$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

$$|\nu_\mu\rangle = -\sin\alpha\,|\nu_1\rangle + \cos\alpha\,|\nu_2\rangle$$

$$|\nu_\mu(t)\rangle = -\sin\alpha\, e^{-ip_1x}\,|\nu_1\rangle + \cos\alpha\, e^{-ip_2x}\,|\nu_2\rangle$$

$$\sin\alpha\,|\nu_e\rangle + \cos\alpha\,|\nu_\mu\rangle$$

$$\cos\alpha\,|\nu_e\rangle - \sin\alpha\,|\nu_\mu\rangle$$

$$\langle\nu_e|\nu_\mu\rangle = -\sin\alpha\cos\alpha\, e^{-ip_1x} + \sin\alpha\cos\alpha\, e^{-ip_2x}$$

$$P(\nu_\mu \to \nu_e) = \sin^2\alpha\,\cos^2\alpha\,\left|-e^{-ip_1x} + e^{-ip_2x}\right|^2$$

$$p_i x = E_i t - p_i L$$

$$\text{same } E \rightarrow p_i = E - \frac{m_i^2}{2E}$$

$$e^{-ip_ix} = e^{-i(Et - EL)} \cdot e^{i\frac{m_i^2}{2E}L}$$

$$P(\nu_\mu \to \nu_e) = \sin^2\alpha \cos^2\alpha \, \underbrace{\left|e^{-i(Et - EL)}\right|^2}_{1} \, \underbrace{\left| e^{\frac{i m_2 L}{2E}} + e^{\frac{i m_1 L}{2E}} \right|^2}_{e^{-i\frac{(m_1^2 + m_2^2)L}{4E}} \left(e^{i\frac{(m_2^2 - m_1^2)L}{4E}} - e^{-i\frac{(\ )L}{4E}} \right)}$$

$$= \sin^2\alpha \cos^2\alpha \, \underbrace{\left| e^{i\frac{\Delta m^2 L}{4E}} - e^{-i\frac{\Delta m^2 L}{4E}} \right|^2}_{\left|2\sin\left(\frac{\Delta m^2 L}{4E}\right)\right|} \qquad \Delta m^2 = m_2^2 - m_1^2$$

$$= \sin^2(2\alpha) \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

b

$$\pi^+ \longrightarrow \mu^+ \ \nu_\mu$$

$$\mu^+ \longrightarrow e^+ + \nu_e + \bar{\nu}_\mu$$

$$N = 2$$

c

more energy, more β → live longer

$E = \gamma m$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

distance travelled

$\gamma c \tau$

$\uparrow$ life time

$$\frac{N(d)}{N(0)} = e^{-d/\gamma c \tau}$$

number decayed after travelling a distance d

d/

$$5 \frac{\Delta m^2 L}{4E} \sim 1$$

10 GeV

$$\Delta m^2 \approx \frac{E}{L} \approx 10^{-3}\ \mathrm{eV}^2$$

10^4 km

e/

same above

Dirac mass term

2 a/

$$\begin{pmatrix} 0 & m \\ m & M \end{pmatrix}$$

Majorana mass term

$$m_2 \approx M$$

$$m_1 \approx \frac{m^2}{M}$$

$$.01\ \mathrm{eV} < \frac{m_\mu^2}{M} < 1\ \mathrm{eV} \Rightarrow$$

$$M \le \frac{m_e^2}{.01\ \mathrm{eV}}$$

$$M \geq \frac{m_e^2}{1\ \mathrm{eV}}$$

b -

$$n_B = 2.5 \; 10^{-7} / cm^3$$

number density of baryons

$$n_\gamma = 10^{-10} \; n_B = 2.5 \; 10^{3} / cm^3$$

species

$$n_\nu = 2.5 \; 10^{2} / cm^3 \times 3$$

$$= 7,5 \; 10^{2} / cm^3$$

(non relativistic)

$$\rho = m \; n$$

critical density

$$\rho_c = h^2 \; 10^4 \; eV / cm^3 \approx .5 \; 10^4 \; eV / cm^3$$

say at most $\rho_\nu \approx \frac{\rho_c}{100} \approx \frac{1}{2} \; 10^2 \; eV / cm^3$

$$50 \; eV \approx m_\nu \; 7,5 \; 10^2$$

$$\boxed{m_\nu \approx 0,07 \; eV}$$