Beyond the Standard Model

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- What do we know so far?
- Supersymmetry
- Extra dimensions
- Di-photons and other excesses

Physical Motivation for BSM

Let us review some relevant facts about the universe we live in.

Microscopically we have *quantum mechanics* and *special relativity* as two fundamental theories.

A consistent framework incorporating these two theories is *quantum field theory (QFT)*. In this theory the fundamental entities are quantum fields. Their excitations correspond to the physically observable elementary particles which are the basic constituents of matter as well as the mediators of all the known interactions.

Therefore, fields have a particle-like character. Particles can be classified in two general classes: bosons (spin $s = n \in Z$) and fermions ($s = n + \frac{1}{2} \in Z + \frac{1}{2}$). Bosons and fermions have very different physical behaviour. The main difference is that fermions can be shown to satisfy the PAULI "exclusion principle", which states that two identical fermions cannot occupy the same quantum state, and therefore explaining the vast diversity of atoms.

All elementary matter particles: the leptons (including electrons and neutrinos) and quarks (that make protons, neutrons and all other hadrons) are fermions. Bosons on the other hand include the photon (particle of light and mediator of electromagnetic interaction), and the mediators of all the other interactions. They are not constrained by the Pauli principle.

Basic principle: symmetry

If QFT is the basic framework to study elementary processes, one tool to learn about these processes is the concept of *symmetry*.

A symmetry is a transformation that can be made to a physical system leaving the physical observables unchanged. Throughout the history of science symmetry has played a very important role to better understand nature.

Classes of symmetries

For elementary particles, we can define two general classes of symmetries:

- *Space-time symmetries:* These symmetries correspond to transformations on a field theory acting explicitly on the space-time coordinates,

$$x^\mu \quad \mapsto \quad x'^\mu\left(x^\nu\right) \forall \mu, \nu = 0, 1, 2, 3 \ .$$

  Examples are rotations, translations and, more generally, *Lorentz- and Poincaré transformations* defining special relativity as well as *general coordinate transformations* that define *general relativity*.

- *Internal symmetries:* These are symmetries that correspond to transformations of the different fields in a field theory,

$$\Phi^a(x) \mapsto M^a{}_b \Phi^b(x) \ .$$

Roman indices a, b label the corresponding fields. If $M^a{}_b$ is constant then the symmetry is a *global symmetry*; in case of space-time dependent $M^a{}_b(x)$ the symmetry is called a *local symmetry*.

Importance of symmetries

Symmetry is important for various reasons:

- *Labelling and classifying particles:* Symmetries label and classify particles according to the different conserved quantum numbers identified by the space-time and internal symmetries (mass, spin, charge, colour, etc.). In this regard symmetries actually "define" an elementary particle according to the behaviour of the corresponding field with respect to the different symmetries.
- Symmetries determine the *interactions* among particles, by means of the *gauge principle*, for instance. It is

important that *most QFTs of vector bosons are sick: they are non-renormalisable*. The counter example to this is *gauge* theory, where vector bosons are *necessarily in the adjoint representation* of the gauge group. As an illustration, consider the Lagrangian

$$\mathcal{L} = \partial_\mu \phi \, \partial^\mu \phi^* \ - \ V(\phi, \phi^*)$$

which is invariant under rotation in the complex plane

$$\phi \mapsto \exp(i\alpha) \, \phi,$$

as long as α is a constant (global symmetry). If $\alpha =$

$\alpha(x)$, the kinetic term is no longer invariant:

$$\partial_\mu \phi \mapsto \exp(i\alpha)\left(\partial_\mu \phi + i(\partial_\mu \alpha)\phi\right).$$

However, the covariant derivative D_μ, defined as

$$D_\mu \phi = \partial_\mu \phi + iA_\mu \phi,$$

transforms like ϕ itself, if the gauge - potential A_μ transforms to $A_\mu - \partial_\mu \alpha$:

$$\begin{aligned} D_\mu \phi &\mapsto \exp(i\alpha)\left(\partial_\mu \phi + i(\partial_\mu \alpha)\phi + i(A_\mu - \partial_\mu \alpha)\,\phi\right) \\ &= \exp(i\alpha)\, D_\mu \phi, \end{aligned}$$

so we rewrite the Lagrangian to ensure gauge -

invariance:

$$\mathcal{L} = D_\mu \phi \, D^\mu \phi^* - V(\phi, \phi^*) \,.$$

The scalar field ϕ couples to the gauge - field A_μ via $A_\mu \phi A^\mu \phi$, similarly, the Dirac Lagrangian

$$\mathcal{L} = \overline{\Psi} \, \gamma^\mu \, D_\mu \Psi$$

has an interaction term $\overline{\Psi} A_\mu \Psi$. This interaction provides the three point vertex that describes interactions of electrons and photons and illustrate how photons mediate the electromagnetic interactions.

- Symmetries can hide or be *spontaneously broken:*

Consider the potential $V(\phi, \phi^*)$ in the scalar field Lagrangian above.

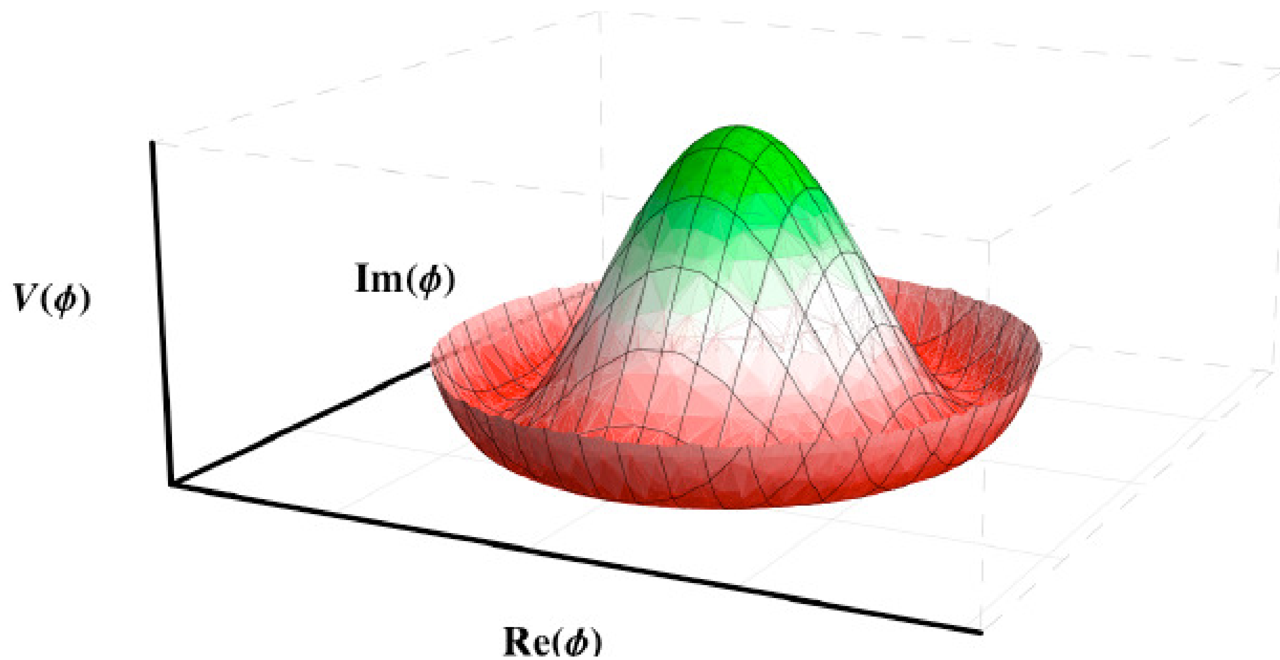


Figure 1: The Mexican hat potential for $V = \left(a - b\,|\phi|^2\right)^2$ with $a, b \geq 0$.

If $V(\phi, \phi^*) = V(|\phi|^2)$, then it is symmetric for $\phi \mapsto$

$\exp(i\alpha)\phi$. If the potential is of the type

$$V = a\left|\phi\right|^2 + \ b\left|\phi\right|^4 \forall a, b \geq 0,$$

then the minimum is at $\langle\phi\rangle = 0$ (here $\langle\phi\rangle \equiv \langle 0|\phi|0\rangle$ denotes the *vacuum expectation value (VEV)* of the field ϕ). The vacuum state is then also symmetric under the symmetry since the origin is invariant. However if the potential is of the form

$$V = \left(a - b\left|\phi\right|^2\right)^2 \forall a, b \geq 0,$$

the symmetry of V is lost in the ground state $\langle\phi\rangle \neq 0$. The existence of hidden symmetries is important for at least two reasons:

(i) This is a natural way to introduce an energy scale in the system, determined by the non vanishing VEV. In particular, we will see that for the standard model $M_{\rm ew} \approx 10^3$ GeV, defines the basic scale of mass for the particles of the standard model, the electroweak gauge bosons and the matter fields, through their Yukawa couplings, obtain their mass from this effect.

(ii) The existence of hidden symmetries implies that the fundamental symmetries of nature may be huge despite the fact that we observe a limited amount of symmetry. This is because the only manifest symmetries we can observe are the symmetries of the vacuum we live in and not those of the full underlying theory.

This opens-up an essentially unlimited resource to consider physical theories with an indefinite number of symmetries even though they are not explicitly realised in nature. The standard model is the typical example and supersymmetry and theories of extra dimensions are further examples.

The Standard Model

The Standard Model is well-defined and currently well confirmed by experiments.

- space-time symmetries: Poincaré in 4 dimensions
- gauged G_{SM}=SU(3)$_c$×SU(2)$_L$×U(1)$_Y$ symmetry, where SU(3)$_c$ defines the strong interactions. SU(2)$_L$×U(1)$_Y$ is spontaneously broken by the *Higgs* mechanism to U(1)$_{em}$. The gauge fields are spin-1 bosons, for example the photon A^μ, or gluons $G^{a=1,\ldots,8}$. Matter fields (quarks and leptons) have spin 1/2 and come in three 'families' (successively heavier copies). The Higgs

boson is the spin zero particle that spontaneously breaks the SU(2)$_L\times$U(1)$_Y$. The $W^\pm$ and Z particles get a mass via the Higgs mechanism and therefore the weak interactions are short range. This is also the source of masses for all quarks and leptons. The sub-index L in $SU(2)_L$ refers to the fact that the Standard Model does not preserve parity and differentiates between left-handed and right-handed particles. In the Standard Model only left-handed particles transform non-trivially under $SU(2)_L$. The gauge particles have all spin $s = 1\hbar$ and mediate each of the three forces: photons (γ) for $U(1)$ electromagnetism, gluons for $SU(3)_C$ of strong interactions, and the massive $W^\pm$ and Z for the weak interactions.

Problems of the Standard Model

The Standard Model is one of the cornerstones of all science and one of the great triumphs of the past century. It has been carefully experimentally verified in many ways, especially during the past 20 years. However, there are still some unresolved issues or mysteries:

- Quantum Gravity: The Standard Model describes three of the four fundamental interactions at the quantum level and therefore microscopically. However, gravity is only treated classically and any quantum discussion of gravity has to be considered as an effective field theory valid at scales smaller than the Planck scale ($M_{\rm pl} = \sqrt{\frac{Gh}{c^3}} \approx 10^{19}{\rm GeV}$). At this scale quantum effects of gravity have to be included and then Einstein theory has the problem of being non-renormalizable and therefore it cannot provide proper answers to observables beyond this scale.

- Why $G_{\rm SM} = SU(3) \otimes SU(2) \otimes U(1)$? Why there are four interactions and three families of fermions? Why $3+1$ spacetime - dimensions? Why there are some 20 parameters (masses and couplings between particles) in the Standard Model for which their values are only determined to fit experiment without any theoretical understanding of these values?

- The strong CP problem: There is a coupling in the Standard Model of the form $\theta F^{\mu\nu}\tilde{F}_{\mu\nu}$ where θ is a parameter, $F^{\mu\nu}$ refers to the field strength of quantum chromodynamics (QCD) and $\tilde{F}_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$. This term breaks the symmetry CP (charge conjugation followed by parity). The problem refers to the fact that the parameter θ is unnaturally small $\theta < 10^{-8}$. A parameter can be made naturally small by the T'HOOFT "naturalness criterion" in which a parameter is naturally small if setting it to zero implies there is a symmetry protecting its value. For this problem, there is a concrete proposal due to PECCEI and QUINN in which, adding a new particle, the *axion* a, with coupling $aF^{\mu\nu}\tilde{F}_{\mu\nu}$, then the corresponding Lagrangian will be symmetric under $a \to a + c$ which is the *PQ symmetry*. This solves the strong CP problem because non-perturbative QCD effects introduce a potential for a with minimum at $a = 0$ which would correspond to $\theta = 0$.

- Baryogenesis. Why didn't the anti-matter and matter all annihilate into photons? Andrei Sakharov postulated three necessary conditions required to produce more of one than the other in interactions:

1. Baryon number B violation

2. C violation: so that any interactions producing more baryons than anti-baryons won't be counterbalanced by interactions which produce more anti-baryons than baryons. CP violation because otherwise equal numbers of left-handed baryons and right-handed anti-baryons would be produced, as well as equal numbers of left-handed anti-baryons and right-handed baryons.
3. Interactions out of *thermal equilibrium*, otherwise CPT symmetry assures compensation between processes increasing and decreasing the baryon number.

The Standard Model does not produce enough B violation. Also, the Higgs phase transition must be a first-order phase transition, since otherwise sphalerons wipe away any baryon asymmetry happening up to the phase transition (and after that, the amount of baryon non-conservation is negligible).

- The hierarchy problem. The Higgs mass is $m_h \approx 126$ GeV, whereas the gravitational scale is $M_{Planck} \sim \sqrt{G} \sim 10^{19}$ GeV. The 'hierarchy problem' is: why is $m_h/M_{Planck} \sim 10^{-17}$ so much smaller than 1? In a fundamental theory, one might expect them to be the same order. In QFT, one sees that quantum corrections (loops) to m_h are expected to be of order of the heaviest scale in the theory divided by 4π. The question of why the hierarchy is stable with respect to the quantum corrections is called the *technical hierarchy problem*, and is arguably the main motivation for weak-scale supersymmetry.

- The cosmological constant (Λ) problem: probably the biggest problem in fundamental physics. Λ is the energy density of free space time. Why is $(\Lambda/M_{Planck})^4 \sim 10^{-120} \ll 1$?

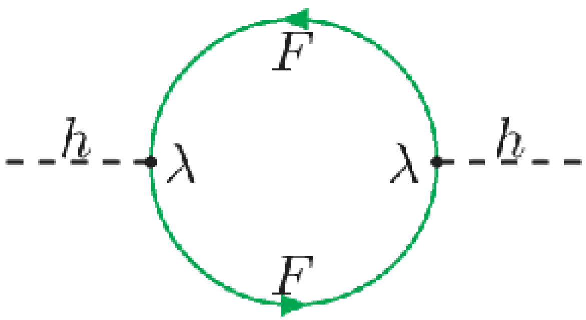
- The Standard Model has around 20 parameters, which must be measured then set 'by hand'. This is particularly obvious in the *fermion mass parameters*: there is a curious pattern, with a factor of a million in between the heaviest known fundamental charged fermion (the top quark) and the lightest (the neutrino). The problem only gets worse if you include the neutrinos.

- What particle constitutes the dark matter observed in the universe? It is not contained in the Standard Model.

We wish to find extensions that could solve some or all of the problems mentioned above in order to generalise the Standard Model. See the Part III Standard Model course for more details. Experiments are a traditional way of making progress in science. We need experiments to explore energies above the currently attainable scales and discover new particles and underlying principles that generalise the Standard Model. This approach is of course being followed at the LHC. The experiment will explore physics at the 10^3 GeV scale and new physics beyond the Standard Model. Notice that directly exploring energies closer to the Planck scale $M_{Planck} \approx 10^{19}$ GeV is out of the reach for many years to come.

A Problem With the Higgs Boson

The Higgs boson mass receives quantum corrections from heavy particles in the theory:



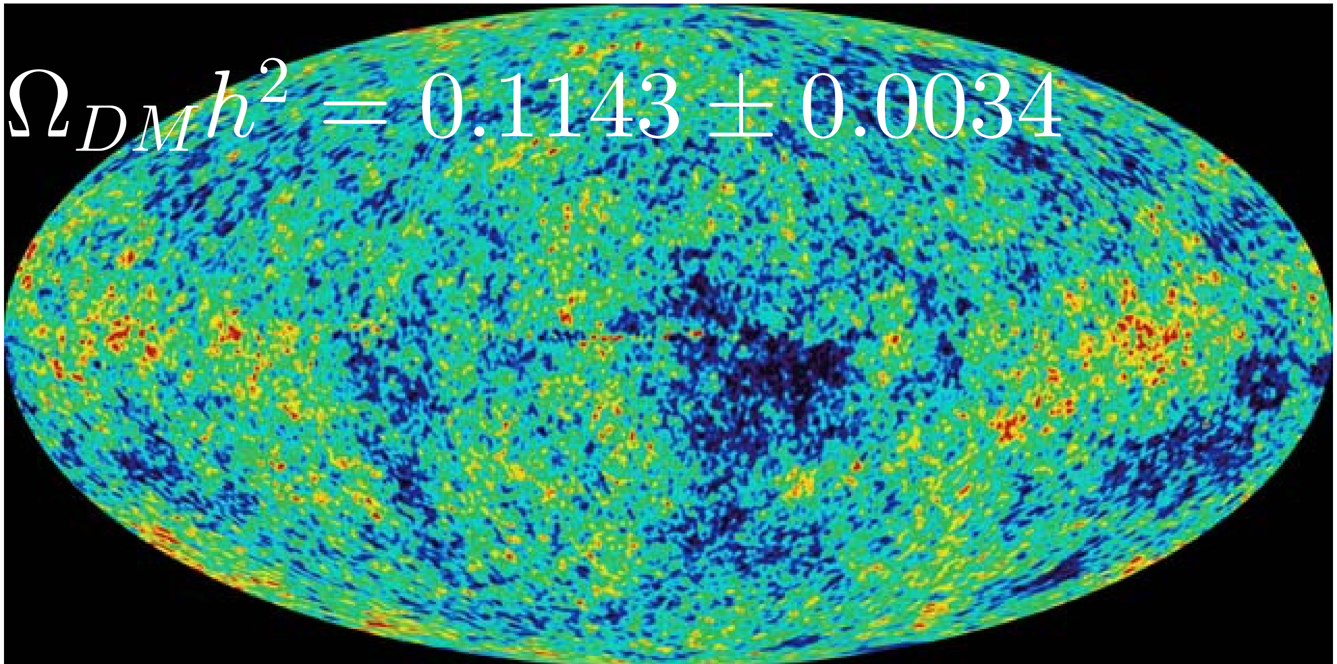
$$\sim -\frac{a\lambda^2}{16\pi^2}\int \frac{d^n k}{k^2 - m_F^2} + \ldots$$

Quantum correction to Higgs mass:

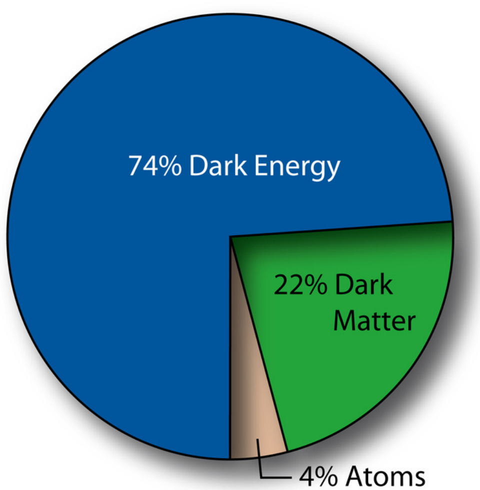
$$m_h^{phys^2} = (125 \text{ GeV}/c^2)^2 = m_h^{tree^2} + \mathcal{O}(m_F^2/(16\pi^2)).$$

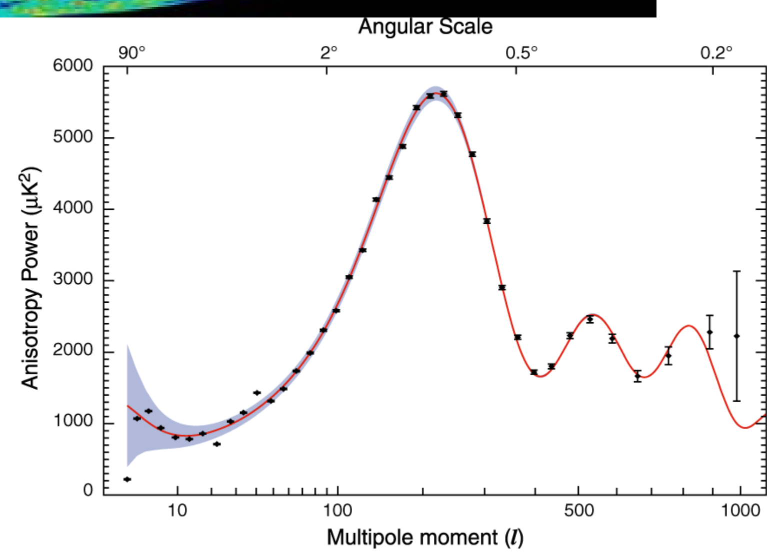
$m_F \sim 10^{19}$ GeV$/c^2$ is *heaviest mass scale* present.

Cosmological Fits



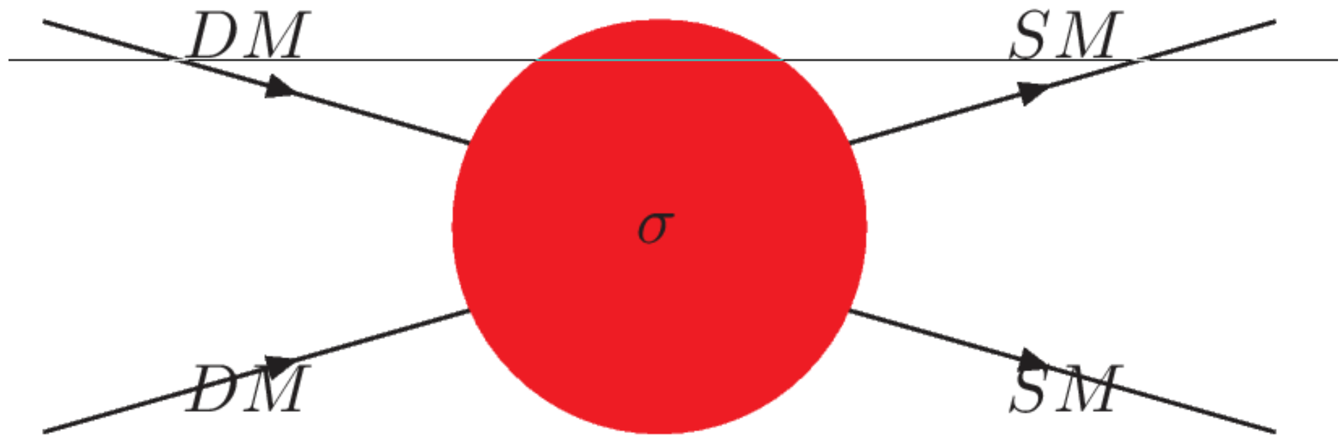
$$\Omega_{DM}h^2 = 0.1143 \pm 0.0034$$





Power law ΛCDM fit

Dark Matter



- For $t \rightarrow$, this describes annihilation - tells us how much is thermally produced. It also allows dark matter indirect detection.
- For $t \leftarrow$, it's production at (e.g.) the LHC
- For $t \uparrow$, it's direct detection

Modifications of the Standard Model

In order to go beyond the Standard Model we can follow several avenues, for example:

- Add new particles and/or interactions (e.g. a dark matter particle).

- More symmetries. For example,

(i) internal symmetries, for example *grand unified theories (GUTs)* in which the symmetries of the Standard Model are themselves the result of the breaking of a yet larger symmetry group.

$$G_{\rm GUT} \stackrel{M\approx 10^{16}{\rm GeV}}{\longrightarrow} G_{\rm SM} \stackrel{M\approx 10^{2}{\rm GeV}}{\longrightarrow} SU(3)_c \times U(1)_Y,$$

This proposal is very elegant because it unifies, in one single symmetry, the three gauge interactions of the Standard Model. It leaves unanswered most of the open questions above, except for the fact that it reduces the number of independent parameters due to the fact that there is only one gauge coupling at large energies. This is expected to "run" at low energies and give rise to the three different couplings of the Standard Model (one corresponding to each group factor). Unfortunately, with our present precision understanding of the gauge couplings and spectrum of the Standard Model, the running of the three gauge couplings does **not** unify at a single coupling at higher energies but they cross each other at different energies.

(ii) *Supersymmetry*. Supersymmetry is an external, or space-time symmetry. Supersymmetry solves the technical hierarchy problem due to cancellations between the contributions of bosons and fermions to the electroweak scale, defined by the Higgs mass. Combined with the GUT idea, it also solves the unification of the three gauge couplings at one single point at larger energies. Supersymmetry also provides the most studied example for dark matter candidates. Moreover, it provides well defined QFTs in which issues of strong coupling can be better studied than in the non-supersymmetric models.

(iii) Extra spatial dimensions. More general space-time symmetries open up many more interesting avenues. These can be of two types. First we can add more dimensions to space-time, therefore the Poincaré symmetries of the Standard Model and more generally the general coordinate transformations of general relativity, become substantially enhanced. This is the well known *Kaluza Klein theory* in which our observation of a 4 dimensional universe is only due to the fact that we have limitations about ”seeing” other dimensions of space-time that may be hidden to our experiments. In recent years this has been extended to the *brane world scenario* in which our 4 dimensional universe is only a brane or surface inside a higher dimensional universe. These ideas may lead to a different perspective of the hierarchy problem and also may help unify internal and space-time symmetries.

- Beyond QFT: A QFT with Supersymmetry and extra dimensions does not address the problem of quantising gravity. For this purpose, the current best hope is string theory which goes beyond our basic framework of QFT. It so happens that for its consistency, string theory requires supersymmetry and extra dimensions also. This gives a further motivation to study supersymmetry.

Grand Unified Theories

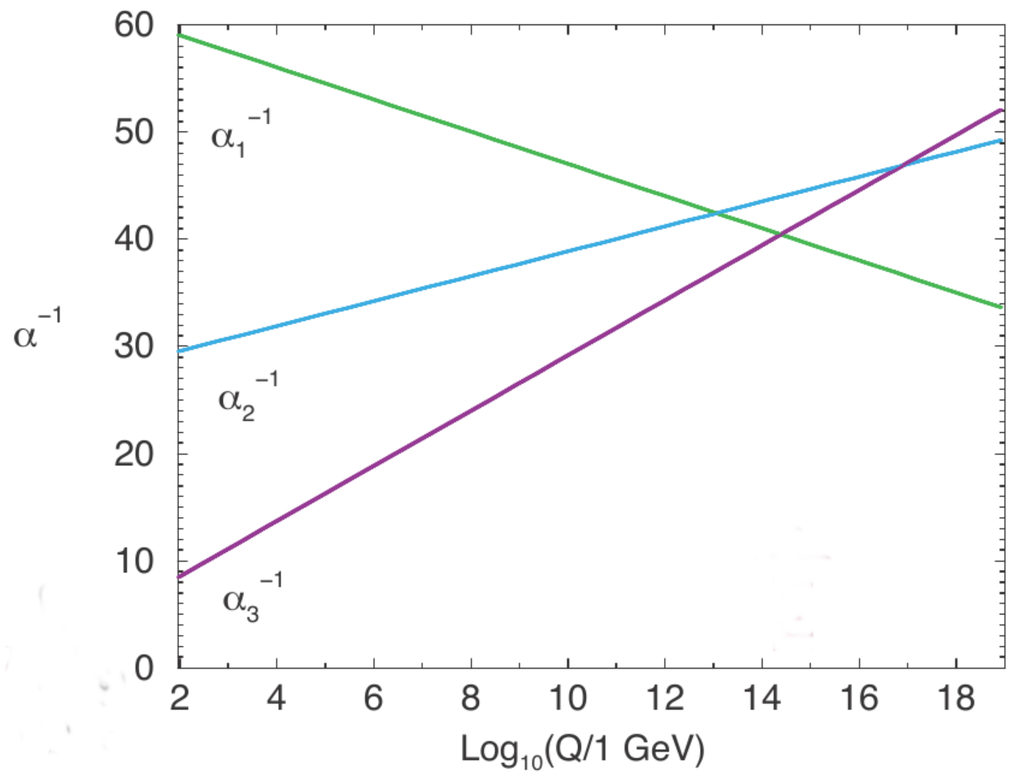
These make a simpler group out of G_{SM}, for example $SU(5)$:

$$\underline{5} = \begin{pmatrix} d \\ d \\ d \\ e^+ \\ \bar{\nu}_e \end{pmatrix}_R , \qquad \underline{10} = \begin{pmatrix} 0 & \bar{u} & -\bar{u} & -u & -d \\ & 0 & \bar{u} & -u & d \\ & & 0 & -u & d \\ & & & 0 & e^+ \\ & & & & 0 \end{pmatrix}_L \quad (1)$$

There are some problems with Grand Unified Theories, though.

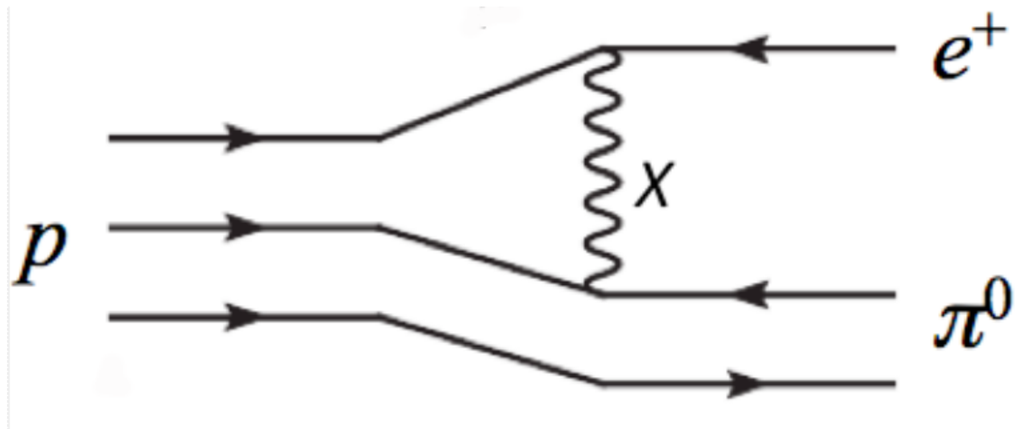
Problems with GUTs

Gauge unification doesn't immediately work:



Because leptons and quarks are unified in the same multiplet, they predict e.g. $m_e(M_{GUT}) = m_d(M_{GUT})$, which also doesn't work.

They predict *proton decay*, which isn't observed at super-Kamiokande: $\tau_{p\rightarrow e^+\pi^0} > 10^{34}$ years.



$$\tau \approx \frac{M_X^4}{\alpha^2 m_p^5} = 4.5 \times 10^{29\pm1.7} \text{ years}$$

Prediction of the strong coupling

$\alpha_{1,2}(M_Z)$ are known with *good* accuracy, so use them to predict $\alpha_S(M_Z)$.

$\mathcal{Q}$: What's the prediction of $\alpha_S(M_Z)$in the MSSM?

$\mathcal{A}$: 0.129 ± 0.002

The experimental number is: 0.119 ± 0.002, so the naive prediction is 5σ out!

GUT threshold corrections could explain the difference (just).

Coleman-Mandula Theorem

$\mathcal{Q}$: What kind of symmetries can you impose on a field theory and still have non-zero scattering?

Coleman and Mandula say there are only 2 classes of conserved quantities:

External	Internal
(Poincaré symmetry)	(Gauge symmetries)
Energy-momentum p^μ	Electric charge e
Angular momentum $M^{\mu\nu}$	Colour charge

However, there's a loop hole

Loop Hole: Fermionic Generators

Standard model gauge symmetry is *internal*, but supersymmetry is a space-time symmetry. We call extra supersymmetry generators $Q, \bar{Q}$.

$$
\begin{aligned}
Q|\text{fermion}\rangle &\rightarrow |\text{boson}\rangle \\
Q|\text{boson}\rangle &\rightarrow |\text{fermion}\rangle
\end{aligned}
$$

In the simplest form of supersymmetry, we have multiplets

$$\begin{pmatrix} \text{spin } 0\hbar \\ \text{spin } 1/2\hbar \end{pmatrix}, \qquad \begin{pmatrix} \text{spin } 1/2\hbar \\ \text{spin } 1\hbar \end{pmatrix},$$

where each spin component in the multiplet should have identical properties (except for spin).

Motivation

- It's perturbative, ie calculable
- It's a new kind of symmetry: Coleman-Mandula Theorem
- Unification
  - Superstrings
  - Grand Unified Theories
- Why visible at low energy? Solution to hierarchy problem

Masslessness

Let us now think some more about the technical hierarchy problem. In the Standard Model we know that:

- Vector bosons are massless due to gauge invariance, that means, a direct mass term for the gauge particles $M^2 A_\mu A^\mu$ is not allowed by gauge invariance ($A_\mu \to A_\mu + \partial_\mu \alpha$ for a $U(1)$ field, for example).
- Chiral fermion masses $m\psi\psi$ are also forbidden for all quarks and leptons by gauge invariance. $\mathcal{Q}$: Which symmetry bans say $m e_R e_R$? Recall that these particles receive a mass only through

the Yukawa couplings to the Higgs (e.g. $H\bar{\psi}_L\psi_R$ giving a Dirac mass to ψ after H gets a non-zero value[1]).

- The Higgs is the only scalar particle in the Standard Model. There is no symmetry banning its mass term $m_H^2 H^\dagger H$ in the Standard Model Lagrangian.

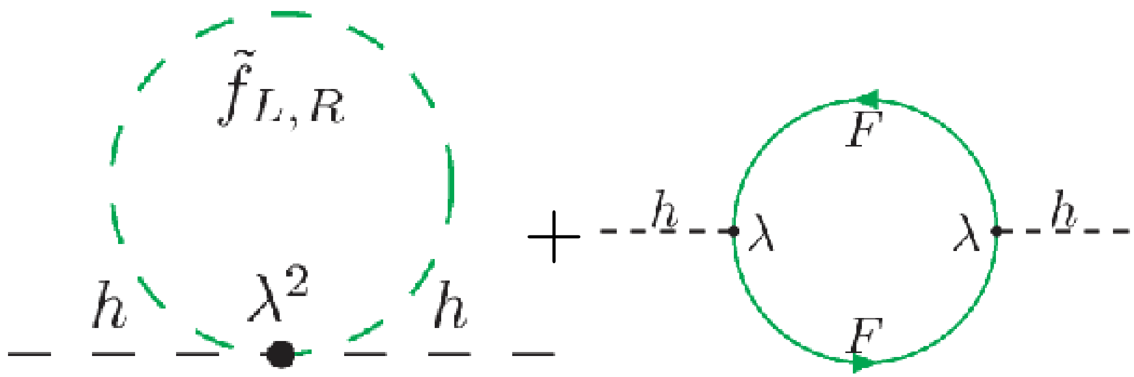
[1]Notice that with $R-$parity, the MSSM does not give neutrinos mass. Thus one must augment the model in some way.

Supersymmetric Solution

Exact supersymmetry adds 2 spin $0\hbar$ particles $\tilde{f}_{L,R}$ for every massive spin $1/2\hbar$ particle with

$$m^2_{\tilde{f}_{L,R}} = m^2_F$$

and they couple to h with the same strength:



$$\sim \mathcal{O}\left(\frac{m_h^2 \log(M_Z/m_F)}{16\pi^2}\right)$$

$\mathcal{Q}$: Where are the spin $0\hbar$ copies of electrons?

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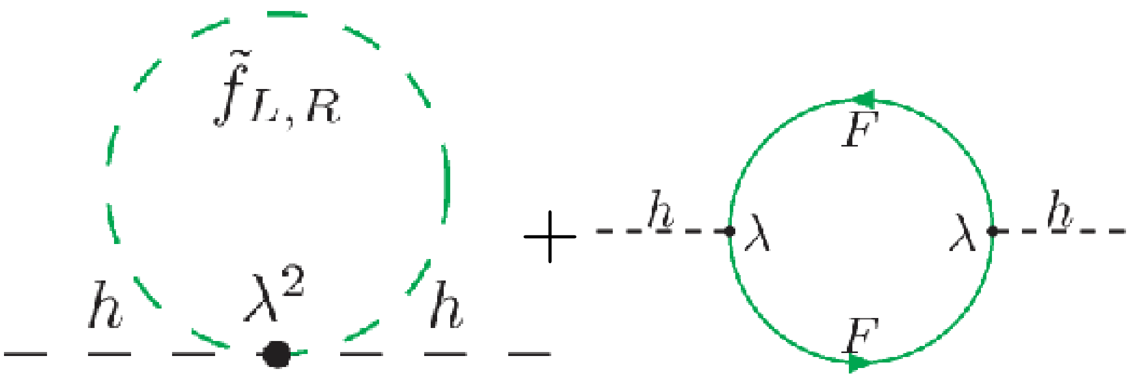


$$\sim \mathcal{O}\left(\frac{m_h^2 \log(M_Z/m_F)}{16\pi^2}\right)$$

$\mathcal{Q}$: Where are the spin $0\hbar$ copies of electrons?

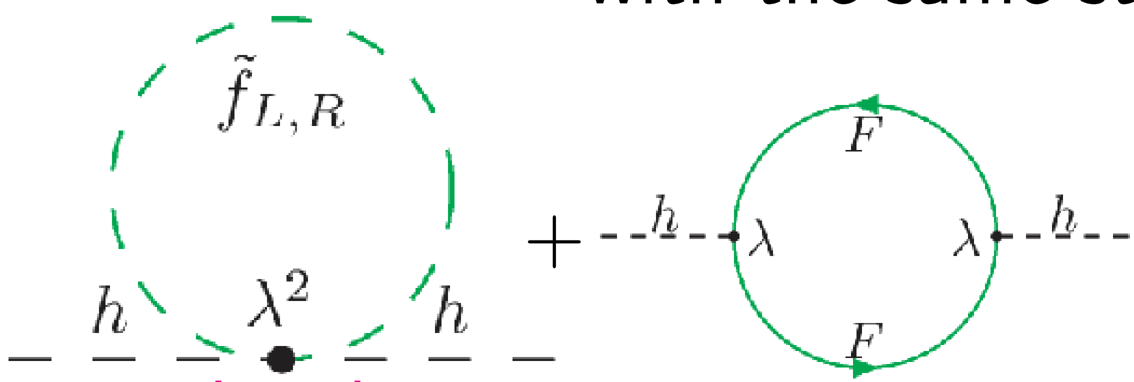
$\mathcal{A}$: supersymmetry must be softly broken.

SUSY Broken Solution

Exact supersymmetry adds 2 spin $0\hbar$ particles $\tilde{f}_{L,R}$ for every massive spin $1/2\hbar$ particle with

$$m^2_{\tilde{f}_{L,R}} = m^2_F + \Delta m^2$$

and they couple to h with the same strength:



$$\sim \mathcal{O}\left(\frac{\Delta m^2}{16\pi^2}\right)$$

When we break supersymmetry, we must make sure that we don't reintroduce the Higgs mass problem: "soft breaking".

SUSY Broken Solution

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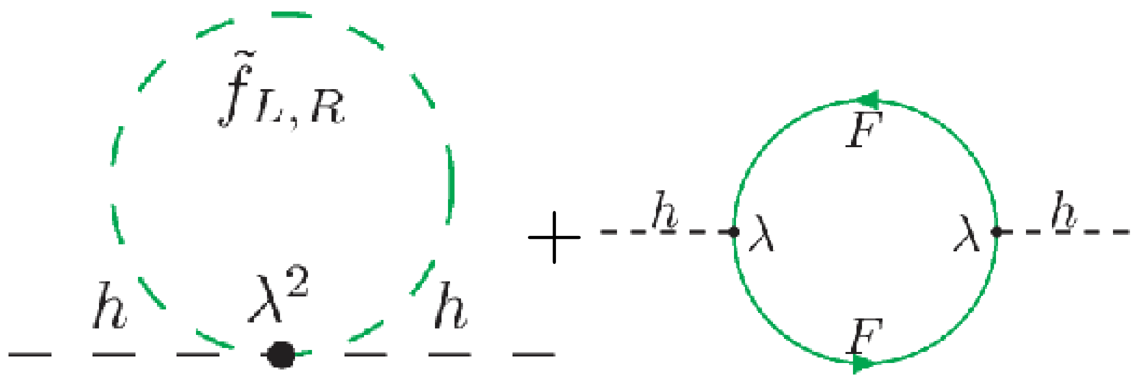
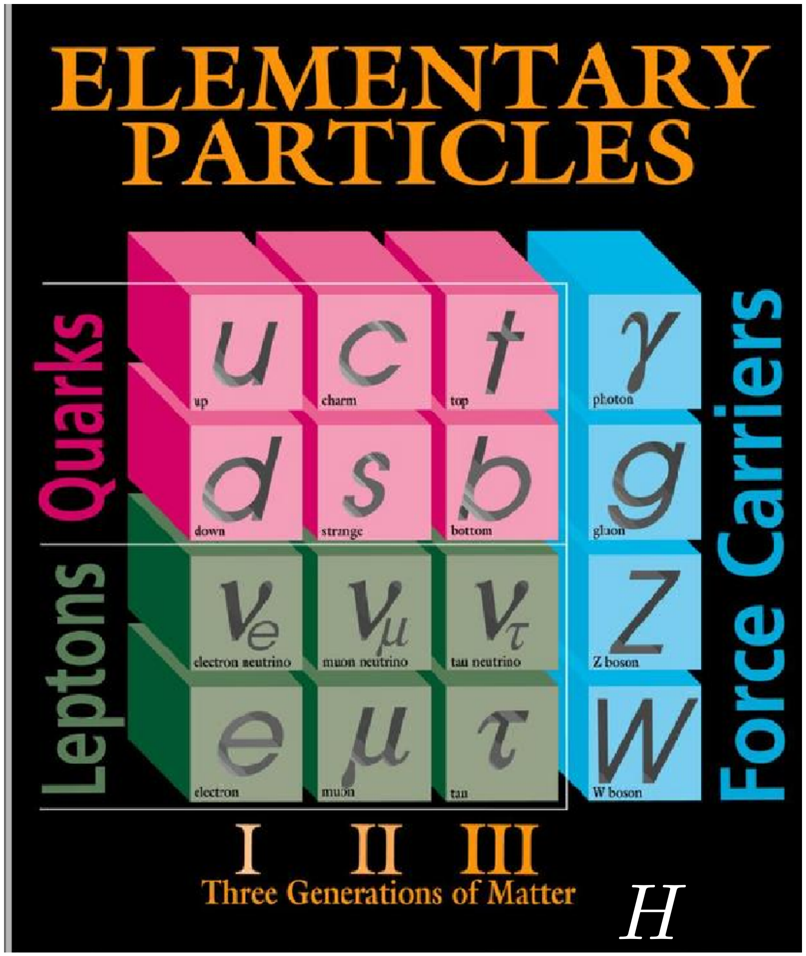


$\sim \mathcal{O}\left(\frac{\Delta m^2}{16\pi^2}\right)$

This only works if the supersymmetric particles are not too heavy: the LHC should see them

Supersymmetric Copies



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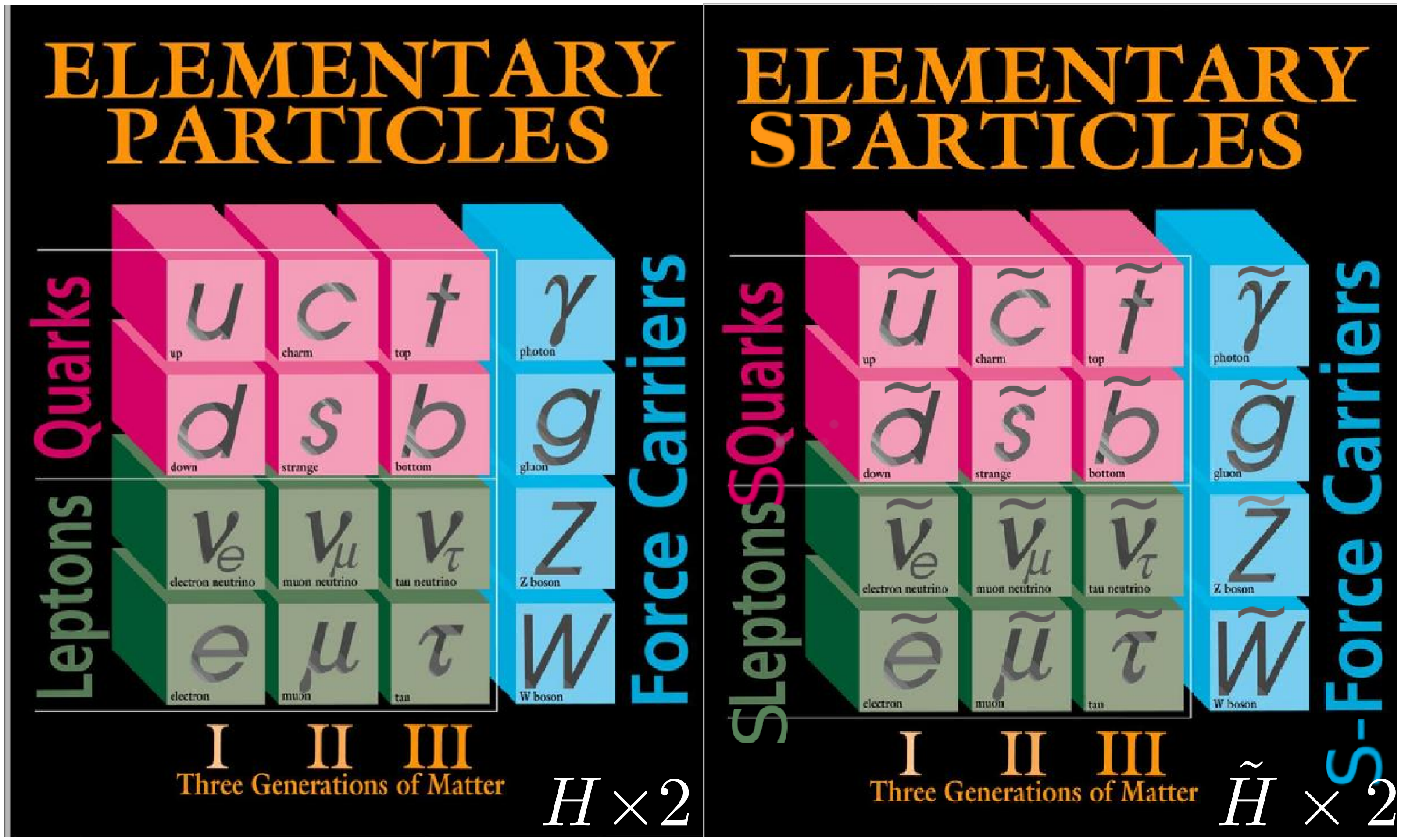


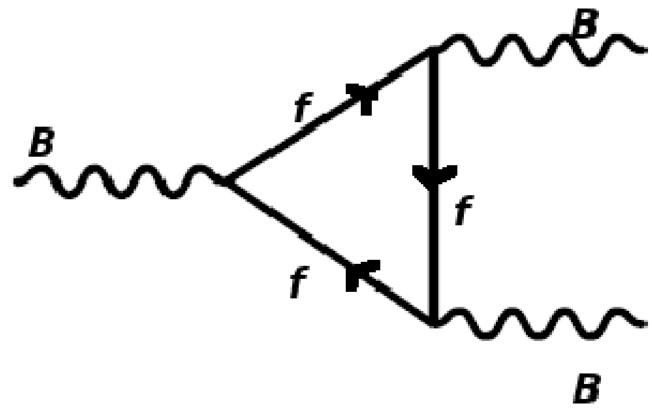
Two Higgs Doublet Model

$$(SU(3), SU(2), U(1)_Y),\ H_2 = \left(1, 2, \frac{1}{2}\right),\ H_1 = \left(1, 2, -\frac{1}{2}\right)$$

the second of which is a new Higgs doublet not present in the Standard Model. Thus, the MSSM is a *two Higgs doublet model*. The extra Higgs doublet is needed in order to avoid a gauge anomaly, and to give masses to down-type quarks and leptons.

Note that after the breaking of electroweak symmetry, the electric charge generator is $Q = T_3^{SU(2)_L} + Y/2$.

Chiral fermions may generate an *anomaly* in the theory



This is where a symmetry that is present in the tree-level Lagrangian is broken by quantum corrections. Here, the symmetry is $U(1)_Y$: all chiral fermions in the theory travel in the loop, and yield a logarithmic divergence proportional to

$$A = \sum_{LH\ f_i} Y_i^3 - \sum_{RH\ f_i} Y_i^3$$

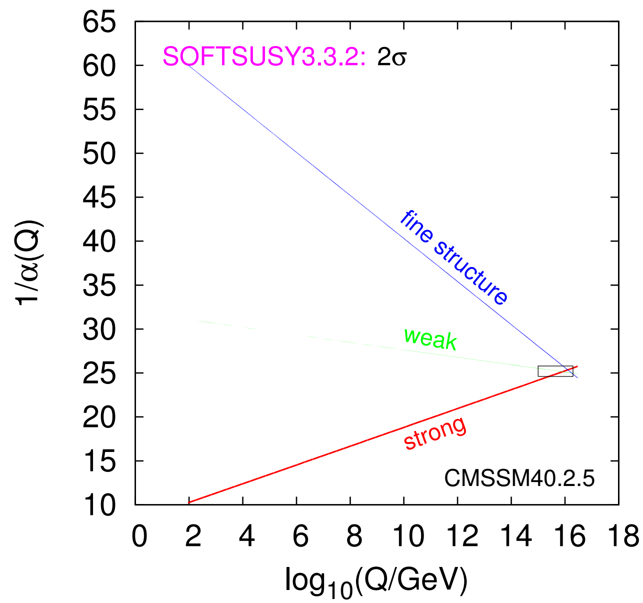
multiplied by some kinematic factor which is the same for each fermion. If $A \neq 0$, one must renormalise the diagram away by adding a $B_\mu B_\nu B_\rho$ counter term in the Lagrangian. But this breaks $U(1)_Y$, meaning that $U(1)_Y$ would not be a consistent symmetry at the quantum level. Fortunately, $A = 0$ for each fermion family in the Standard Model.
$\mathcal{Q}$: Can you show that $A = 0$ in a Standard Model family?

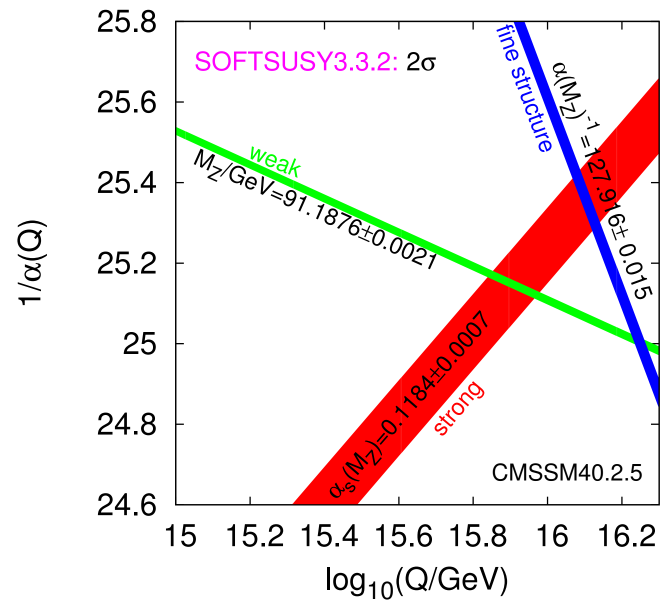
In SUSY, we add the Higgsino doublet $\tilde{H}_1$, which yields a non-zero contribution to A. This must be cancelled by another Higgsino doublet with opposite Y: $\tilde{H}_2$.

Unification

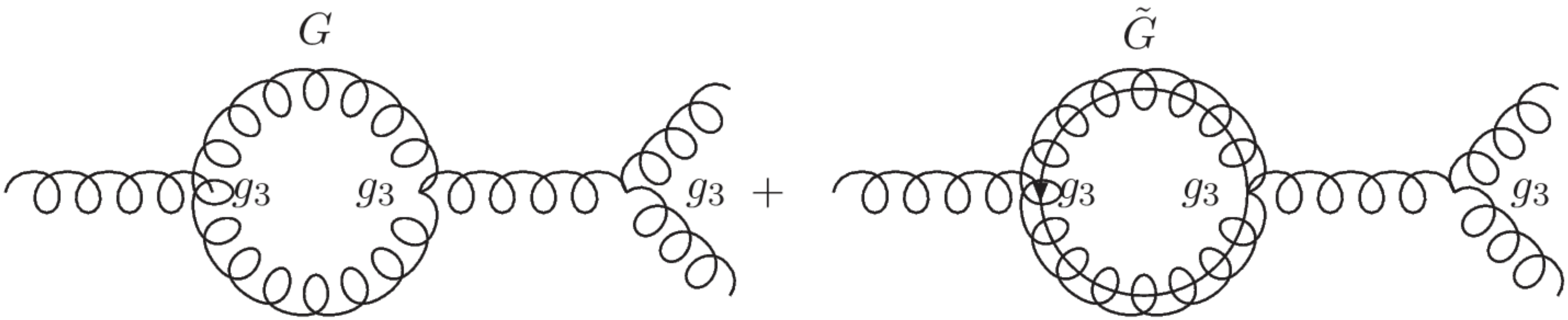
- Superstrings tend to lead to effective theories which are supersymmetric.

- Grand Unified Theories predict that the gauge couplings should unify at some energy scale $M_{GUT} \approx 10^{16}$ GeV.





Gauge Unification in the Minimal Supersymmetric Standard Model



Example Feynman diagrams leading to renormalisation.

Gauge couplings are renormalised, which ends up giving them *renormalisation scale dependence*, which matches onto dependence upon the energy scale at which one is probing them:

$$\mu \frac{dg_a(\mu)}{d\mu} = \beta_a g_a^3(\mu), \Rightarrow g_a^{-2}(\mu) = g_a^{-2}(\mu_0) - 2\beta_a \ln \frac{\mu}{\mu_0} \quad (2)$$

where β_a is a constant determined by which particles travel in the loop in the theory. For ordinary QCD it is $\beta_3 = -7/(16\pi^2)$ whereas for the MSSM, it is $\beta_3 = -3/(16\pi^2)$ because of additional contributions from squarks and gluinos to the loops.

SUSY Breaking

Parameterisation of soft breaking terms:

- Scalar masses e.g. $V = m^2_{\tilde{t}_L} |\tilde{t}_L|^2$
- Gaugino masses e.g. $V = m_{\tilde{g}} \bar{\tilde{g}} \tilde{g}$
- Bi-linear scalar mixing e.g. $V = \mu B H_1 H_2$
- Trilinear scalar interactions e.g. $V = A_t h_t \tilde{t}_L H_2 \tilde{t}_R$

General prescription: add all possible terms consistent with symmetries. If the massive parameters are $\stackrel{<}{\sim} O(1)$ TeV, the model has no hierarchy problem.

(Haber) There are $\sim$100 parameters here.

Soft breaking

Make all spin $0\hbar$ partners heavier than ordinary matter:

$$m^2_{\tilde{f}_{L,R}} = m_F^2 + \delta^2$$

Then we find a quantum correction to m_h of (Drees)

$$\Delta m_h^2 \sim \frac{\lambda^2}{16\pi^2}\left(4\delta^2 + 2\delta^2 \ln \frac{m_F^2}{\mu^2}\right) + O(\delta^4).$$

So, if $\delta \lesssim 1000$ GeV$/c^2$, there's no fine tuning in m_h. *We should see supersymmetric particles in the Large Hadron Collider.*

Universality

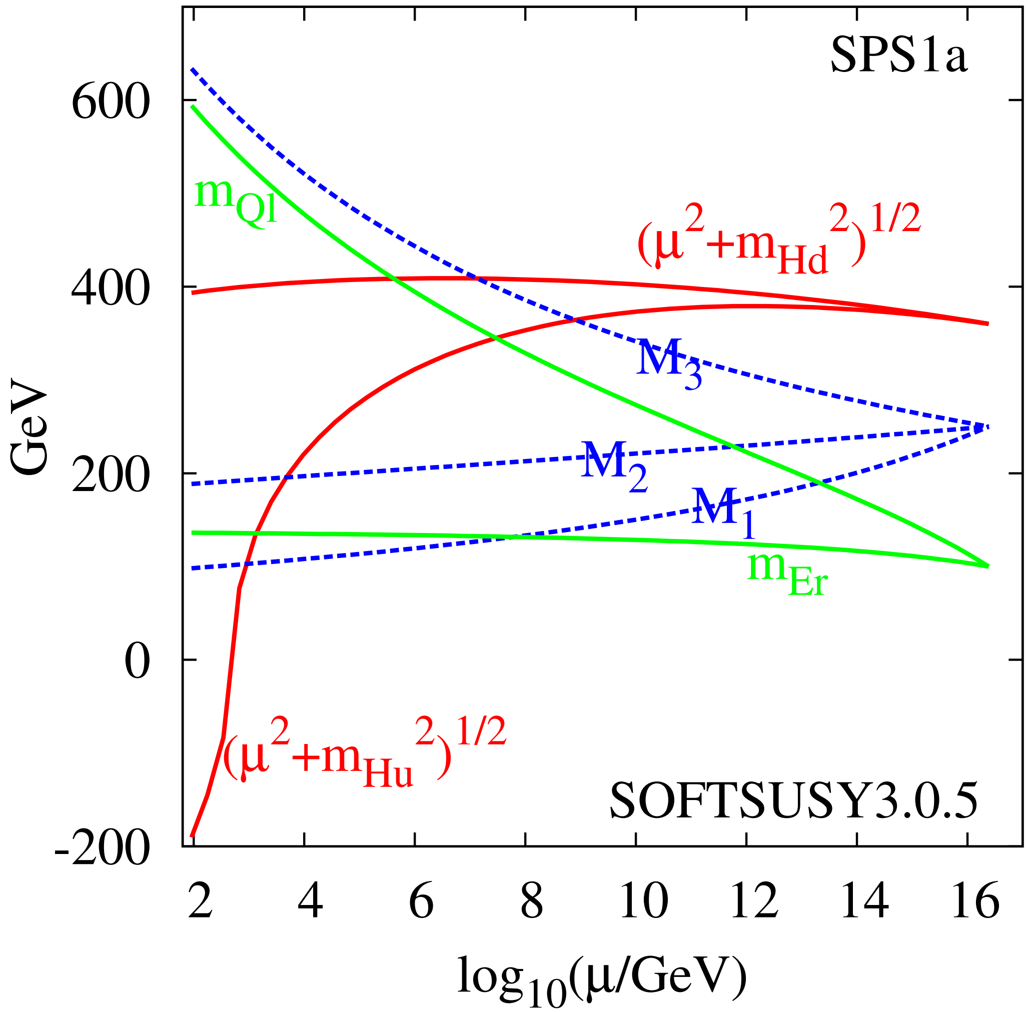
Reduces number of SUSY breaking parameters to 3:

- $\tan\beta \equiv v_2/v_1$
- m_0, the common scalar mass (flavour).
- $M_{1/2}$, the common gaugino mass (GUT/string).
- A_0, the common trilinear coupling (flavour).

These conditions should be imposed at $M_X \sim O(10^{16-18})$ GeV and receive radiative corrections

$$\propto \; 1/(16\pi^2)\ln(M_X/M_Z).$$

Also, Higgs potential parameter sgn(μ)=$\pm$1.



MSSM evolution of soft terms

Supersymmetry Breaking Models

- Give masses $\sim O(M_Z) - 1$ TeV$/c^2$ to the supersymmetric particles.
- Reduce 100 supersymmetry breaking parameters.

The models have 3 sectors:

- Observable Contains standard model fields and interactions. No direct supersymmetry breaking.
- Hidden Separate fields and interactions. Direct supersymmetry breaking.

- Messenger Supersymmetry breaking is transferred to observable sector through gravitation, quantum effects or new forces.

MSSM Neutral h Potential

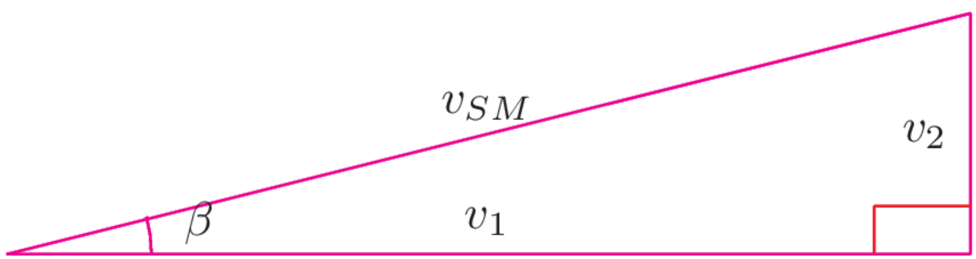
$$
\begin{aligned}
V &= (|\mu|^2 + m_{H_2}^2)|H_2^0|^2 + (|\mu|^2 + m_{H_1}^2)|H_1^0|^2) \\
&\quad -\mu B(H_2^0 H_1^0 + c.c.) \\
&\quad +\frac{1}{8}(g^2 + g'^2)(|H_2^0|^2 - |H_1^0|^2)^2, \\
&\quad \frac{\partial V}{\partial H_2^0} = \frac{\partial V}{\partial H_1^0} = 0 \\
\Rightarrow \mu B &= \frac{\sin 2\beta}{2}(\bar{m}_{H_1}^2 + \bar{m}_{H_2}^2 + 2\mu^2), \\
\mu^2 &= \frac{\bar{m}_{H_1}^2 - \bar{m}_{H_2}^2 \tan^2\beta}{\tan^2\beta - 1} - \frac{M_Z^2}{2}.
\end{aligned}
$$

Electroweak Breaking

Both Higgs get vacuum expectation values:

$$\begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} \to \begin{pmatrix} v_1 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} \to \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

and to get M_W correct, match with $v_{SM} = 246$ GeV:



$$\tan\beta = \frac{v_2}{v_1}$$

$$\mathcal{L} \;=\; h_t \bar{t}_L H_2^0 t_R + h_b \bar{b}_L H_1^0 b_R + h_\tau \bar{\tau}_L H_1^0 \tau_R$$

$$\Rightarrow \frac{m_t}{\sin\beta} = \frac{h_t v_{SM}}{\sqrt{2}}, \qquad \frac{m_{b,\tau}}{\cos\beta} = \frac{h_{b,\tau} v_{SM}}{\sqrt{2}}.$$

Broken Symmetry

3 components of the Higgs particles are eaten by $W^\pm, Z^0$, leaving us with 5 physical states:

$$h^0, H^0\text{(CP+)}, \qquad A^0\text{(CP-)}, \qquad H^\pm$$

SUSY breaking and electroweak breaking imply particles with identical quantum numbers mix:

$$\begin{aligned}
(\tilde{B}, \tilde{W}_3, \tilde{H}_1^0, \tilde{H}_2^0) &\rightarrow \chi^0_{1,2,3,4} \\
(\tilde{t}_L, \tilde{t}_R) &\rightarrow \tilde{t}_{1,2} \\
(\tilde{b}_L, \tilde{b}_R) &\rightarrow \tilde{b}_{1,2}
\end{aligned}$$

$$
\begin{aligned}
(\tilde{\tau}_L, \tilde{\tau}_R) &\rightarrow \tilde{\tau}_{1,2} \\
(\tilde{W}^{\pm}, \tilde{H}^{\pm}) &\rightarrow \chi^{\pm}_{1,2}
\end{aligned}
$$