Standard Model of Particle Physics

Gauge Theory based on the group:

$$SU(3) \times SU(2) \times U(1)$$

$SU(3) \Rightarrow$ Quantum Chromodynamics

Strong Force (Quarks and Gluons)

$SU_L(2) \times U(1) \Rightarrow$ ElectroWeak Interactions broken to $U_{EM}(1)$

by HIGGS

$SU_L(2) \times U_Y(1) \Rightarrow U_{EM}(1)$

Force Carriers: $W^{\pm}$, Z^0 and γ masses: 80, 91 and 0 GeV

quark, SU(2) doublets: $\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L$

up-quark, SU(2) singlets: u_R, c_R, t_R

down-quark, SU(2) singlets: d_R, s_R, b_R

lepton, SU(2) doublets: $\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$

neutrino, SU(2) singlets: $- - -$

charge lepton, SU(2) singlets: e_R, μ_R, τ_R

Electron mass

comes from a term of the form

$\bar{L}\phi e_R$

Absence of ν_R

forbids such a mass term (dim 4)

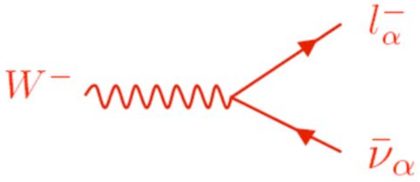
for the Neutrino

Therefore in the SM neutrinos are massless

and hence travel at speed of light.

Interactions:

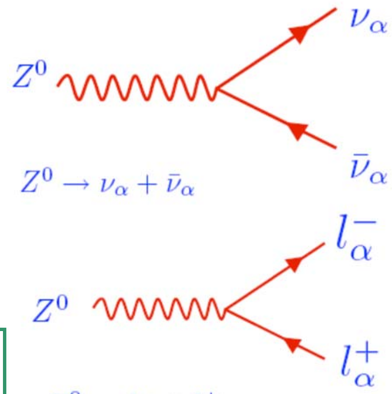
Charge Current (CC)



$W^- \rightarrow l_\alpha^- + \bar{\nu}_\alpha$

$$\Gamma(Z^0 \rightarrow f + \bar{f}) = K\frac{g_Z^2 M_Z}{48\pi}[|c_V^f|^2 + |c_A^f|^2]$$

Neutral Current (NC)



$Z^0 \rightarrow \nu_\alpha + \bar{\nu}_\alpha$

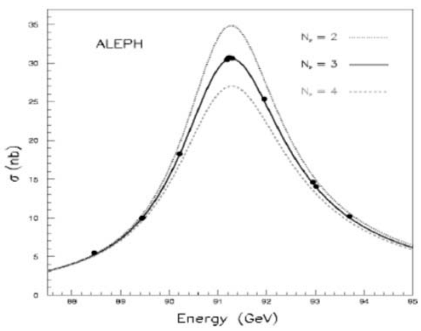
$Z^0 \rightarrow l_\alpha^- + l_\alpha^+$

$\alpha = e, \mu, \ or \ \tau$

Invisible width of Z plus other data from LEP:

$Z^0 \rightarrow \nu\bar{\nu}$

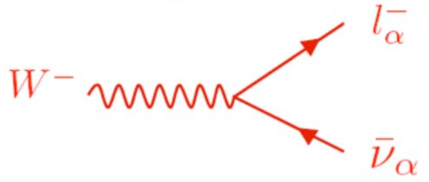
Implies $N_\nu = 2.99 \pm 0.01$



Three Active Neutrinos!!!

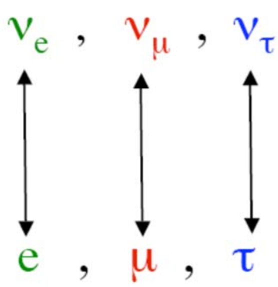
Sterile Neutrinos don't couple to Z^0

Note That

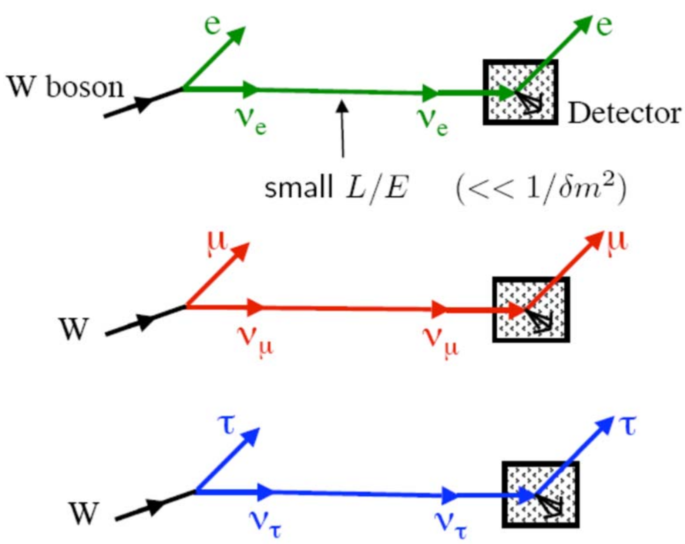


$W^- \to l_\alpha^- + \bar{\nu}_\alpha$

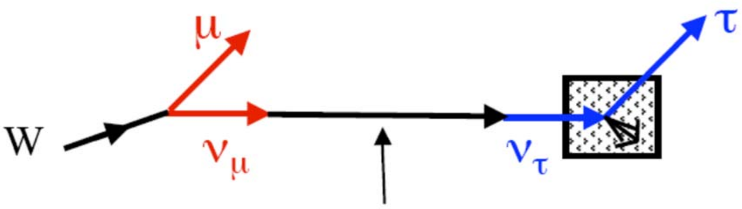
Implies



Observed

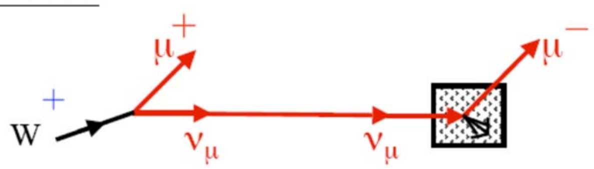


Not Observed



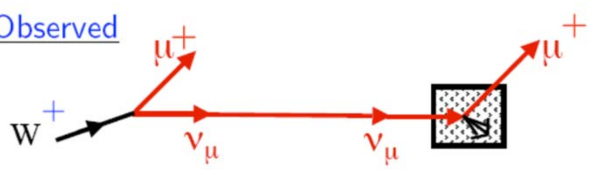
small L/E $\quad (<< 1/\delta m^2)$

Observed



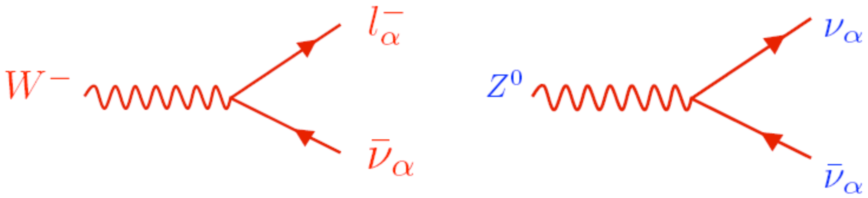
neutrino beam (not anti-neutrino beam)

Not Observed



large E $(>> m_\nu)$

Standard Model

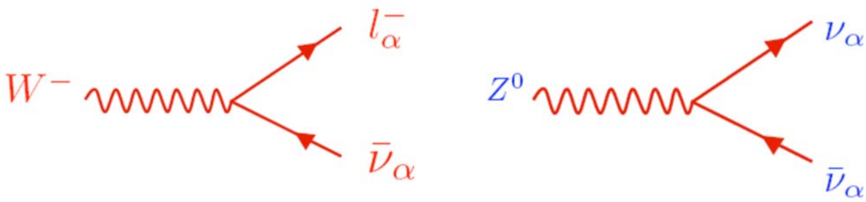


couplings conserve the Lepton Number L defined by—

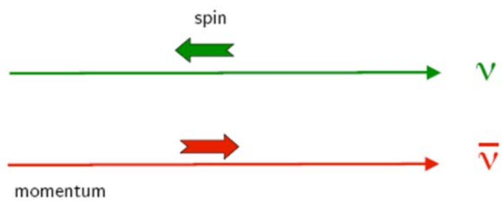
$$L(\nu) = L(\ell^-) = -L(\bar{\nu}) = -L(\ell^+) = 1.$$

Actually L_e, L_μ, and L_τ separately

Left Handed Nature of The Neutrino

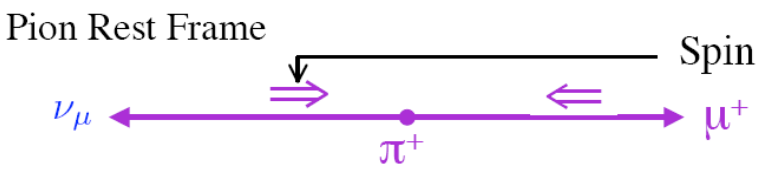


Produce Left-Handed Neutrinos
and Right-Handed Anti-Neutrinos



What about the RH neutrinos and LH anti-neutrino ????

π^+ decay



Left Handed anti-fermion

Suppressed by powers of m_f

Why $\pi^+ \to \mu^+ \nu_\mu$ >99% and $\pi^+ \to e^+ \nu_e$ is 0.01%.

There exist three fundamental and discrete transformations in nature:

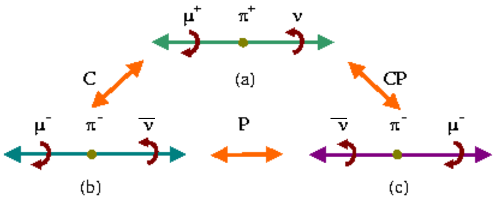
• **Parity**	$\mathcal{P}$	$\vec{x} \to -\vec{x}$
• **Time reversal**	$\mathcal{T}$	$t \to -t$
• **Charge conjugation**	$\mathcal{C}$	$q \to -q$

$\mathcal{P}$, $\mathcal{T}$ and $\mathcal{C}$ are conserved in the classical theories of mechanics and electrodynamics!

$\mathcal{CPT} \leftrightarrow$ Lorentz invariance $\oplus$ unitarity: is an essential building block of field theory

$\mathcal{CPT}$: L particle $\leftrightarrow$ R antiparticle

Neutrinos in the MSM are massless and exist only in two states: particle with negative helicity and antiparticle with positive one: Weyl fermion



$\mathcal{P}$: L particle $\leftrightarrow$ R particle

Parity violation is nowhere more obvious than in the neutrino sector: the reflection of a left-handed neutrino in a mirror is nothing !

Summary of ν's in SM:

Three flavors of massless neutrinos

$$W^- \to l_\alpha^- + \bar{\nu}_\alpha$$

$$W^+ \to l_\alpha^+ + \nu_\alpha$$

$$\alpha = e, \mu, \; or \; \tau$$

Anti-neutrino, $\bar{\nu}_\alpha$, has +ve helicity, Right Handed

Neutrino, ν_α, has -ve helicity, Left Handed

ν_L and $\bar{\nu}_R$ are CPT conjugates

massless implies helicity = chirality

Beyond the SM

What if Neutrino have a MASS?

speed is less than c therefore time can pass

and

Neutrinos can change character!!!

What are the stationary states?

How are they related to the interaction states?

NEUTRINO OSCILLATIONS:

Two Flavors

flavor eigenstates $\neq$ mass eigenestates

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

W's produce ν_μ and/or ν_τ's

but ν_1 and ν_2 are the states

that change by a phase over time, mass eigenstates.

$$|\nu_j\rangle \rightarrow e^{-ip_j \cdot x}|\nu_j\rangle \qquad p_j^2 = m_j^2$$

$\alpha, \beta \ldots$ flavor index $\qquad i, j \ldots$ mass index

Production:

$$|\nu_\mu\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$$

Propogation:

$$\cos\theta e^{-ip_1\cdot x}|\nu_1\rangle + \sin\theta e^{-ip_2\cdot x}|\nu_2\rangle$$

Detection:

$$|\nu_1\rangle = \cos\theta|\nu_\mu\rangle - \sin\theta|\nu_\tau\rangle$$

$$|\nu_2\rangle = \sin\theta|\nu_\mu\rangle + \cos\theta|\nu_\tau\rangle$$

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$$P(\nu_\mu \to \nu_\tau) = |\cos\theta(e^{-ip_1\cdot x})(-\sin\theta) + \sin\theta(e^{-ip_2\cdot x})\cos\theta|^2$$

$$P(\nu_\mu \to \nu_\tau) = |\cos\theta(e^{-ip_1\cdot x})(-\sin\theta) + \sin\theta(e^{-ip_2\cdot x})\cos\theta|^2$$

Same E, therefore $p_j = \sqrt{E^2 - m_j^2} \approx E - \frac{m_j^2}{2E}$

$$e^{-ip_j\cdot x} = e^{-iEt}e^{-ip_jL} \approx e^{-i(Et-EL)} \quad e^{-im_j^2 L/2E}$$

$$P(\nu_\mu \to \nu_\tau) = \sin^2\theta\cos^2\theta|e^{-im_2^2L/2E} - e^{-im_1^2L/2E}|^2$$

$$P(\nu_\mu \to \nu_\tau) = \sin^2 2\theta \sin^2\frac{\delta m^2 L}{4E}$$

$\delta m^2 = m_2^2 - m_1^2$ and $\frac{\delta m^2 L}{4E} \equiv \Delta$ kinematic phase:

$$P(\nu_\mu \to \nu_\tau) = |\cos\theta(e^{-ip_1\cdot x})(-\sin\theta) + \sin\theta(e^{-ip_2\cdot x})\cos\theta|^2$$

Same E, therefore $p_j = \sqrt{E^2 - m_j^2} \approx E - \frac{m_j^2}{2E}$

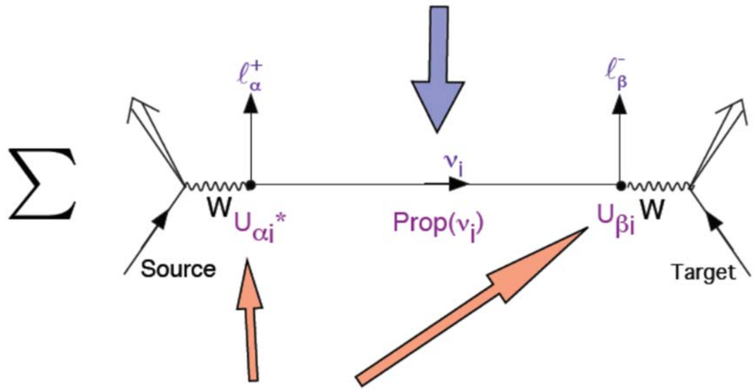
$$e^{-ip_j\cdot x} = e^{-iEt}e^{-ip_jL} \approx e^{-i(Et-EL)} \quad e^{-im_j^2L/2E}$$

$$P(\nu_\mu \to \nu_\tau) = \sin^2\theta\cos^2\theta|e^{-im_2^2L/2E} - e^{-im_1^2L/2E}|^2$$

$$P(\nu_\mu \to \nu_\tau) = \sin^2 2\theta \sin^2 \frac{\delta m^2 L}{4E}\frac{c^4}{hc}$$

Amplitude

$$e^{-im_j^2L/2E}$$



$$U_{\alpha j} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

Appearance:

$$P(\nu_\mu \rightarrow \nu_\tau) = \sin^2 2\theta \sin^2 \frac{\delta m^2 L}{4E}$$

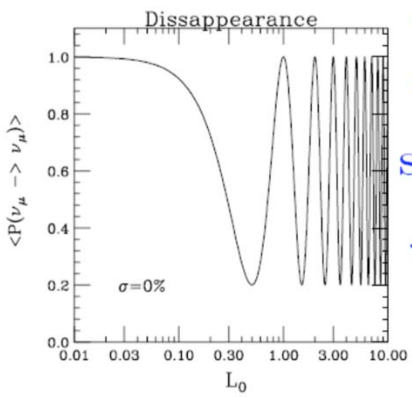
Disappearance:

$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2 2\theta \sin^2 \frac{\delta m^2 L}{4E}$$

$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2 2\theta \sin^2 \frac{\delta m^2 L}{4E}$$

Oscillation Length $L_0 = 4\pi E/\delta m^2$

Fixed E_ν

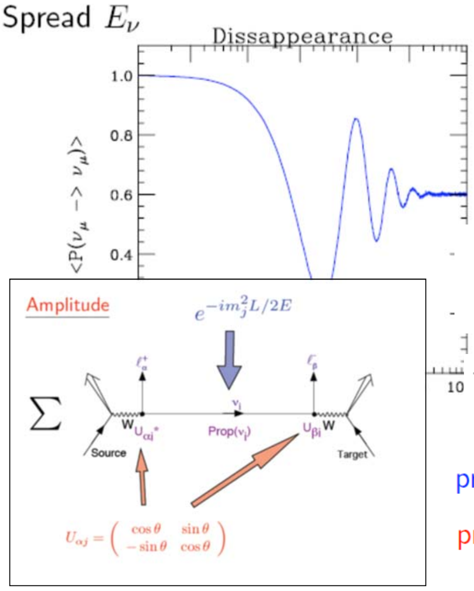


Amplitude of Oscillation

$\Uparrow$ $\sin^2 2\theta$ $\Downarrow$

$$\langle P(\nu_\mu \to \nu_\mu)\rangle = 1 - \sin^2 2\theta \left\langle \sin^2 \frac{\delta m^2 L}{4E} \right\rangle$$

Spread E_ν



effectively incoherent mass eigenstates

$1 - \sin^2 2\theta(\frac{1}{2}) = \cos^4\theta + \sin^4\theta$

- $W^+ \to \mu^+ + \nu_1$ probability $\cos^2\theta$
- $W^+ \to \mu^+ + \nu_2$ probability $\sin^2\theta$

flavour fractions $|\nu_1\rangle$ and $|\nu_2\rangle$ during propagation remain unchanged

probability ν_1 contains ν_μ is $\cos^2\theta$

probability ν_2 contains ν_μ is $\sin^2\theta$

Using the unitarity of the mixing matrix: ($W_{\alpha\beta}^{jk} \equiv [V_{\alpha j} V_{\beta j}^* V_{\alpha k}^* V_{\beta k}]$)

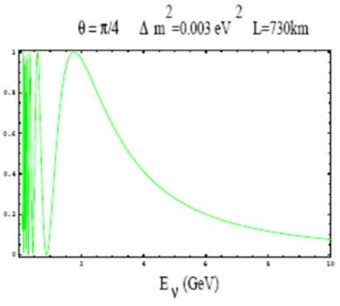
$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{k>j} \mathrm{Re}[W_{\alpha\beta}^{jk}] \sin^2\left(\frac{\Delta m_{jk}^2 L}{4E_\nu}\right)$$

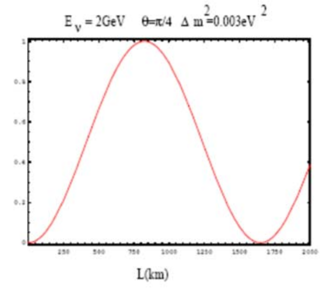
$$\pm \; 2 \sum_{k>j} \mathrm{Im}[W_{\alpha\beta}^{jk}] \sin\left(\frac{\Delta m_{jk}^2 L}{2E_\nu}\right)$$

For 2 families: $V_{MNS} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

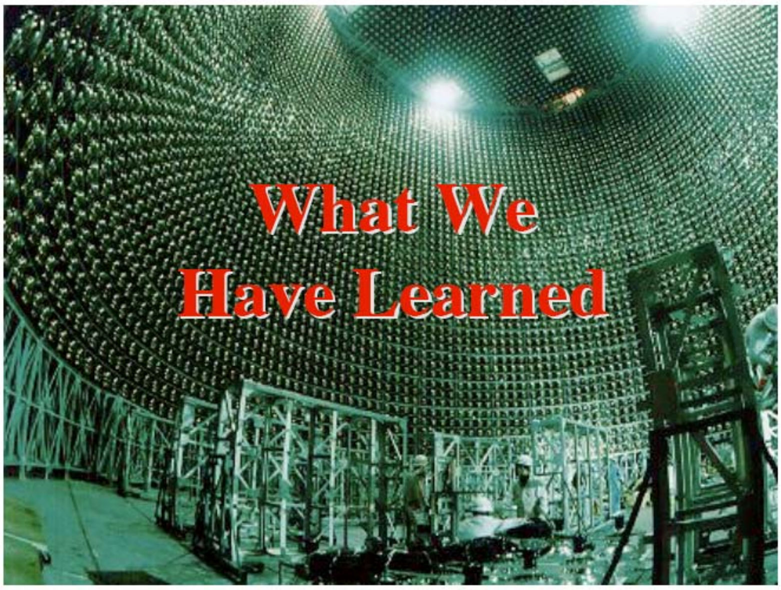
$$P_{\alpha\beta} = \sin^2 2\theta \; \sin^2\left(\frac{\Delta m^2 L}{4E_\nu}\right) \to \text{appearance}$$

$$P_{\alpha\alpha} = 1 - P_{\alpha\beta} < 1 \to \text{disappearance}$$





Oscillation probabilities show the expected GIM suppression of any flavour changing process: they vanish if the neutrinos are degenerate



Probability for Neutrino Oscillation in Vacuum

$$P(\nu_\alpha \to \nu_\beta) = |\mathrm{Amp}(\nu_\alpha \to \nu_\beta)|^2 =$$

$$P_{\alpha\beta} = \sin^2 2\theta \;\; \sin^2\left(\frac{\Delta m^2 L}{4E_\nu}\right) \to \text{appearance}$$

$$P_{\alpha\alpha} = 1 - P_{\alpha\beta} < 1 \to \text{disappearance}$$

Probability for Neutrino Oscillation in Vacuum

$$P(\nu_\alpha \to \nu_\beta) = |\mathrm{Amp}(\nu_\alpha \to \nu_\beta)|^2 =$$

$$P_{\alpha\beta} = \sin^2 2\theta \;\; \ldots \text{pearance}$$

$$P_{\alpha\alpha} = 1 - P_{\alpha\beta} \ldots \text{nce}$$

$$\left(\frac{\Delta m^2\, L}{4\, E}\right)$$

$$\left(1.27\, \frac{\Delta m^2\, (eV^2)\, L(km)}{E(GeV)}\right)$$

L/E becomes crucial !!!

Evidence for Flavor Change:

⋆⋆⋆ Atmospheric and Accelerator Neutrinos with L/E = 500 km/GeV

⋆⋆⋆ Solar and Reactor Neutrinos with L/E = 15 km/MeV

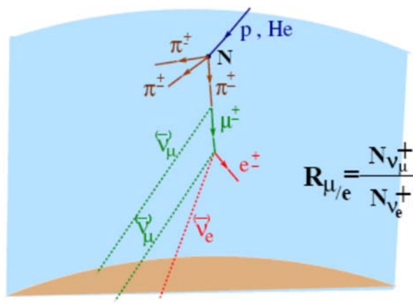
Neutrinos from Stopped muons L/E= 2m/MeV (Unconfirmed)

Atmospheric neutrinos

- Atmospheric neutrinos are produced by the interaction of *cosmic rays* (p, He, …) with the Earth's atmosphere:

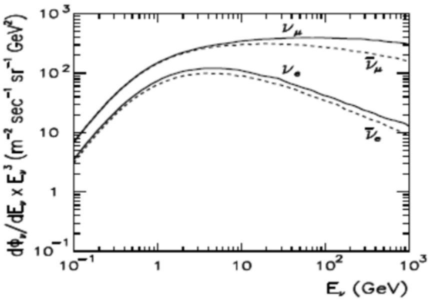
1. $A_{cr} + A_{air} \rightarrow \pi^{\pm}, K^{\pm}, K^{0}, \ldots$
2. $\pi^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu},$
3. $\mu^{\pm} \rightarrow e^{\pm} + \nu_{e} + \nu_{\mu};$

- at the detector, some ν interacts and produces a **charged lepton**, which is observed.



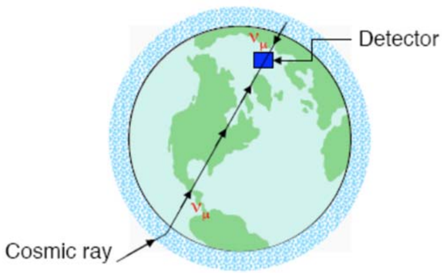
$$R_{\mu/e} = \frac{N_{\nu_\mu} + N_{\bar{\nu}_\mu}}{N_{\nu_e} + N_{\bar{\nu}_e}} \sim 2$$

ν are produced in the atmosphere when primary cosmic rays impinge on it producing K, π which subsequently decay:



A deficit was observed in the ratio μ/e events: Soudan2, IMB, Kamiokande

Atmospheric Neutrinos

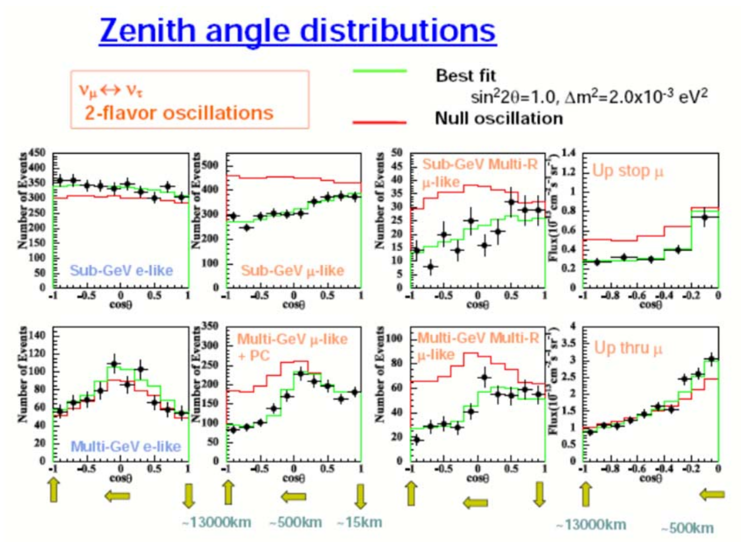


Isotropy of the $\gtrsim 2$ GeV cosmic rays + Gauss' Law + No ν_μ disappearance

$$\Rightarrow \frac{\phi_{\nu_\mu}(\text{Up})}{\phi_{\nu_\mu}(\text{Down})} = 1 \, .$$

But Super-Kamiokande finds for $E_\nu > 1.3$ GeV

$$\frac{\phi_{\nu_\mu}(\text{Up})}{\phi_{\nu_\mu}(\text{Down})} = 0.54 \pm 0.04 \, .$$



Half of the upward-going, long-distance-traveling ν_μ are disappearing.

Voluminous atmospheric neutrino data are well described by —

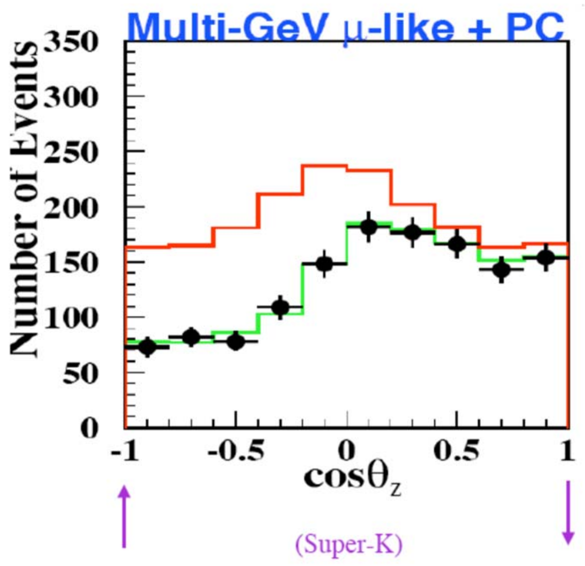
$$\nu_\mu \longrightarrow \nu_\tau$$

with —

$$\Delta m_{atm}^2 \cong 2.4\ 10^{-3}\ eV^2$$

and —

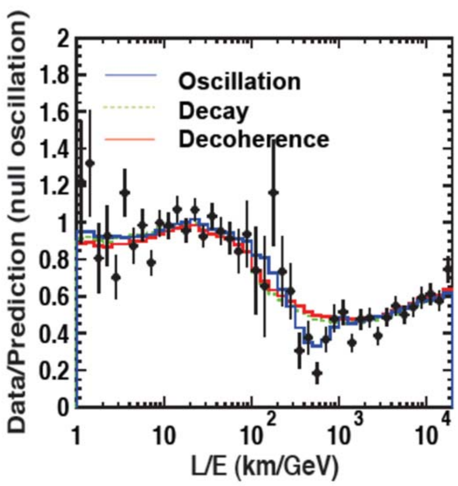
$$\sin^2 2\theta_{atm} \cong 1$$



L/E Analysis

❖Oscillation, decay and decoherence models tested

χ^2_{osc} = 83.9/83
χ^2_{dcy} = 107.1/83, $\Delta\chi^2$ = 23.2(**4.8σ**)
χ^2_{dec} = 112.5/83, $\Delta\chi^2$ = 27.6(**5.3σ**)



Confirmation of the oscillation hypothesis in "man-made" ν sources

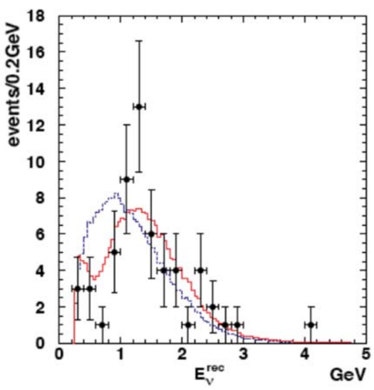
$$|\Delta m^2_{\rm atmos}| \sim \frac{E_\nu(1-10GeV)}{L(10^2-10^3km)}$$

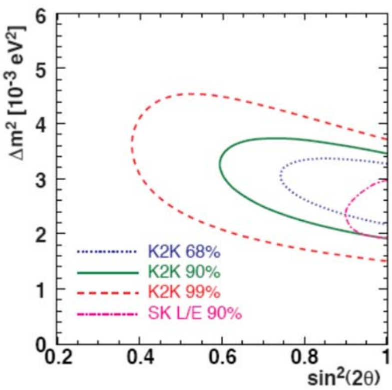
ν beams in this energy range can easily be produced at accelerators

$$p \;\rightarrow\; \text{Target} \rightarrow \pi^+, K^+ \rightarrow \quad \nu_\mu\ (\%\ \nu_e,\ \bar{\nu}_\mu,\ \bar{\nu}_e)$$
$$\nu_\mu \rightarrow \nu_x$$

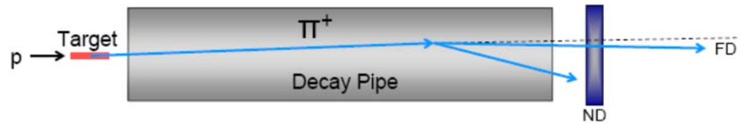
Three such conventional beams KEK-Kamioka (235km), Fermilab-Soudan (730km), CERN-Gran Sasso (730km) are looking for the disappearance of ν_μ or appearance of ν_τ:

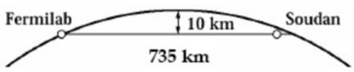
K2K





The MINOS Experiment





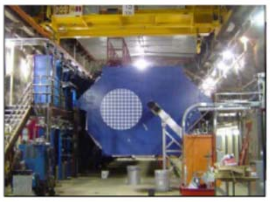
Near Detector

1 kton

$3.8 \times 4.8 \times 15$ m^3

282 steel planes

153 scintillator planes

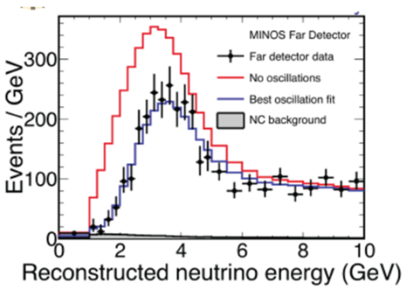


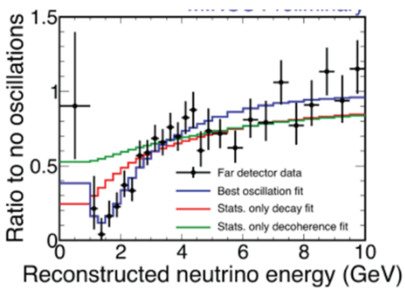
Far Detector

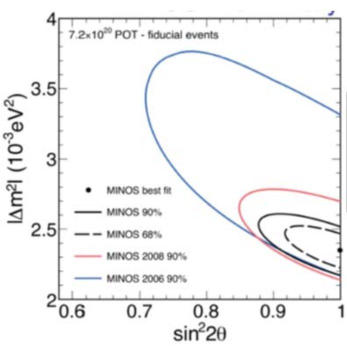
5.4 kton

$8 \times 8 \times 30$ m^3

484 planes







$$\left|\Delta m^2_{atm}\right| = 2.35^{+0.11}_{-0.08} \times 10^{-3}\ \mathrm{eV}^2$$

$$\sin^2(2\theta_{23}) = 1$$

$$\sin^2(2\theta_{23}) > 0.91\ (90\%\ \mathrm{C.L.})$$

"Atmospheric" Neutrino Summary

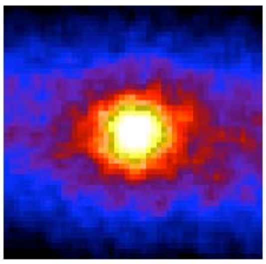
$\nu_\mu \rightarrow \nu_\tau$

no evidence of ν_e involvement:

$\delta m^2_{atm} = 2.7^{+0.4}_{-0.3} \times 10^{-3} eV^2$ $\quad L/E = 500\ km/GeV$

$\sin^2 2\theta_{atm} > 0.92$ $\quad \Rightarrow 0.35 < \sin^2\theta_{atm} < 0.65$

Solar δm^2



Solar Engine:

$$4p + 2e^- \rightarrow {}^4He + 2\nu_e + 26.7MeV$$

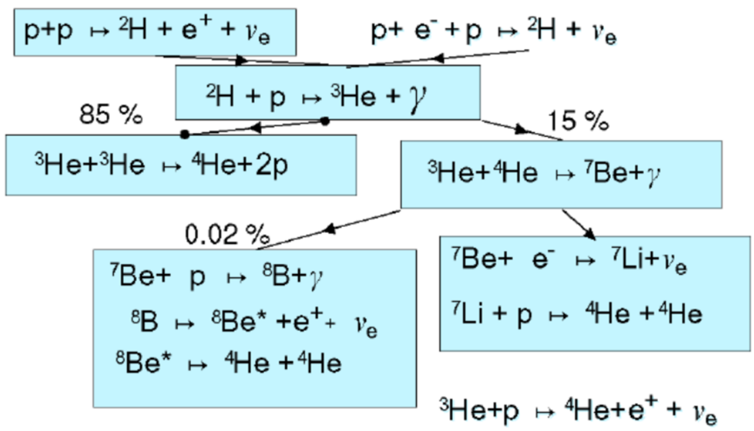
$$E = mc^2$$

1 ν_e for every 13.4 MeV (=2.1 $\times 10^{-12}$ J)

$\mathcal{L}_\odot$ at earth's surface 0.13 watts/cm^2

$$\phi_\nu = \frac{0.13}{2.1 \times 10^{-12}} = 6 \times 10^{10}/cm^2/sec$$

This corresponds to an average of 2 ν's per cm^3
since they are going at speed c.



Solar Spectrum:

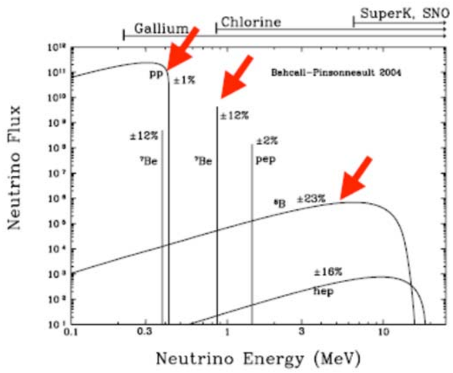


Figure 1. The predicted solar neutrino energy spectrum. The figure shows the energy spectrum of solar neutrinos predicted by the BP04 solar model [22]. For continuum sources, the neutrino fluxes are given in number of neutrinos $cm^{-2}\,s^{-1}\,MeV^{-1}$ at the Earth's surface. For line sources, the units are number of neutrinos $cm^{-2}\,s^{-1}$. Total theoretical uncertainties taken from column 2 of table 1 are shown for each source. To avoid complication in the figure, we have omitted the difficult-to-detect CNO neutrino fluxes (see table 1).

$$p + p \rightarrow {}^2H + e^+ + \nu_e$$

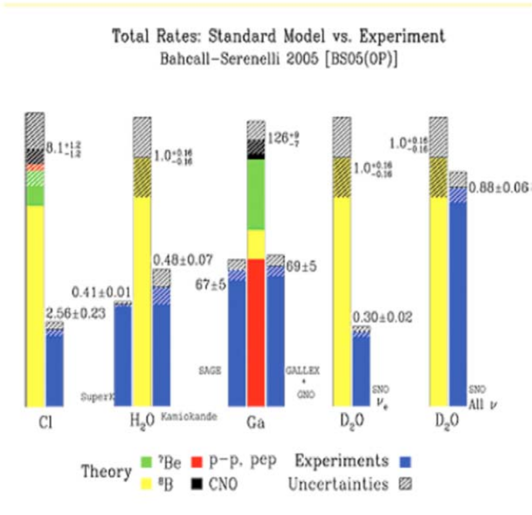
$$\phi_{pp} = 5.94(1 \pm 0.01) \times 10^{10} cm^{-2} sec^{-1}$$

$$^7Be + e^- \rightarrow {}^7Li + \nu_e$$

$$\phi_{^7Be} = 4.86(1 \pm 0.12) \times 10^{9} cm^{-2} sec^{-1}$$

$$^7Be + p \rightarrow {}^8B \rightarrow {}^8Be^* + e^- + \nu_e$$

$$\phi_{^8B} = 5.82(1 \pm 0.23) \times 10^{6} cm^{-2} sec^{-1}$$

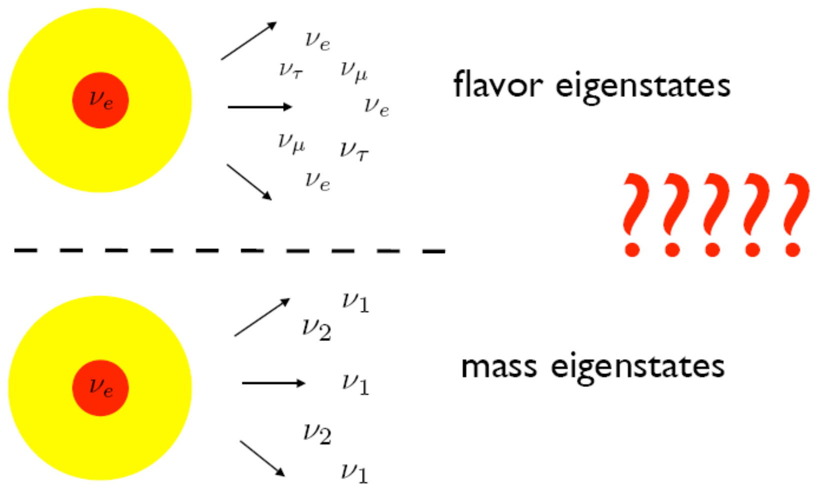


Ray Davis & John Bahcall

Theory v Exp.

Neutrino Flavor Transistions!!!

Identical Solar Twins:



Kinematical Phase:

$\delta m^2_\odot = 8.0 \times 10^{-5} eV^2$

$\sin^2 \theta_\odot = 0.31$

$$\Delta_\odot = \frac{\delta m^2_\odot L}{4E} = 1.27 \ \frac{8\times10^{-5} \ eV^2 \ \cdot \ 1.5\times10^{11} \ m}{0.1-10 \ MeV}$$

$$\Delta_\odot \approx 10^{7\pm1}$$

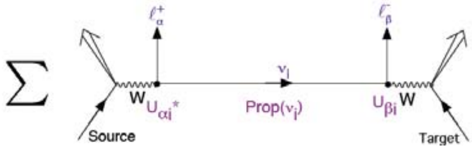
Effectively Incoherent !!!

Vacuum ν_e Survival Probability:

$$\langle P_{ee} \rangle = f_1 \cos^2\theta_\odot + f_2 \sin^2\theta_\odot$$

where f_1 and f_2 are the fraction of ν_1 and ν_2 at production.

In vacuum $f_1 = $ co

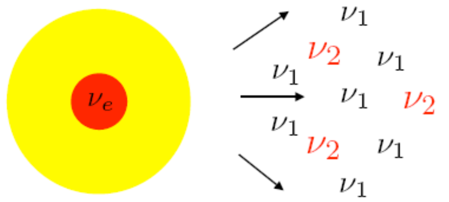


$$\langle P_{ee} \rangle = \cos^4\theta_\odot + \sin^4\theta_\odot = 1 - \tfrac{1}{2}\sin^2 2\theta_\odot$$

for pp and ^{7}Be this is approximately THE ANSWER.

$f_1 \sim 69\%$ and $f_2 \sim 31\%$ and $\langle P_{ee} \rangle \approx 0.6$

pp and ^{7}Be



$f_1 \sim 69\%$

$f_2 \sim 31\%$

$$\langle P_{ee} \rangle \approx 0.6$$

$f_3 = \sin^2\theta_{13} < 4\%$

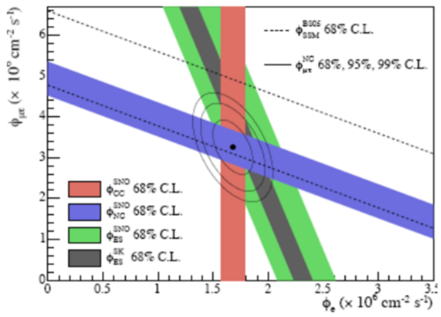
What about ^{8}B ?

SNO's CC/NC

CC: $\nu_e + d \rightarrow e^- + p + p$

NC : $\nu_x + d \rightarrow \nu_x + p + n$

ES: $\nu_\alpha + e^- \rightarrow \nu_\alpha + e^-$

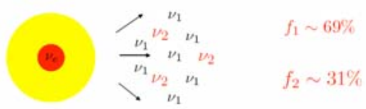


$$\frac{CC}{NC} = \langle P_{ee} \rangle = f_1 \cos^2\theta_\odot + f_2 \sin^2\theta_\odot$$

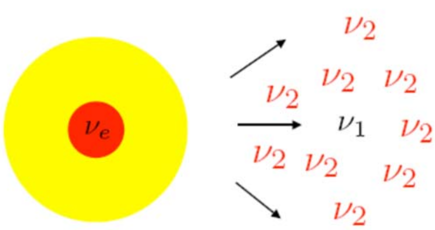
$$f_1 = \left(\frac{CC}{NC} - \sin^2\theta_\odot\right) / \cos 2\theta_\odot$$

$$= (0.35 - 0.31)/0.4 \approx 10$$

pp and ^{7}Be



^{8}B



$f_2 \sim 90\%$

$f_1 \sim 10\%$

$$\langle P_{ee} \rangle = \sin^2\theta + f_1 \cos 2\theta_\odot \approx \sin^2\theta_\odot = 0.31$$

Wow!!! How did that happen???

energy dependence!!!