Cosmology

I. Tkachev

Institute for Nuclear Research, Moscow

24-27 June 2016, Norway

Cosmology

- Subject of cosmology:
  Origin, evolution, and eventual fate of the Universe.
- Modern cosmology is based on:
  - Assumption that the Universe is homogeneous and isotropic on large scales
  - Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G \, T_{\mu\nu}$$

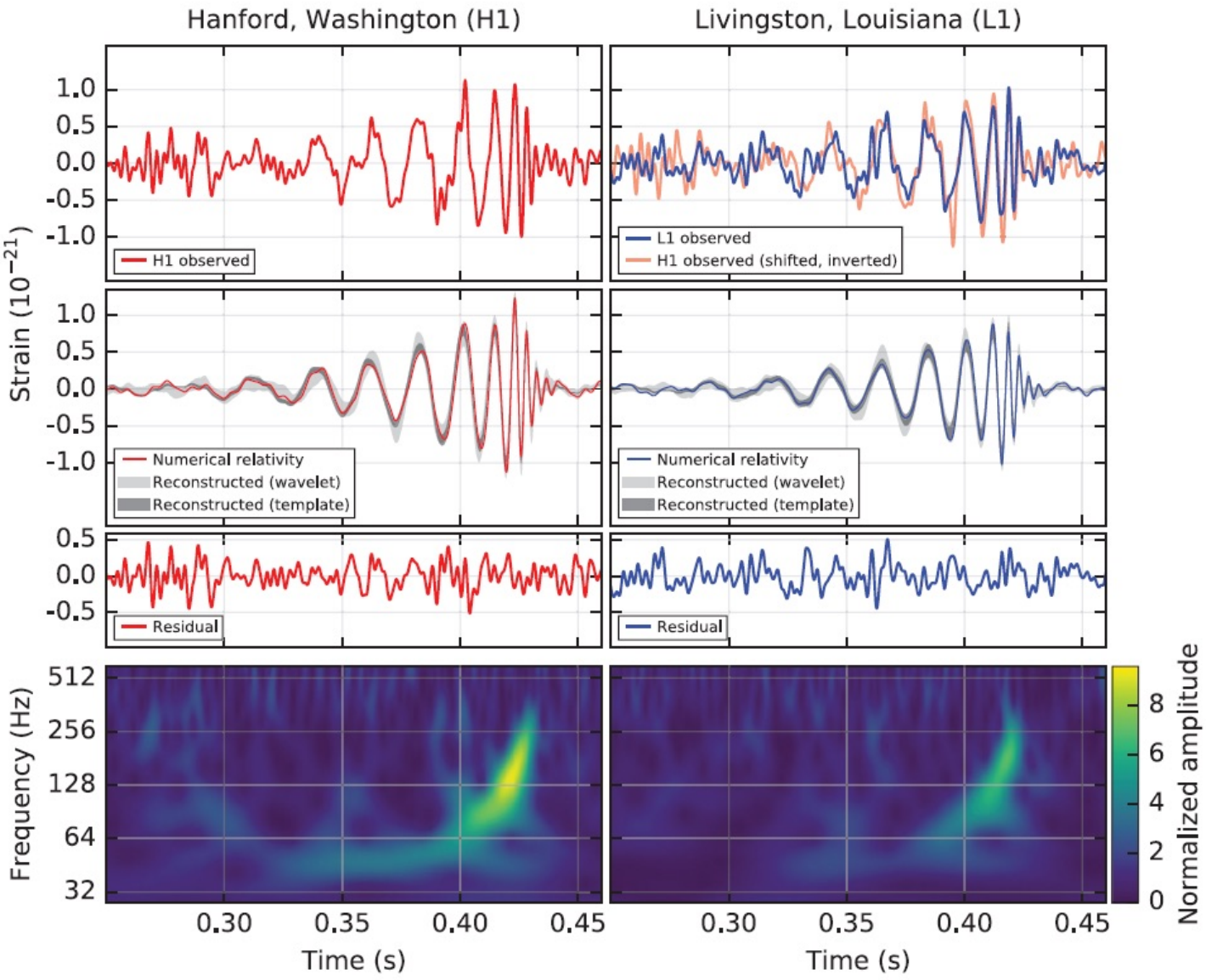
  - Modern particle physics which gives content to $T_{\mu\nu}$

Natural Units: $c = \hbar = k_B = 1$

$$G = M_{\rm Pl}^{-2}$$

$$M_{\rm Pl} = 1.22 \times 10^{19} {\rm GeV}$$

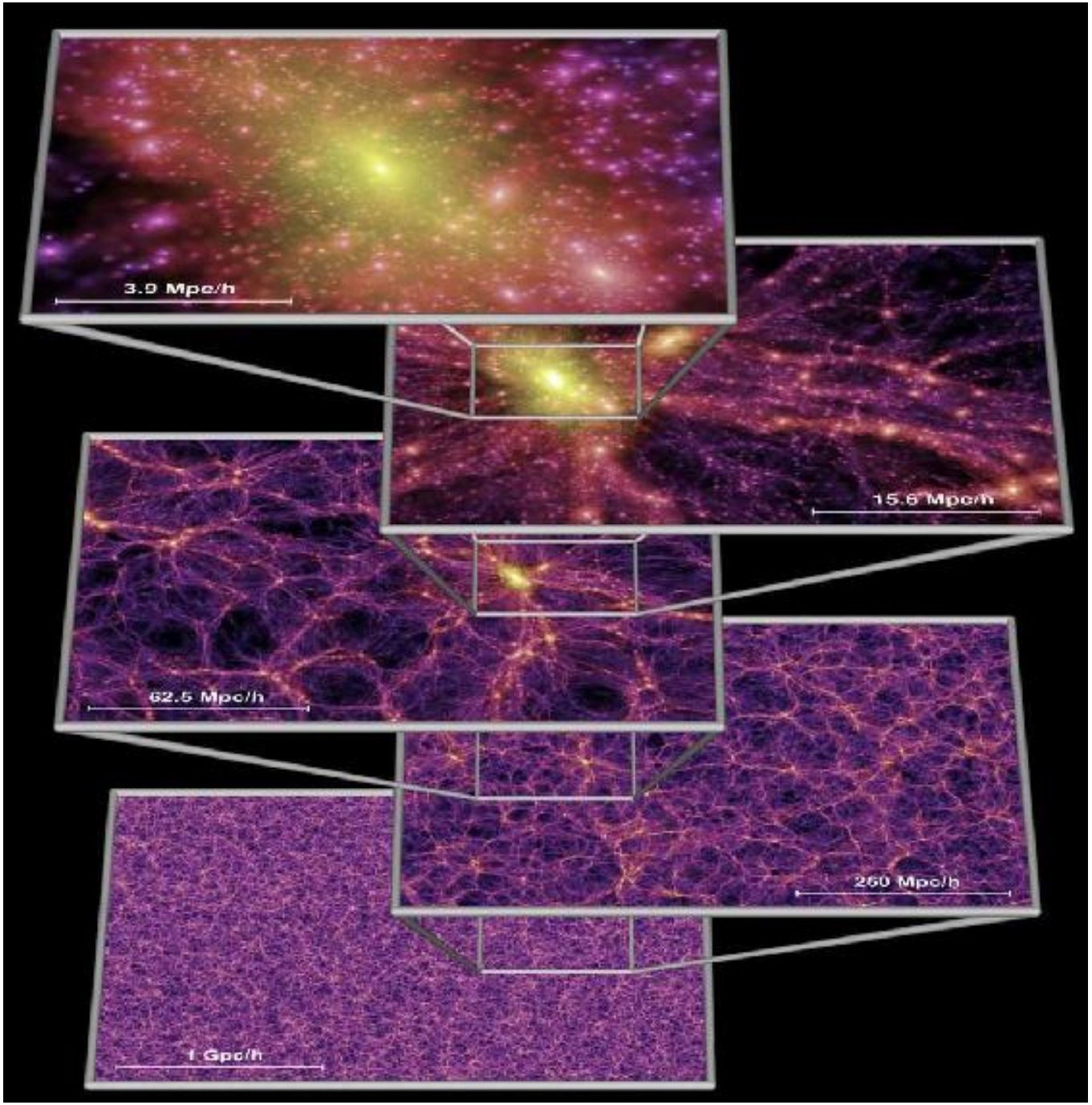
Gravitational waves are discovered!



Einstein theory *is* correct.

Event GW150914 Sept. 14, 2015 at 09:50:45 UTC

Universe is homogeneous

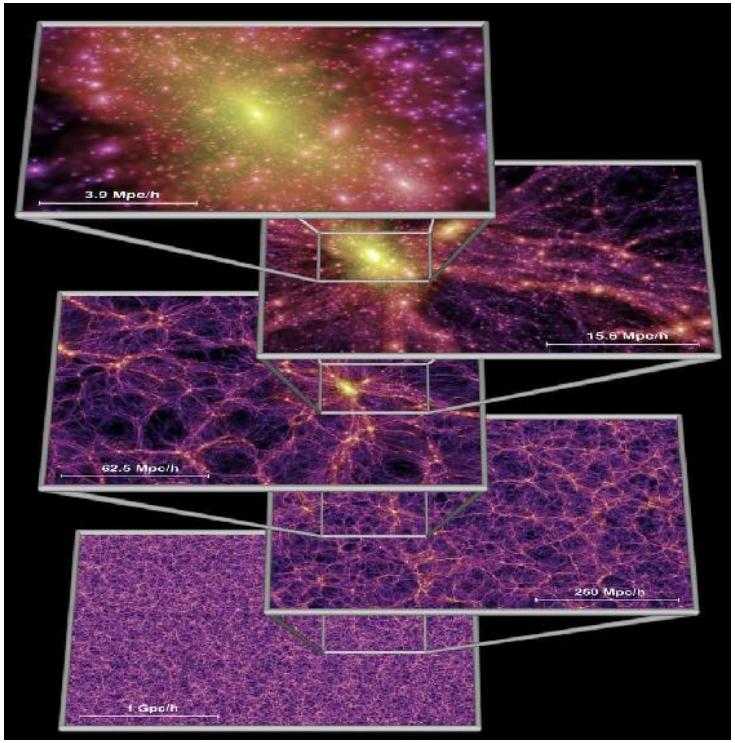


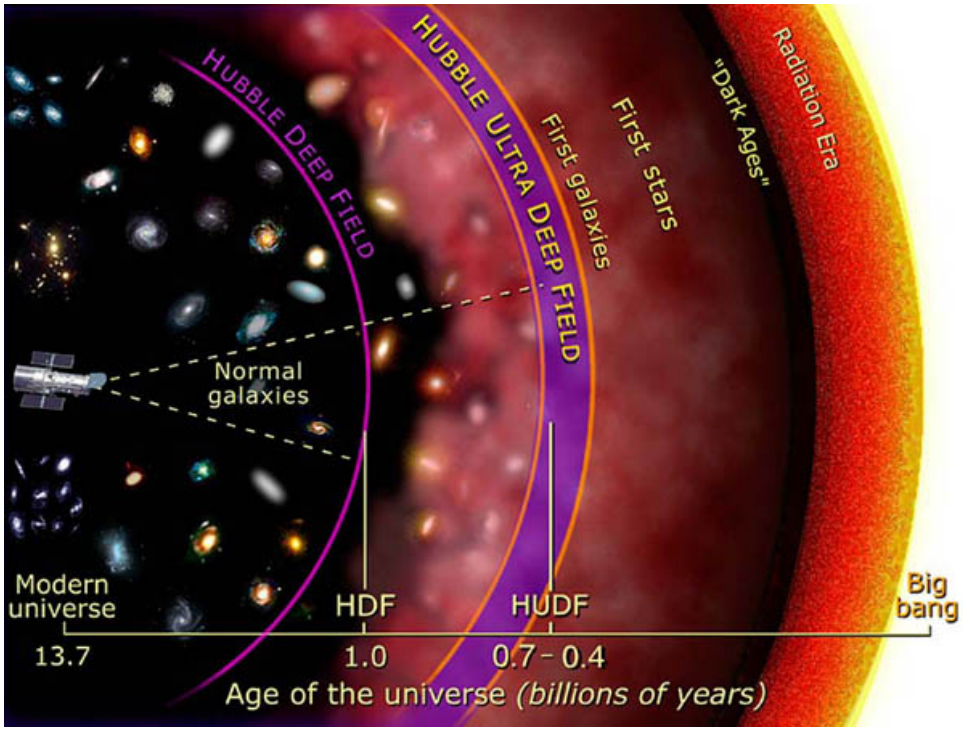
$1 \text{ Mpc} = 3.26 \times 10^6 \text{ yrs}$

Horizon $\sim$ age:

$t = 13.8 \text{ Gyr} = 4.2 \text{ Gpc} = 10^{28} \text{ cm}$

Looking at distant objects we are looking back in time

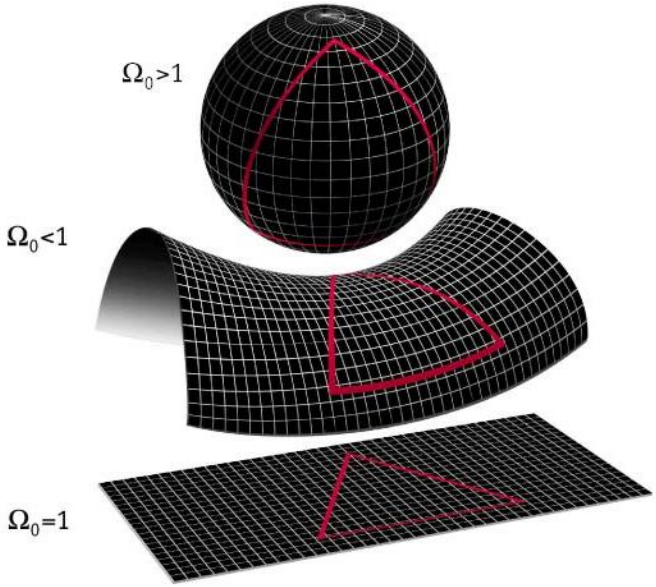




At the age of 380,000 years the Universe was homogeneous to the level of 10^{-5}

We cannot see past this surface even if the Universe is infinite

Space-time metric



$$ds^2 = dt^2 - a^2(t)\, dl^2$$

All encoded in the *scale factor* $a(t)$.

There are three possible realizations of a homogeneous isotropic space:

$$dl^2 = \frac{dr^2}{1 - k\, r^2} + r^2\, (d\theta^2 + \sin^2\theta\, d\phi^2)$$

$${}^{(3)}R = \frac{6k}{a^2(t)} \qquad \begin{cases} k = -1 & \text{Open} \\ k = 0 & \text{Flat} \\ k = +1 & \text{Closed} \end{cases}$$

Friedmann equations

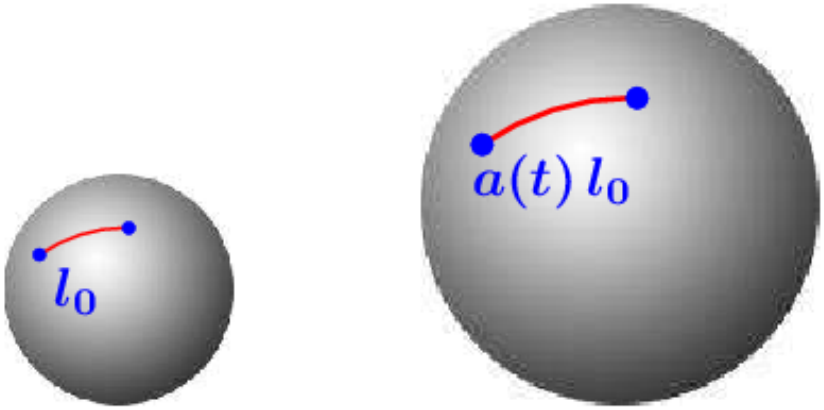
Assume ideal fluid for the energy momentum tensor

$$T_{\mu\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

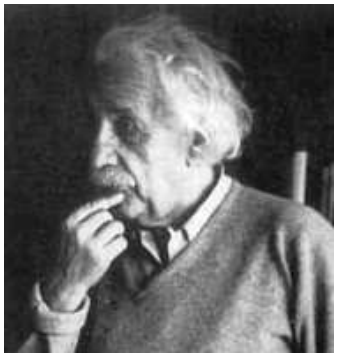
Einstein equations written for homogeneous isotropic world

$$\boxed{\begin{aligned} \left(\frac{\dot{a}}{a}\right)^2 &= \frac{8\pi G}{3}\rho - \frac{k}{a^2} \\ \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}(\rho + 3\,p) \end{aligned}}$$

Solution: the Universe should be expanding



$$ds^2 = dt^2 - a^2(t)\, dl^2$$



- In 1917 Einstein added cosmological constant to his equations thinking it will provide static solutions.

- In 1922 Friedmann had shown that the Universe **must** expand as a whole. After some debate Einstein admitted mistake and called the introduction of a cosmological constant "the greatest blunder of my life".

Friedmann equations: the physics behind

One of the two Friedmann equations can be excluded in favour of

$$\boxed{\frac{d\rho}{dt} + 3\,\frac{\dot{a}}{a}\,(\rho + p) = 0}$$

which is nothing but energy-momentum conseravtion $T^{\mu\nu}{}_{;\nu} = 0$

and corresponds to the First Law of thermodynamics with $dS = 0$.

$$\boxed{dE + p\,dV = T\,dS}$$

Here $E = \rho\,V = \rho\,a^3$ is energy and S is entropy.

Isentropic expansion

Friedmann expansion driven by an ideal fluid is isentropic, $dS = 0$.
Dissipation is negligible most of the time.
Entropy in thermal equilibrium:

$$S = \frac{2\pi^2}{45} g_* T^3 a^3 = \text{const} ,$$

$$g_* = \sum_{i=\text{bosons}} g_i + \frac{7}{8} \sum_{j=\text{fermions}} g_j .$$

In the past the Universe was hot

$$\boxed{T \propto \frac{1}{a}}$$

Energy and matter content in the Universe

- Radiation. (Relativistic degrees of freedom)
  Major energy fraction at early times.
- Baryonic matter. (Stars)
  Observable world today.
- Dark matter. (Should be there)
  Major matter fraction today. New physics.
- Dark energy. (Vacuum)
  Major energy fraction today. Likely new physics.

Equation of state

Definition: $$\boxed{w \equiv \frac{p}{\rho}}$$

If $w = \mathrm{const}$, the energy-momentum conseravtion

$$\frac{d\rho}{dt} + 3\,\frac{\dot{a}}{a}\,(\rho + p) = 0$$

gives

$$\rho = a^{-3(1+w)}\,\rho_0$$

In particular:

- Radiation ..... $w = \frac{1}{3}$ ..... $\rho = a^{-4}\,\rho_0$
- Matter ..... $w = 0$ ..... $\rho = a^{-3}\,\rho_0$
- Vacuum ..... $w = -1$ ..... $\rho = \mathrm{const}$

Law of expansion

First Friedmann equation in spatially flat Universe

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

with $\rho = a^{-3(1+w)}\,\rho_0$ gives

$$a = \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}}$$

In particular:

- Radiation ..... $w = \frac{1}{3}$ ..... $a = (t/t_0)^{1/2}$
- Matter ..... $w = 0$ ..... $a = (t/t_0)^{2/3}$
- Vacuum ..... $w = -1$ ..... $a = \exp(H_0 t)$

We know that the Universe is spatially flat practically.

Geometry versus Universe future

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

$k = +1$	Universe will recollapce	*(Valid if dark energy is absent)*
$k = -1$	Universe will expand forever	
$k = 0$	Critical density	$\rho_c \equiv \frac{3}{8\pi G}\left(\frac{\dot{a}}{a}\right)^2$

Critical density is related to very important cosmological parameter

$$H \equiv \frac{\dot{a}}{a} \quad - \text{the Hubble constant}$$

Note: $H = \frac{2}{3(1+w)}\frac{1}{t}$. For radiation or matter $H^{-1} \sim$ horizon.

Hubble law

Distance between two points $r(t) = a(t) r_0$ increases as

$$v \equiv \dot{a} r_0 = \frac{\dot{a}}{a} a r_0 = H r$$

This gives the Hubble law $v = H\, r$

For convenience "small h" is introduced as $H = 100\, h \ \mathrm{km\ s^{-1} Mpc^{-1}}$

Numerically

$$\rho_c \equiv \frac{3H^2}{8\pi G} = 1.05 \times 10^{-5}\, h^2\, \frac{\mathrm{GeV}}{\mathrm{cm}^3}$$

ρ_c - *approximately 5 hydrogen atoms per cubic meter*

Friedmann equation as H(z)

Friedmann equation for a spatially flat Universe

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \sum_i \rho_i$$

can be re-written as

$$H^2 = H_0^2 \sum_i \Omega_i \, (1+z)^{3(1+w_i)}$$

where

$$\rho_i = \rho_{i,0} \, a^{-3(1+w_i)} = \rho_c \, \Omega_i \, (1+z)^{3(1+w_i)}$$

and

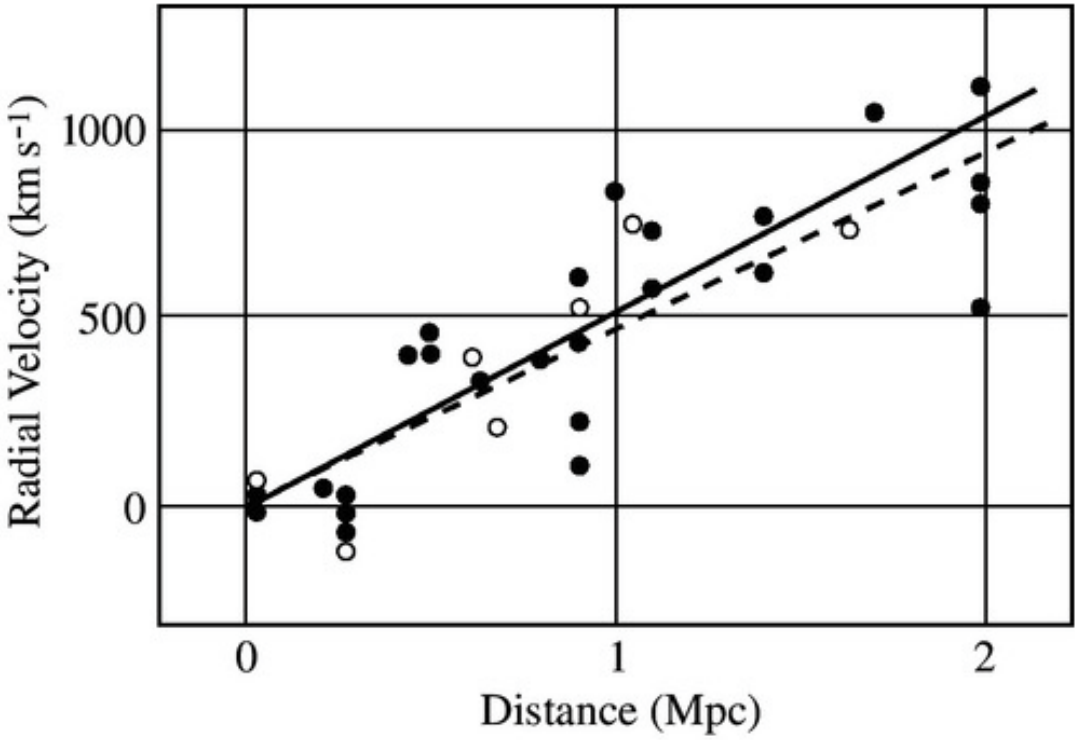
$$\Omega_i \equiv \frac{\rho_{i,0}}{\rho_c}$$

Ω_i are important cosmological parameters derived from observations.

Note that $\sum \Omega_i = 1$

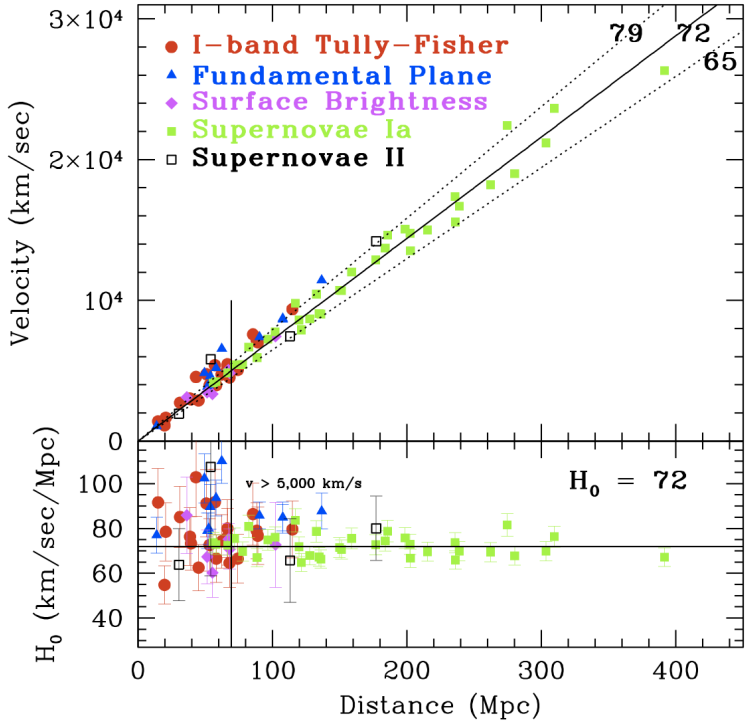
Hubble law

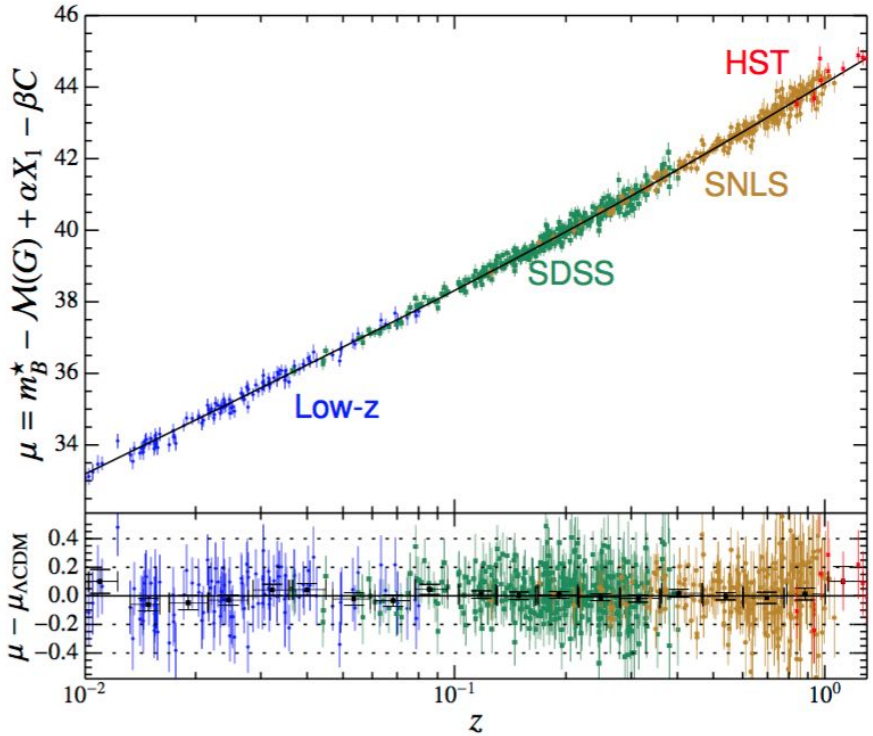
Original Hubble diagram (1929), $v = H\,r$



Hubble law

Modern Hubble diagrams





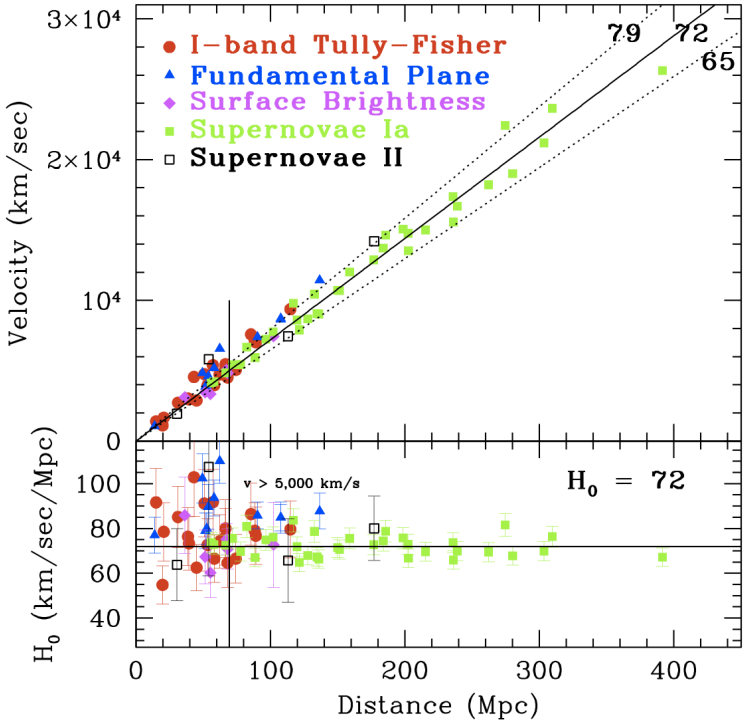
Latest value from direct observations:

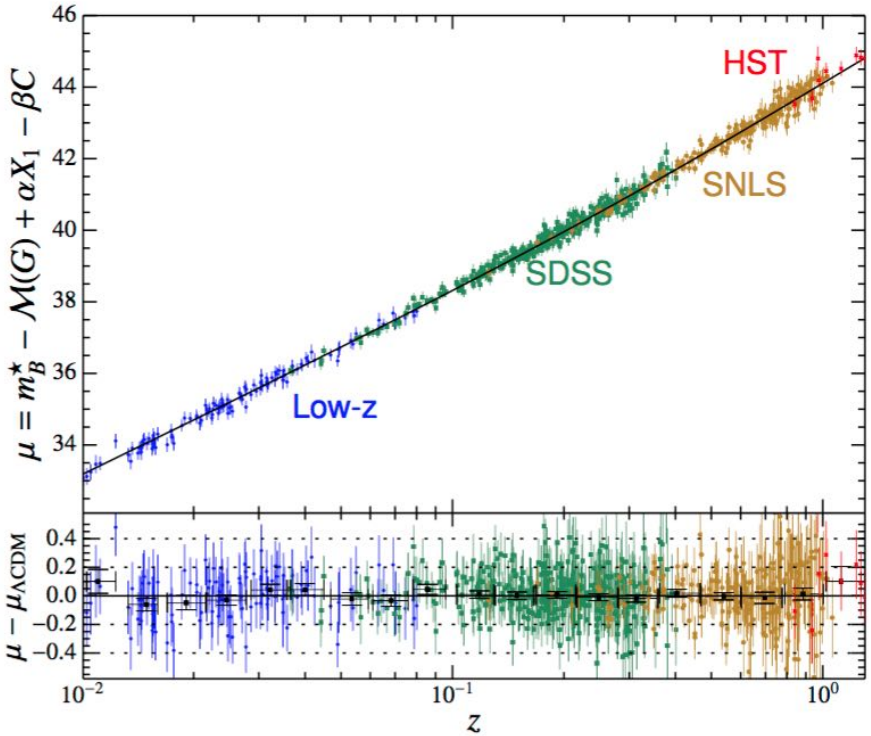
$$H_0 = (73.00 \pm 1.75)\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$$

arxiv: 1604.01424

Hubble law

Modern Hubble diagrams





But what is shown on the right?

Cosmography

Looking at distant objects we see just light they emit. How then cosmological information is derived? E.g. distance and velocity?

1. Doppler effect

Velocity <=> Redshift

$$z \equiv \frac{\omega_0 - \omega}{\omega}$$

2. Standard Candles

Distance <=> Flux$^{-1/2}$

$$D_L^2 \equiv \frac{L}{4\pi F}$$

Redshift

Photon trajectory is given by $ds^2 = dt^2 - a^2 d\chi^2 = 0$.

Define conformal time η as $d\eta = dt/a$. Then

$$ds^2 = a^2(d\eta^2 - d\chi^2) = 0$$

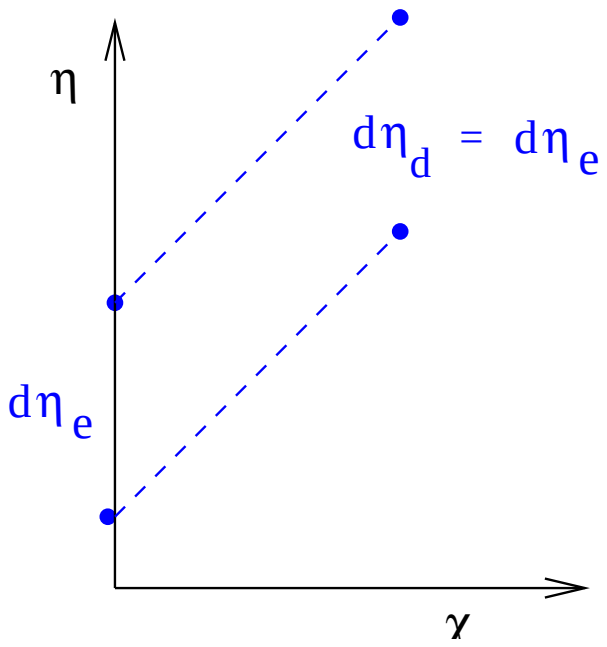
Solution: $\chi = \pm\eta + \text{const}$, and for signal emission and detection

$$\frac{dt}{a}|_{\text{detection}} = \frac{dt}{a}|_{\text{emission}}$$

which is $\boxed{\omega\, a_d = \omega_0\, a_e}$

We can say that wavelength stretches with expansion, but $d\eta_d = d\eta_e$ is more general.

Equivalently $1 + z = \frac{a_d}{a_e}$, where $z \equiv \frac{\omega_0 - \omega}{\omega}$.



This also gives $dz = -H d\chi$ (local form of the Hubble law).

Luminosity distance

Background geometry $ds^2 = a^2\,(d\eta^2 - d\chi^2 - \chi^2\,d\Omega)$

Bolometric flux

$$F = \frac{L}{4\pi\,\chi^2(1+z)^2}$$

- Surface area at the point of detection $(\,a = 1\,)$ is $4\pi\,\chi^2$
- Energy and arrival rates are redshifted. This reduces the flux by $(1+z)^2$

Luminosity distance:

$$D_L \equiv \sqrt{\frac{L}{4\pi F}} = (1+z)\,\chi$$

Comoving distance:

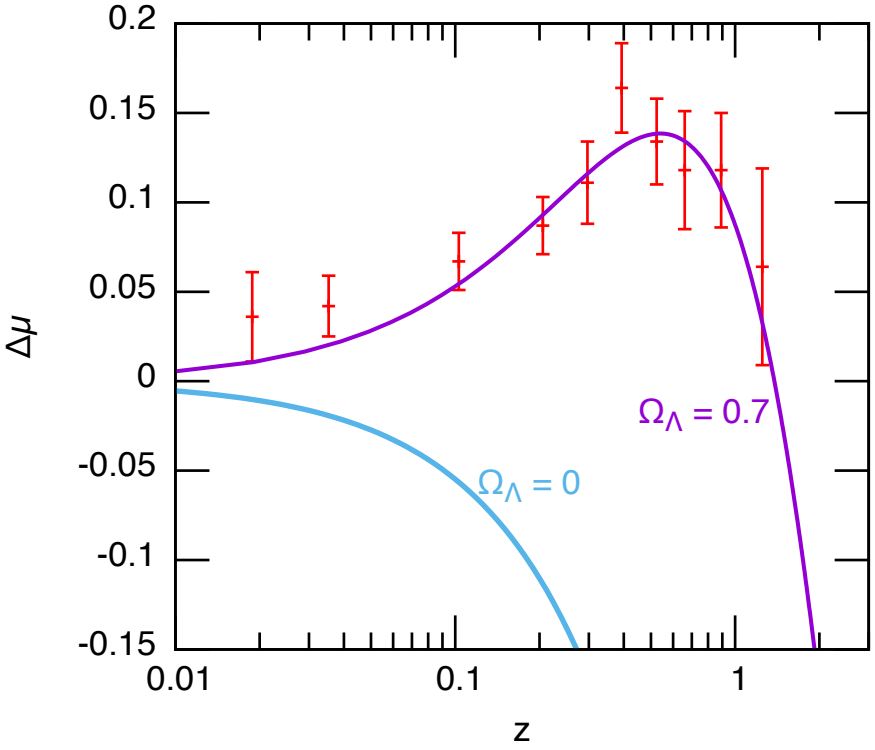
$$\chi(z) = \int_0^z \frac{dz'}{H(z')}$$

Astronomers measure flux in magnitudes, $\mu \propto 5\log_{10} F$, which gives

$$\Delta\mu = \mu - \mu_{\rm base} = 5\log_{10}\left[\frac{\chi(z)}{\chi(z)_{\rm base}}\right]$$

Supernovae: direct probe of Dark energy

$$D_L \equiv \sqrt{\frac{L}{4\pi F}}$$



Dark energy is a dominant component today, $\Omega_\Lambda \approx 0.7$

Ancient evidence for Dark energy

Age of the Universe

During mater dominated expansion $a \propto t^{2/3}$. Therefore

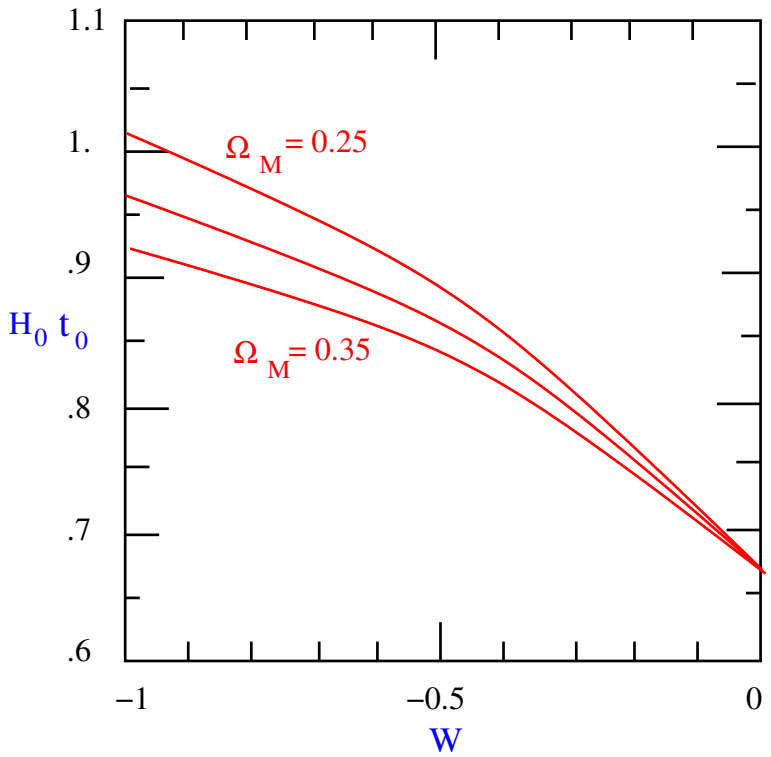
$$H_0\, t_0 = 2/3$$

Previous millennium measurments:

$$H_0 = 70 \pm 7 \text{ km sec}^{-1} \text{ Mpc}^{-1}, \qquad t_0 = 13 \pm 1.5 \text{ Gyr}$$

and $H_0\, t_0 = 0.93 \pm 0.15$. The Age Problem.

However, for two components, usual matter and dark energy:



OK with $\Omega_\Lambda \approx 0.7$

Georgy Antonovich Gamov

Nuclear physicist and cosmologist

- Used his knowledge of nuclear reactions to interpret stellar evolution and element formation in stars (1938).

- In 1946 realized that $^4\mathrm{He}$ could not have been produced in stars. He suggested, as a way out, that the early Universe itself was the Oven in which light elements were cooked up.
- He also calculated the left-over heat which should be measured today as 5° K CMBR.

The Hot Big Bang theory was born.

Cosmic Microwave Background Radiation

Predicted by G. Gamov in 1946: $5\ K^\circ$
Observed by T. Shmaonov in 1957: $(3.7 \pm 3.7)\ K^\circ$
Measured by A. Penzias and R. Wilson in 1965: $(3.5 \pm 1)\ K^\circ$

- $2.7255 \pm 0.0006\ \mathrm{K}^\circ$ above absolute zero
- mm-cm wavelength
- 410 photons per cubic centimeter
- 10 trillion photons per second per squared centimeter

Cosmic Microwave Background Radiation

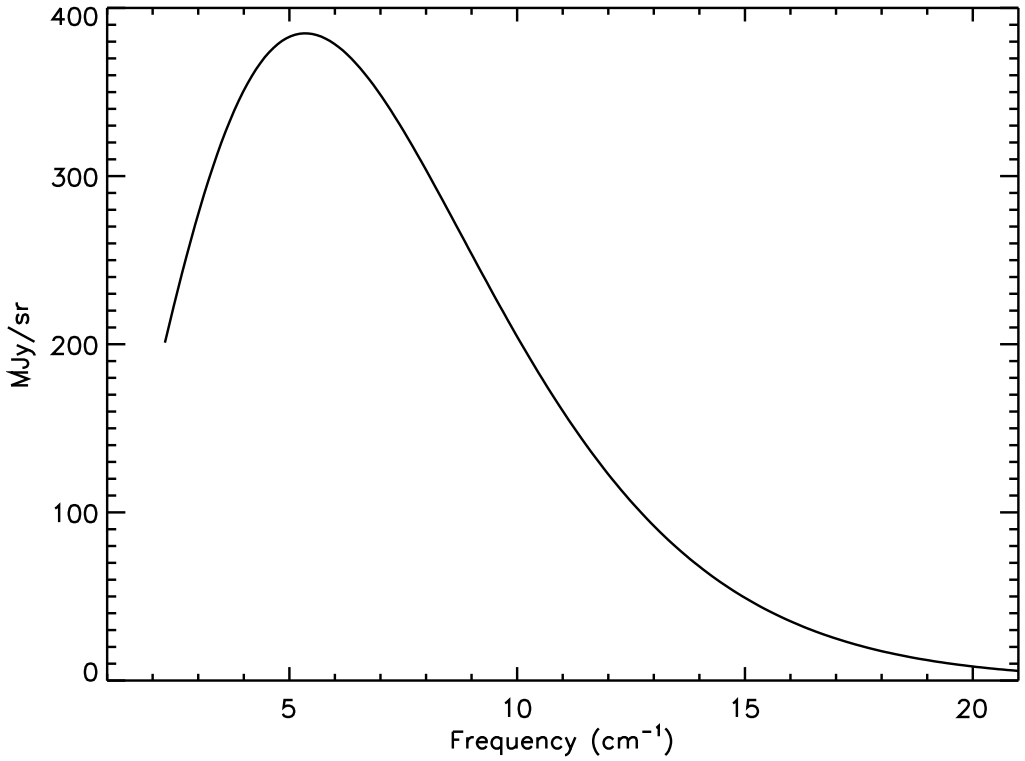
Predicted by G. Gamov in 1946: $5\ K^\circ$
Observed by T. Shmaonov in 1957: $(3.7 \pm 3.7)\ K^\circ$
Measured by A. Penzias and R. Wilson in 1965: $(3.5 \pm 1)\ K^\circ$

- $2.7255 \pm 0.0006\ \mathrm{K}^\circ$ above absolute zero
- mm-cm wavelength
- 410 photons per cubic centimeter
- 10 trillion photons per second per squared centimeter

Few percent of TV "snow"

Perfect Blackbody

The CMBR spectrum is strictly blackbody, $T = 2.7255$ K



Bose-Einstein distribution

$$n = \frac{1}{\exp(E/T) - 1}$$

COBE (1994)

There is no explanation to it but the hot Big Bang

Isentropic expansion

Friedmann expansion driven by an ideal fluid is isentropic, $dS = 0$.
Dissipation is negligible most of the time.
Entropy in thermal equilibrium:

$$S = \frac{2\pi^2}{45} g_* T^3 a^3 = \text{const} ,$$

$$g_* = \sum_{i=\text{bosons}} g_i + \frac{7}{8} \sum_{j=\text{fermions}} g_j .$$

In the past the Universe was hot

$$\boxed{T \propto \frac{1}{a}}$$

Bits of microphysics

For particles in thermal equilibrium phase-space distribution functions:

$$f_i(k) = \frac{1}{\mathrm{e}^{(k_0-\mu)/T} \mp 1} \qquad (\mathrm{Bose}, \mathrm{Fermi})$$

where $k_0 = \sqrt{\vec{k}^2 + m_i^2}$.

Number density:

$$n_i = \frac{g_i}{(2\pi)^3} \int d^3k f_i(k),$$

where e.g. $g_\gamma = g_{e^-} = g_{e^+} = 2, \quad g_\nu = g_{\bar{\nu}} = 1.$

Energy momentum tensor:

$$T_{\mu\nu}(i) = \frac{g_i}{(2\pi)^3} \int d^3k \frac{k_\mu k_\nu}{k_0} f_i(k),$$

This gives energy density and pressure as $\rho = T_{00}$ and $p = -\frac{1}{3} T^i_i$.

Entropy density:

$$s = \frac{\rho + p - \mu n}{T}$$

Bits of microphysics

Non-relativistic particles:

$$
\begin{aligned}
n_i &= g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} \mathrm{e}^{(\mu_i - m_i)/T} \\
\rho &= m_i\, n_i + \frac{3}{2} T\, n_i \\
p &= T\, n_i
\end{aligned}
$$

With $\mu \neq 0$ it tells e.g. that anti-baryons are virtually extinct now.

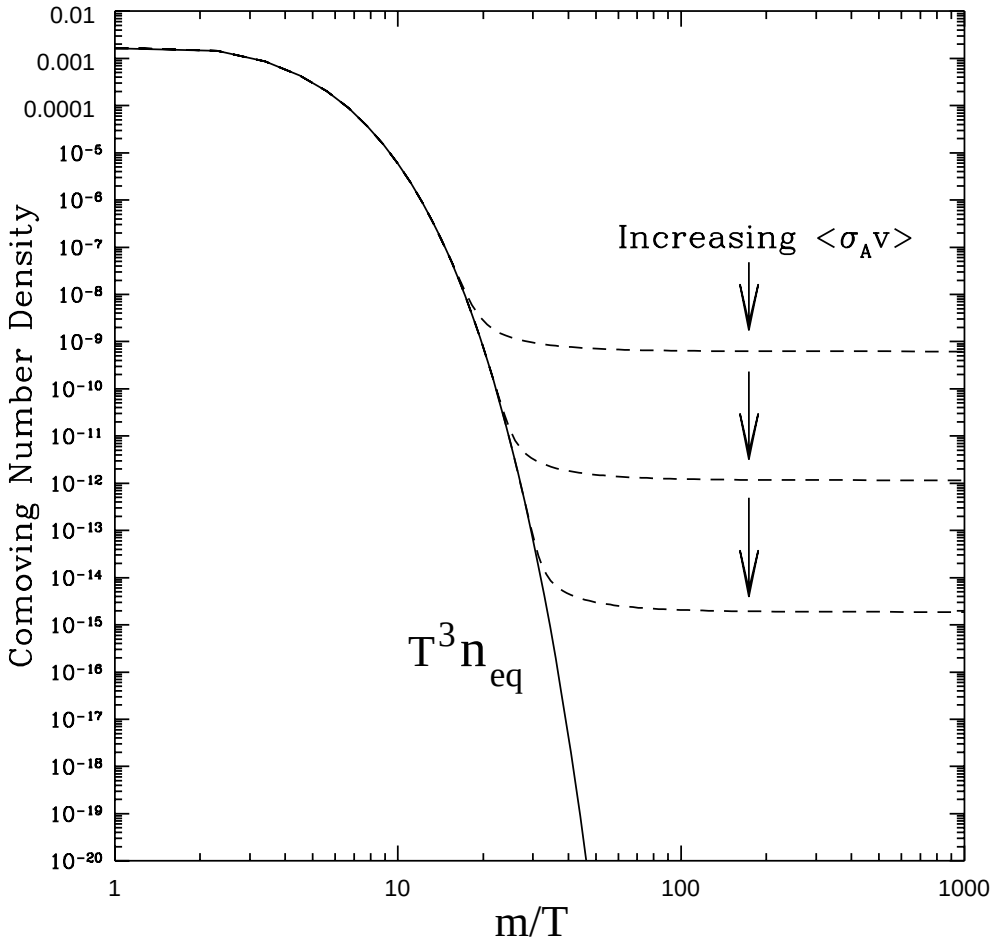
With $\mu = 0$ the expression for n_i makes in particular, basis for Saha equation. It gives the ratio of atoms to ions, or protons to neutrons, etc. E.g.

$$
\frac{n_n}{n_p} \approx \mathrm{e}^{-(m_n - m_p)/T}
$$

until reactions $n \leftrightarrow p$ go out of equilibrium. It has many other applications, e.g. determines the amount of thermally produced dark matter.

Freeze-out temperature

In the expanding Universe particle concentrations are in equilibrium as long as $\sigma n v > H$. After that distributions do not change in a co-moving volume, i.e. "freeze-out".



- Electromagnetic interactions are in equilibrium at $T > 1$ eV
- Neutrino concentration "freeze-out" at $T \sim 1$ MeV
- WIMP "freeze-out" at $T \sim M/25$

Bits of microphysics

Relativistic plasma without chemical potentials, $p = \rho/3$:

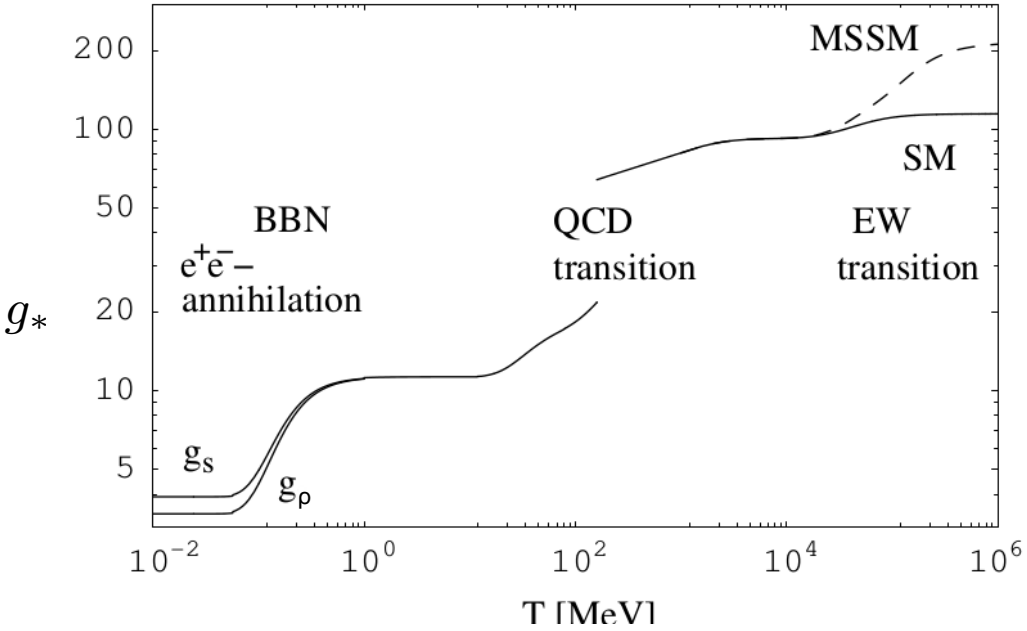
$$
\begin{aligned}
n &= g'_* \frac{\zeta(3)}{\pi^2} T^3 \\
\rho &= g_* \frac{\pi^2}{30} T^4 \\
s &= g_* \frac{2\pi^2}{45} T^3
\end{aligned}
$$

where

$$
\begin{aligned}
g'_* &= \sum_{\text{bosons}} g_i + \frac{3}{4} \sum_{\text{fermions}} g_i \,. \\
g_* &= \sum_{\text{bosons}} g_i + \frac{7}{8} \sum_{\text{fermions}} g_i \,.
\end{aligned}
$$

Particles with $m \ll T$ should be counted only, i. e. g_* and g'_* are functions of the temperature.

Radiation dominated phase



Substituting

$$\rho = g_* \frac{\pi^2}{30} T^4$$

into first Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \rho$$

we obtain during radiation dominated phase

$$H \simeq 1.66 \sqrt{g_*} \frac{T^2}{M_{\rm Pl}}$$

This integrates into time-temperature relation

$$t(s) = \frac{2.42}{\sqrt{g_*}} \left(\frac{\rm MeV}{T}\right)^2$$

Notice that g_s and g_ρ separate at $T < \mathrm{MeV}$. That's because at lower temperatures we have two gases with different temperatures, γ and ν.

Neutrino freeze-out

In the expanding Universe particle concentrations are in equilibrium as long as $\sigma n v > H$. After that distributions do not change in a comoving volume, i.e. "freeze-out".

For neutrino:

$$\sigma_W n \sim G_F^2 T^2 \cdot T^3, \qquad H \sim \frac{T^2}{M_{\rm Pl}},$$

and rate of weak processes $\approx$ Hubble expansion rate when

$$G_F^2 M_{\rm Pl} T^3 \approx 1$$

or

$$T \approx 1 \text{ MeV}$$

Neutrino temperature

At $T \approx m_e \approx 0.5\ \mathrm{MeV}$ electron-positron pairs start to annihilate.

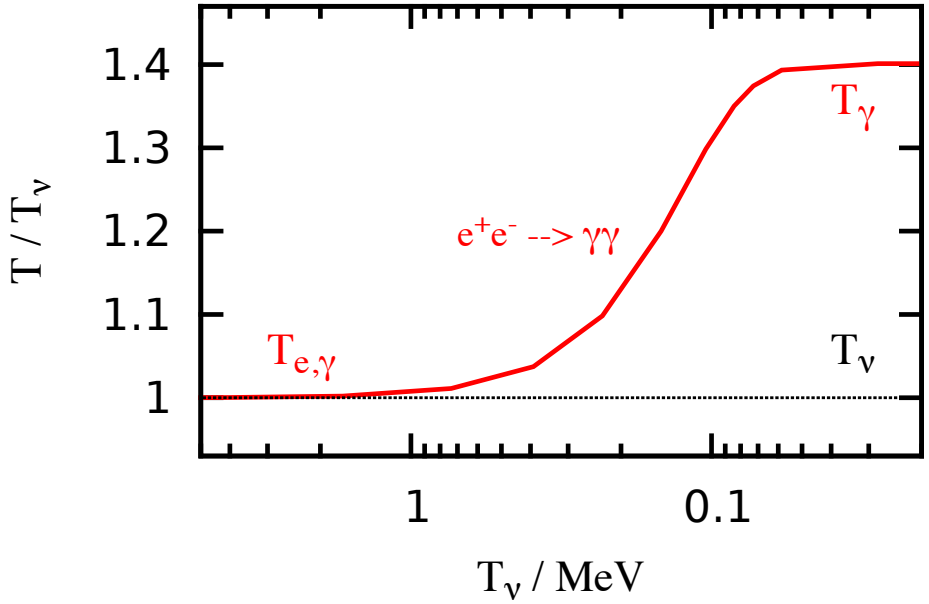
Entropy conservation is a key

Entropy in comoving volume is conserved, $g_* \, T^3 = \mathrm{const}$.

Before annihilation $(g_\gamma + g_e)\, T^3 = [2 + 4 \cdot (7/8)]\, T^3 = (11/2)\, T^3$.

After annihilation $g_\gamma \, T^3 = 2\, T^3$.

Neutrinos are decoupled already and do not participate in these relations.

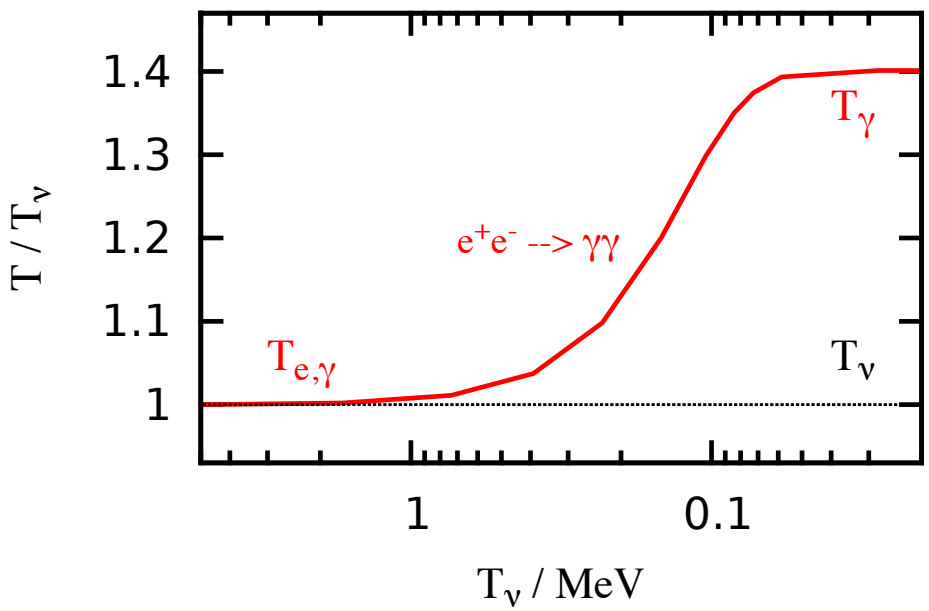


As a consequence of e^+e^- annihilation temperature of photons increases,

$$T_\gamma = \left(\frac{11}{4}\right)^{1/3} T_\nu$$

Cosmological neutrino density

Present day photon temperature $T_\gamma = 2.7255\ K$.



Neutrino temperature is lower

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma = 1.9454\ K$$

Neutrino number density

$$n_\nu = 115\ \mathrm{cm}^{-3}$$

Since $\rho_\nu = \sum_i m_{\nu i}\, n_{\nu i} < \Omega_m\, \rho_c$ we already have a constraint

$$\sum_i m_{\nu i} < 93\ \Omega_m h^2\ \mathrm{eV} \approx 10\ \mathrm{eV}$$

CMB constraints are much stronger (see later in the lectures)

Cosmological neutrino density

After "freeze-out"and e^+e^- annihilation neutrino contribution into cosmological radiation background is parametrized as

$$\rho_r = \rho_\gamma + \rho_\nu = \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{eff}\right]\rho_\gamma$$

Actually $N_{eff} \neq 3$, neither it is integer. Namely:

- When e^+e^- annihilate, neutrino are not decoupled completely. This leads to somewhat larger T_ν, which instead is parametrised as larger N,

$$N_{eff} = 3.046$$

- There can be other contributions into radiation
  e.g. Light sterile neutrinos, Goldstones or some other very light particles
- These contributions are called "dark radiation"and are searched for in modern data as a signal $N_{eff} > 3.046$ of the new physics

N_{eff} - important cosmological parameter

Cosmological neutrino density

After "freeze-out"and e^+e^- annihilation neutrino contribution into cosmological radiation background is parametrized as

$$\rho_r = \rho_\gamma + \rho_\nu = \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{eff}\right]\rho_\gamma$$

Actually $N_{eff} \neq 3$, neither it is integer. Namely:

- When e^+e^- annihilate, neutrino are not decoupled completely. This leads to somewhat larger T_ν, which instead is parametrised as larger N,

$$N_{eff} = 3.046$$

- There can be other contributions into radiation
  e.g. Light sterile neutrinos, Goldstones or some other very light particles
- These contributions are called "dark radiation"and are searched for in modern data as a signal $N_{eff} > 3.046$ of the new physics

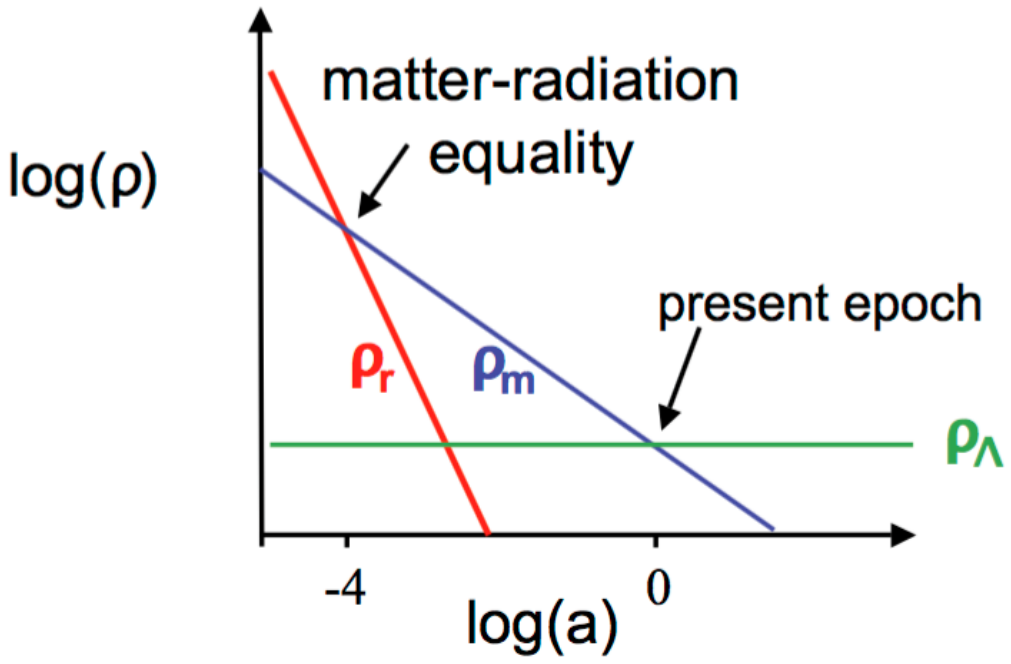
N_{eff} - important cosmological parameter

Expansion history

$$\rho_\gamma = \frac{\pi^2}{15} T^4 \quad \text{with} \quad T = 2.7255 \text{ K} \quad \text{gives} \quad \rho_\mathrm{r} = 4.41 \times 10^{-10} \text{ GeV cm}^{-3}$$

Recall $\rho_c = 1.05 \times 10^{-5}\, h^2 \text{ GeV cm}^{-3}$ to get:

$$\Omega_r = 4.2 \times 10^{-5}\, h^{-2}$$



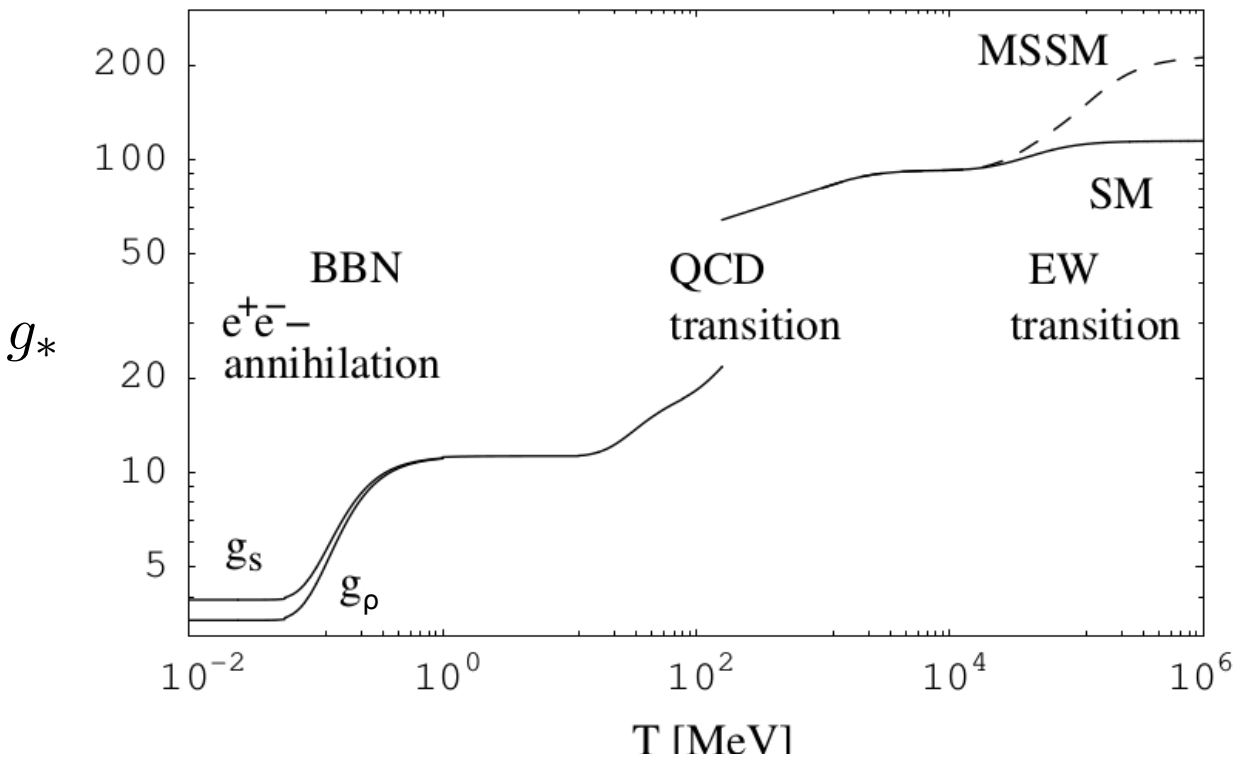
Radiation $\rho_r \propto a^{-4}$
Matter $\rho_m = a^{-3}$
Vacuum $\rho_\Lambda = \text{const}$

$$(1 + z_\Lambda)^3 = \frac{\Omega_\Lambda}{\Omega_m} \approx 2.3$$

$$1 + z_\mathrm{eq} = \frac{\Omega_m}{\Omega_r} = 3372 \pm 23$$

$$1 + z = \frac{1}{a} \propto \frac{T}{T_0}$$

Relativistic degrees of freedom



$$g_\rho(0) = 2 + 3.046 \left(\frac{7}{4}\right) \left(\frac{4}{11}\right)^{4/3} = 3.38$$

$$g_s(0) = 2 + 3.046 \left(\frac{7}{4}\right) \left(\frac{4}{11}\right) = 3.94$$