Rare FCNC radiative leptonic decays $B \to \gamma ll$

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1. **Motivation: tensions with SM predictions in FCNC $b \to s, d$ decays**
2. **H_{eff} for $b \to s, d$ and the $\langle \gamma l^+ l^- | H_{\text{eff}} | B \rangle$ amplitude**
3. **$B \to \gamma$ form factors and light resonance contributions**
4. **Charming loops**
5. **Conclusions and outlook**

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FCNC $b \to s$ and $b \to d$ transitions do not occur at the tree level in SM and proceed via loops. As the result, BRs of FCNC decays are small; on the other hand, new particles may show up virtually in the loops. Therefore, FCNC decays are most popular candidates for indirect search of physics BSM.

Tensions between SM predictions and observations in FCNC $b \to s$ transitions:

In SL decays:

- $\mathcal{R}_K = \frac{\mathcal{B}(B^+\to K^+\mu^+\mu^-)}{\mathcal{B}(B^+\to K^+e^+e^-)} = 0.745^{+0.090}_{-0.074}(\text{stat}) \pm 0.036(\text{syst})$ **(2.6σ)**
- $\mathcal{B}(B^+ \to K^+\mu^+\mu^-)_{\text{SM}} = (1.75^{+0.60}_{-0.29}) \times 10^{-7}$

$\mathcal{B}(B^+ \to K^+\mu^+\mu^-)_{\text{exp}} = (1.19 \pm 0.03 \pm 0.06) \times 10^{-7}$

- $\mathcal{R}_{K^{*0}} = 0.69\,^{+\,0.11}_{-\,0.07}\,(\text{stat}) \pm 0.05\,(\text{syst})$ **for** $1.1 < q^2 < 6.0\text{GeV}^2$
- **Same for** $\mathcal{B}(B^+ \to \phi\mu^+\mu^-)$ **$(> 3\sigma)$**

In leptonic decays:

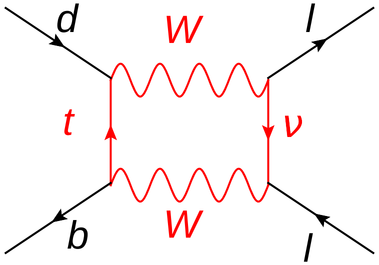
- $\frac{\mathcal{B}(B_s^0\to\mu^+\mu^-)_{\text{exp}}}{\mathcal{B}(B_s^0\to\mu^+\mu^-)_{\text{SM}}} = 0.76^{+0.20}_{-0.18}$ **(1.2σ)**

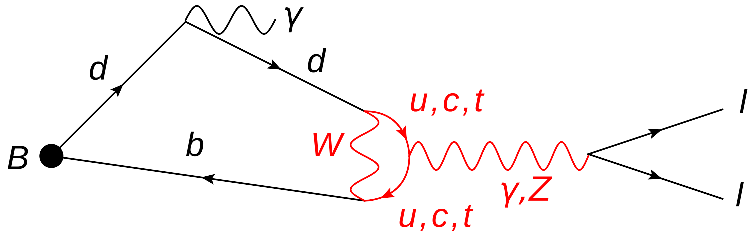
Expectations for *radiative leptonic decays*:

$\frac{\mathcal{B}(B_s^0\to\ell^+\ell^-\gamma)}{\mathcal{B}(B_s^0\to\ell^+\ell^-)} \sim \left(\frac{M_{B^0}}{m_\ell}\right)^2 \frac{\alpha_{em}}{4\pi} \sim 1$ **for muons.**

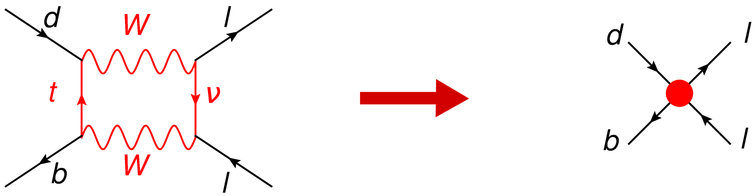
Effective Hamiltonian for FCNC B – decays

FCNC transitions are forbidden in SM and proceed via boxes and penguins





For B-decays W, Z, t are integrated out



$$\mathcal{H}_{\rm eff}^{b\to q} = \frac{G_F}{\sqrt{2}} V_{tb} V_{tq}^* \sum_i C_i(\mu) O_i(\mu),$$

$C_i(\mu)$ - **Wilson coefficients**, $O_i(\mu)$ - **basis operators.**

Contributions of top and W to $\mathcal{H}_{\rm eff}$:

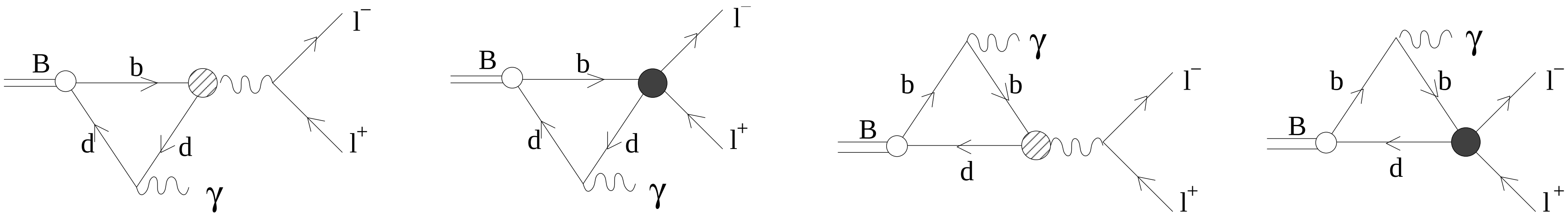
$$\mathcal{H}_{\rm eff}^{b\to d\ell^+\ell^-} = \frac{G_F}{\sqrt{2}} \frac{\alpha_{\rm em}}{2\pi} V_{tb} V_{tq}^* [-2 i m_b \frac{C_{7\gamma}(\mu)}{q^2} \cdot \bar{d} \sigma_{\mu\nu} q^\nu (1+\gamma_5) b \cdot \bar{\ell}\gamma^\mu \ell +$$
$$+ \; C_{9V}(\mu) \cdot \bar{d}\gamma_\mu (1-\gamma_5) b \cdot \bar{\ell}\gamma^\mu \ell + C_{10A}(\mu) \cdot \bar{d}\gamma_\mu (1-\gamma_5) b \cdot \bar{\ell}\gamma^\mu \gamma_5 \ell]$$

But c, u, d, s-quarks are dynamical! Calculate the amplitude $\langle \gamma l^+ l^- | H_{\rm eff} | B \rangle$ + add CKM suppressed contributions of c and u quarks, $C_{9V}(\mu) \to C_{9V}^{\rm eff}(\mu, q^2)$.

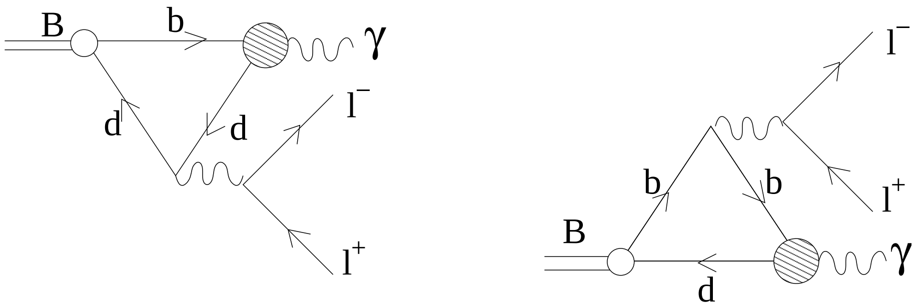
The $\langle \gamma l^+ l^- | H_{\rm eff} | B \rangle$ amplitude can be parameterized via form factors ($0 < q^2 < M_B^2$):

$$
\begin{aligned}
\langle \gamma(k,\,\epsilon)|\bar d \gamma_\mu \gamma_5 b|B(p)\rangle &= i\,e\,\epsilon^*_\alpha \left(g_{\mu\alpha}\, pk - p_\alpha k_\mu\right) \frac{F_A(q^2)}{M_B},\\
\langle \gamma(k,\,\epsilon)|\bar d \gamma_\mu b|B(p)\rangle &= e\,\epsilon^*_\alpha\, \epsilon_{\mu\alpha\xi\eta} p_\xi k_\eta \frac{F_V(q^2)}{M_B},\\
\langle \gamma(k,\,\epsilon)|\bar d \sigma_{\mu\nu} \gamma_5 b|B(p)\rangle\,(p-k)^\nu &= e\,\epsilon^*_\alpha \left[g_{\mu\alpha}\, pk - p_\alpha k_\mu\right] F_{TA}(q^2),\\
\langle \gamma(k,\,\epsilon)|\bar d \sigma_{\mu\nu} b|B(p)\rangle\,(p-k)^\nu &= i\,e\,\epsilon^*_\alpha \epsilon_{\mu\alpha\xi\eta} p_\xi k_\eta\, F_{TV}(q^2).
\end{aligned}
$$

Diagrams with *real* photon emission from valence quarks (no poles in the range $0 < q^2 < M_B^2$):

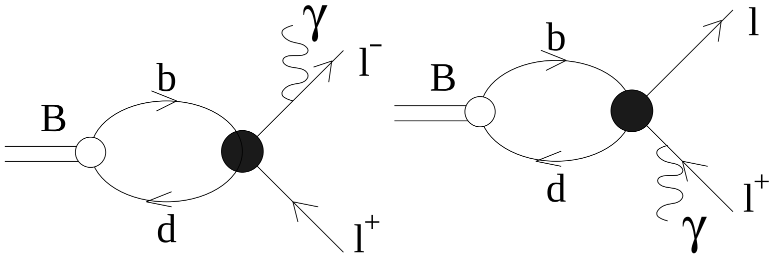


Diagrams with *virtual* photon emission from valence quarks (pole in the physical q^2-range):

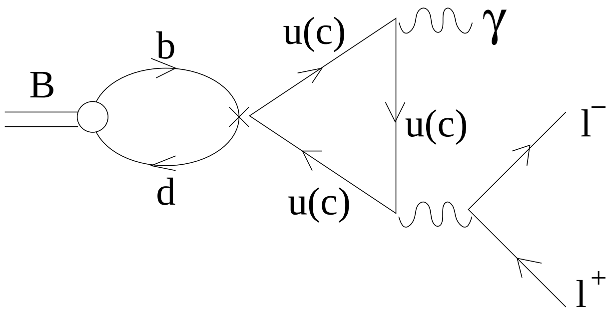


Dashed blob: penguin operator $O_{7\gamma}$; full blob: four-fermion operators O_{9V} and O_{10A}.

Bremsstrahlung:



Weak annihilation:



Form factor calculation

For form factor calculations at q^2 below the resonance region, we make use of dispersion approach based on constituent quark picture: All hadron observables are given by dispersion representations in terms of the hadron relativistic wave functions and the spectral densities of Feynman diagrams with constituent quarks in the loops.

- **Decay constants:**

$$f_{P,V} = \int ds \phi_{P,V}(s) \rho_{P,V}(s)$$

- **Meson-meson transition form factors:**

$$F_{M_1 \to M_2}(q^2) = \int ds_1 \phi_1(s_1) ds_2 \phi_2(s_2) \Delta(s_1, s_2, q^2)$$

- **Meson-photon transition form factors into photon of virtuality k^2:**

$$\phi_\gamma(s) = 1/(s - k^2)$$

- **The wave function is normalized via the electromagnetic form factor at $q^2 = 0$:**

$$F_{el}(q^2 = 0) = \text{charge}$$

How these form factors agree with the known properties form QCD in different limits?

The spectral representations:

- **The meson-meson transition form factors satisfy constraints from HQET for heavy-to-heavy transitions**
- **The meson-meson and meson-photon form factors satisfy constraints from LEET for heavy-to-light transitions**

Fixing numerical parameters:

- **For relativistic wave functions a simple one-parameter Gaussian Ansatz is taken (k^2 is the known function of s)**

$$\phi(s) \sim e^{-\frac{k^2}{2\beta^2}}$$

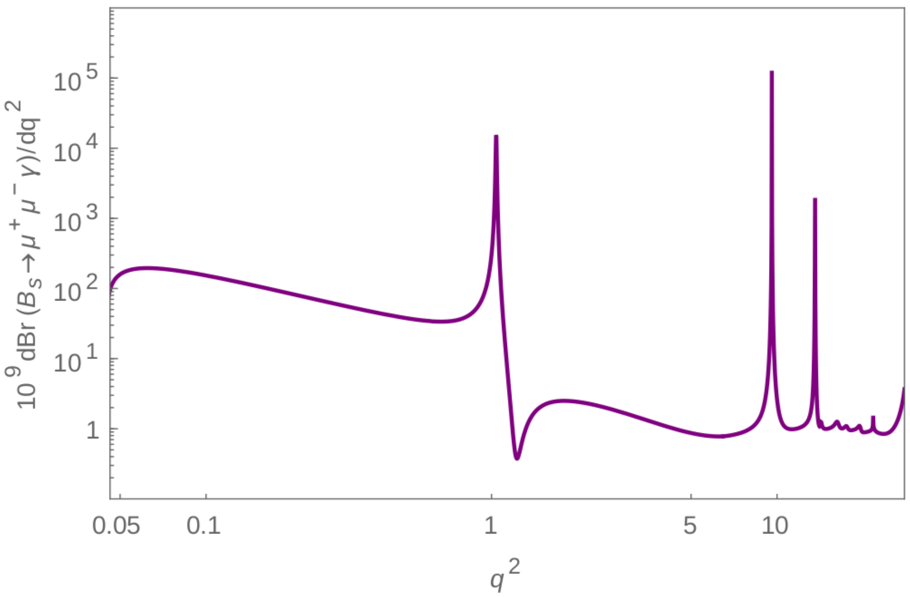
- **Constituent quark masses and the parameter of the wave function are fixed by the condition that the known leptonic decay constants of the mesons involved + a few well-measured values of the form factors at large q^2 from lattice QCD at large q^2.**

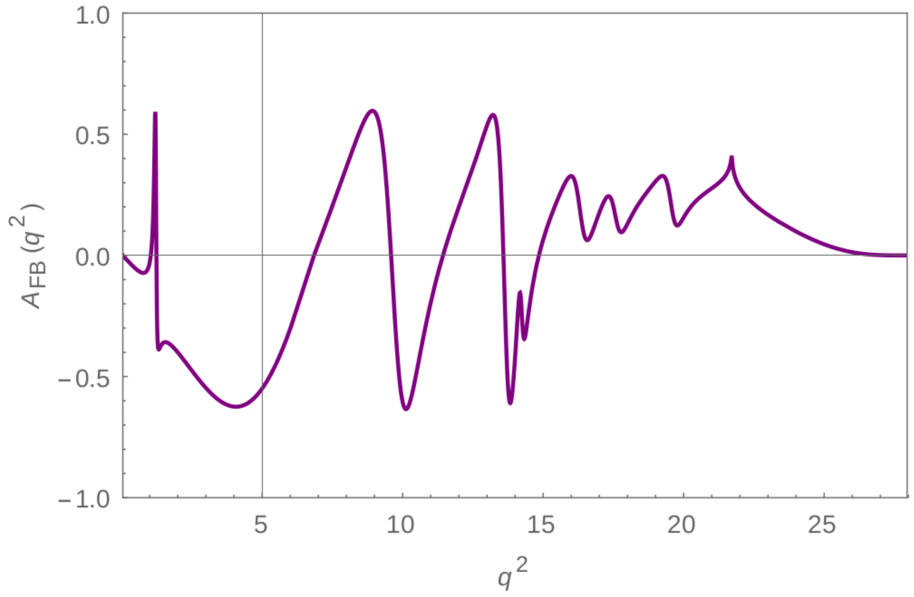
For subprocesses with resonances in the physical q^2-range, we have calculated form factors at q^2 below the resonances via dispersion approach. For larger values of q^2 we make use of the vector-meson dominance

$$F(q^2) = F(0) + q^2 \frac{f_V/M_V}{M_V^2 - q^2 - i\Gamma_V M_V}$$

Numerical results

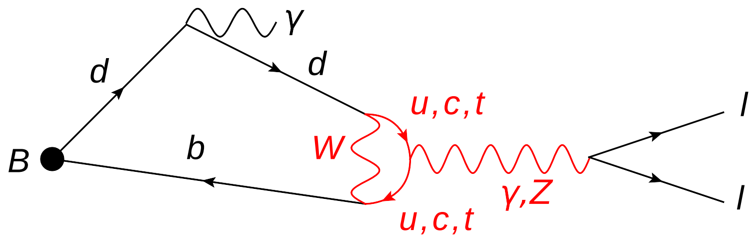
	This work	MN'2004
$Br\,(B \to e^+e^-\gamma)\,\times\,10^{10}$	4.84	3.95
$Br\,(B \to \mu^+\mu^-\gamma)\,\times\,10^{10}$	1.60	1.31
$Br\,(B_s \to e^+e^-\gamma)\,\times\,10^{9}$	18.8	24.6
$Br\,(B_s \to \mu^+\mu^-\gamma)\,\times\,10^{9}$	12.2	18.8





Differential distributions in $B_s \to \gamma l^+ l^-$ vs $q^2[GeV^2]$.

Difficulty arising from the charming loops:



Conclusions and outlook

• We calculated all $B \to \gamma$ form factors using the dispersion approach. The form factors from dispersion aprroach satisfy rigorous constraints (e-m gauge invariance, LEET, HQET). For the form factors at timelike momentum transfers, where light vector resonances appear, we used gauge-invariant version of VMD. Parameters of the meson wave functions have been fixed by reproducing their lepton decay constants. Comparison of form factors from our approach with results from lattice QCD and QCD sum rules (where such results are available) suggest that we have the form factor accuracy at the level not worse than 10%.

• We obtained predictions for various differential distributions. The distributions potentially have the sensitivity to the precise values of the Wilson coefficients, i.e. have sensitivity to new physics.

• The main open problem is the contribution of charming loops which "pollute" the differential distributions, including also the asymmetries, at the level of 5-10% at q^2 far beyound J/ψ, ψ'. We are working on improving the accuracy and on a search for new observables free from the theoretical uncertainties induced by charming loops.