

# Nonperturbative determination of form factors for semileptonic $B_s$ meson decays

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# RBC- and UKQCD collaborations

## BNL/RBRC

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Taku Izubuchi  
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## U Connecticut

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Chris Sachrajda

## U Liverpool

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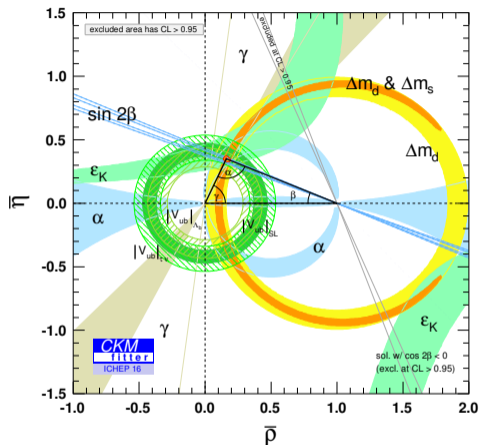
## U Liverpool

Nicolas Garron

introduction

## Why $B_s$ meson decays?

- ▶ Alternative, tree-level determination of  $|V_{cb}|$  and  $|V_{ub}|$  from  $B_s \rightarrow D\ell\nu$  and  $B_s \rightarrow K\ell\nu$
- ▶ Commonly used  $B \rightarrow \pi\ell\nu$  and  $B \rightarrow D^{(*)}\ell\nu$
- ▶ Longstanding  $2 - 3\sigma$  discrepancy between exclusive ( $B \rightarrow \pi\ell\nu$ ) and inclusive ( $B \rightarrow X_u\ell\nu$ )
- ▶  $B \rightarrow \tau\nu$  has larger error
- ▶ Alternative, exclusive ( $\Lambda_b \rightarrow p\ell\nu$ ) determination  
[Detmold, Lehner, Meinel, PRD92 (2015) 034503]

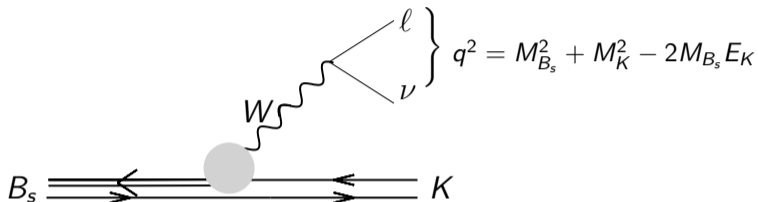


[<http://ckmfitter.in2p3.fr>]

## Why $B_s$ meson decays?

- ▶ Not (yet) experimentally measured with sufficient precision
- ▶  $B$ -factories typically run at the  $\Upsilon(4s)$  threshold
  - $B$  but no  $B_s$  mesons are produced
- ▶ At the LHC energies are large enough to produce sufficient  $B_s$  mesons
- ▶ LHCb is working on the analysis
  - Absolute normalization is challenging; ratios are preferred
  - Determine  $|V_{cb}|/|V_{ub}|$
- ▶ strange-quarks are easier on the lattice

# $|V_{ub}|$ from exclusive semileptonic $B_s \rightarrow K\ell\nu$ decay



► Conventionally parametrized by

$$\frac{d\Gamma(B_s \rightarrow K\ell\nu)}{dq^2} = \frac{G_F^2}{192\pi^3 M_{B_s}^3} \left[ (M_{B_s}^2 + M_K^2 - q^2)^2 - 4M_{B_s}^2 M_K^2 \right]^{3/2} \times |f_+(q^2)|^2 \times |V_{ub}|^2$$

experiment

known

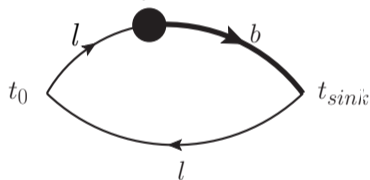
nonperturbative input

CKM

## $B_s \rightarrow Kl\nu$ form factors

- ▶ Parametrize the hadronic matrix element for the flavor changing vector current  $V^\mu$  in terms of the form factors  $f_+(q^2)$  and  $f_0(q^2)$

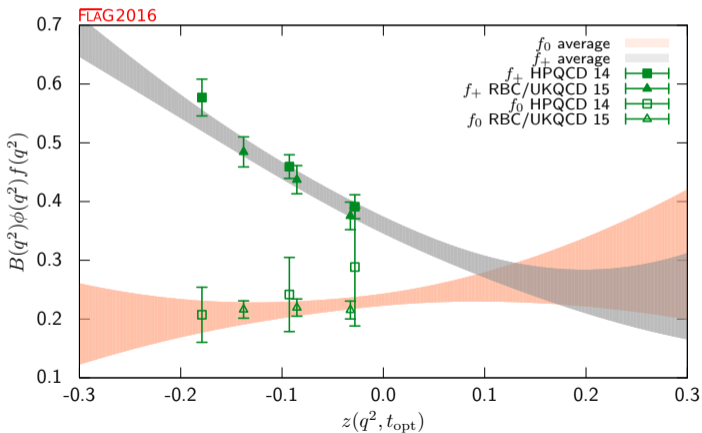
$$\langle K | V^\mu | B_s \rangle = f_+(q^2) \left( p_{B_s}^\mu + p_K^\mu - \frac{M_{B_s}^2 - M_K^2}{q^2} q^\mu \right) + f_0(q^2) \frac{M_{B_s}^2 - M_K^2}{q^2} q^\mu$$



- ▶ Calculate 3-point function by
  - Inserting a quark source for a “light” propagator at  $t_0$
  - Allow it to propagate to  $t_{sink}$ , turn it into a sequential source for a  $b$  quark
  - Use another “light” quark propagating from  $t_0$  and contract both at  $t$

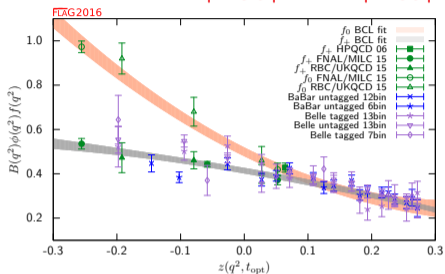
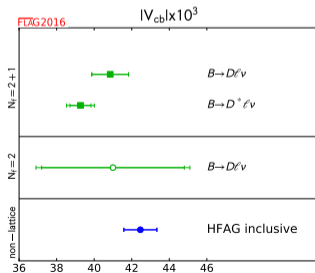
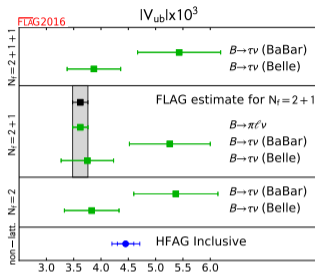
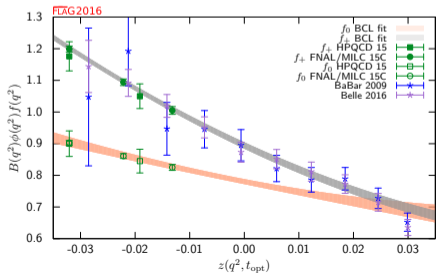


# Lattice determinations of $B_s \rightarrow K\ell\nu$ form factors



[FLAG2016]

# Lattice determinations of $|V_{ub}|$ and $|V_{cb}|$

 $B \rightarrow \pi l \nu$ 

 $B \rightarrow D l \nu$ 


[FLAG2016]

RBC-UKQCD's project

# Target quantities

- ▶ Decay constants  $f_B$  and  $f_{B_s}$
  - ▶  $B^0 - \bar{B}^0$  mixing matrix elements
  - ▶ Semileptonic form factors with charged and neutral flavor changing currents  
 $B \rightarrow \pi l \nu$ ,  $B_s \rightarrow K l \nu$ ,  $B \rightarrow D^{(*)} l \nu$ ,  $B_s \rightarrow D_s^{(*)} l \nu$ , ...  
 $B \rightarrow K^{(*)} l^+ l^-$ ,  $B_s \rightarrow \phi l^+ l^-$ , ...
- Ratios  $R(D^{(*)})$ ,  $R(K^{(*)})$ , ...

# Set-up

- ▶ RBC-UKQCD's 2+1 flavor domain-wall fermion and Iwasaki gauge action ensembles
  - Three lattice spacings  $a \sim 0.11$  fm, 0.08 fm, 0.07 fm; one ensemble with physical pions  
[PRD 78 (2008) 114509][PRD 83 (2011) 074508][PRD 93 (2016) 074505][arXiv:1701.02644]
- ▶ Unitary and partially quenched domain-wall up/down quarks  
[Kaplan PLB 288 (1992) 342], [Shamir NPB 406 (1993) 90]
- ▶ Domain-wall strange quarks at/near the physical value
- ▶ Charm: Möbius domain-wall fermions optimized for heavy quarks [Boyle et al. JHEP 1604 (2016) 037]
  - Simulate 3 or 2 charm-like masses then extrapolate/interpolate
- ▶ Effective relativistic heavy quark (RHQ) action for bottom quarks  
[Christ et al. PRD 76 (2007) 074505], [Lin and Christ PRD 76 (2007) 074506]
  - Builds upon Fermilab approach [El-Khadra et al. PRD 55 (1997) 3933]
  - Allows to tune the three parameters ( $m_0 a$ ,  $c_P$ ,  $\zeta$ ) nonperturbatively [PRD 86 (2012) 116003]
  - Smooth continuum limit; heavy quark treated to all orders in  $(m_b a)^n$

## Determining $B_s \rightarrow K\ell\nu$ form factors $f_+$ and $f_0$ on the lattice

- ▶ Updating calculation [PRD 91 (2015) 074510] with new values for  $a^{-1}$  and RHQ parameters
- ▶ On the lattice we prefer using the  $B_s$ -meson rest frame and compute

$$f_{\parallel}(E_K) = \langle K|V^0|B_s\rangle/\sqrt{2M_{B_s}} \quad \text{and} \quad f_{\perp}(E_K)p_K^i = \langle K|V^i|B_s\rangle/\sqrt{2M_{B_s}}$$

- ▶ Both are related by

$$f_0(q^2) = \frac{\sqrt{2M_{B_s}}}{M_{B_s}^2 - M_K^2} [(M_{B_s} - E_K)f_{\parallel}(E_K) + (E_K^2 - M_K^2)f_{\perp}(E_K)]$$

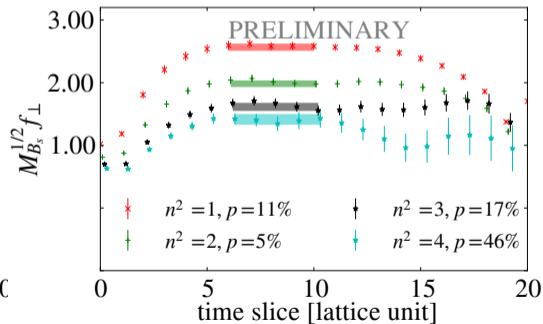
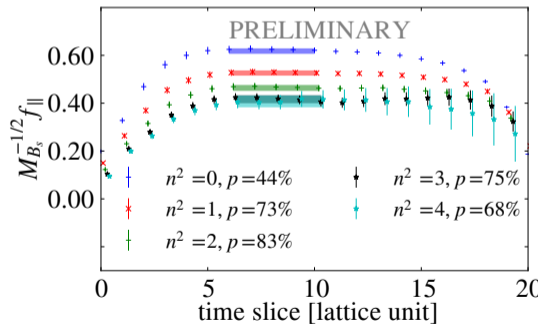
$$f_+(q^2) = \frac{1}{\sqrt{2M_{B_s}}} [f_{\parallel}(E_K) + (M_{B_s} - E_K)f_{\perp}(E_K)]$$

# Lattice results for form factors $f_{\parallel}$ and $f_{\perp}$ for $B_s \rightarrow K\ell\nu$

$$R_{\mu}^{B_s \rightarrow K}(t, t_{\text{sink}}) = \frac{C_{3,\mu}^{B_s \rightarrow K}(t, t_{\text{sink}})}{C_2^K(t) C_2^{B_s}(t_{\text{sink}} - t)} \sqrt{\frac{4M_{B_s} E_K}{e^{-E_K t} e^{-M_{B_s}(t_{\text{sink}} - t)}}$$

$$f_{\parallel} = \lim_{t, t_{\text{sink}} \rightarrow \infty} R_0^{B_s \rightarrow K}(t, t_{\text{sink}})$$

$$f_{\perp} = \lim_{t, t_{\text{sink}} \rightarrow \infty} \frac{1}{p_{\pi}^i} R_i^{B_s \rightarrow K}(t, t_{\text{sink}})$$

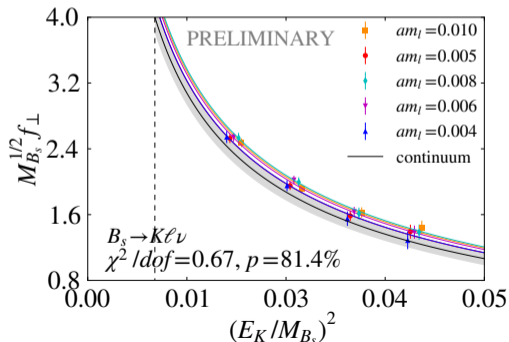
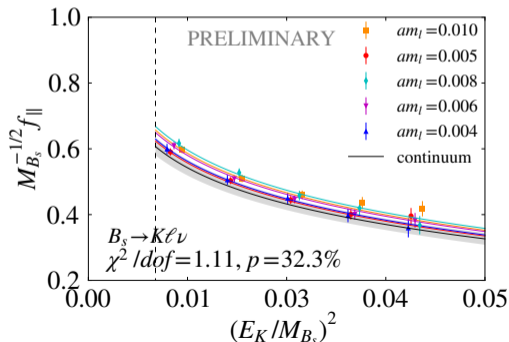


# Chiral-continuum extrapolation using SU(2) hard-kaon $\chi$ PT

$$f_{\parallel}(M_K, E_K, a^2) = \frac{1}{E_K + \Delta} c_{\parallel}^{(1)} \left[ 1 + \left( \frac{\delta f_{\parallel}}{(4\pi f)^2} + c_{\parallel}^{(2)} \frac{M_K^2}{\Lambda^2} + c_{\parallel}^{(3)} \frac{E_K}{\Lambda} + c_{\parallel}^{(4)} \frac{E_K^2}{\Lambda^2} + c_{\parallel}^{(5)} \frac{a^2}{\Lambda^2 a_{32}^4} \right) \right]$$

$$f_{\perp}(M_K, E_K, a^2) = \frac{1}{E_K + \Delta} c_{\perp}^{(1)} \left[ 1 + \left( \frac{\delta f_{\perp}}{(4\pi f)^2} + c_{\perp}^{(2)} \frac{M_K^2}{\Lambda^2} + c_{\perp}^{(3)} \frac{E_K}{\Lambda} + c_{\perp}^{(4)} \frac{E_K^2}{\Lambda^2} + c_{\perp}^{(5)} \frac{a^2}{\Lambda^2 a_{32}^4} \right) \right]$$

with  $\delta f$  non-analytic logs of the kaon mass and hard-kaon limit is taken by  $M_K/E_K \rightarrow 0$

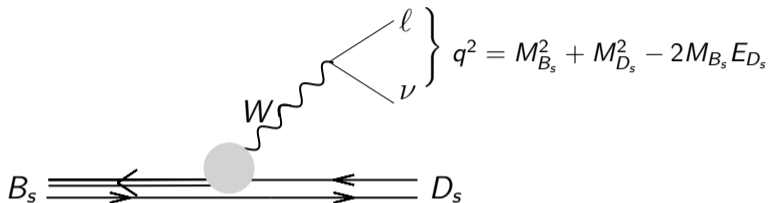




## Next steps

- ▶ Estimate full systematic errors for three “synthetic” data points
- ▶ Perform  $z$ -expansion and polynomial fits
- ▶ Comparison with other result(s) [HPQCD PRD90 (2014) 054506]

# $|V_{cb}|$ from exclusive semileptonic $B_s \rightarrow D_s \ell \nu$ decay



► Conventionally parametrized by

$$\frac{d\Gamma(B_s \rightarrow D_s \ell \nu)}{dq^2} = \frac{G_F^2}{192\pi^3 M_{B_s}^3} \left[ (M_{B_s}^2 + M_{D_s}^2 - q^2)^2 - 4M_{B_s}^2 M_{D_s}^2 \right]^{3/2} \times |f_+(q^2)|^2 \times |V_{cb}|^2$$

experiment

known

nonperturbative input

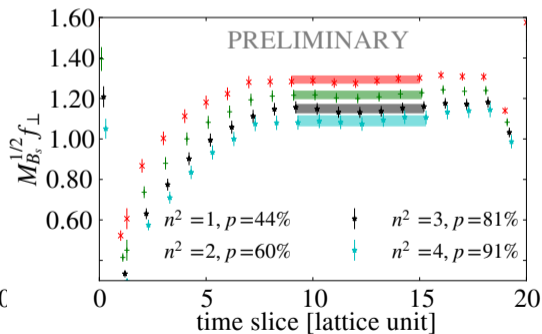
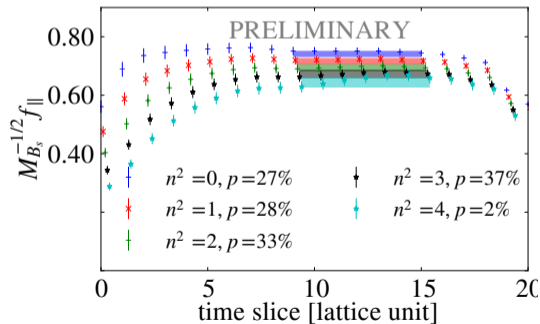
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# Lattice results for form factors $f_{\parallel}$ and $f_{\perp}$ for $B_s \rightarrow D_s \ell \nu$

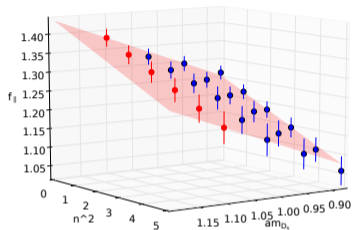
$$R_{\mu}^{B_s \rightarrow D_s}(t, t_{\text{sink}}) = \frac{C_{3,\mu}^{B_s \rightarrow D_s}(t, t_{\text{sink}})}{C_2^{D_s}(t) C_2^{B_s}(t_{\text{sink}} - t)} \sqrt{\frac{4M_{B_s} E_{D_s}}{e^{-E_{D_s} t} e^{-M_{B_s}(t_{\text{sink}} - t)}}$$

$$f_{\parallel} = \lim_{t, t_{\text{sink}} \rightarrow \infty} R_0^{B_s \rightarrow D_s}(t, t_{\text{sink}})$$

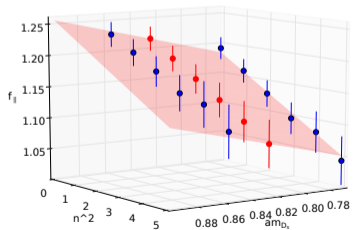
$$f_{\perp} = \lim_{t, t_{\text{sink}} \rightarrow \infty} \frac{1}{p_{\pi}^i} R_i^{B_s \rightarrow D_s}(t, t_{\text{sink}})$$



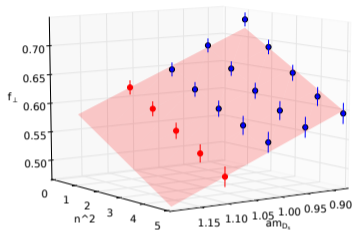
# Charm extra-/interpolation for $B_s \rightarrow D_s \ell \nu$



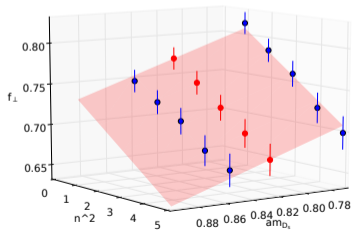
PRELIMINARY



PRELIMINARY



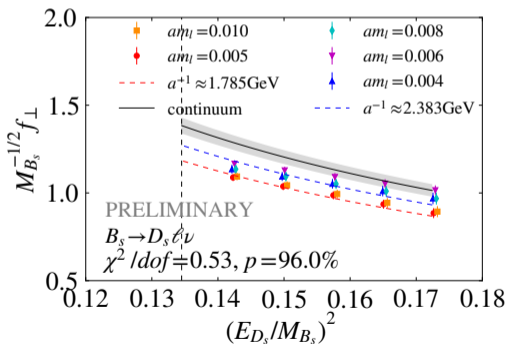
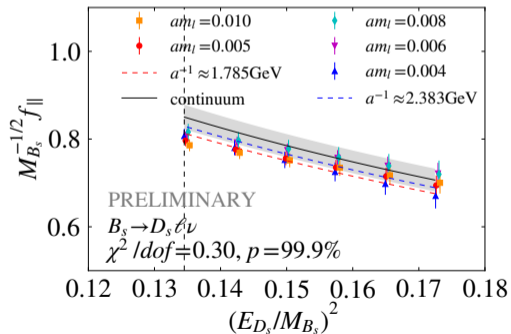
PRELIMINARY



PRELIMINARY

# Chiral-continuum extrapolation for $B_s \rightarrow D_s \ell \nu$

$$f(q, a) = \frac{c_0 + c_1(\Lambda_{\text{QCD}} a)^2}{1 + c_2(q/M_{B_c})^2}$$



## Next steps

- ▶ Estimate full systematic errors for three “synthetic” data points
- ▶ Perform z-expansion and polynomial fits
- ▶ Comparison with other result(s) [HPQCD 2017]
- ▶ Explore advantageous ratios

conclusion

# Conclusion

- ▶ About to complete calculation for  $B_s \rightarrow K l \nu$  and  $B_s \rightarrow D_s l \nu$ 
  - Finalizing systematic error estimates and kinematic extrapolations
  
- ▶ Not enough time to cover  $B_s \rightarrow \phi l^+ l^-$  (→ see appendix)
  
- ▶ We have more data for
  - $B \rightarrow \pi l \nu$ ,  $B \rightarrow \pi l^+ l^-$
  - $B \rightarrow K^* l^+ l^-$
  - $B \rightarrow D^{(*)} l \nu$
  - $B_s \rightarrow D_s^* l \nu$
  - ...



# Resources and Acknowledgments

**USQCD:** Ds, Bc, and  $\pi_0$  cluster (Fermilab), qcd12s cluster (Jlab)

**RBC** qcdcl (RIKEN) and cuth (Columbia U)

**UK:** ARCHER (EPCC) and DiRAC (UKQCD)



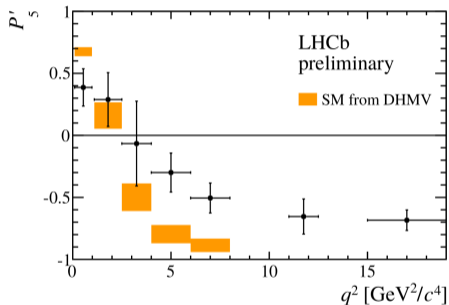
appendix

# flavor changing neutral currents

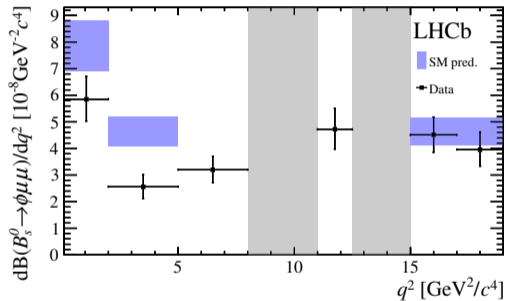
(loop-level in the Standard Model)

## Rare $B$ decays (FCNC)

- ▶ GIM suppressed in the Standard Model  $\Rightarrow$  sensitive to new physics
- ▶ Angular observable  $P'_5$  in  $B \rightarrow K^* \mu^+ \mu^-$  received a lot of attention



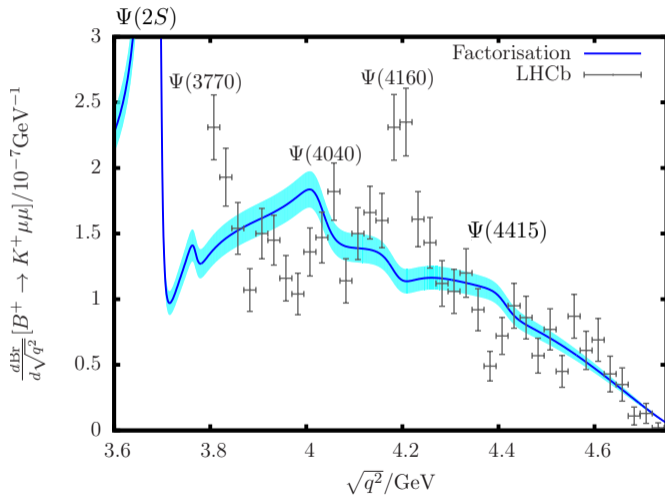
[LHCb-CONF-2015-002]



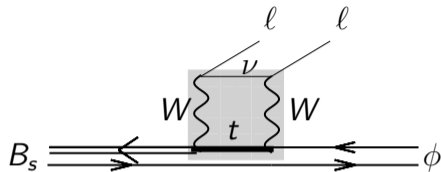
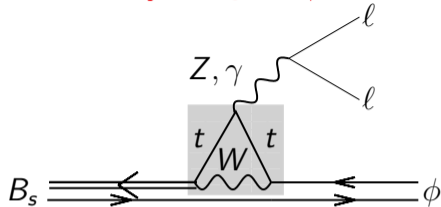
[LHCb JHEP 1509 (2015) 179]

- ▶ Lattice QCD: [Horgan et al. PRD 89 (2013) 094501]

► Charm resonances under control? [Lyon and Zwicky, arXiv:1406.0566]



## Rare $B$ decays: $B_s \rightarrow \phi l^+ l^-$



- ▶ Pseudoscalar or vector final state (narrow width approximation)
- ▶ Effective Hamiltonian

$$\mathcal{H}_{\text{eff}}^{b \rightarrow s} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_i^{10} C_i O_i^{(r)}$$

- ▶ Leading contributions at short distance

$$O_7^{(r)} = \frac{m_b e}{16\pi^2} \bar{s} \sigma^{\mu\nu} P_{R(L)} b F_{\mu\nu}$$

$$O_9^{(r)} = \frac{e^2}{16\pi^2} \bar{s} \gamma^\mu P_{L(R)} b \bar{l} \gamma_\mu l$$

$$O_{10}^{(r)} = \frac{e^2}{16\pi^2} \bar{s} \gamma^\mu P_{L(R)} b \bar{l} \gamma_\mu \gamma^5 l$$

## Seven form factors

$$\langle \phi(k, \lambda) | \bar{s} \gamma^\mu b | B_s(p) \rangle = f_V(q^2) \frac{2i \epsilon^{\mu\nu\rho\sigma} \epsilon_\nu^* k_\rho p_\sigma}{M_{B_s} + M_\phi}$$

$$\langle \phi(k, \lambda) | \bar{s} \gamma^\mu \gamma_5 b | B_s(p) \rangle = f_{A_0}(q^2) \frac{2M_\phi \epsilon^* \cdot q}{q^2} q^\mu$$

$$+ f_{A_1}(q^2) (M_{B_s} + M_\phi) \left[ \epsilon^{*\mu} - \frac{\epsilon^* \cdot q}{q^2} q^\mu \right]$$

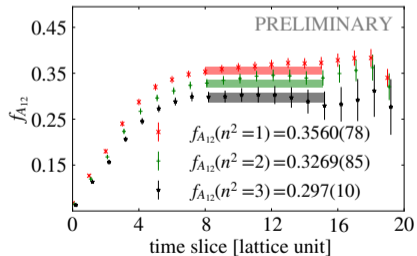
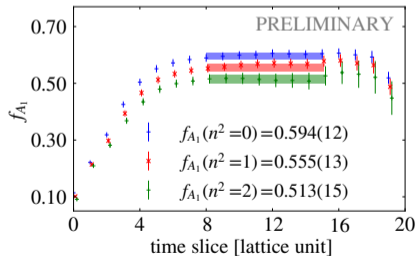
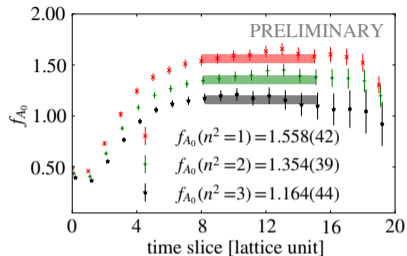
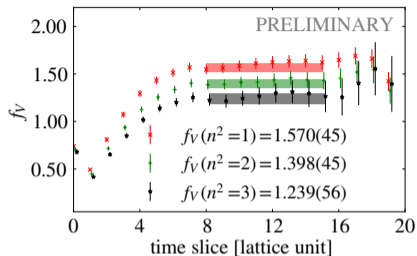
$$- f_{A_2}(q^2) \frac{\epsilon^* \cdot q}{M_{B_s} + M_\phi} \left[ k^\mu + p^\mu - \frac{M_{B_s}^2 - M_\phi^2}{q^2} q^\mu \right]$$

$$q_\nu \langle \phi(k, \lambda) | \bar{s} \sigma^{\nu\mu} b | B_s(p) \rangle = 2f_{T_1}(q^2) \epsilon^{\mu\rho\tau\sigma} \epsilon_\rho^* k_\tau p_\sigma ,$$

$$q_\nu \langle \phi(k, \lambda) | \bar{s} \sigma^{\nu\mu} \gamma^5 b | B_s(p) \rangle = if_{T_2}(q^2) \left[ \epsilon^{*\mu} (M_{B_s}^2 - M_\phi^2) - (\epsilon^* \cdot q)(p + k)^\mu \right]$$

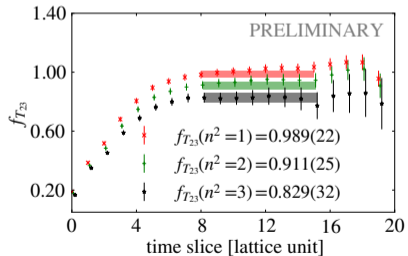
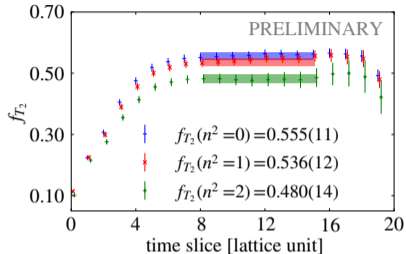
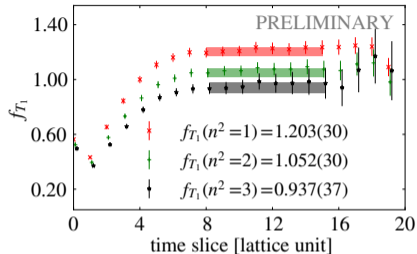
$$+ if_{T_3}(q^2) (\epsilon^* \cdot q) \left[ q^\mu - \frac{q^2}{M_{B_s}^2 - M_\phi^2} (p + k)^\mu \right]$$

# Seven form factors

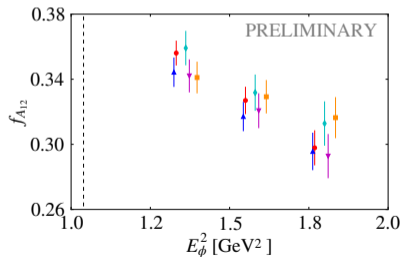
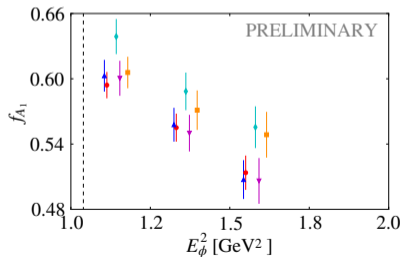
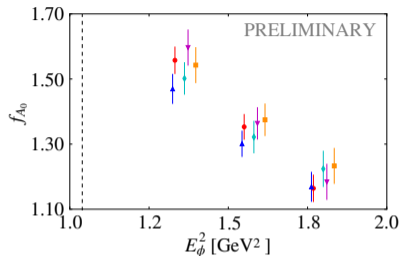
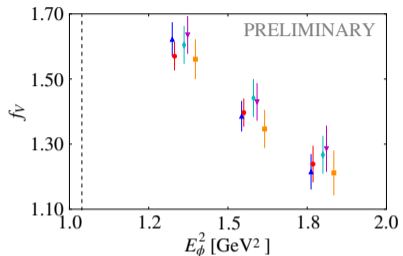




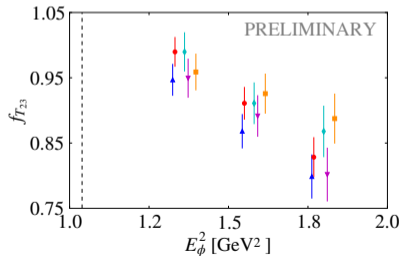
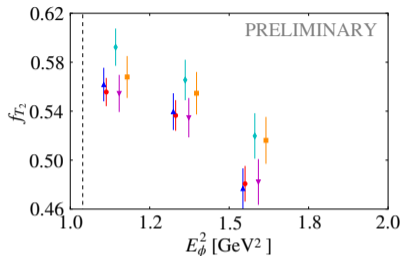
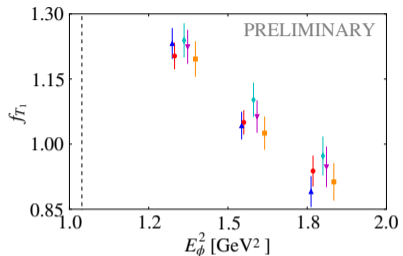
# Seven form factors



# Seven form factors vs. $q^2$



# Seven form factors vs. $q^2$



## 2+1 Flavor Domain-Wall Iwasaki ensembles

L	$a^{-1}(\text{GeV})$	$am_l$	$am_s$	$M_\pi(\text{MeV})$	# configs.	#sources	
24	1.784	0.005	0.040	338	1636	1	[PRD 78 (2008) 114509]
24	1.784	0.010	0.040	434	1419	1	[PRD 78 (2008) 114509]
32	2.383	0.004	0.030	301	628	2	[PRD 83 (2011) 074508]
32	2.383	0.006	0.030	362	889	2	[PRD 83 (2011) 074508]
32	2.383	0.008	0.030	411	544	2	[PRD 83 (2011) 074508]
48	1.730	0.00078	0.0362	139	40	81/1*	[PRD 93 (2016) 074505]
64	2.359	0.000678	0.02661	139	—	—	[PRD 93 (2016) 074505]
48	2.774	0.002144	0.02144	234	70	24	[arXiv:1701.02644]

\* All mode averaging: 81 “sloppy” and 1 “exact” solve [Blum et al. PRD 88 (2012) 094503]

► Lattice spacing determined from combined analysis [Blum et al. PRD 93 (2016) 074505]

►  $a$ :  $\sim 0.11$  fm,  $\sim 0.08$  fm,  $\sim 0.07$  fm