II. Electromagnetic (EM) and strong interactions of baryons in NRCQM, quark model selection rules and symmetry breakings

Baryon EM transitions

Supposing the constituent quark has a mass m_i (≈ 330 MeV for the *u* and *d*), the magnetic moment of the constituent quark thus can be expressed as

$$\mu_i = g\frac{e_i}{2m_i} = \mu_0\frac{e_i}{e}$$

where $\mu_0 = e/2m_q = \sqrt{4\pi\alpha_e}/2m_q = \sqrt{4\pi} \times 0.13\ \text{GeV}^{-1}$ is the proton magnetic moment, and e_i is the fractional charge of the quark.

The electromagnetic field of a photon in form of the second quantization is

$$\mathbf{A}_k(\mathbf{r}_i) = \frac{1}{\sqrt{2\omega_\gamma}}\epsilon_\gamma(a_k^\dagger e^{i\mathbf{k}\cdot\mathbf{r}_i} + a_k e^{-i\mathbf{k}\cdot\mathbf{r}_i})$$

Taking the direction of the photon momentum *k* as the *z*-axis, and taking the advantage of the permutation symmetry of the baryon wavefunctions, we can express the EM interaction Hamiltonian as

$$\begin{aligned} & \langle\Psi_f|H_1+H_2+H_3|\Psi_i\rangle \\ = & \langle\Psi_f|P_{13}H_3P_{13}^{-1}+P_{23}H_3P_{23}^{-1}+H_3|\Psi_i\rangle \\ = & 3\langle\Psi_f|H_3|\Psi_i\rangle \ , \end{aligned}$$

where H_i is the single quark-photon interaction. The factor 3 implies that the photon couples to the three constituent the same way when the baryon total wavefunctions are antisymmetrized. One thus can "define" that in the EM excitation, the photon "only" couples to the third quark. Explicitly, the EM interaction can be expressed as:

$$\begin{aligned} H_{em} &= \sum_{i=1}^{3} H_i = 3H_3 \\ &= \frac{3}{\sqrt{2\omega_\gamma}}\mu_0\sqrt{2}q_3 e^{ikz_3}\left(kS_{3+}+\frac{1}{g}P_{3+}\right) \ , \end{aligned} \tag{55}$$

where $S_{3+} \equiv S_x + iS_y$ and $P_{3+} \equiv P_x + iP_y$.

We can then express the above equation into a more compact form:

$$H_{em} = q_3(AL_+ + BS_+) \ , \tag{56}$$

where quantity A and B are determined by the transitions in the spatial space:

$$A = \frac{3}{\sqrt{2\omega_\gamma}}\mu_0\sqrt{2}\frac{1}{g}\langle\Psi_f|e^{ikz_3}P_{3+}|\Psi_i\rangle \ , \tag{57}$$

and

$$B = \frac{3}{\sqrt{2\omega_\gamma}}\mu_0\sqrt{2}k\langle\Psi_f|e^{ikz_3}|\Psi_i\rangle \ . \tag{58}$$

Now given the initial and final state baryon wavefunctions, we can compute the EM transition amplitudes between these two states. The amplitudes can be expressed as two independent helicity amplitudes. Namely,

$$\begin{cases} A_{1/2} = \langle\Psi_f(J^f, J^f_z = +\frac{1}{2})|H_{em}|\Psi_i(J^i, J^i_z = -\frac{1}{2})\rangle \\ A_{3/2} = \langle\Psi_f(J^f, J^f_z = +\frac{3}{2})|H_{em}|\Psi_i(J^i, J^i_z = +\frac{1}{2})\rangle \end{cases}$$

For the initial photon-nucleon helicity parallel or anti-parallel, respectively.

We show below an example for the transition $\gamma p \rightarrow S_{11}(1535)$. Including the spin and isospin wavefunctions explicitly, we have

$$\begin{aligned} A_{\frac{1}{2}} &= \sum_{L_z+S_z=1/2} \langle 1L_z, \frac{1}{2}S_z|J\frac{1}{2}\rangle \frac{1}{2}[(\phi_p^\rho\chi_{\frac{1}{2}}^\lambda + \phi_p^\lambda\chi_{\frac{1}{2}}^\rho)\psi_{11L_z}^\rho(\mathbf{R},\boldsymbol{\rho},\boldsymbol{\lambda}) \\ &\quad +(\phi_p^\rho\chi_{\frac{1}{2}}^\rho - \phi_p^\lambda\chi_{\frac{1}{2}}^\lambda)\psi_{11L_z}^\lambda(\mathbf{R},\boldsymbol{\rho},\boldsymbol{\lambda})]\; q_3(AL_+ + BS_+) \\ &\quad \times\frac{1}{2}(\phi_p^\rho\chi_{-\frac{1}{2}}^\rho + \phi_p^\lambda\chi_{-\frac{1}{2}}^\lambda)\psi_{000}^s(\mathbf{R},\boldsymbol{\rho},\boldsymbol{\lambda}) \\ &= \frac{1}{2\sqrt{2}}\Big[\langle 11, \frac{1}{2}\; -\frac{1}{2}|J\frac{1}{2}\rangle A(\langle\phi_p^\rho|q_3|\phi_p^\rho\rangle - \langle\phi_p^\lambda|q_3|\phi_p^\lambda\rangle) \\ &\quad +\langle 10, \frac{1}{2}\frac{1}{2}|J\frac{1}{2}\rangle B(\langle\phi_p^\rho|q_3|\phi_p^\rho\rangle\langle\chi_{\frac{1}{2}}^\rho|S_{3+}|\chi_{-\frac{1}{2}}^\rho\rangle \\ &\quad -\langle\phi_p^\lambda|q_3|\phi_p^\lambda\rangle\langle\chi_{\frac{1}{2}}^\lambda|S_{3+}|\chi_{-\frac{1}{2}}^\lambda\rangle)\Big]\;, \end{aligned} \tag{61}$$

where we have adopted the factor:

$$\langle\phi^\rho|\hat{O}_3|\psi^\lambda\rangle \equiv \langle\phi^\lambda|\hat{O}_3|\psi^\rho\rangle \equiv 0\;, \tag{62}$$

since the operator $\hat{O}_3$ does not change the symmetry properties of quark 1 and 2.

Given the spin and flavor wavefunctions, one can easily compute the following elements in spin and flavor space:

$$\begin{cases} \langle\chi^{\rho}_{\frac{1}{2}}|S_{3+}|\chi^{\rho}_{-\frac{1}{2}}\rangle &= 1 \\ \langle\chi^{\lambda}_{\frac{1}{2}}|S_{3+}|\chi^{\lambda}_{-\frac{1}{2}}\rangle &= -\frac{1}{3} \end{cases} \quad \begin{cases} \langle\phi^{\rho}_{p}|q_3|\phi^{\rho}_{p}\rangle &= \frac{2}{3} \\ \langle\phi^{\lambda}_{p}|q_3|\phi^{\lambda}_{p}\rangle &= 0\,. \end{cases} \quad \begin{cases} \langle\phi^{\rho}_{n}|q_3|\phi^{\rho}_{n}\rangle &= -\frac{1}{3} \\ \langle\phi^{\lambda}_{n}|q_3|\phi^{\lambda}_{n}\rangle &= \frac{1}{3}\,. \end{cases}$$

Consequently, we have the helicity amplitude for $\gamma p \to S_{11}$:

$$\begin{aligned} A_{\frac{1}{2}} &= \frac{1}{3\sqrt{2}}\left[A\langle 11,\frac{1}{2}\ -\frac{1}{2}|J\frac{1}{2}\rangle + B\langle 10,\frac{1}{2}\frac{1}{2}|J\frac{1}{2}\rangle\right] \\ &= \frac{1}{3\sqrt{3}}A - \frac{1}{3\sqrt{6}}B\,, \end{aligned} \tag{66}$$

where for the S_{11}, $J = 1/2$. A and B can be worked out by explicitly introducing the spatial wavefunctions (See Appendix II for details). We thus have:

$$A_{\frac{1}{2}} = \mu_0 \frac{1}{\sqrt{2\omega_\gamma}}\frac{2}{3}\left(\frac{\alpha_h}{g} + \frac{\mathbf{k}^2}{2\alpha_h}\right) e^{-\mathbf{k}^2/6\alpha_h^2}\,. \tag{67}$$

Some numerical results

In the $S_{11}(1535)$ mass system, substituting $\alpha_h = 0.41$ GeV, $\mu_0 = \sqrt{4\pi} \times 0.13$ GeV^{-1}, and the kinematical variables $\omega_\gamma = |\mathbf{k}| = (M_{S_{11}}^2 - M_p^2)/2M_{S_{11}} = 0.48$ GeV, into Eq. (67) gives $A_{\frac{1}{2}} \approx 0.172$ GeV$^{-\frac{1}{2}}$. Compare to the experimental value $A_{\frac{1}{2}}^{exp.} = 0.090 \pm 0.030$ GeV$^{-\frac{1}{2}}$ (See PDG [4]), the naive quark model has overestimated the experimental values significantly.

Relativistic corrections (I)

Taking into account the Lorentz contracting effects, more realistic number can be obtained by introducing a Lorentz boost factor for the spatial part, i.e., $\frac{1}{\gamma_k} \equiv \frac{M_p}{E_p} = \frac{M_p}{\sqrt{M_p^2+\mathbf{k}^2}}$. The spatial part changes as, $f(|\mathbf{k}|) \to \frac{1}{\gamma_k^2} f\left(\frac{|\mathbf{k}|}{\gamma_k}\right)$. In this way, we have

$$A_{\frac{1}{2}} = \mu_0 \frac{1}{\sqrt{2\omega_\gamma}} \frac{2}{3\gamma_k^2} \left(\frac{\alpha_h}{g} + \frac{\mathbf{k}^2}{2\alpha_h \gamma_k} \right) e^{-\mathbf{k}^2/6\alpha_h^2\gamma_k^2} , \tag{68}$$

which gives $A_{\frac{1}{2}} = 0.125$ GeV$^{-\frac{1}{2}}$, which is significantly reduced.

Relativistic corrections (II)

Including O(v^2/c^2) terms in the EM operator and QCD mixings:

Multiplet states	A_J^N	A^{NR}	A^{RE}	A_1^M	A_2^M	A^{expt}
$(70,1^-)_1 S_{11}(1535)$	$A_{1/2}^p$	17[illegible]	163	142	162	73 ± 14
	$A_{1/2}^n$	−12[illegible]	−106	−77	−90	-76 ± 32

F.E. Close and Z.-P. Li, Phys. Rev. D **42**, 2194 (1990).

Z.-P. Li and F.E. Close, Phys. Rev. D **42**, 2207 (1990).

Relativistic quark models

See review:

S. Capstick and W. Roberts, Prog. Part. Nucl. Phys. **45**, 5241 (2000).

Aznauryan, Burkert and Lee, arXiv:0810.0997 [nucl-th].

Possible five-quark component inside S11(1535)?

See e.g. Prof. B.S. Zou's talk.

Some other results from NRCQM symmetry

- The Delta resonance $P_{33}(1232)$ is classified as the ground state of isospin 3/2 baryons. Since it has the same spatial wavefunction as the nucleons, its excitation from the nucleons can only occur via the magnetic interaction, i.e., its helicity amplitudes $A_{\frac{1}{2}}$ and $A_{\frac{3}{2}}$ are both proportional to B. The quark model predicts:

$$\frac{A_{\frac{3}{2}}}{A_{\frac{1}{2}}} = \sqrt{3}\,, \tag{69}$$

  while the PDG gives 1.67 [4].

- The Roper resonance $P_{11}(1440)$ is classified as the nucleon radial excitations with $N = 2$ and $L = 0$. Since compared to the proton and neutron, the $P_{11}(1470)$ has only the spatial wavefunction different from the nucleons, the quark model predicts

$$\frac{A^p_{\frac{1}{2}}(P_{11})}{A^n_{\frac{1}{2}}(P_{11})} = \frac{\mu_p}{\mu_n} = -\frac{3}{2}\,, \tag{70}$$

  which is the ratio of the magnetic moments between the proton and neutron.

- The resonance $P_{13}(1720)$ and $F_{15}(1680)$ are assigned to representation $|\mathbf{56},^2\mathbf{8},2,2,J\rangle$, which has the same spin and flavor wavefunctions as the ground state nucleons. Since the total spin for this representation is 1/2, in order to have spin 3/2 for the P_{13} and spin 5/2 for the F_{15}, the total orbital angular momentum projection $L_z \neq 0$. The transition can thus occur only via the A mode which is proportional to the charge operator. For the neutron transition, the transition element in the flavor space vanishes which results in $A^n_{\frac{3}{2}} \equiv 0$. This selection rule based on only quark model symmetry is consistent with the experimental observation.

Quark model selection rules and symmetry breakings

Baryons in SU(6)⊗O(3) symmetric quark model

Basic assumptions:

i) Chiral symmetry spantaneous breaking leads to the presence of massive constituent quarks as effective degrees of freedom inside hadrons;

ii) Hadrons can be viewed as quark systems in which the gluon fields generate effective potentials that depend on the spins and positions of the massive quarks.

Color	$SU(3)$	$\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{3}$	$=\mathbf{10}_s+\mathbf{8}_\rho+\mathbf{8}_\lambda+\boxed{\mathbf{1}_a}$
Spin	$SU(2)$	$\mathbf{2}\otimes\mathbf{2}\otimes\mathbf{2}$	$=\mathbf{4}_s+\mathbf{2}_\rho+\mathbf{2}_\lambda,$
Flavor	$SU(3)$	$\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{3}$	$=\mathbf{10}_s+\mathbf{8}_\rho+\mathbf{8}_\lambda+\mathbf{1}_a,$
Spin-flavor	$SU(6)$	$\mathbf{6}\otimes\mathbf{6}\otimes\mathbf{6}$	$=\mathbf{56}_s+\mathbf{70}_\rho+\mathbf{70}_\lambda+\mathbf{20}_a,$
Spatial	$\mathrm{O}(3)$	L^P	s, ρ, λ, a

Baryon wavefunction as representation of 3-dimension permutation group:

$$\phi_c|\mathrm{SU}(6)\otimes\mathrm{O}(3)\rangle=\phi_c|\mathbf{N}_6,\ {}^{2S+1}\mathbf{N}_3,\ N,L,J\rangle$$

symmetric

Spin wavefunction

$$\left\{\begin{array}{rcl} \chi^s(S_z = \frac{3}{2}) & = & \uparrow\uparrow\uparrow \\ \chi^\rho(S_z = \frac{1}{2}) & = & \frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) \\ \chi^\lambda(S_z = \frac{1}{2}) & = & \frac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow) \end{array}\right.$$

Flavor wavefunction

$$\phi^\lambda(\mathbf{8}) = \left\{\begin{array}{ll} \frac{1}{\sqrt{6}}(2uud - duu - udu) & \text{for } p \\ \frac{1}{\sqrt{6}}(dud + udd - 2ddu) & \text{for } n \\ \frac{1}{2}(sud + usd - sdu - dsu) & \text{for } \Lambda \end{array}\right.$$

$$\phi^\rho(\mathbf{8}) = \left\{\begin{array}{ll} \frac{1}{\sqrt{2}}(udu - duu) & \text{for } p \\ \frac{1}{\sqrt{2}}(udd - dud) & \text{for } n \\ \frac{1}{2\sqrt{3}}(usd + sdu - sud - dsu - 2dus + 2uds) & \text{for } \Lambda \end{array}\right.$$

Spin-Flavor wavefunctions

$$SU(6) \quad \mathbf{6} \otimes \mathbf{6} \otimes \mathbf{6} \quad = \mathbf{56}_s + \mathbf{70}_\rho + \mathbf{70}_\lambda + \mathbf{20}_a$$

$$\boxed{|\mathbf{56},{}^2\,\mathbf{8}\rangle^s \;=\; \frac{1}{\sqrt{2}}(\phi^\rho\chi^\rho + \rho^\lambda\chi^\lambda)\ ,}$$

$$|\mathbf{56},{}^4\,\mathbf{10}\rangle^s \;=\; \phi^s\chi^s\ ,$$

$$|\mathbf{20},{}^2\,\mathbf{8}\rangle^a \;=\; \frac{1}{\sqrt{2}}(\phi^\rho\chi^\lambda - \phi^\lambda\chi^\rho)\ ,$$

$$|\mathbf{20},{}^4\,\mathbf{1}\rangle^a \;=\; \phi^a\chi^s\ .$$

$$|\mathbf{70},{}^2\,\mathbf{8}\rangle^\rho \;=\; \frac{1}{\sqrt{2}}(\phi^\rho\chi^\lambda + \phi^\lambda\chi^\rho)\ ,$$

$$\boxed{|\mathbf{70},{}^4\,\mathbf{8}\rangle^\rho \;=\; \phi^\rho\chi^s,}$$

$$|\mathbf{70},{}^2\,\mathbf{10}\rangle^\rho \;=\; \phi^s\chi^\rho\ ,$$

$$|\mathbf{70},{}^2\,\mathbf{1}\rangle^\rho \;=\; \phi^a\chi^\lambda\ ,$$

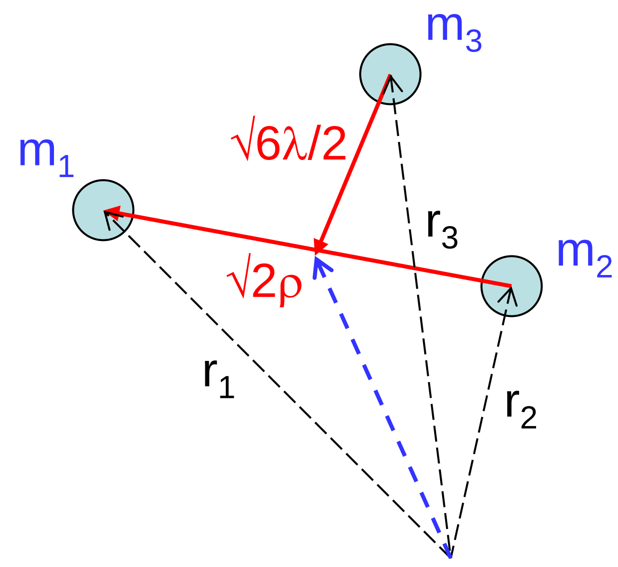
$$\boxed{|\mathbf{70},{}^4\,\mathbf{8}\rangle^\lambda \;=\; \phi^\lambda\chi^s\ ,}$$

$$|\mathbf{70},{}^2\,\mathbf{10}\rangle^\lambda \;=\; \phi^s\chi^\lambda\ ,$$

$$|\mathbf{70},{}^2\,\mathbf{1}\rangle^\lambda \;=\; \phi^a\chi^\rho\ ,$$

1. Spin-independent potential

Hamiltonian $$H = H_{si} + H_{sd}$$



Jacobi coordinate

$$\begin{aligned} H_{si} &= \sum_i \left(m_i + \frac{\mathbf{p}_i^2}{2m_i}\right) + \sum_{i<j}\left(\frac{1}{2}br_{ij} + c - \frac{2\alpha_s}{3r_{ij}}\right) \\ &\equiv \sum_i \left(m_i + \frac{\mathbf{p}_i^2}{2m_i}\right) + \sum_{i<j}\left(\frac{1}{2}kr_{ij}^2 + U(r_{ij})\right) \\ &\equiv H_0 + \sum_{i<j} U(r_{ij}) \end{aligned}$$

$$\psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \frac{1}{(2\pi)^{3/2}} e^{i\mathbf{P}_R\cdot\mathbf{R}} \psi^{\sigma}_{NLL_z}(\boldsymbol{\rho}, \boldsymbol{\lambda})$$

$$\sigma = s, \rho, \lambda, a$$

Nucleon $$|56,\ {}^2 8, 0, 0, \frac{1}{2}\rangle = \frac{1}{\sqrt{2}}[\phi^\rho \chi^\rho_{S_z} + \phi^\lambda \chi^\lambda_{S_z}]\psi^s_{000},$$

N* $$|70,\ {}^4 8, N, L, J\rangle = \sum_{L_z+S_z=J_z} \langle LL_z, \frac{3}{2}S_z|JJ_z\rangle \frac{1}{\sqrt{2}}[\phi^\rho \chi^s_{S_z}\psi^\rho_{NLL_z} + \phi^\lambda \chi^s_{S_z}\psi^\lambda_{NLL_z}].$$

Isgur, and Karl, PLB109, 72(1977); PRD18, 4187(1978); PRD19, 2653(1979)

Some selection rules in the symmetric quark model

• **Moorhouse selection rule (Moorhouse, PRL16, 771 (1966))**

$$\gamma + p(|\mathbf{56}, {}^2\mathbf{8}; 0, 0, 1/2\rangle) \not\leftrightarrow \quad N^*(|\mathbf{70}, {}^4\mathbf{8}\rangle)$$

$$\gamma + n(|\mathbf{56}, {}^2\mathbf{8}; 0, 0, 1/2\rangle) \leftrightarrow N^*(|\mathbf{70}, {}^4\mathbf{8}\rangle)$$

Explicitly, the amplitude has the following expression:

$$A_{S_z} = \frac{1}{2} B \langle 10, \frac{3}{2} S_z | J S_z \rangle \langle \phi_p^\lambda | q_3 | \phi_p^\lambda \rangle \langle \chi_{S_z}^s | S_{3+} | \chi_{S_z - 1}^\lambda \rangle \ ,$$

where we only keep the term of $B \sim \langle \psi_f^\lambda | e^{ikz_3} | \psi_i^s \rangle$ since the other one $\langle \psi_f^\rho | e^{ikz_3} | \psi_i^s \rangle \equiv 0$. Notice that $\langle \phi_p^\lambda | q_3 | \phi_p^\lambda \rangle \equiv 0$, it leads to $A_{S_z} = 0$.

Some selection rules in the symmetric quark model

$$\gamma + p(|\mathbf{56}, {}^2\mathbf{8}; 0, 0, 1/2\rangle) \not\leftrightarrow \quad N^*(|\mathbf{70}, {}^4\mathbf{8}\rangle)$$

$$\gamma + n(|\mathbf{56}, {}^2\mathbf{8}; 0, 0, 1/2\rangle) \leftrightarrow N^*(|\mathbf{70}, {}^4\mathbf{8}\rangle)$$

EM transition of 0^+ ($1/2^+$) $\to$ 1^- ($1/2^-$, $3/2^-$, $5/2^-$)

$|\mathbf{70},\ {}^2\mathbf{8},\ 1, 1, J\rangle$ • $S_{11}(1535)$ (****), $D_{13}(1520)$ (****);

$|\mathbf{70},\ {}^4\mathbf{8},\ 1, 1, J\rangle$ • $S_{11}(1650)$ (****), $D_{13}(1700)$ (***), $\boxed{D_{15}(1675)\ (****)}$;

$N(1675) \to p\gamma$, helicity-1/2

VALUE (GeV$^{-1/2}$)
+0.019±0.008 OUR ESTIMATE

$N(1675) \to p\gamma$, helicity-3/2

VALUE (GeV$^{-1/2}$)
+0.015±0.009 OUR ESTIMATE

$N(1675) \to n\gamma$, helicity-1/2

VALUE (GeV$^{-1/2}$)
−0.043±0.012 OUR ESTIMATE

$N(1675) \to n\gamma$, helicity-3/2

VALUE (GeV$^{-1/2}$)
−0.058±0.013 OUR ESTIMATE

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- **Λ selection rule (Zhao & Close, PRD74, 094014(2006)) in strong decays**

$$N^*(|\mathbf{70}, {}^4\mathbf{8}\rangle) \not\leftrightarrow \quad K(K^*) + \Lambda$$

N* (p,n) $$|70,\ {}^48, N, L, J\rangle = \sum_{L_z+S_z=J_z} \langle LL_z, \frac{3}{2}S_z|JJ_z\rangle \frac{1}{\sqrt{2}}[\phi^\rho \chi^s_{S_z} \psi^\rho_{NLL_z} + \phi^\lambda \chi^s_{S_z} \psi^\lambda_{NLL_z}].$$

Λ $$|56,\ {}^28, 0, 0, \frac{1}{2}\rangle = \frac{1}{\sqrt{2}}[\phi^\rho \chi^\rho_{S_z} + \phi^\lambda \chi^\lambda_{S_z}]\psi^s_{000},$$

$$\langle \phi^\lambda_\Lambda | \hat{I}_3 | \phi^\lambda_p \rangle \ = \ 0$$

- **Faiman-Hendry selection rule (Faiman & Hendry, PR173, 1720 (1968)).**

$$\Lambda^*(|\mathbf{70}, {}^4\mathbf{8}\rangle) \not\leftrightarrow \quad N(|\mathbf{56}, {}^2\mathbf{8}; 0, 0, 1/2\rangle) + \bar{K}$$

Isgur, Karl, & Koniuk, PRL41, 1269(1978)

2. Spin-dependent potential from one-gluon-exchange (OgE) and SU(6)⊗O(3) symmetry breaking

$$H_{hyper} = \frac{2\alpha_s}{3m_i m_j}\left[\frac{8\pi}{3}\mathbf{S}_i\cdot\mathbf{S}_j\delta^3(\mathbf{r}_{ij}) + \frac{1}{r_{ij}^3}\left(\frac{3(\mathbf{S}_i\cdot\mathbf{r}_{ij})(\mathbf{S}_j\cdot\mathbf{r}_{ij})}{r_{ij}^2} - \mathbf{S}_i\cdot\mathbf{S}_j\right)\right]$$

Introduces mass splittings and configuration mixings in the SU(6) multiplets.

Nucleon:

$$\begin{aligned} |N\rangle \;=\; & 0.90|\mathbf{56},{}^2\mathbf{8};0,0,1/2\rangle - 0.34|\mathbf{56},{}^2\mathbf{8};2,0,1/2\rangle \\ & \boxed{-0.27|\mathbf{70},{}^2\mathbf{8};2,0,1/2\rangle} - 0.06|\mathbf{70},{}^4\mathbf{8};2,2,1/2\rangle \end{aligned}$$

Selection rules must be violated !

$$|N\rangle \xrightarrow{-0.27|\mathbf{70},{}^2\mathbf{8};2,0,1/2\rangle} |70,\ {}^4 8, N, L, J\rangle$$

Selection rule violations

TABLE I. Violations of some SU(6) rules.

Quantity	SU(6) (Relative values)	This calculation (Relative values)	Experiment (Various units)	PDG2006
$A_{3/2}^{n}(D_{15}\to n\gamma)$	$-\alpha$	$-\alpha$	-60 ± 33 [a]	**–58 ± 13**
$A_{1/2}^{n}(D_{15}\to n\gamma)$	-0.71α	-0.71α	-33 ± 25 [a]	**–43 ± 12**
$A_{3/2}^{p}(D_{15}\to p\gamma)$	0	$+0.31\alpha$	$+20\pm 13$ [a]	**+15 ± 9**
$A_{1/2}^{p}(D_{15}\to p\gamma)$	0	$+0.22\alpha$	$+19\pm 14$ [a]	**+19 ± 8**
$A(D_{15}\to \bar{K}N)$	β	β	$+0.41\pm 0.03$ [b]	
$A(D_{05}\to \bar{K}N)$	0	-0.28β	-0.09 ± 0.04 [c]	
$\langle\sum e_i r_i^2\rangle_p$	γ	γ	$+0.82\pm 0.02$ [d]	
$\langle\sum e_i r_i^2\rangle_n$	0	-0.16γ	-0.12 ± 0.01 [e]	

Isgur, Karl, & Koniuk, PRL41, 1269(1978)

Λ selection rule violation

Hyperfine interaction in Λ(uds) is relatively suppressed compared with that in nucleon due to the heavier s-quark

$$H_{hyper} = \frac{2\alpha_s}{3m_i m_j}\left[\frac{8\pi}{3}\mathbf{S}_i \cdot \mathbf{S}_j \delta^3(\mathbf{r}_{ij}) + \frac{1}{r_{ij}^3}\left(\frac{3(\mathbf{S}_i \cdot \mathbf{r}_{ij})(\mathbf{S}_j \cdot \mathbf{r}_{ij})}{r_{ij}^2} - \mathbf{S}_i \cdot \mathbf{S}_j\right)\right]$$

In the constitute regime, the Λ wavefunction will be dominated by |56, ^{2} 8>. One expects relatively smaller configuration mixings from |70, ^{2} 8>. Hence

$$\boxed{N^*(|\mathbf{70}, {}^4\mathbf{8}\rangle) \not\leftrightarrow \quad K(K^*) + \Lambda}$$

Approximately hold !

D15(1675) →? KΛ

F17(1990) →? KΛ

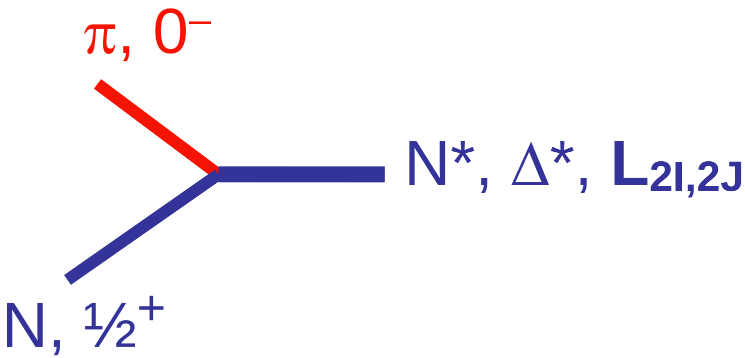
PDG2006

$(\Gamma_i\Gamma_f)^{1/2}/\Gamma_{total}$ **in** $N\pi \to N(1675) \to \Lambda K$ $(\Gamma_1\Gamma_3)^{1/2}/\Gamma$

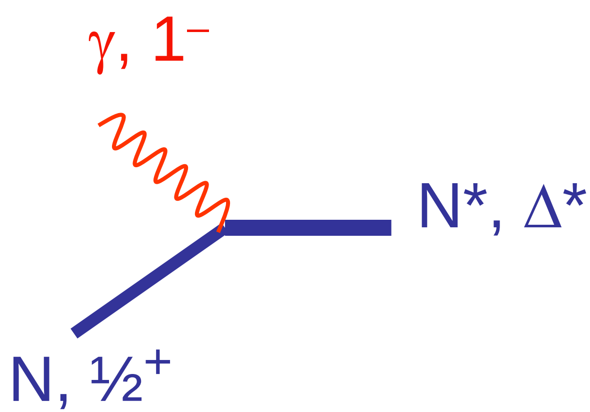
VALUE	DOCUMENT ID		TECN	COMMENT
±0.04 to ±0.08 OUR ESTIMATE				
−0.01	BELL	83	DPWA	$\pi^- p \to \Lambda K^0$
+0.036	[5] SAXON	80	DPWA	$\pi^- p \to \Lambda K^0$
• • • We do not use the following data for averages, fits, limits, etc. • • •				
−0.034±0.006	DEVENISH	74B		Fixed-t dispersion rel.

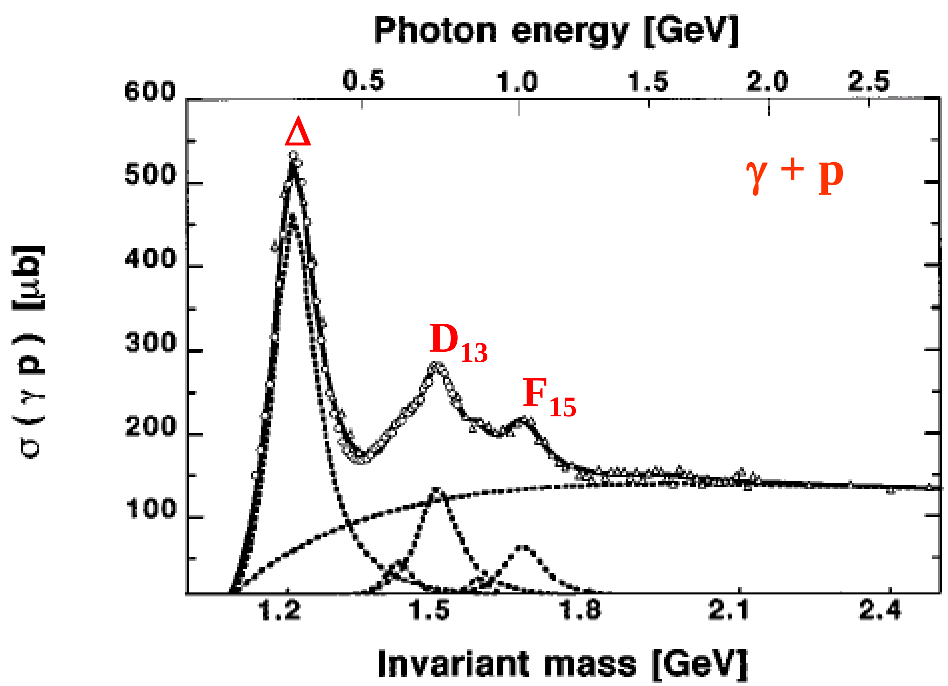
If the Λ selection rule holds approximately, one would expect a relatively small number of baryons contributing to Λ production channel. – Ideal place to disentangle baryon resonances

• Baryons excitations via strong and EM probes



- $P_{33}(1232)$ Δ
- $P_{11}(1440)$
- $S_{11}(1535)$
- $D_{13}(1520)$
- …





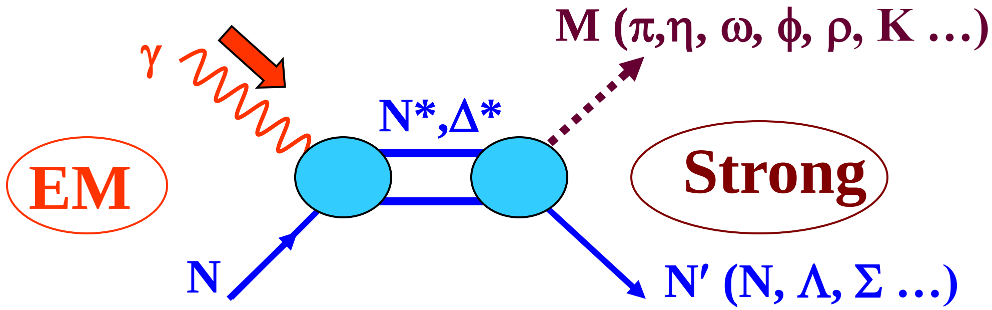
Talks by V. Burkert, F. Klein, P. Cole

PDG2006: 22 nucleon resonances

Particle	$L_{2I\cdot 2J}$	Overall status	Status as seen in — $N\pi$	$N\eta$	ΛK	ΣK	$\Delta\pi$	$N\rho$	$N\gamma$
$N(939)$	P_{11}	****							
$N(1440)$	P_{11}	****	****	*			***	*	***
$N(1520)$	D_{13}	****	****	*			****	****	****
$N(1535)$	S_{11}	****	****	****			*	**	***
$N(1650)$	S_{11}	****	****	*	***	**	***	**	***
$N(1675)$	D_{15}	****	****	*	*		****	*	****
$N(1680)$	F_{15}	****	****				****	****	****
$N(1700)$	D_{13}	***	***	*	**	*	**	*	**
$N(1710)$	P_{11}	***	***	**	**	*	**	*	***
$N(1720)$	P_{13}	****	****	*	**	*	*	**	**
$N(1900)$	P_{13}	**	**					*	
$N(1990)$	F_{17}	**	**	*	*	*			*
$N(2000)$	F_{15}	**	**	*	*	*	*	**	
$N(2080)$	D_{13}	**	**	*	*				*
$N(2090)$	S_{11}	*	*						
$N(2100)$	P_{11}	*	*	*					
$N(2190)$	G_{17}	****	****	*	*	*		*	*
$N(2200)$	D_{15}	**	**	*	*				
$N(2220)$	H_{19}	****	****	*					
$N(2250)$	G_{19}	****	****	*					
$N(2600)$	$I_{1\,11}$	***	***						
$N(2700)$	$K_{1\,13}$	**	**						

Moorhouse and Λ selection rule violated

(**) not well-established



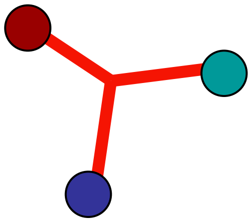
$\gamma\, n \to N^* (\Delta^*) \to \pi\, N$	27 states
$\gamma\, p \to N^* (\Delta^*) \to \pi\, N$	19 states
$\gamma\, n \to N^* \to \eta\, N$	16 states
$\gamma\, p \to N^* \to \eta\, N$	8 states
$\gamma\, n \to N^* \to K\, \Lambda$ $\gamma\, p \to N^* \to K\, \Lambda$	**8 states**

$P_{11}(1440)$
$D_{13}(1520)$
$S_{11}(1535)$
$F_{15}(1680)$
$P_{11}(1710)$
$P_{13}(1720)$
$\boxed{P_{13}(1900)}$
$\boxed{F_{15}(2000)}$

The Λ selection rule can be tested at JLAB.

• Excitation of 20-plets due to QCD mixing

Existence of scalar [ud] diquark in color $\bar{3}$?



√ 20-plets ×

[20, 1^+]

$$\psi^a_{211}(\boldsymbol{\rho},\boldsymbol{\lambda}) = \sqrt{2}(\rho_+\lambda_z - \lambda_+\rho_z)\frac{\alpha_h^5}{\pi^{3/2}}e^{-\alpha_h^2(\boldsymbol{\rho}^2+\boldsymbol{\lambda}^2)/2}$$

$$\text{where } \rho_\pm \equiv \mp(\rho_x \pm i\rho_y)/\sqrt{2} \text{ and } \lambda_\pm \equiv \mp(\lambda_x \pm i\lambda_y)/\sqrt{2},$$

QCD admixture of [70, $^2 8$; 2, 0, ½] in the nucleon enables the excitation of 20-plets in 3-q model.

Evidence for 20-plets will be able to distinguish 3-q and quark-diquark model for baryon.

[70, ^{2}8; 2, 0, ½] → [20, ^{2}8; 2, 1, J], J=1/2, 3/2

Non-zero photon transition occurs via orbital-flip “electric” term.

$$A = 6\sqrt{\frac{\pi}{k_0}}\mu_0 \frac{1}{g}\langle \psi^a_{211}| e^{ikr_{3z}} p_{3+} |\psi^\rho_{200}\rangle$$
$$= -6\sqrt{\frac{\pi}{k_0}}\mu_0 \frac{1}{g} \times \frac{2k}{3\sqrt{3}} e^{-k^2/6\alpha_h^2}$$

In comparison with D15 → γ p
(Moorhouse violated)

$$B = 6\sqrt{\frac{\pi}{k_0}}\mu_0 k \langle \psi^\rho_{110}| e^{ikr_{3z}} |\psi^\rho_{200}\rangle$$
$$= 6\sqrt{\frac{\pi}{k_0}}\mu_0 k \times (-i) \frac{k}{3\alpha_h} e^{-k^2/6\alpha_h^2}$$

$10^3 \times \mathrm{GeV}^{1/2}$

Final state	$A^p_{1/2}$	$A^p_{3/2}$
P_{11}	$-\frac{1}{3\sqrt{3}}A$	-
P_{13}	$\frac{1}{3\sqrt{6}}A$	$\frac{1}{3\sqrt{2}}A$
D_{15}	$\frac{1}{3}\sqrt{\frac{1}{15}}B$	$\frac{1}{3}\sqrt{\frac{2}{15}}B$
Theo.	14	20
Exp.	19 ± 8	15 ± 9

Do we have spectroscopy evidence for 20-plets?

Without 20-plets:

$|\mathbf{56},\ {}^{\mathbf{2}}\mathbf{8},\ 2,0,\frac{1}{2}\rangle$ $\quad P_{11}(1440)$ (Roper resonance) (****)

$|\mathbf{70},\ {}^{\mathbf{2}}\mathbf{8},\ 2,0,\frac{1}{2}\rangle$ $\quad P_{11}(1710)$ (***)

$|\mathbf{70},\ {}^{\mathbf{4}}\mathbf{8},\ 2,2,J\rangle$ $\quad P_{11}(2100)$ (*) $\qquad P_{13}(1900)$ (**), $F_{15}(2000)$ (**), $F_{17}(1990)$ (**)

$|\mathbf{70},\ {}^{\mathbf{4}}\mathbf{8},\ 2,0,\frac{3}{2}\rangle$ $\quad \boxed{P_{13}(1900)\ (**)}$

$|\mathbf{56},\ {}^{\mathbf{2}}\mathbf{8},\ 2,2,J\rangle$ $\quad P_{13}(1720)$ (****), $F_{15}(1680)$ (****)

$|\mathbf{70},\ {}^{\mathbf{2}}\mathbf{8},\ 2,2,J\rangle$ $\quad \boxed{P_{13}(1900)\ (**),\ F_{15}(2000)\ (**)}$

With 20-plets, another P11 and P13 states are needed.

The present data are not sufficient for determine the observed P11(1710), P13(1720), and P13(1900).

Helicity amplitudes for different quark model assignments for P11 and P13

Mixing angle included

$10^3 \times \mathrm{GeV}^{1/2}$

Reso.	Heli. amp.	$[\mathbf{56},^2 8; 2^+]$	$[\mathbf{70},^2 8; 0^+]$	$[\mathbf{70},^2 8; 2^+]$	$[\mathbf{70},^4 8; 0^+]$	$[\mathbf{70},^4 8; 2^+]$	$[\mathbf{20},^2 8; 1^+]$	Exp. data
$P_{11}(1710)$	$A^p_{1/2}$	*	32	*	*	-8	-15	$+9 \pm 22$
$P_{13}(1720)$	$A^p_{1/2}$	100	*	-71	17	7	-11	$+18 \pm 30$
	$A^p_{3/2}$	30	*	-21	29	12	-18	-19 ± 20

Moorhouse violated amplitudes

The experimental values are compatible with the Moorhouse-violated and 20-plet assignment of P11(1710) and P13(1720) !

Some general features about the 20-plets

- Since $20 \not\to 56 \otimes 35$, the two-body decay of the 20-plets, $B(20) \to B(56) \otimes M(35)$, is suppressed.

- The 20-plet decay will favor $B(20) \to B(70) + M(35) \to B(56) + M(35) + M(35)$.

$$\left\{\begin{array}{l} \text{B.R.}(P11(1710) \to N\pi\pi) \sim 40\text{-}90\% \\ \\ \text{B.R.}(P13(1720) \to N\pi\pi) \sim >70\% \end{array}\right.$$

Should be examined by precise measurement !

Summary

- The QCD spin-dependent forces result in configuration mixings in baryon resonances, and violation of naïve quark model selection rules.
- However, the Λ selection rule turns to be robust due to the heavier mass of s-quark in $N^* \rightarrow K\Lambda$. Experimental test of this seems possible at JLab.
- As a consequence of the QCD mixings, one expects that 20-plet excitations are possible if the 3-quark-baryon scenario is dominant, while a dominant quark-diquark picture does not lead to [20, 1+]. Precise measurement of the P11 and P13 helicity amplitudes is important.
- More theoretical studies are needed.