Neutrino physics: problem set

1. **Neutrino oscillations**

Suppose that neutrinos have mass. Then, as for quarks, we expect that the weak charge eigenstates of leptons are not the same as the mass eigenstates. Similarly to quarks, we can always align the charged lepton weak eigenstates with their mass eigenstates. But then the neutrinos will be mixtures. Consider for simplicity only 2 lepton doublets (instead of all 3). Write the weak doublets as

$$\begin{pmatrix} \nu_1 \cos\alpha + \nu_2 \sin\alpha \\ e \end{pmatrix} \quad \begin{pmatrix} \nu_2 \cos\alpha - \nu_1 \sin\alpha \\ \mu \end{pmatrix}$$

where α is the neutrino mixing angle and ν_1, ν_2 are the mass eigenstates with masses m_1, m_2.

If I produce a ν_1 neutrino with four momentum p at a time $t = 0$ and let it propagate freely, its wavefunction at later times is just

$$\nu_1(t) = e^{-ip.x}\nu_1(0) \tag{1}$$

(a) Suppose I produce a muon neutrino ν_μ with energy E. Assume that muon neutrinos can oscillate to only one other species of neutrino (x). Assume that the neutrinos are highly relativistic, i.e. $E \sim p \gg m$.

Show that the probability that the muon has oscillated to the other species, after travelling a distance L, is approximately

$$P(\nu_\mu \longrightarrow \nu_x) \simeq \sin^2(2\alpha)\ \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \tag{2}$$

where Δm^2 is the difference of the square of the masses of the two neutrino mass eigenstates.

Large number of muon and electron neutrinos are produced by cosmic rays hitting the Earth's atmosphere.

(b) Assuming that the primary source of these neutrinos is pion and muon decays, estimate the ratio

$$\frac{N(\nu_\mu + \bar{\nu}_\mu)}{N(\nu_e + \bar{\nu}_e)} \tag{3}$$

(c) This ratio starts to increase for neutrinos with energies above $\sim$ 1 GeV. Why? (Hint: assume that the Earth's atmosphere is 20 km deep, and use Special Relativity.) The SuperK experiment measured the ratio above for 1 GeV neutrinos and got an answer only about 60% of what you computed. They also found that the ratio varied with the zenith angle: it was larger for neutrinos coming from directly overheard $\sim$ 20 km and smaller for neutrinos that hat to pass through the Earth $\sim$ 12000 km.

(d) Assuming this result is due to muon neutrino oscillating into some other kind of neutrino, and assuming that the mixing angle is large, estimate Δm^2 for atmospheric neutrino oscillations.

(e) use this to make an argument why the MINOS detector, situated 700 km from Fermilab, ougth to see the same oscillation effects. (Use the fact that you can make a nice neutrino beam starting with the 120 GeV protons in the Fermilab Main Injector.

2. **Neutrino masses** We said in the lectures that Majorana masses are bad because they would violate charge conservation. However if right-handed neutrinos exist, they do not carry any electric charge or weak charge. So I can give them a Majorana mass and the only penalty is that I violate lepton number.

   (a) Assume that neutrinos get both a Dirac mass and that the right-handed neutrinos get a Majorana mass. For one neutrino species, diagonalize this 2 by 2 mass matrix. Assume that the three neutrino species that we see have Dirac masses equal to the masses of e, μ and τ leptons, compute the required Majorana masses to get three light mass eigenstates in the range .01 to 1 eV.

   (b) Obtain an upper bound on the masses on neutrinos by assuming that the number density of each neutrino species is about 1/10 the number density of photons, in our Universe. Use the fact that the number density of baryons is measured to be about 10^{-10} that of photons.