ON THE UNIVERSAL STRUCTURE OF HIGGS AMPLITUDES MEDIATED BY HEAVY PARTICLES

F. Driencourt-Mangin[1], G. Rodrigo[1] and G. Sborlini[1,2]

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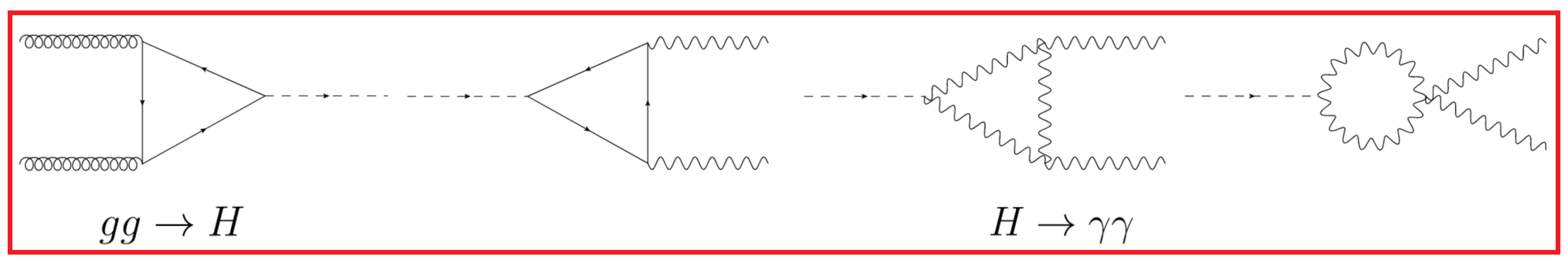
[1]Instituto de Física Corpuscular, Universitat de Valencia - Consejo Superior de Investigaciones Científicas, Parc Cientific, E-46980 Paterna, Valencia, Spain
[2]Dipartimento di Física, Università di Milano and INFN Sezione Milano, I-20133 Milan, Italy

Introduction

The one-loop amplitudes for Higgs to massless gauge bosons are finite because there is not a tree-level contribution in the Standard Model. However, the expressions involved are individually ill-defined and the four-dimensional limit has to be carefully considered. In other terms, a finite result does not guarantee that we can leave aside the regularization framework. In fact, a non-consistent four-dimensional implementation would produce misleading results.

Since the Higgs amplitudes exhibit this pathological behaviour, we used them as a case of study for our method: the *four-dimensional unsubtraction (FDU)* framework.



One-loop diagrams involved

We center the discussion in two processes: (1) Higgs production through gluon fusion and (2) Higgs decay into a photon pair. Whilst in the first case only massive coloured fermions can be included in the loop, the diphoton decay also involves W bosons. We show in Ref. [1] that the structure of all these contributions is universal, i.e. independent of the nature of the particle inside the loop. However, the specific UV behaviour of the different contributions is slightly different: this leads to potential discrepancies when naively considering the computation in 4-dimensions.

In particular, there are some subtleties with the treatment of the algebra in D-dimensions and the presence of singularities. As discussed in the literature [2], the substitution

$$\ell^\mu \ell^\nu \to \frac{1}{n} g^{\mu\nu} \ell^2 \quad \text{with} \quad n = 4 \quad \text{or } n = 4 - 2\epsilon$$

could lead to an inconsistent result if the associated tensor integral is not finite: an indetermination is produced due to misleading order ε terms. This problem can be easily solved within the FDU framework because we achieve a *fully local regularization*.

Loop-tree duality (LTD) and *Four-Dimensional Unsubtraction* (FDU)

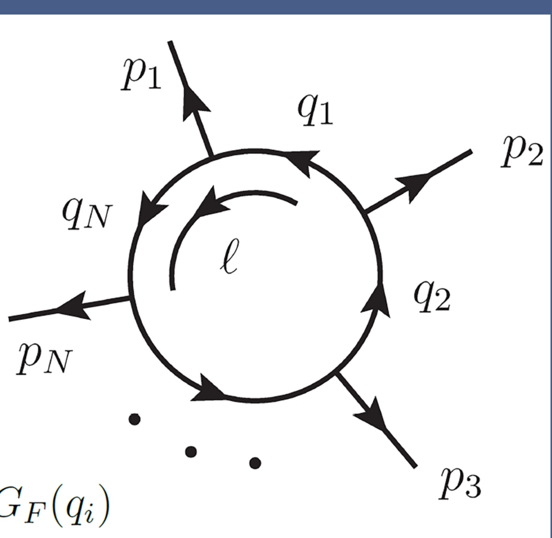
The FDU framework is based in two key ideas: (1) the loop-tree duality (LTD) theorem, and (2) the introduction of local counter-terms. At one-loop level, the LTD [3] establishes that loop amplitudes can be expressed as the sum of dual amplitudes, which are build starting from tree-level like objects and replacing the loop measure with a phase-space one. The theorem is valid beyond one-loop and, also, when multiple powers of the propagators are present [4].

$$k_i = p_1 + \ldots + p_i$$
$$q_i = \ell + k_i$$

Momenta definition

$$G_F(q_i) = \frac{1}{q_i^2 - m_i^2 + \imath 0}$$

Feynman propagator



$$L^{(1)}(p_1, \ldots, p_N) = \int_\ell \prod_{i=1}^{N} G_F(q_i)$$

Generic one-loop scalar integral

$$\int_\ell = -\imath \mu^{4-d} \int \frac{d^d\ell}{(2\pi)^d} \Rightarrow \int_\ell \tilde{\delta}(q_i) \equiv 2\pi\,\imath \int_\ell \theta(q_{i,0})\, \delta(q_i^2 - m_i^2)$$

Loop measure → Phase-space measure

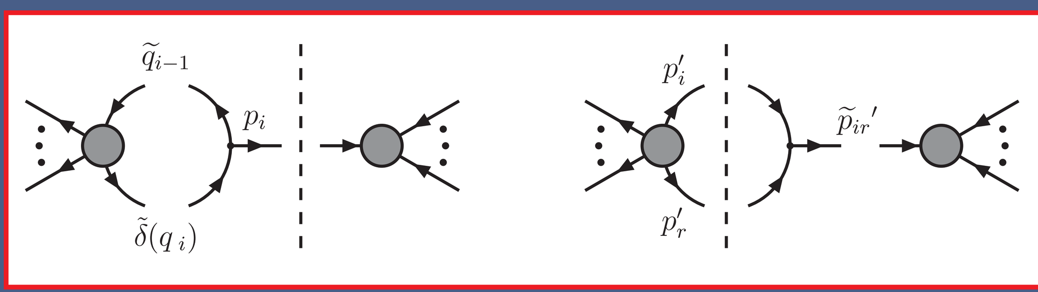
$$G_F(q_i) \Rightarrow G_D(q_i; q_j) = \frac{1}{q_j^2 - m_j^2 - \imath 0\, \eta \cdot k_{ji}}$$

Feynman propagator → Dual propagator (modified prescription)

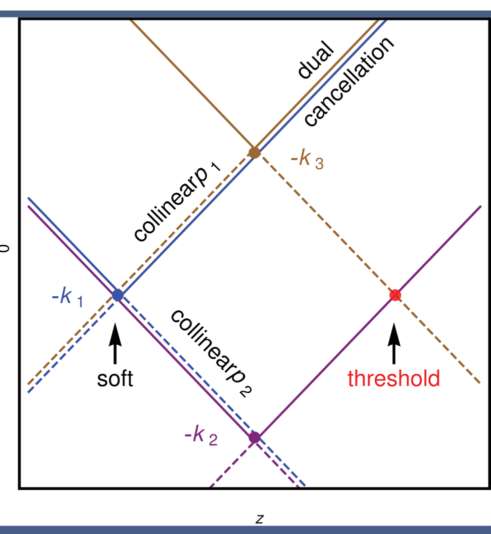
Transformations induced by *cutting lines* (i.e. putting on-shell) in LTD

$$L^{(1)}(p_1, \ldots, p_N) = -\sum_{i=1}^{N} \int_\ell \tilde{\delta}(q_i) \prod_{j \neq i} G_D(q_i; q_j)$$

Master formula for scalar integrals at one-loop



Topological matching between dual (left) and real (right) contributions. The picture describes the configuration i//r.



Location of singularities in the dual space.

On the other hand, the method relies in the possibility of regularizing the expressions at integrand-level and in 4-dimensions. To deal with UV singularities, we need to obtain integrand-level formulae for the renormalization factors. However, the real-emission is directly used as a *local counter-term* in the IR region through the application of a proper momentum mapping [5,6]. This mapping is motivated by the universal factorization of IR singularities in the scattering amplitudes [7].

REAL EMISSION MOMENTA *(m+1 particles)*

$$p_r'^\mu = q_i^\mu$$
$$p_i'^\mu = (1-\alpha_i)\,\hat{p}_i^\mu + (1-\gamma_i)\,\hat{p}_j^\mu - q_i^\mu$$
$$p_j'^\mu = \alpha_i\,\hat{p}_i^\mu + \gamma_i\,\hat{p}_j^\mu$$
$$p_k'^\mu = p_k^\mu\,, \qquad k \neq i, j, r$$

DUAL KINEMATICS *(m particles plus 1 on-shell vector -loop-)*

The free parameters introduced in the mapping are fixed to fulfil on-shell conditions for the real-emission kinematics.

Results and discussion

In order to apply the LTD/FDU to the Higgs amplitudes, we introduced a tensor basis and a set of projectors to extract the physically relevant scalar coefficients. Because of Ward's identities, only the first element of the basis contributes to the complete amplitude. The second element is also relevant because it vanishes *only* after integration *within DREG*; the integrand is non-trivial.

$$\mathcal{A}_{\mu\nu}^{(1,f)} = \sum_{i=1}^{5} \mathcal{A}_i^{(1,f)}\, T_{\mu\nu}^i \quad \text{with} \quad T_i^{\mu\nu} = \left\{ g^{\mu\nu} - \frac{2\,p_1^\nu\,p_2^\mu}{s_{12}}, g^{\mu\nu}, \frac{2\,p_1^\mu\,p_2^\nu}{s_{12}}, \frac{2\,p_1^\mu\,p_1^\nu}{s_{12}}, \frac{2\,p_2^\mu\,p_2^\nu}{s_{12}} \right\}$$

$$P_1^{\mu\nu} = \frac{1}{d-2}\left(g^{\mu\nu} - \frac{2\,p_1^\nu p_2^\mu}{s_{12}} - (d-1)\,\frac{2\,p_1^\mu p_2^\nu}{s_{12}} \right)$$
$$P_2^{\mu\nu} = \frac{2\,p_1^\mu p_2^\nu}{s_{12}}$$

Projectors

$$\Rightarrow P_i^{\mu\nu}\, \mathcal{A}_{\mu\nu}^{(1,f)} = \mathcal{A}_i^{(1,f)}$$

Scalar coefficients

Then, we apply the LTD decomposition into dual contributions. In this case, there are four different propagators, so we obtain four dual terms. Since the internal on-shell momenta can be related by unifying the coordinate system, we can add the four contributions and rewrite them as a single integral.

Momenta associated to the internal lines: $q_1 = \ell + p_1,\, q_2 = \ell + p_{12},\, q_3 = \ell$ and $q_4 = \ell + p_2$

Unified dual contributions:

$$\mathcal{A}_1^{(1,f)} = g_f \int_\ell \tilde{\delta}(\ell) \left[\left(\frac{\ell_0^{(+)}}{q_{1,0}^{(+)}} + \frac{\ell_0^{(+)}}{q_{4,0}^{(+)}} + \frac{2\,(2\ell\cdot p_{12})^2}{s_{12}^2 - (2\ell\cdot p_{12} - \imath 0)^2} \right) \left(\frac{s_{12}\,M_f^2}{(2\,\ell\cdot p_1)(2\,\ell\cdot p_2)}\, c_1^{(f)} + c_2^{(f)} \right) + \frac{2\,s_{12}^2}{s_{12}^2 - (2\ell\cdot p_{12} - \imath 0)^2}\, c_3^{(f)} \right]$$

$$\mathcal{A}_2^{(1,f)} = g_f\, \frac{c_3^{(f)}}{2} \int_\ell \tilde{\delta}(\ell) \left(\frac{\ell_0^{(+)}}{q_{1,0}^{(+)}} + \frac{\ell_0^{(+)}}{q_{4,0}^{(+)}} - 2 \right) \quad \text{with} \quad f = \phi, t, W$$

On-shell loop energies (unified coordinate system)

$$q_{1,0}^{(+)} = \sqrt{(\boldsymbol{\ell} + \mathbf{p}_1)^2 + M_f^2}\,,$$
$$q_{4,0}^{(+)} = \sqrt{(\boldsymbol{\ell} + \mathbf{p}_2)^2 + M_f^2}\,,$$
$$\ell_0^{(+)} = q_{2,0}^{(+)} = q_{3,0}^{(+)} = \sqrt{\boldsymbol{\ell}^2 + M_f^2}\,.$$

IMPORTANT: This choice leads to a complete integrand-level cancellation of singularities among dual contributions

Finally, we recombine both scalar coefficients exploiting the fact that the second one vanishes (after integration). It is worth appreciating that the functional structure is universal; the process information is encoded within the coefficients *c*.

$$\mathcal{A}_1^{(1,f)} = g_f \int_\ell \tilde{\delta}(\ell) \left[\left(\frac{\ell_0^{(+)}}{q_{1,0}^{(+)}} + \frac{\ell_0^{(+)}}{q_{4,0}^{(+)}} + \frac{2\,(2\ell\cdot p_{12})^2}{s_{12}^2 - (2\ell\cdot p_{12} - \imath 0)^2} \right) \frac{s_{12}\,M_f^2}{(2\,\ell\cdot p_1)(2\,\ell\cdot p_2)}\, c_1^{(f)} + \frac{2\,s_{12}^2}{s_{12}^2 - (2\ell\cdot p_{12} - \imath 0)^2}\, c_{23}^{(f)} \right]$$

Regular and integrable in four-dimensions!!

The function is UV singular, but the coefficient vanishes in 4-dimensions!!

List of coefficients

$$c_i^{(f)} = c_{i,0}^{(f)} + r_f\, c_{i,1}^{(f)} \text{ with } r_f = s_{12}/M_f^2 \quad \text{and} \quad c_2^{(f)} = c_{23}^{(f)} - c_3^{(f)}$$

$$\Rightarrow g_f = \frac{2\,M_f^2}{\langle v \rangle\, s_{12}}\,, \quad c_{1,0}^{(f)} = \left(\frac{2}{d-2}, \frac{8}{d-2}, \frac{4(d-1)}{d-2} \right), \quad c_{1,1}^{(f)} = \left(0, -1, \frac{2(5-2d)}{d-2} \right)$$

$$c_{3,0}^{(f)} = (d-2)\, c_{1,0}^{(f)}\,, \quad c_{3,1}^{(f)} = 0\,, \quad c_{23,0}^{(f)} = (d-4)\frac{c_{1,0}^{(f)}}{2}\,, \quad c_{23,1}^{(f)} = \left(0, 0, \frac{d-4}{d-2} \right),$$

Local renormalization

Even if the Higgs amplitude is UV finite and does not need to be renormalized, the integrand is locally singular in the high-energy region. This leads to ambiguous results if we naively consider the four-dimensional limit. Thus, we applied the FDU framework to introduce a local UV counter-term.

$$\mathcal{A}_{1,\mathrm{UV}}^{(1,f)} = -g_f \int_\ell \frac{\tilde{\delta}(\ell)\, \ell_0^{(+)}\, s_{12}}{2(q_{\mathrm{UV},0}^{(+)})^3} \left(1 + \frac{1}{(q_{\mathrm{UV},0}^{(+)})^2} \frac{3\,\mu_{\mathrm{UV}}^2}{d-4} \right) c_{23}^{(f)}$$

$$\mathcal{A}_{1,\mathrm{UV}}^{(1)} = 0 \text{ in } d\text{-dimensions}$$

$$\text{with} \quad q_{\mathrm{UV},0}^{(+)} = \sqrt{\boldsymbol{\ell}^2 + \mu_{\mathrm{UV}}^2}$$

$$c_{23}^{(f)} \propto d - 4$$

$$\mathcal{A}_{1,\mathrm{R}}^{(1,f)}\Big|_{d=4} = \left(\mathcal{A}_1^{(1,f)} - \mathcal{A}_{1,\mathrm{UV}}^{(1,f)} \right)_{d=4}$$

The subleading term in the counter-term is adjusted to reproduce the traditional expressions in DREG (i.e. a vanishing counter-term). Notice that the effect of this contribution is similar to the Dyson prescription (when the last one is well-defined).

Asymptotic expansions

The application of LTD transforms the original D-dimensional Minkowski integration domain into a (D-1)-dimensional one with Euclidean metric. In consequence, many of the difficulties associated with asymptotic expansions at integrand-level in Minkowski spaces are avoided. To illustrate this idea, we used the Higgs process as a benchmark test.

$$\tilde{\delta}(q_3)\; G_D(q_3; q_2) = \frac{\tilde{\delta}(q_3)}{s_{12} + 2q_3\cdot p_{12} - \imath 0} \quad \xrightarrow{M_f^2 \gg s_{12}} \quad \tilde{\delta}(q_3)\; G_D(q_3; q_2) = \frac{\tilde{\delta}(q_3)}{2q_3\cdot p_{12}} \sum_{n=0}^{\infty} \left(\frac{-s_{12}}{2q_3\cdot p_{12}} \right)^n$$

Standard series expansion

$$\text{with} \quad q_3\cdot p_{12} = q_{3,0}^{(+)} \sqrt{s_{12}} \quad \text{and} \quad q_{3,0}^{(+)} \geq M_f$$

Asymptotic expansion for the renormalized Higgs amplitude (all orders!)

$$\mathcal{A}_{1,\mathrm{R}}^{(1,f)}(s_{12} < 4M_f^2)\Big|_{d=4} = \frac{M_f^2}{\langle v \rangle} \int_\ell \tilde{\delta}(\ell) \left[\frac{3\,\mu_{\mathrm{UV}}^2\, \ell_0^{(+)}}{(q_{\mathrm{UV},0}^{(+)})^5}\, \hat{c}_{23}^{(f)} + \frac{M_f^2}{(\ell_0^{(+)})^4} \left(\sum_{n=0}^{\infty} Q_n(z) \left(\frac{s_{12}}{(2\ell_0^{(+)})^2} \right)^n \right) c_1^{(f)} \right]$$

$$\text{with } z = (2\boldsymbol{\ell}\cdot\mathbf{p}_1)/(\ell_0^{(+)}\sqrt{s_{12}}) \text{ and } Q_n(z) = \frac{1}{1-z^2}\,(P_{2n}(z) - 1)$$

The integral of the expansion agrees, order by order, with the known results!

$\mathcal{A}_{1,\mathrm{R}}^{(1,\phi)}\Big\vert_{d=4} = \frac{s_{12}}{8\pi^2\langle v \rangle}\frac{1}{12}\left(1 + \frac{2\,r_\phi}{15} + \mathcal{O}(r_\phi^2)\right)$	Scalar	
$\mathcal{A}_{1,\mathrm{R}}^{(1,t)}\Big\vert_{d=4} = \frac{s_{12}}{8\pi^2\langle v \rangle}\frac{2}{3}\left(1 + \frac{7\,r_t}{120} + \mathcal{O}(r_t^2)\right)$	Fermion	
$\mathcal{A}_{1,\mathrm{R}}^{(1,W)}\Big\vert_{d=4} = \frac{s_{12}}{8\pi^2\langle v \rangle}\frac{1}{2}\left(7 + \frac{11\,r_W}{30} + \mathcal{O}(r_W^2)\right)$	W-boson	

Conclusions

In this work, we applied the LTD/FDU framework to tackle the computation of one-loop scattering amplitudes for Higgs bosons. The compact expressions obtained for the dual contributions allowed to achieve a universal integrand-level representation of the complete virtual amplitude. The information related with the nature of the virtual particles inside the loop is encoded in constant coefficients but the functional form is shared. This universality might be present even at higher-orders due to the properties of the LTD representation.

Also, we manage to remove the scheme ambiguities by implementing a local renormalization inspired in the FDU method. All the previously known results were recovered within a pure four-dimensional framework. Another important result of our work is the possibility of an straightforward implementation of asymptotic expansions at integrand-level. This is due to the fact that the dual integration is performed in an Euclidean space, i.e. namely the loop three-momentum space.

Finally, we expect further simplifications of higher-order computations and asymptotic expansions by exploiting the four-dimensional nature of the LTD/FDU approach.

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