

$$\begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix} = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

$$|\nu_\mu\rangle = -\sin \alpha |\nu_1\rangle + \cos \alpha |\nu_2\rangle$$

$$|\nu_\mu(t)\rangle = -\sin \alpha e^{-i p_1 x} |\nu_1\rangle + \cos \alpha e^{-i p_2 x} |\nu_2\rangle$$

\swarrow
 $\cos \alpha |\nu_e\rangle - \sin \alpha |\nu_\mu\rangle$

\swarrow
 $\sin \alpha |\nu_e\rangle + \cos \alpha |\nu_\mu\rangle$

$$\langle \nu_e | \nu_\mu \rangle = -\sin \alpha \cos \alpha e^{-i p_1 x} + \sin \alpha \cos \alpha e^{-i p_2 x}$$

$$P(\nu_\mu \rightarrow \nu_e) = \sin^2 \alpha \cos^2 \alpha \left| -e^{-i p_1 x} + e^{-i p_2 x} \right|^2$$

$$p_i x = E_i t - p_i L$$

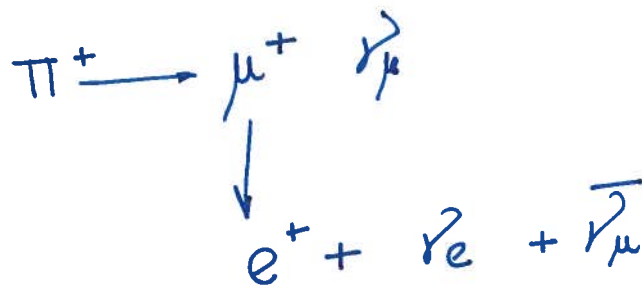
same $E \rightarrow p_i = E - \frac{m_i^2}{2E}$

$$e^{-i p_i x} = e^{-i (E t - E L)} \cdot e^{\frac{i m_i^2 L}{2E}}$$

$$\begin{aligned}
 P(\nu_\mu \rightarrow \nu_e) &= \sin^2 \alpha \cos^2 \alpha \left| e^{-i(Et - EL)} \right|^2 \left| e^{\frac{i m_1 L}{2E}} + e^{\frac{i m_2 L}{2E}} \right|^2 \\
 &= \sin^2 \alpha \cos^2 \alpha \left| \underbrace{e^{-i \frac{(m_1^2 + m_2^2)L}{4E}}}_{\uparrow \downarrow} \left(e^{\frac{i (m_2^2 - m_1^2)L}{4E}} - e^{-i \frac{(\quad)L}{4E}} \right) \right|^2 \\
 &= \sin^2 \alpha \cos^2 \alpha \left| \underbrace{e^{i \frac{\Delta m^2 L}{4E}} - e^{-i \frac{\Delta m^2 L}{4E}}}_{\uparrow \downarrow} \right|^2 \\
 &= \sin^2(2\alpha) \sin^2\left(\frac{\Delta m^2 L}{4E}\right)
 \end{aligned}$$

$\Delta m^2 = m_2^2 - m_1^2$

b



$$N = 2$$

c

more energy, more $\beta \Rightarrow$ live longer

$$E = \gamma m \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

distance travelled

$$\gamma c \tau$$

↑ lifetime

$$\frac{N(d)}{N(0)} = e^{-d/\gamma c \tau}$$

number decayed after travelling a distance d

d

$$5 \frac{\Delta m^2 L}{4E} \sim 1$$

$$\Delta m^2 \approx \frac{E}{L}$$

$$10^4 \text{ km}$$

$$10 \text{ GeV}$$

$$\approx 10^{-3} \text{ eV}^2$$

e

same above

$$\begin{pmatrix} 0 & m \\ m & M \end{pmatrix}$$

↑ Dirac mass term

↑ Majorana mass term

$$m_1 \approx \frac{m^2}{M}$$

$$m_2 \approx M$$

(2) a

$$.01 \text{ eV} < \frac{m \mu^2}{M} < 1 \text{ eV} \Rightarrow$$

$$M \leq \frac{m e^2}{.01 \text{ eV}}$$

$$M \geq \frac{m e^2}{1 \text{ eV}}$$

b -

$$n_B = 2.5 \cdot 10^{-7} / \text{cm}^3$$

✓

number density of baryons

$$n_{\gamma} = 10^{-10} n_B = 2.5 \cdot 10^3 / \text{cm}^3$$

$$n_{\gamma} = 2.5 \cdot 10^2 / \text{cm}^3 \times \textcircled{3} \quad \swarrow \text{species}$$

$$= 7.5 \cdot 10^2 / \text{cm}^3$$

$\rho = n m$ (non relativistic)

critical density $\rho_c = h^2 \cdot 10^4 \text{ eV} / \text{cm}^3 \approx 0.5 \cdot 10^4 \text{ eV} / \text{cm}^3$

say at most $\rho_{\gamma} \approx \frac{\rho_c}{100} \approx \frac{1}{2} \cdot 10^2 \text{ eV} / \text{cm}^3$

$$50 \text{ eV} \approx n_{\gamma} \cdot 7.5 \cdot 10^2$$

$$n_{\gamma} \approx 0.07 \text{ eV}$$