{e\mu} & c^{TT}_{\mu\mu} & c^{TT}_{\mu\tau} \\ c^{TT *}_{e\tau} & c^{TT *}_{\mu\tau} & c^{TT}_{\tau\tau} \end{pmatrix}$$

$$H^{\bar\nu}_{\rm LV} = -\begin{pmatrix} a^{T}_{ee} & a^{T}_{e\mu} & a^{T}_{e\tau} \\ a^{T *}_{e\mu} & a^{T}_{\mu\mu} & a^{T}_{\mu\tau} \\ a^{T *}_{e\tau} & a^{T *}_{\mu\tau} & a^{T}_{\tau\tau} \end{pmatrix}^{*} - \frac{4E}{3}\begin{pmatrix} c^{TT}_{ee} & c^{TT}_{e\mu} & c^{TT}_{e\tau} \\ c^{TT *}_{e\mu} & c^{TT}_{\mu\mu} & c^{TT}_{\mu\tau} \\ c^{TT *}_{e\tau} & c^{TT *}_{\mu\tau} & c^{TT}_{\tau\tau} \end{pmatrix}^{*}$$

Sun-centered celestial equatorial frame

$(T,\ X,\ Y,\ Z)$ Let us first focus on $a^{T}_{\alpha\beta}$

3. LORENTZ VIOLATIONS AND CURRENT ICECUBE RESULTS ON ASTROPHYSICAL NEUTRINO FLAVOR COMPOSITIONS



E_ν is between 25 TeV and 2.8 PeV

$H_{SM} \approx \Delta m^2_{31}/2E_\nu$

Hence H_{SM} is between

$$5 \times 10^{-26}\ \mathrm{GeV}$$

and

$$4.5 \times 10^{-28}\ \mathrm{GeV}$$

M. G. Aartsen et al. [IceCube Collaboration], Astrophys. J. 809, no. 1, 98 (2015)

Can Lorentz violation play role in this data?

CURRENT BOUNDS ON LORENTZ VIOLATION PARAMETERS SUPER-KAMIOKANDE MEASUREMENTS

LV parameter		Limit at 95% C.L.	Best fit	No LV $\Delta\chi^2$	Previous limit
$e\mu$	$\mathrm{Re}(a^T)$	1.8×10^{-23} GeV	1.0×10^{-23} GeV	1.4	4.2×10^{-20} GeV [61]
	$\mathrm{Im}(a^T)$	1.8×10^{-23} GeV	4.6×10^{-24} GeV		
	$\mathrm{Re}(c^{TT})$	8.0×10^{-27}	1.0×10^{-28}	0.0	9.6×10^{-20} [61]
	$\mathrm{Im}(c^{TT})$	8.0×10^{-27}	1.0×10^{-28}		
$e\tau$	$\mathrm{Re}(a^T)$	4.1×10^{-23} GeV	2.2×10^{-24} GeV	0.0	7.8×10^{-20} GeV [62]
	$\mathrm{Im}(a^T)$	2.8×10^{-23} GeV	1.0×10^{-28} GeV		
	$\mathrm{Re}(c^{TT})$	9.3×10^{-25}	1.0×10^{-28}	0.3	1.3×10^{-17} [62]
	$\mathrm{Im}(c^{TT})$	1.0×10^{-24}	3.5×10^{-25}		
$\mu\tau$	$\mathrm{Re}(a^T)$	6.5×10^{-24} GeV	3.2×10^{-24} GeV	0.9	...
	$\mathrm{Im}(a^T)$	5.1×10^{-24} GeV	1.0×10^{-28} GeV		
	$\mathrm{Re}(c^{TT})$	4.4×10^{-27}	1.0×10^{-28}	0.1	...
	$\mathrm{Im}(c^{TT})$	4.2×10^{-27}	7.5×10^{-28}		

$$H_{\mathrm{SM}} < 5 \times 10^{-26}\ \mathrm{GeV}$$

K. Abe *et al.* [Super-Kamiokande Collaboration], Phys. Rev. D 91, no. 5, 052003 (2015).

Significant room for H_{LV} to play an important role

SPECIAL STRUCTURES OF H_{LV} AND THE RESULTING ASTROPHYSICAL NEUTRINO FLAVOR COMPOSITION ON EARTH

(A) only $a^T_{e\mu}(a^{T*}_{e\mu})$ are non-vanishing

$$H^{\nu}_{\mathrm{LV}} = \begin{pmatrix} 0 & a^T_{e\mu} & 0 \\ a^{T*}_{e\mu} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad P = \begin{pmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

For anti-neutrinos,

$$a^T_{e\mu} \to -a^{T*}_{e\mu}$$

Large breaking of μτ symmetry

$$(P_{e\mu} - P_{e\tau}) = (P_{\mu\mu} - P_{\mu\tau}) = 1/2$$

$$(P_{\mu\tau} - P_{\tau\tau}) = -1$$

$$(f_e, f_\mu, f_\tau) = (1/2, 1/2, 0)$$

SPECIAL STRUCTURES OF H_{LV} AND THE RESULTING ASTROPHYSICAL NEUTRINO FLAVOR COMPOSITION ON EARTH

(B) only $a^T_{e\tau}(a^{T*}_{e\tau})$ are non-vanishing

$$H^{\nu}_{\mathrm{LV}} = \begin{pmatrix} 0 & 0 & a^T_{e\tau} \\ 0 & 0 & 0 \\ a^{T*}_{e\tau} & 0 & 0 \end{pmatrix} \quad P = \begin{pmatrix} 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \end{pmatrix}$$

For anti-neutrinos,

$$a^T_{e\tau} \to -a^{T*}_{e\tau}$$

$$(P_{e\mu} - P_{e\tau}) = (P_{\mu\tau} - P_{\tau\tau}) = -1/2$$

$$(P_{\mu\mu} - P_{\mu\tau}) = 1$$

$$(f_e,\ f_\mu,\ f_\tau) = (1/6,\ 2/3,\ 1/6)$$

Large breaking of μτ symmetry

(C) only $a^T_{\mu\tau}(a^{T*}_{\mu\tau})$ are non-vanishing

$$H^{\nu}_{\rm LV} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & a^T_{\mu\tau} \\ 0 & a^{T*}_{\mu\tau} & 0 \end{pmatrix} \qquad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \end{pmatrix}$$

For anti-neutrinos,

$$a^T_{\mu\tau} \to -a^{T*}_{\mu\tau}$$

$$(P_{e\mu} - P_{e\tau}) = (P_{\mu\mu} - P_{\mu\tau}) = (P_{\mu\tau} - P_{\tau\tau}) = 0$$

μτ symmetric case

$$(f_e,\ f_\mu,\ f_\tau) = (1/3,\ 1/3,\ 1/3)$$

SPECIAL STRUCTURES OF H_{LV} AND THE RESULTING ASTROPHYSICAL NEUTRINO FLAVOR COMPOSITION ON EARTH

(D) only $a^T_{\mu\mu}, a^T_{\tau\tau}$ are non-vanishing, $a^T_{\mu\mu} \neq a^T_{\tau\tau}$

$$H^{\nu}_{\mathrm{LV}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & a^T_{\mu\mu} & 0 \\ 0 & 0 & a^T_{\tau\tau} \end{pmatrix} \quad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

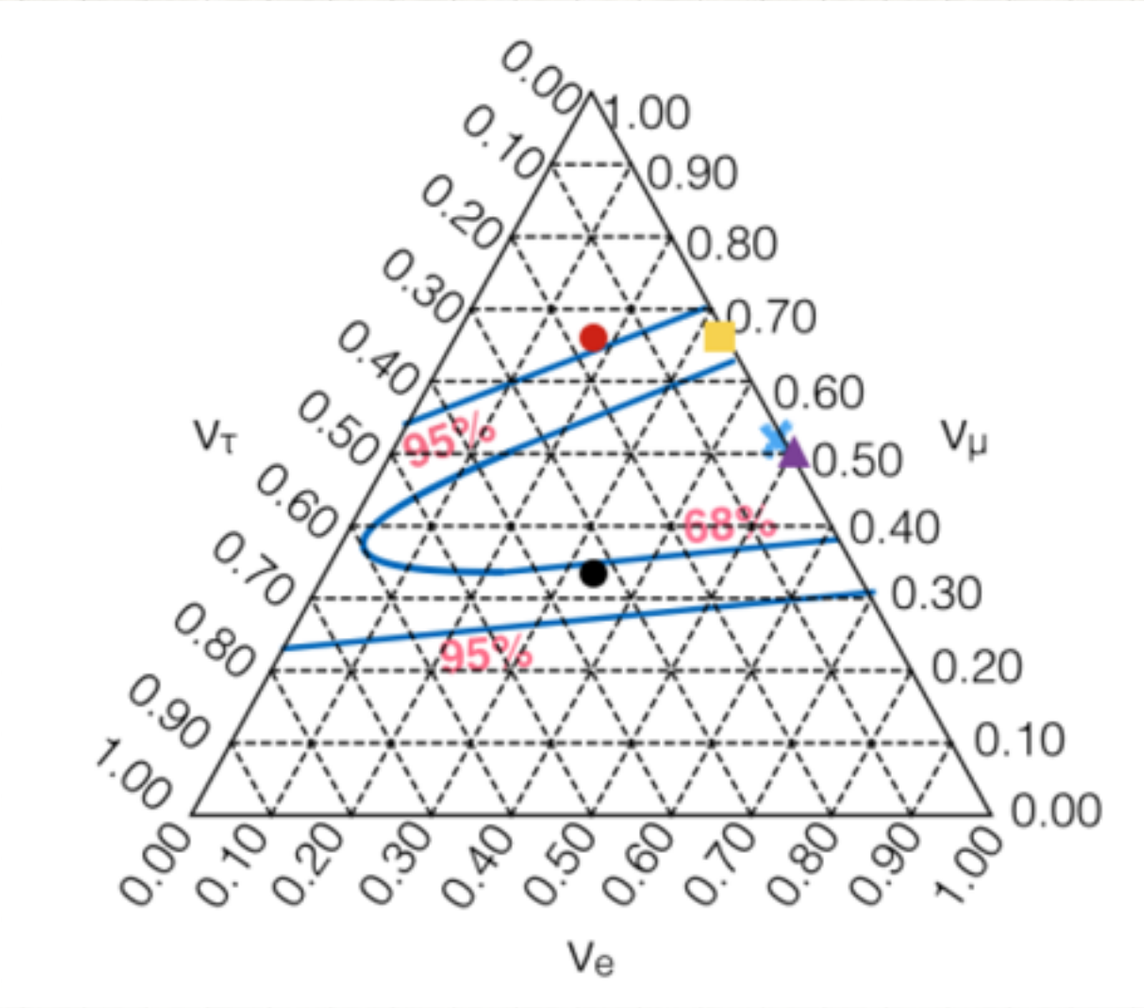
$$(P_{e\mu} - P_{e\tau}) = 0$$

Large breaking of μτ symmetry

$$(P_{\mu\mu} - P_{\mu\tau}) = 1 \quad (P_{\mu\tau} - P_{\tau\tau}) = -1$$

$$(f_e,\ f_\mu,\ f_\tau) = (1/3,\ 2/3,\ 0)$$

COMPARISONS OF SPECIAL CASES WITH RECENT ICECUBE MEASUREMENT OF ASTROPHYSICAL NEUTRINO FLAVOR COMPOSITION



Red: $a^T_{e\tau}, a^{T*}_{e\tau} \neq 0$

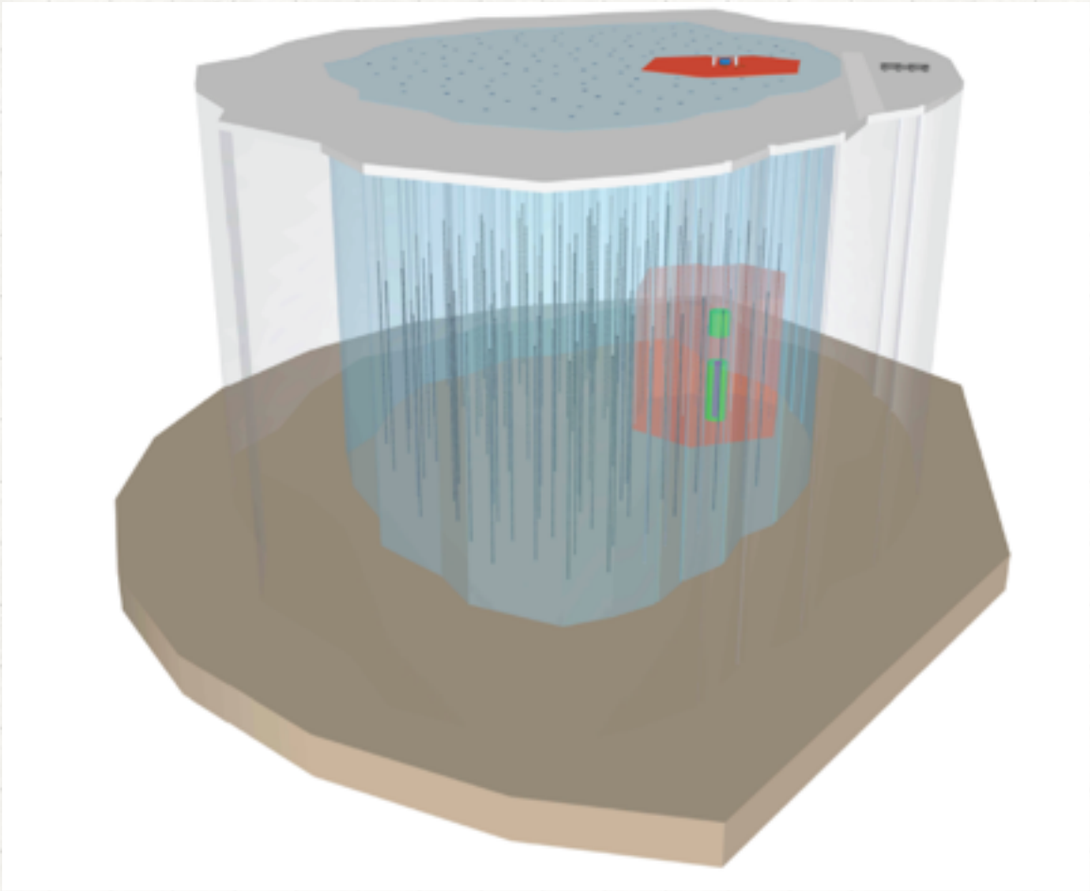
Yellow: $a^T_{\mu\mu}, a^T_{\tau\tau} \neq 0$

Purple: $a^T_{e\mu}, a^{T*}_{e\mu} \neq 0$

Black: $a^T_{\mu\tau}, a^{T*}_{\mu\tau} \neq 0$

All cases fall into 2σ region as other elements grow from zero

4. ICECUBE GEN2 AND ITS POTENTIAL OF CONSTRAINING LORENTZ VIOLATION HAMILTONIAN



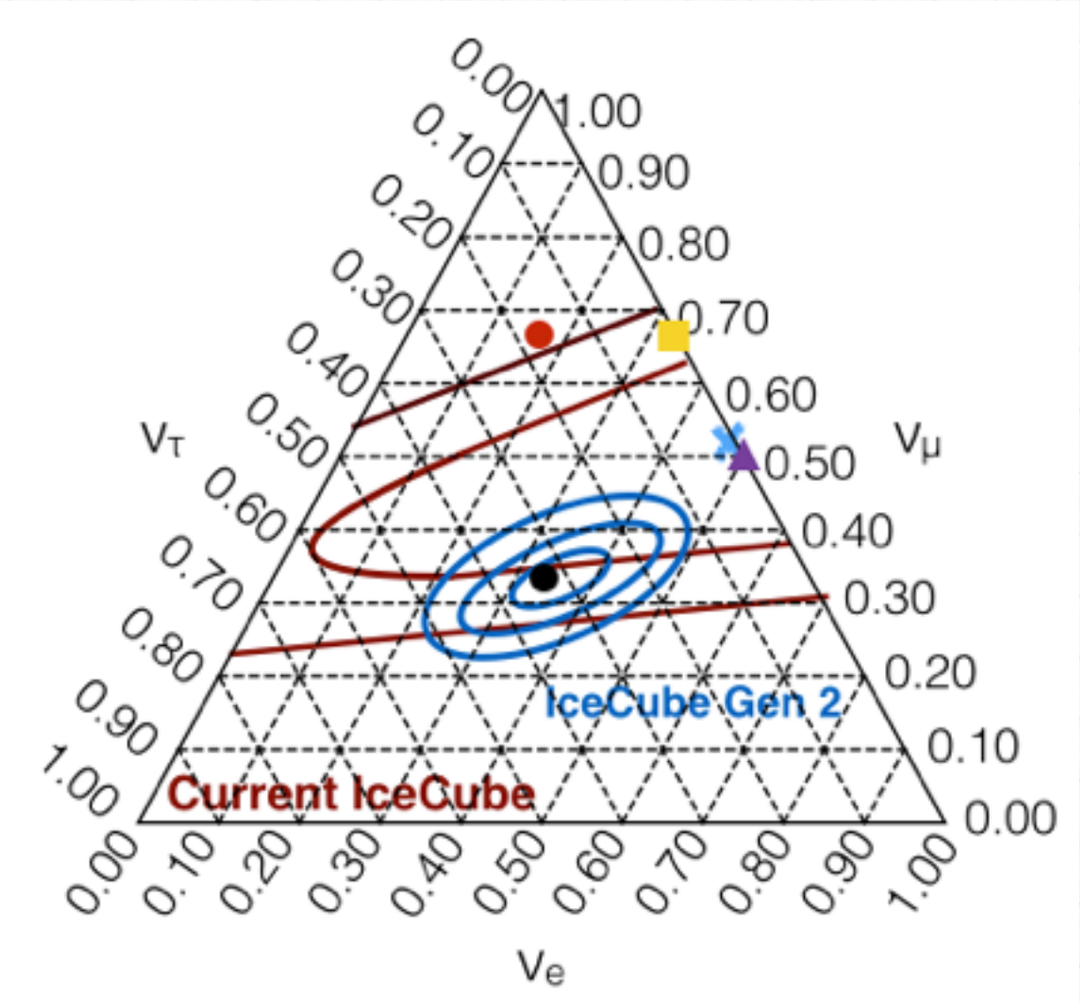
IceCube Collaboration (M.G. Aartsen (Adelaide U.) et al.), arXiv:1412.5106

~10 km^3 instrumented volume

~250 m spacing of photo sensors

(1) A possible IceCube-Gen2 configuration.
(2) IceCube, in red, and the infill sub-detector DeepCore, in green.
(3) blue volume shows the full instrumented next-generation detector, with PINGU displayed in grey as a denser infill extension within DeepCore.

SENSITIVITIES OF ICECUBE-GEN2 ON ASTROPHYSICAL NEUTRINO FLAVOR COMPOSITIONS



$$\Phi_\nu(E) = \Phi_0 \left(\frac{100 \text{ TeV}}{E} \right)^\gamma$$

$$\gamma = 2.2 \pm 0.2$$

$$\Phi_0 = (5.1 \pm 1.8) \times 10^{-18} \text{GeV}^{-1} \text{cm}^{-2} \text{s}^{-1} \text{sr}^{-1}$$

Pion source from *pp* collision is assumed

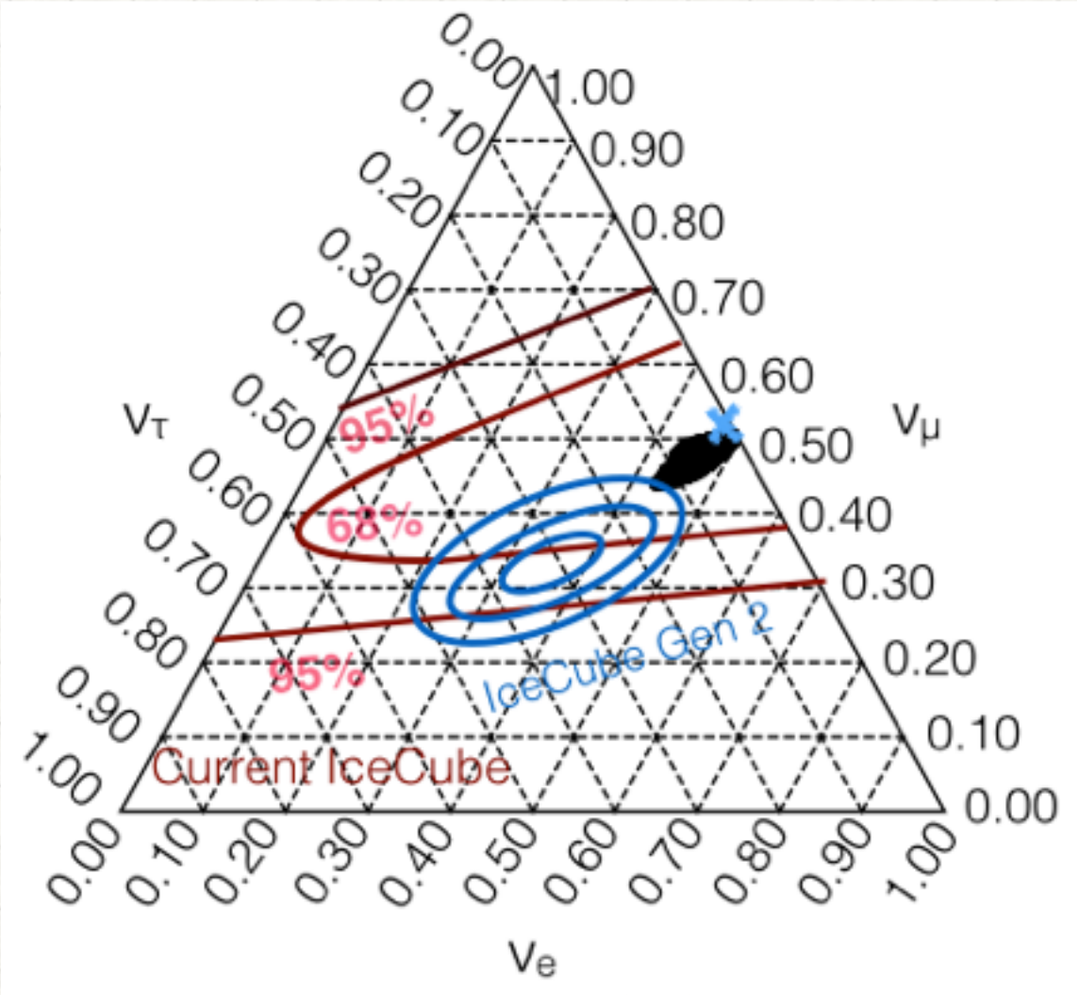
$$E_{\text{th}} = 100 \text{ TeV}$$

10 years of exposure

$$1\sigma, \ 2\sigma, \ \text{and } 3\sigma \text{ regions}$$

I. M. Shoemaker and K. Murase, Phys. Rev. D 93 085004 (2016) IceCube Gen2 regions

SK 95% C.L. limits: $\mathrm{Re}(a^T_{e\mu}) < 1.8 \times 10^{-23}$ GeV $\mathrm{Im}(a^T_{e\mu}) < 1.8 \times 10^{-23}$GeV



$$H^{\nu}_{\mathrm{LV}} = \begin{pmatrix} 0 & a^T_{e\mu} & 0 \\ a^{T*}_{e\mu} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

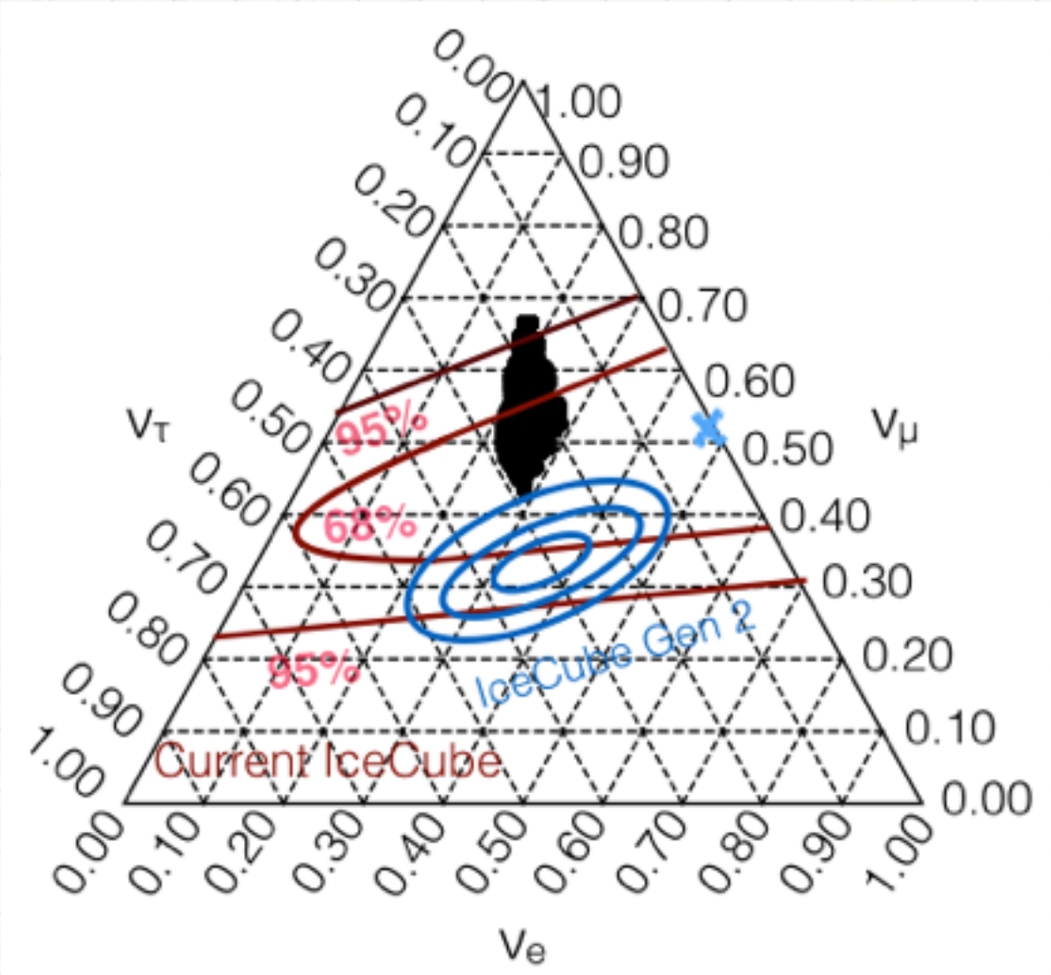
Allow other elements to grow from zero and include the contribution from H_{SM}

$\lvert a^T_{e\mu}\rvert$	5×10^{-24} GeV	5×10^{-25} GeV
$\lvert a^T_{e\tau}\rvert/\lvert a^T_{e\mu}\rvert$	$(0, 0.24)$	$(0, 0.23)$
$\lvert a^T_{\mu\tau}\rvert/\lvert a^T_{e\mu}\rvert$	$(0, 0.24)$	$(0, 0.23)$
$\lvert a^T_{\mu\mu}\rvert/\lvert a^T_{e\mu}\rvert$	$(0, 0.30)$	$(0, 0.26)$
$\lvert a^T_{\tau\tau}\rvert/\lvert a^T_{e\mu}\rvert$	$(0, 0.30)$	$(0, 0.26)$

The parameter ranges in the table predict the black region of flavor fraction—disfavored at 3 σ

$$0 \leq |a^T_{\tau\tau}|/|a^T_{e\mu}| \leq 0.30$$

SK 95% C.L. limits: $\mathrm{Re}(a^T_{e\tau}) < 4.1 \times 10^{-23}\mathrm{GeV}$ $\mathrm{Im}(a^T_{e\tau}) < 2.8 \times 10^{-23}\mathrm{GeV}$

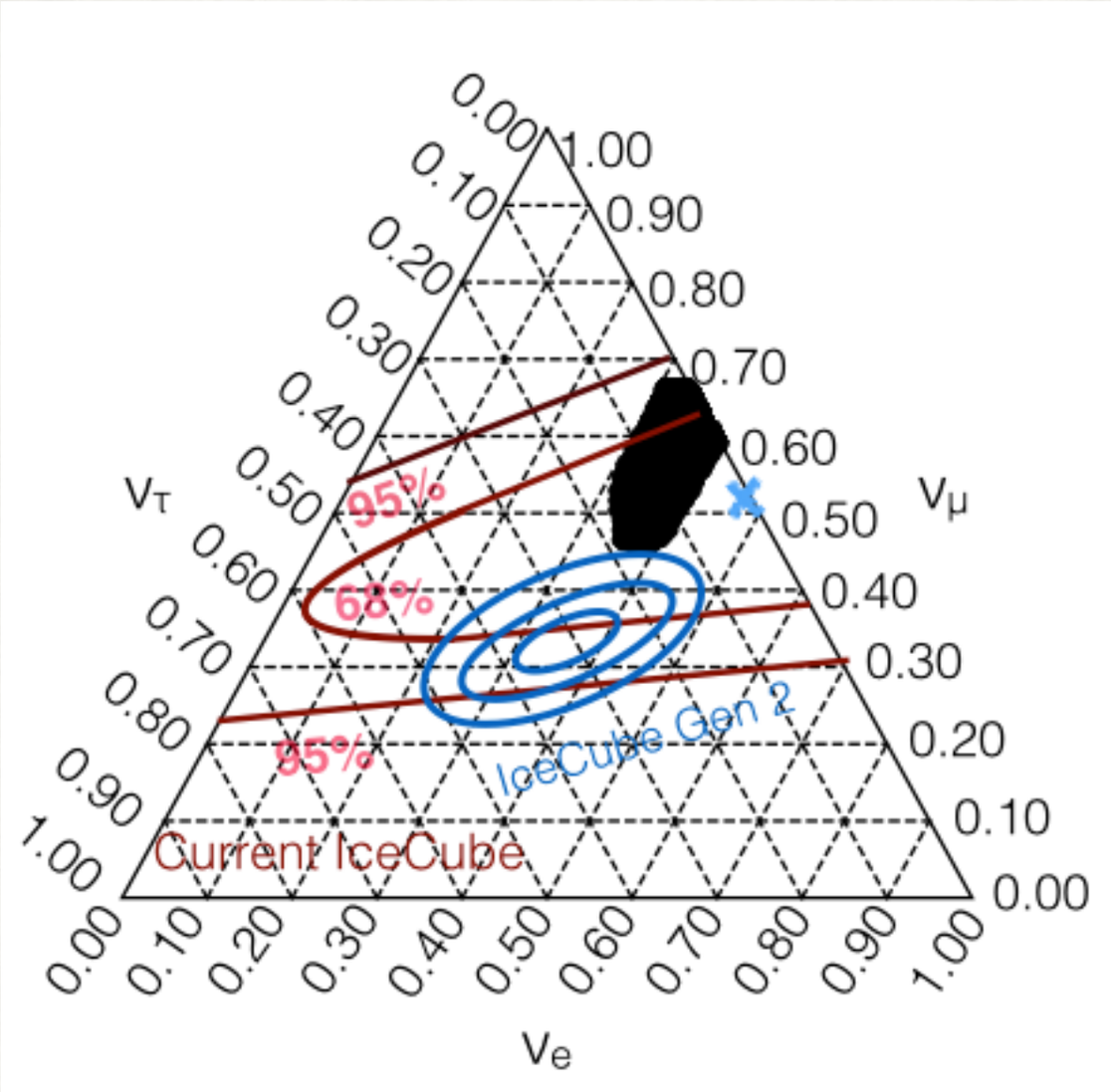


$$H^{\nu}_{\mathrm{LV}} = \begin{pmatrix} 0 & 0 & a^T_{e\tau} \\ 0 & 0 & 0 \\ a^{T*}_{e\tau} & 0 & 0 \end{pmatrix}$$

Allow other elements to grow from zero and include the contribution from H_{SM}

$\lvert a^T_{e\tau}\rvert$	5×10^{-24} GeV	5×10^{-25} GeV
$\lvert a^T_{e\mu}\rvert/\lvert a^T_{e\tau}\rvert$	$(0, 0.42)$	$(0, 0.39)$
$\lvert a^T_{\mu\tau}\rvert/\lvert a^T_{e\tau}\rvert$	$(0, 0.42)$	$(0, 0.39)$
$\lvert a^T_{\mu\mu}\rvert/\lvert a^T_{e\tau}\rvert$	$(0, 0.50)$	$(0, 0.45)$
$\lvert a^T_{\tau\tau}\rvert/\lvert a^T_{e\tau}\rvert$	$(0, 0.50)$	$(0, 0.45)$

The parameter ranges in the table predict the black region of flavor fraction—disfavored at 3 σ



$$H^{\nu}_{\rm LV} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & a^T_{\mu\mu} & 0 \\ 0 & 0 & a^T_{\tau\tau} \end{pmatrix}$$

Allow other elements to grow from zero and include the contribution from H_{SM}

$a^T_{\mu\mu} = 2a^T_{\tau\tau}$	5×10^{-24} GeV	5×10^{-25} GeV
$\lvert a^T_{e\mu}\rvert / \lvert a^T_{\mu\mu}\rvert$	$(0, 0.40)$	$(0, 0.39)$
$\lvert a^T_{e\tau}\rvert / \lvert a^T_{\mu\mu}\rvert$	$(0, 0.40)$	$(0, 0.39)$
$\lvert a^T_{\mu\tau}\rvert / \lvert a^T_{\mu\mu}\rvert$	$(0, 0.40)$	$(0, 0.39)$

The parameter ranges in the table predict the black region of flavor fraction—disfavored at 3 σ

- Each special pattern assumed above has a large μτ symmetry breaking effect to begin with.
- μτ symmetry breaking effect in each case is washed out as other elements in the LV Hamiltonian are growing. At some point, the predicted neutrino flavor fraction is consistent with that given by standard vacuum oscillation alone, i.e., the neutrino flavor fraction is no longer sensitive to LV Hamiltonian.
- It is of interest to study how the constraint on LV energy scale imposed by neutrino flavor measurement varies with the μτ symmetry breaking effect.

Taking the example

$$H^{\nu}_{\rm LV} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & a^T_{\mu\mu} & a^T_{\mu\tau} \\ 0 & a^{T*}_{\mu\tau} & a^T_{\tau\tau} \end{pmatrix}$$

$$= \frac{a^T_{\mu\mu} + a^T_{\tau\tau}}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \frac{1}{2} \begin{pmatrix} a^T_{\mu\mu} + a^T_{\tau\tau} & 0 & 0 \\ 0 & a^T_{\tau\tau} - a^T_{\mu\mu} & -2a^T_{\mu\tau} \\ 0 & -2a^{T*}_{\mu\tau} & a^T_{\mu\mu} - a^T_{\tau\tau}) \end{pmatrix}$$

Relevant

The relevant part of the Hamiltonian can be written as

$$H = H_{\rm SM} + H^{\nu}_{\rm LV}$$

$$H^{\nu}_{\rm LV} = -M \begin{pmatrix} \gamma & 0 & 0 \\ 0 & \cos 2\alpha & e^{i\beta}\sin 2\alpha \\ 0 & e^{-i\beta}\sin 2\alpha & -\cos 2\alpha \end{pmatrix}$$

Here

$$M = \frac{1}{2}\sqrt{(a^T_{\tau\tau} - a^T_{\mu\mu})^2 + 4a^T_{\mu\tau}a^{T*}_{\mu\tau}}$$

$$\gamma = \frac{a^T_{\mu\mu} + a^T_{\tau\tau}}{\sqrt{(a^T_{\tau\tau} - a^T_{\mu\mu})^2 + 4a^T_{\mu\tau}a^{T*}_{\mu\tau}}}$$

$$\cos 2\alpha = \frac{a^T_{\tau\tau} - a^T_{\mu\mu}}{\sqrt{(a^T_{\tau\tau} - a^T_{\mu\mu})^2 + 4a^T_{\mu\tau}a^{T*}_{\mu\tau}}}$$

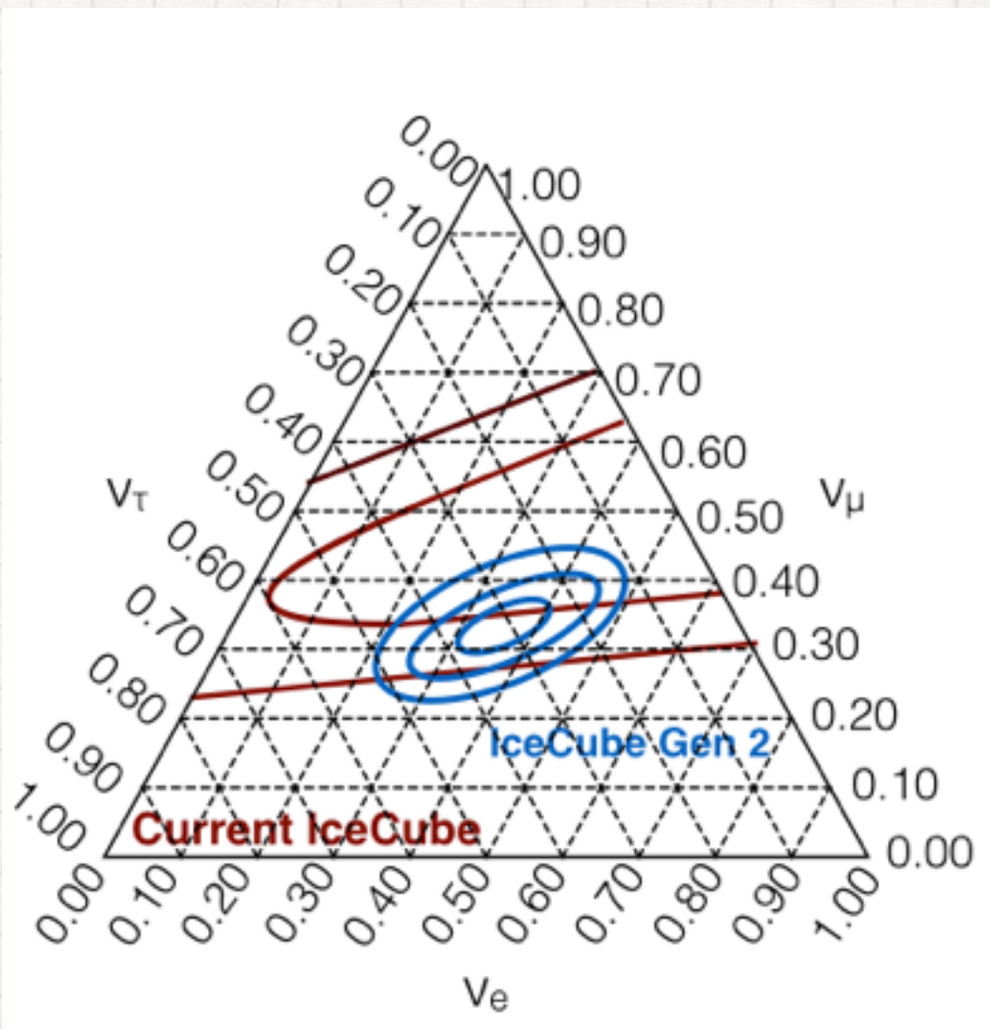
$$\sin 2\alpha = \frac{2|a^T_{\mu\tau}|}{\sqrt{(a^T_{\tau\tau} - a^T_{\mu\mu})^2 + 4a^T_{\mu\tau}a^{T*}_{\mu\tau}}}$$

μτ symmetry limit
$\alpha = \pi/4$

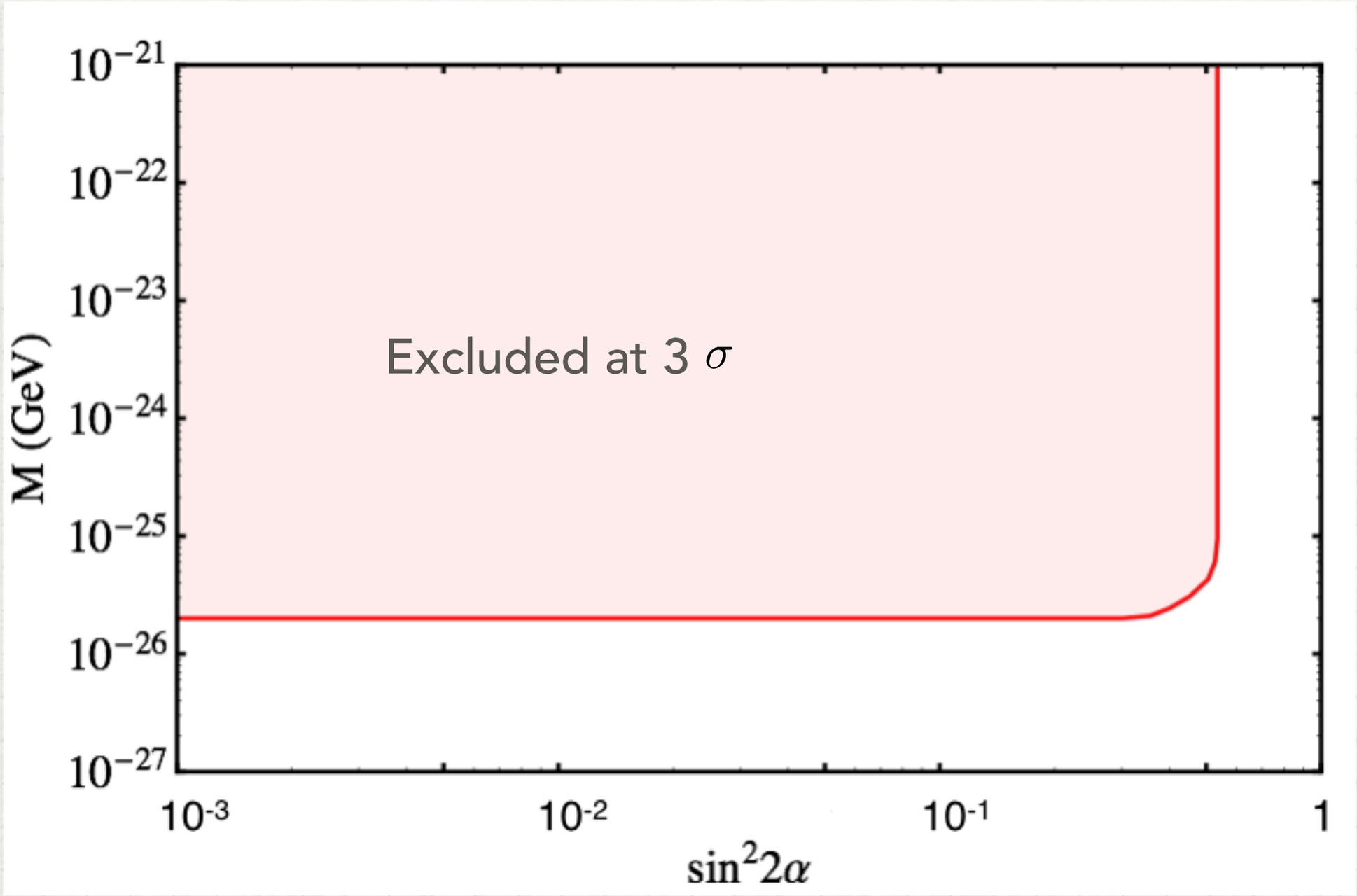
The constraint on *M* depends on α

$$H = H_{\rm SM} + H^{\nu}_{\rm LV}$$

$$H^{\nu}_{\rm LV} = -M \begin{pmatrix} \gamma & 0 & 0 \\ 0 & \cos 2\alpha & e^{i\beta}\sin 2\alpha \\ 0 & e^{-i\beta}\sin 2\alpha & -\cos 2\alpha \end{pmatrix}$$



Increasing M until the predicted flavor fraction is out of the IceCube Gen2 3σ region.



For most values of $\sin^2 2\alpha$, the energy scale *M* is constrained to be less than few times 10^{-26} GeV.

Constraints on $C^{TT}_{\alpha\beta}$

$-4Ec^{TT}_{\alpha\beta}/3$ replaces $a^{T}_{\alpha\beta}$ when the latter is turn off.

If the constraint on *M*, which is made of $a^{T}_{\alpha\beta}$, is few times 10^{-26} GeV, the corresponding constraint on *M'* (dimensionless quantity made of $C^{TT}_{\alpha\beta}$) is about 10^{-31} with *E* chosen as 100 TeV.

Threshold energy

K. Abe *et al.* [Super-Kamiokande Collaboration], Phys. Rev. D 91, no. 5, 052003 (2015).

LV parameter		Limit at 95% C.L.	Best fit	No LV $\Delta\chi^2$	Previous limit
$e\mu$	$\mathrm{Re}(a^T)$	1.8×10^{-23} GeV	1.0×10^{-23} GeV	1.4	4.2×10^{-20} GeV [61]
	$\mathrm{Im}(a^T)$	1.8×10^{-23} GeV	4.6×10^{-24} GeV		
	$\mathrm{Re}(c^{TT})$	8.0×10^{-27}	1.0×10^{-28}	0.0	9.6×10^{-20} [61]
	$\mathrm{Im}(c^{TT})$	8.0×10^{-27}	1.0×10^{-28}		
$e\tau$	$\mathrm{Re}(a^T)$	4.1×10^{-23} GeV	2.2×10^{-24} GeV	0.0	7.8×10^{-20} GeV [62]
	$\mathrm{Im}(a^T)$	2.8×10^{-23} GeV	1.0×10^{-28} GeV		
	$\mathrm{Re}(c^{TT})$	9.3×10^{-25}	1.0×10^{-28}	0.3	1.3×10^{-17} [62]
	$\mathrm{Im}(c^{TT})$	1.0×10^{-24}	3.5×10^{-25}		
$\mu\tau$	$\mathrm{Re}(a^T)$	6.5×10^{-24} GeV	3.2×10^{-24} GeV	0.9	...
	$\mathrm{Im}(a^T)$	5.1×10^{-24} GeV	1.0×10^{-28} GeV		
	$\mathrm{Re}(c^{TT})$	4.4×10^{-27}	1.0×10^{-28}	0.1	...
	$\mathrm{Im}(c^{TT})$	4.2×10^{-27}	7.5×10^{-28}		

5. SUMMARY AND CONCLUSIONS

- We have introduced Lorentz violation Hamiltonian in neutrino sector and discuss its effect on neutrino oscillations.
- Previous experimental search on Lorentz violation with neutrino is introduced. Previous best limit by Super-Kamiokande experiment is summarized.
- We have shown that Lorentz violating Hamiltonian with parameters in the above SK limits can change significantly the flavor transition probabilities of high energy astrophysical neutrinos in TeV to PeV energy range.
- For the pion source induced from *pp* collisions, Lorentz violating Hamiltonian with large $\mu\tau$ symmetry breaking effect is more stringently constrained.
- We show that IceCube-Gen2 can place stringent constraints on the Lorentz violating Hamiltonian.