Cosmology

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Energy budget derived at Lecture 1

Overall energy density at present

$$\rho_c \equiv \frac{3H_0^2}{8\pi G} = 1.05 \times 10^{-5}\, h^2\, \frac{\mathrm{GeV}}{\mathrm{cm}^3}$$

Constituents, $\Omega_i \equiv \rho_i/\rho_c$:

- Radiation $\Omega_r = 4.2 \times 10^{-5}\, h^{-2}$
    - Two components, γ and ν, with $T_\gamma = 2.73$ K and $T_\nu = 1.95\ K$
- Dark energy $\Omega_\Lambda \simeq 0.7$
    - From the Supernova luminosities, age of the Universe, ...
- Matter $\Omega_m \simeq 0.3$ (Just for now since $\sum_i \Omega_i = 1$)
    - Two contributions, from baryons and dark matter, $\Omega_m = \Omega_b + \Omega_c$. What they are separately?

Baryon asymmetry

Important cosmological parameter, baryon to photon ratio

$$\eta_0 = \frac{n_B}{n_\gamma} = (6.1 \pm 0.05) \times 10^{-10}$$

This number should and can be understood dynamically within frameworks of the Big Bang, starting from zero net baryon charge

Baryon asymmetry can be generated if

- Baryon number is not conserved
- C- and CP- are violated
- There were deviations from thermal equilibrium during BAU generation

Sakharov (1967), Kuzmin (1970)

Many models exist which explain this number

Baryon number can be obtained from

- Counting number of stars (not good)
- From Big Bang Nucleosynthesis (not very precise baryometer)
- From CMBR measurements (best baryometer)

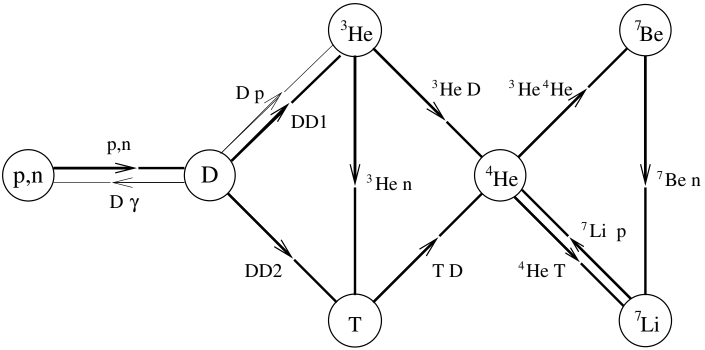
BBN

Ratios of chemical elements produced depend on competition between reaction rates and expansion rate. The answer is sensitive to baryon to photon ratio η and to N_{eff} *($\simeq 3$ in the Standard Model)*.

Neutrinos are playing two fold role in BBN:

- Initially keep neutrons and protons in equilibrium
- Change expansion rate while contributing to ρ_r

Big Bang Nucleosynthesis

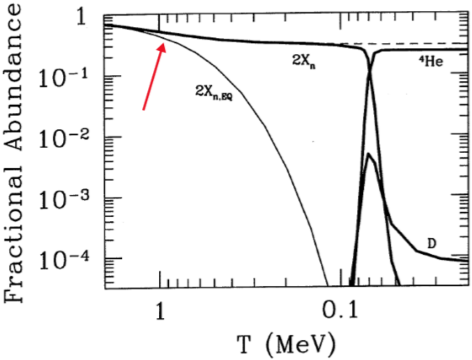


Involves numerical solution of coupled kinetic equations

$$\frac{dn_i}{dt} = I(n_1, n_2, \dots)$$

for the concentrations of elements in the expanding Universe, but $^4\mathrm{He}$ abundance can be estimated reliably in a simple way.

Helium abundace



$$X_n = \frac{n_n}{n_n + n_p}$$

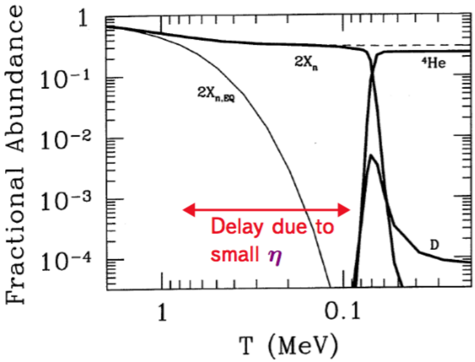
Chemical equilibrium between protons and neutrons is maintained by weak interactions $n + \nu \rightleftarrows p + e^-$, $n + e^+ \rightleftarrows p + \bar{\nu}$, which get out of thermal equilibrium at $T_f \sim 0.8$ MeV. At freeze-out (Saha equation for neutrons)

$$\frac{n_n}{n_p} \approx \mathrm{e}^{-\Delta m/T_f} \approx 0.19$$

where $\Delta m \equiv m_n - m_p = 1.29$ MeV.

We are almost done, but D should still form out of n and p. The Universe is 1 sec old.

Helium abundace



$$X_n = \frac{n_n}{n_n + n_p}$$

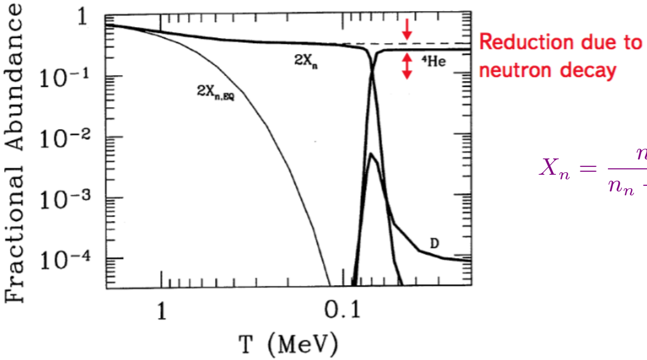
Saha equation for equilibrium deuterium concentration $n_D/n_n n_p$ gives

$$\frac{n_D}{n_n} = \frac{3}{4}\,\frac{2}{\pi^2}\,\eta \left(\frac{4\pi T}{m_p}\right)^{3/2} e^{B/T}$$

where $B = m_n + m_p - m_D = 2.22\ \mathrm{MeV}$.
This ratio becomes of order one at $T \ll B$ due to smallness of η.

Now we have to take into account that some neutrons decayed during this time delay.

Helium abundace



$$X_n = \frac{n_n}{n_n + n_p}$$

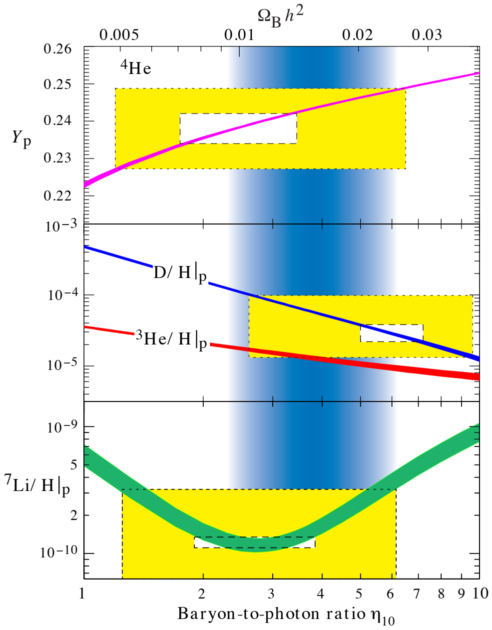
Recall time-temperature relation $t = 132 \text{ sec } (0.1\,\text{MeV}/\text{T})^2$

Deuterium forms at $T \approx 0.08 \text{ MeV}$ or at $t \approx 210 \text{ sec}$
Neutron lifetime $\tau_n = 880 \text{ sec}$. This results in final He mass fraction:

$$Y_{He} = 2X_n\, \mathrm{e}^{-t/\tau_n} \approx 0.25 + 0.01(N_\nu - 3) + 0.01 \ln \eta_{10}$$

see e.g. arXiv:astro-ph/0303073

Element abundances



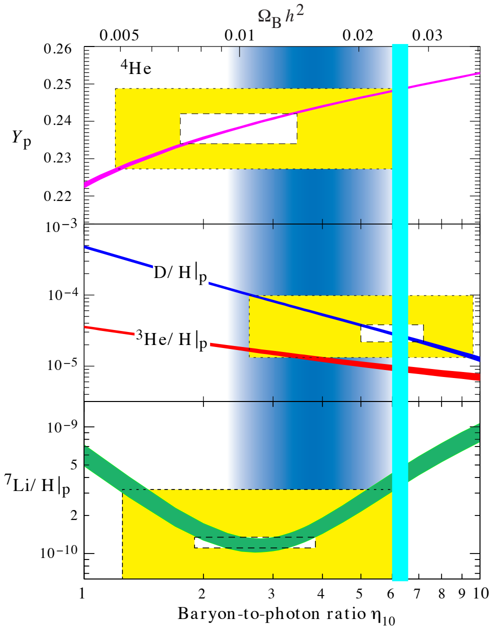
Before CMBR anisotropy were measured:

Baryon-to-photon ratio was derived from BBN.

Yellow bands - observed element abundances ($\sim$ 15 yrs ago).

$N_\nu = 3$ was derived

Element abundances



After WMAP measurements:

- η is derived from CMBR anisotropies

$$10^{10}\,\eta = 274\;\Omega_B h^2$$

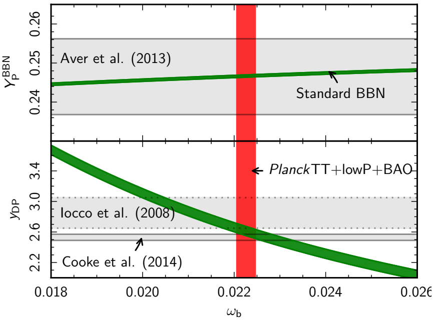
- Element abundances had changed;

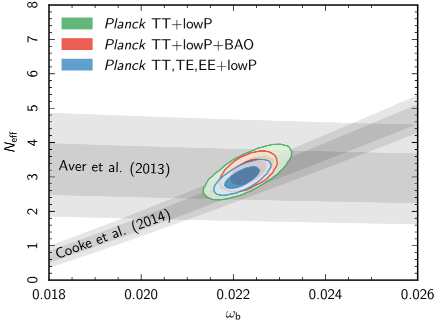
- Neutron life-time had changed;

- $N_\nu = 4$ was a better fit

Element abundances

Life after Planck measurements





$$N_{eff} = 2.91 \pm 0.37$$

Beautiful agreement between CMB and BBN physics.

arXiv:1502.01589

With Planck derived $\Omega_B h^2 \equiv \omega_b$ we find

$$\eta \;=\; (6.1 \pm 0.05) \;\times\; 10^{-10}$$

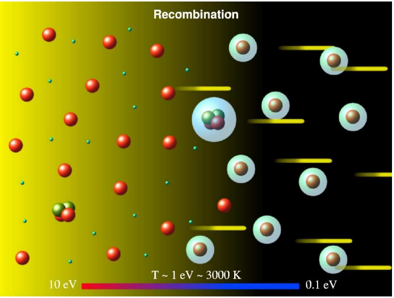
Element abundances

Life after Planck measurements

Great success of Big Bang theory, but
The Primordial Lithium is still a Problem

- Astrophysical systematics in the observations?
- Some resonances missed in nuclear reactions?
- Physics beyond the Standard Model?
  - Decaying Supersymmetric particles? *arXiv:1203.3551*
  - Decaying sterile neutrino? *arXiv:1606.06968*

Last scattering of light



Matter is ionized at temperatures higher than hydrogen ionization energy

$$E_{\text{ion}} = 13.6 \text{ eV}$$

At lower T neutral atoms start to form.

Saha equation for ions/atoms

$$\frac{n_e\, n_p}{n_H} = \left(\frac{m_e T}{2\pi}\right)^{3/2} \mathrm{e}^{-E_{\text{ion}}/T}$$

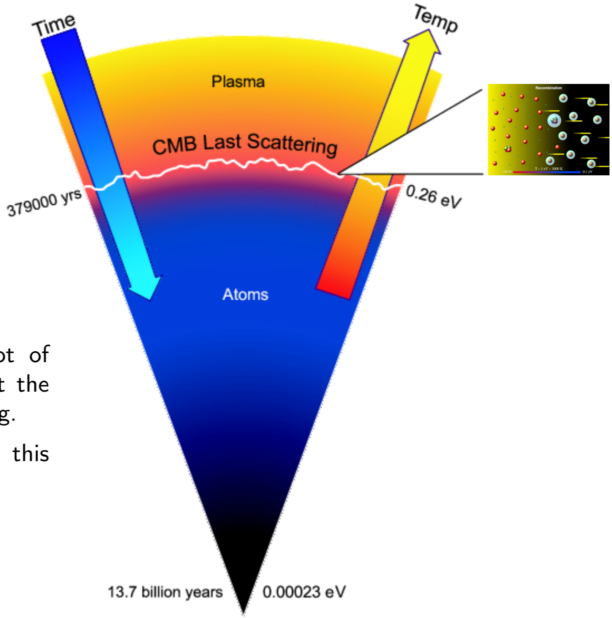
Univerese became transparent for radiation when

$$\sigma_{\gamma e}\, n_e \sim t$$

Here $\sigma_{\gamma e} = 8\pi\alpha^2/3m_e^2$ is the Compton cross-section.

Numerically $T_{\text{ls}} = 0.26 \text{ eV}$

Last scattering of light

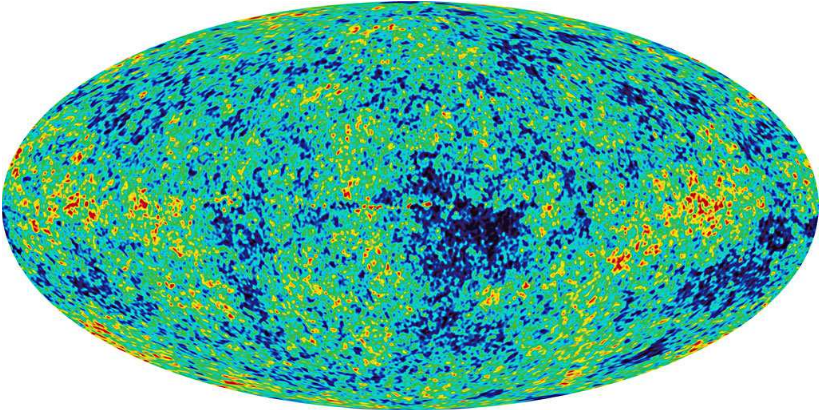


CMBR is a snapshot of the baby Universe at the time of last scattering.

We cannot see past this surface.

Temperature map of the sky

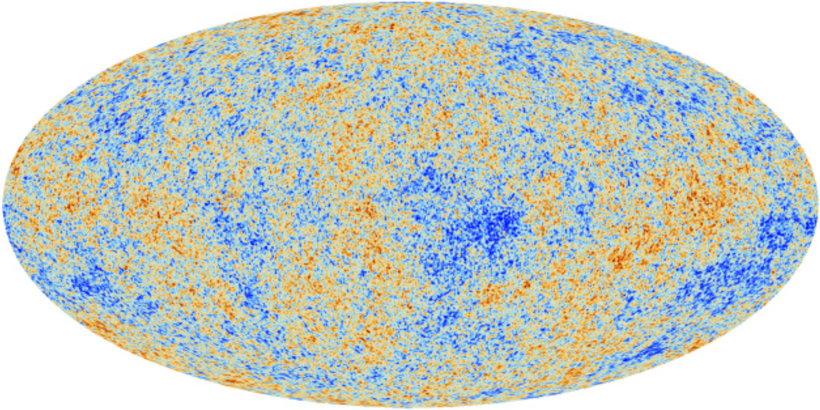
WMAP



- Temperature is slightly different in different patches of the sky - 1 part in 100,000.

Temperature map of the sky

Planck



- Temperature is slightly different in different patches of the sky - 1 part in 100,000.

CMB power spectrum

The temperature anisotropy, $T(\mathbf{n})$, is expanded in a spherical harmonics

$$T(\mathbf{n}) = \sum_{l,m} a_{lm} Y_{lm}(\mathbf{n}).$$

The angular power spectrum, C_l, is defined as

$$C_l = \frac{1}{2l+1} \sum_m |a_{lm}|^2.$$

Assuming random phases, the temperature anisotropy for each multipole moment, ΔT_l, can be associated with the angular spectrum

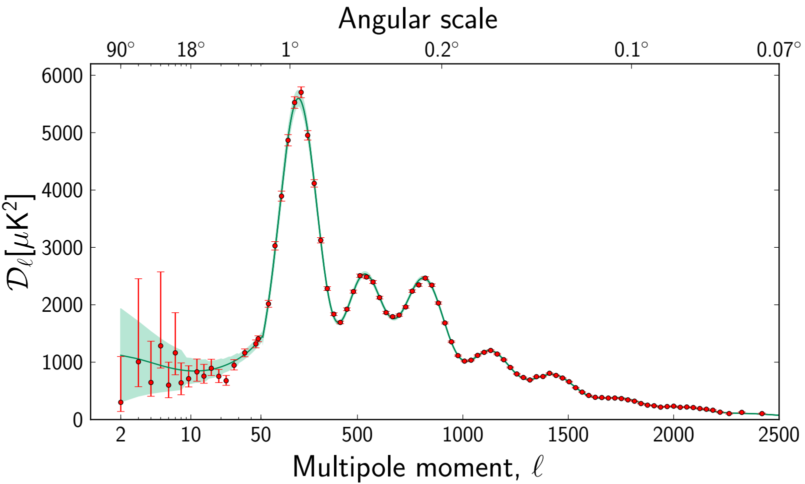
$$\Delta T_l = \sqrt{C_l \; l(l+1)/2\pi} \equiv \sqrt{D_l}.$$

The correlation function is

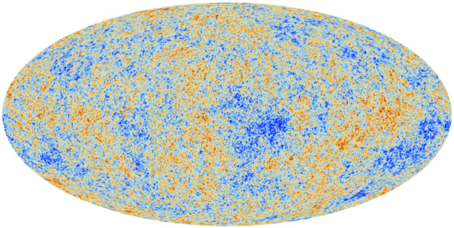
$$C(\theta) = \frac{1}{4\pi} \sum_l (2l+1) C_l \; P_l(cos\ \theta)$$

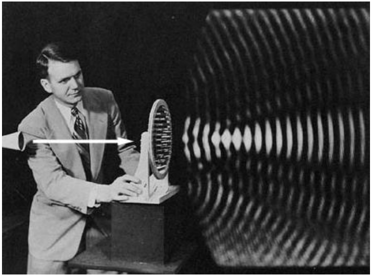
where P_l is the Legendre polynomial or order l.

CMB power spectrum: tool of Precision Cosmology



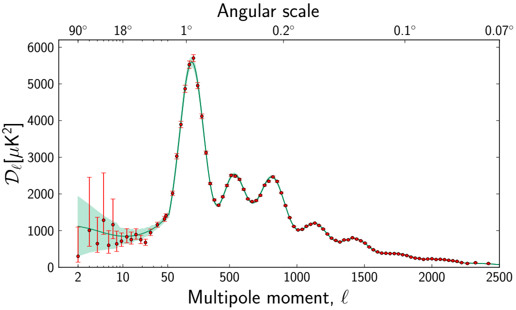
Tool of Precision Cosmology

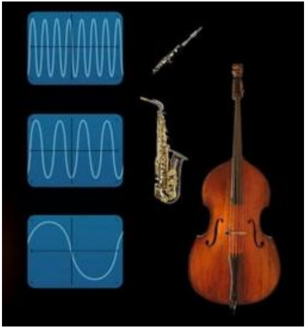




Instant photo of sound waves

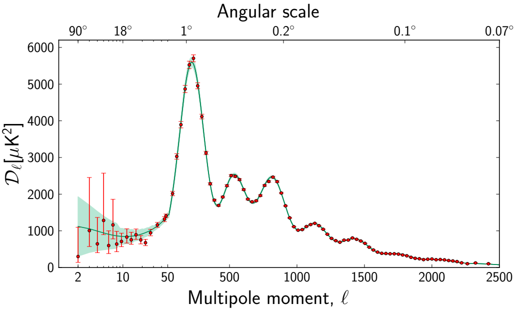
Tool of Precision Cosmology





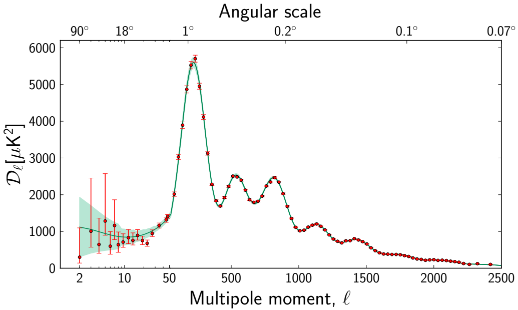
Soundscape of the sky

Tool of Precision Cosmology



Soundscape of the sky

Tool of Precision Cosmology



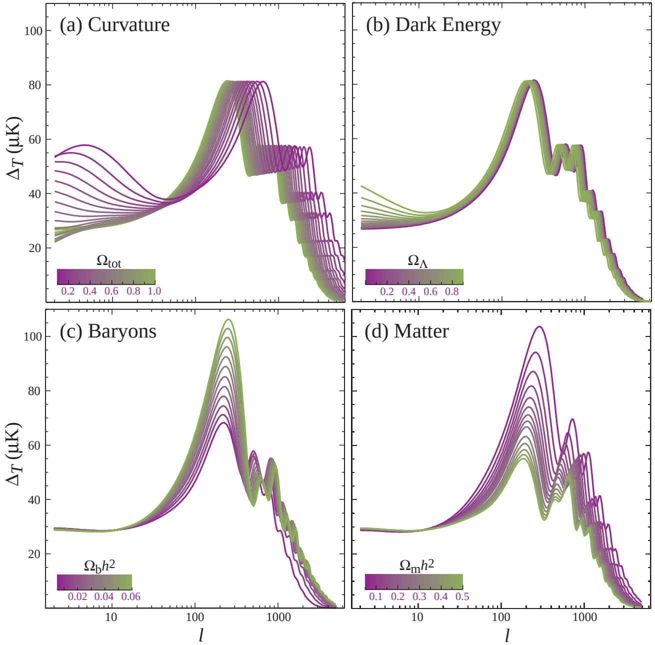
Already the amplitude of fluctuations $\delta T/T \sim 10^{-5}$ gives us unique cosmological information

Shape of the power spectrum is sensitive to

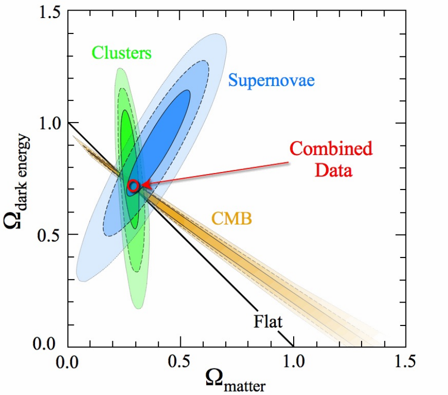
- Shape of primordial perturbations
- Cosmological parameters
- New physics

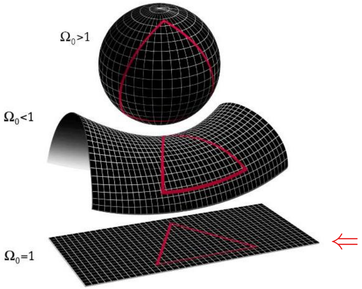
Power spectra measurements gives us a lot of information at the unprecedented precision level

Cosmological parameter sensitivities



Geometry of the Universe



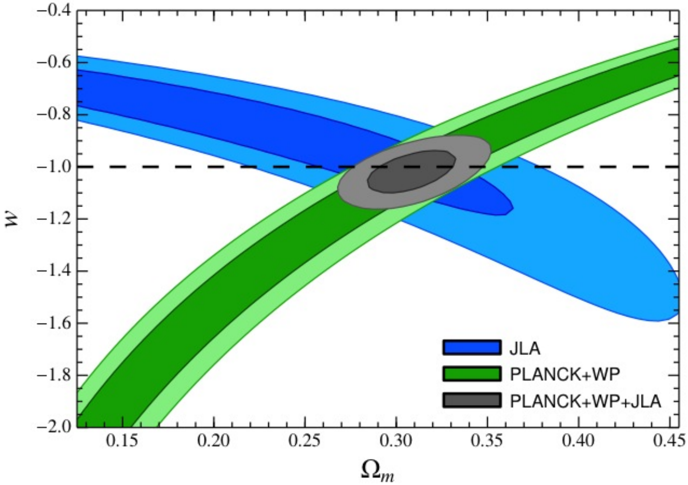


The Universe is spatially flat and is dominated by the dark energy

$$1 - \sum \Omega_i = 0.0008 \pm 0.004$$

arXiv:1502.01589

Dark Energy



Dark energy equation of state is close to vacuum, $w = -1.02 \pm 0.06$.

JLA - most recent Supernova data

arXiv:1401.4064

Some cosmological parameters

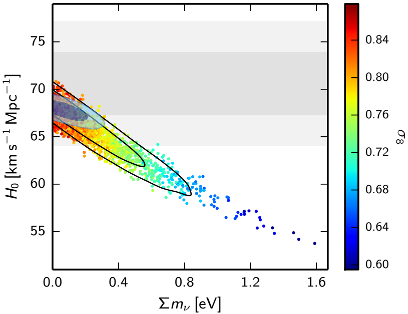
Planck only data on temperature and polarisation power spectra give for the base ΛCDM ($k = 0$, $w = -1$)

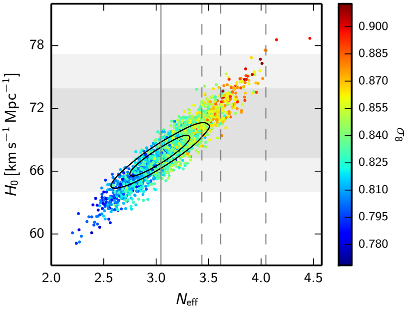
Dark energy density	Ω_Λ	0.684 ± 0.009
Matter density	Ω_m	0.316 ± 0.009
Baryon density	$\Omega_b\, h^2$	0.02225 ± 0.00016
Hubble constant	H_0	67.27 ± 0.66
Fluctuation amplitude at $8h^{-1}$ Mpc	σ_8	0.83 ± 0.013
Age/Gyr	t_u	13.81 ± 0.03
Redshift of equality	z_{eq}	3395 ± 33
Redshift of decoupling	z_*	1090 ± 0.30

arXiv:1502.01589

Constraints on extended models

Planck and external data (BAO+JLA+H_0)



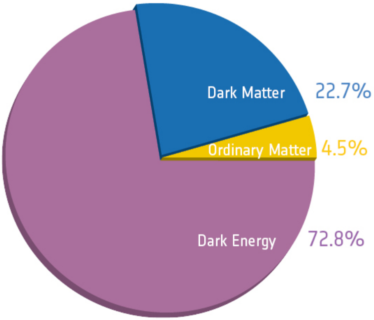


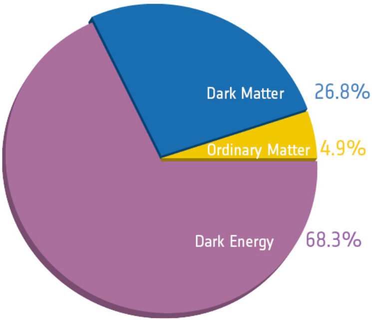
$$\sum m_\nu < 0.2 \text{ eV}$$

$$N_{eff} = 3.04 \pm 0.33$$

arXiv:1502.01589

Cosmic Recipe





Before Planck

$H_0 = 70 \pm 2.5$

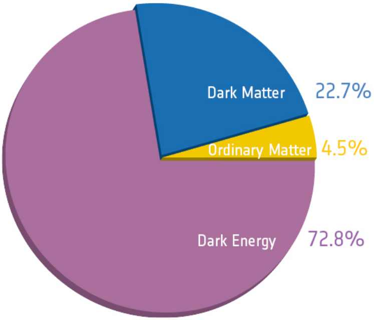
After Planck

$H_0 = 67.3 \pm 0.7$

Grain of salt

But, latest direct measurements give $H_0 = (73.00 \pm 1.75)$.
We have $3.2\,\sigma$ discrepancy. Systematics or hint of new physics?

Cosmic Recipe



Before Planck

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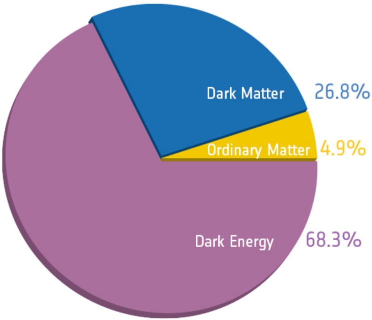

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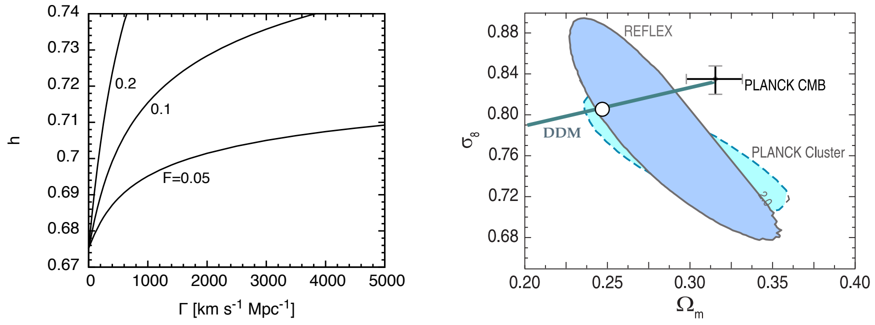
Decaying dark matter?



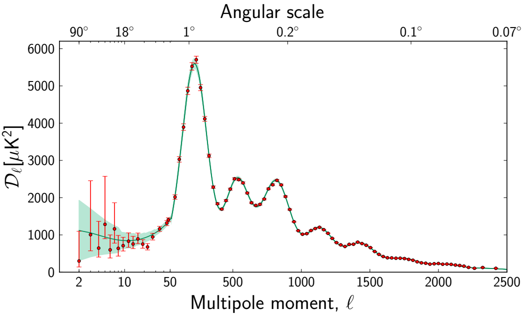
At decoupling physics can be fixed to be identical to ΛCDM, but then part of DM decays leading to different H_0 and σ_8. Low and high z measurements can be reconciled.

Berezhiani, Dolgov, & I.T., arXiv:1505.03644

Everyone can do cosmology using public Monte Carlo codes and e.g. Planck data

Important conceptual analytical stuff

Position of the first peak is determined by the angular size of sound horizon at last scattering. Causal horizon is larger by a factor of $\sqrt{3}$.



Comoving distance χ traveled by light

$$ds^2 = a^2(d\eta^2 - d\chi^2) = 0$$

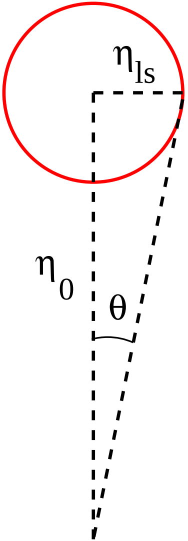
equals to a comoving time interval

$$\eta(z) = \int_z^\infty \frac{dz'}{H(z')}$$

E.g, comoving horizon size in a matter dominated Universe

$$\eta(z) \propto \frac{1}{\sqrt{T(z)}}$$

Angular size of horizon



$$ds^2 = a^2(d\eta^2 - d\chi^2 - \chi^2 d\theta^2) = 0$$

Angular size of horizon at last scattering in a matter dominated universe would be

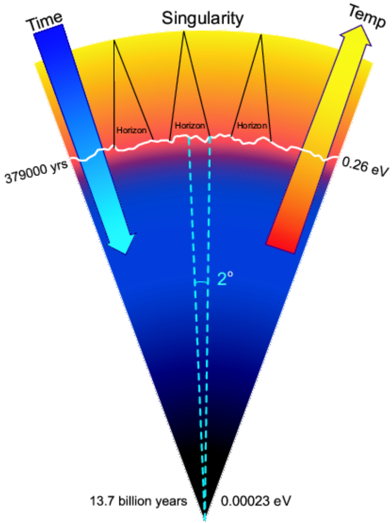
$$\theta_h = \frac{\eta_{\rm ls}}{\eta_0} = \sqrt{\frac{T_0}{T_{\rm ls}}} \approx 1.7^\circ$$

Sound horizon angular scale is tightly constrained by Planck:

$$\theta_* = 0.59648^\circ \pm 0.00018^\circ$$

- This gives strong constraints on combinations of relevant cosmological parameters
- Tells us that there were 10^4 causally disconnected regions at the surface of last scattering

Horizon problem



The microwave sky should not be homogeneous on scales $\Delta\theta > 2^\circ$

Yet CMB is isotropic to better than 10^{-4} on all scales

Puzzles of classical cosmology

why the universe

- is so old, big and flat ?
  ($t > 10^{10}$ years)
- homogeneous and isotropic?
  ($\delta T/T \sim 10^{-5}$)
- contains so much entropy?
  ($S > 10^{90}$)
- does not contain unwanted relics?
  (e.g. magnetic monopoles)

can be solved with the hypothesis of Inflation

Puzzles of classical cosmology

Curvature problem and the solution

Friedmann equation
$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\,\rho - \frac{k}{a^2}$$

can be re-written as
$$k = \dot{a}^2\ (\Omega - 1) = \mathrm{const}$$

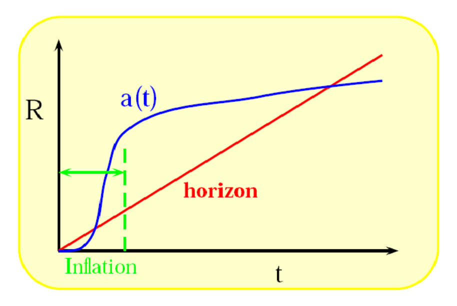
* Problem: During matter or radiation dominated stages $\dot{a}^2$ decreases (since $\ddot{a} < 0$), therefore Ω is driven away from unity. To get $\Omega \sim 1$, today, one needs enormous initial fine-tuning, say at BBN epoch $|\Omega(t_{\mathrm{NS}}) - 1| < 10^{-15}$
* Solution: Accelerated expansion drives Ω to unity. Therefore, the problem can be solved if at early times $\ddot{a} > 0$

Puzzles of classical cosmology

Horizon problem and the solution

Horison: $\propto t$

Physical size: $\propto \quad a(t) \propto t^{\gamma}$



- **Problem:** During matter or radiation dominated stages $\gamma < 1$, therefore the visible universe at early times contains many causially disconnected regions.
- **Solution:** Problem is solved if there was a period with $\gamma > 1$ or $\ddot{a} > 0$

Basics of inflation

Definition

"Inflation" is a period of accelerated universe expansion

$$\ddot{a} > 0$$

Friedmann equations

$$\ddot{a} = -\frac{4\pi}{3} G a (\rho + 3p)$$

We have inflation whenever $p < -\rho/3$

Getting something for nothing

$$T_\mu^\nu = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix}$$

Energy-momentum conseravtion $T^{\mu\nu}{}_{;\nu} = 0$ can be written as

$$\frac{d\rho}{dt} + 3H(\rho + p) = 0$$

Consider stress-energy tensor $T_{\mu\nu}$ for a vacuum. Vacuum has to be Lorentz invariant, hence $T_\mu^\nu = V\,\delta_\mu^\nu$ and we find

$$p = -\rho \quad \Rightarrow \quad \dot{\rho} = 0$$

Energy of the vacuum stays constant despite the expansion !

The Inflaton field

Consider $T_{\mu\nu}$ for a scalar field φ

$$T_{\mu\nu} = \partial_\mu\varphi\,\partial_\nu\varphi - g_{\mu\nu}\,\mathcal{L}$$

with the Lagrangian :

$$\mathcal{L} = \partial_\mu\varphi\,\partial^\mu\varphi - V(\varphi)$$

In a state when all derivatives of φ are zero, the stress-energy tensor of a scalar field is that of a vacuum,

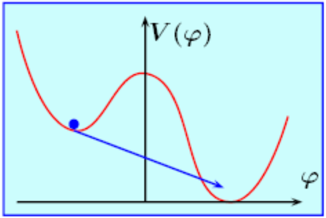
$$T_{\mu\nu} = V(\varphi)\,g_{\mu\nu}$$

or $p \approx -\rho$

Models of Inflation

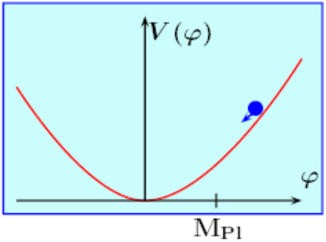
There are two basic ways to arrange $\varphi \approx \text{const}$ and hence to imitate the vacuum-like state.

1. A. Guth (1981): consider potential with two minima



2. A. Linde (1983): consider the simplest potential

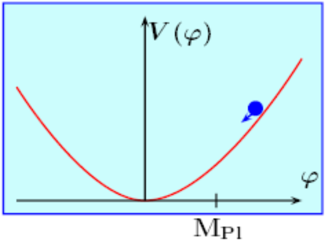
$$V(\varphi) = \frac{1}{2} m^2 \varphi^2$$



Equation of motion

$$\ddot{\varphi} + 3H\dot{\varphi} + m^2\varphi = 0$$

If $H \gg m$ the field rolls down slowly ("slow-roll") .
Since $H \sim m\,\varphi/M_{\rm Pl}$, the slow-roll condition is equivalent to $\varphi > M_{\rm Pl}$.



$\varphi > M_{\rm Pl}$ Inflation
$\varphi < M_{\rm Pl}$ Reheating

During Inflation the Universe is empty, in a vacuum state.

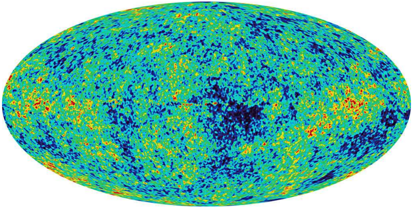
How vacuum has turned into radiation ?

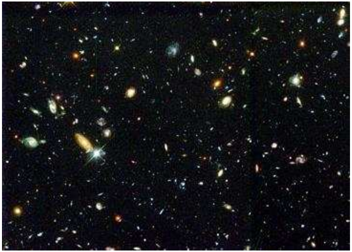
Particle physicist

Cosmologist

Where all matter and seeds for structure formation came from ?

Predictive power of Inflation





Fluctuations in inflaton field

$\Downarrow$

CMBR anisotropy
379,000 years after

$\Downarrow$

Large-scale structure
13.7 billions years after

Unified Theory of Creation

During Inflation the Universe is "empty". Quantum fluctuations of all fields obey

$$\ddot{u}_k \; + \; [k^2 + m^2_{\rm eff}(\tau)]\, u_k = 0$$

But it is not possible to keep fluctuations in vacuum
if $m_{\rm eff}$ is time dependent.

What we see now (as CMBR anisotropies and Large Scale Structure) reflects actually zero mode fluctuations of quantum fields during inflation which were grown then frozen and then blown up to the whole sky.

Moreover, Universe exit from inflation and reheating to Bing Bang is also described by that same equation.

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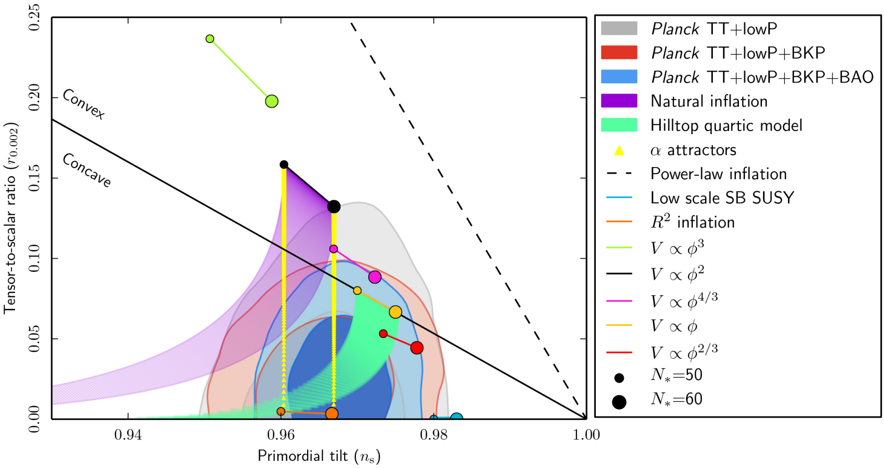
Comparison with CMBR anisotropy measurements



Very first inflationary model by Starobinsky (1980) is the best runner

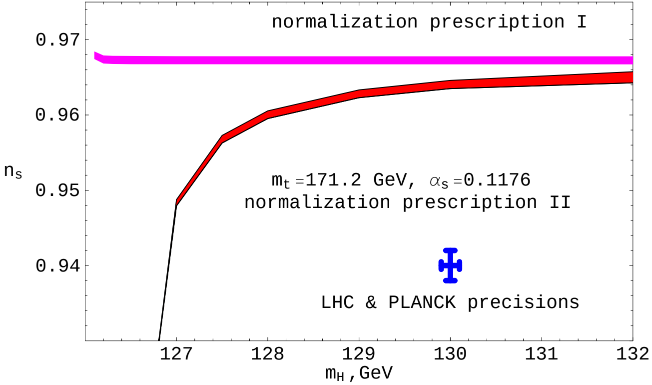
$$\mathcal{L} = \mathcal{L}_{\mathcal{SM}} - \beta R^2$$

Higgs as Inflaton?

Should add to the Standard Model a non-minimal coupling to gravity

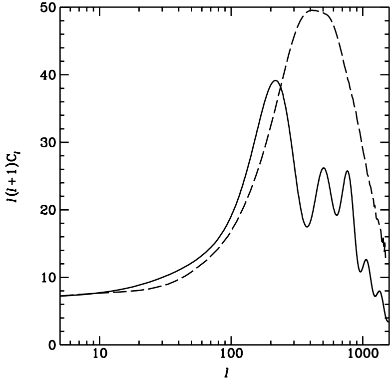
$$\mathcal{L} = \mathcal{L}_{\mathcal{SM}} - \xi R H^{+} H$$

Higgs field can serve as inflaton if $\xi = 44700\sqrt{\lambda}$

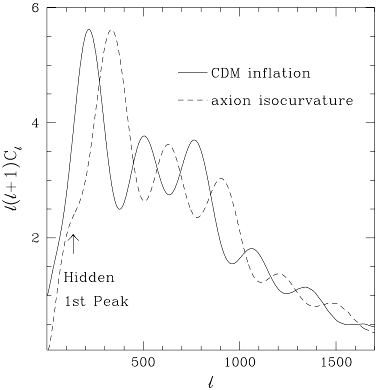


Shaposhnikov, Bezrukov 2008

Inflation vs other models



Models of structure formation by topological defects are ruled out by CMBR data



Isocurvature models are ruled out by CMBR data