The Electroweak Standard Model

1. Structure of the Weak Interactions

M. E. Peskin
European School of
High-Energy Physics
June 2016

The purpose of this course of lectures is to describe the structure of the weak interactions.

This first lecture will discuss the basic elements of the theory - the Lagrangian, the mass spectrum of weak interaction bosons, and their implications for processes involving light fermions.

You all know that the weak interactions are described by a Yang-Mills theory based on the group SU(2)xU(1).

In Yang-Mills theory, the coupling of any field to the vector bosons is determined by the covariant derivative

$$D_\mu = \partial_\mu - igA^a_\mu t^a$$

The gauge charges t^a depends on the quantum numbers of the field.

For SU(2)xU(1), an essential field is the Higgs field $\varphi(x)$, which obtains a constant value throughout space. This nonzero value gives mass to the weak interaction vector bosons and to the quarks and leptons.

The mass spectrum of vector bosons is especially easy to work out. We assign φ the quantum numbers

$$I = \frac{1}{2} \qquad Y = \frac{1}{2}$$

The action of SU(2)xU(1) is

$$\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \to \exp[-i\alpha^a\sigma^a/2 - i\beta/2] \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

Then if φ obtains a nonzero vacuum value, we can write this as

$$\varphi(x) = \exp[-i\alpha^a(x)\sigma^a/2] \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

The covariant derivative on φ is

$$D_\mu\varphi = (\partial_\mu - ig(\sigma^a/2)A^a_\mu - (g'/2)B_\mu)\ \varphi$$

and this forms the kinetic term for φ in the Lagrangian

$$\mathcal{L} = |D_\mu\varphi|^2$$

Replacing φ by its vacuum value, this becomes

$$\frac{1}{8}\begin{pmatrix} 0 & v \end{pmatrix} (g\sigma^a A^a_\mu + g' B_\mu)^2 \begin{pmatrix} 0 \\ v \end{pmatrix}$$

The σ^1 , σ^2 terms give

$$\frac{g^2}{8}[(A^1_\mu)^2 + (A^2_\mu)^2]v^2$$

The remaining terms give $\frac{1}{8}(-gA^3_\mu + B_\mu)^2 v^2$

So we find masses for the vector fields, of the form

$$\mathcal{L} = \frac{1}{2} m^2_{ab} V^a_\mu V^{b\mu}$$

The mass eigenstates are

$$W^\pm = (A^1 \mp iA^2)/\sqrt{2} \qquad m^2_W = g^2 v^2/4$$

$$Z = (gA^3 - g'B)/\sqrt{g^2 + g'^2} \qquad m^2_Z = (g^2 + g'^2)v^2/4$$

$$A = (g'A^3 + gB)/\sqrt{g^2 + g'^2} \qquad m^2_A = 0$$

We introduce the weak mixing angle θ_w , with

$$\cos\theta_w \equiv c_w = g/\sqrt{g^2 + g'^2}$$

$$\sin\theta_w \equiv s_w = g'/\sqrt{g^2 + g'^2}$$

These factors will appear throughout all of the formulae in this course.

An important relation is : $m_W = m_Z \, c_w$

This is a nontrivial consequence of the quantum number assignment for the Higgs field. From the PDG values:

$$80.385 \approx 79.965$$

We will see in the next lecture that, when radiative corrections are included, this relation is satisfied to better than 1 part per mil.

The couplings of quarks and leptons to these vector bosons is also given by the covariant derivative. For a fermion with quantum numbers (I, Y) :

$$D_\mu f = (\partial_\mu - igA^a_\mu \sigma^a/2 - ig' B_\mu Y) f$$

The W couples only to fermions with I = 1/2

$$-i\frac{g}{\sqrt{2}}(W^+_\mu \sigma^+ + W^-_\mu \sigma^-)$$

The diagonal elements give couplings both to Z and A

$$-igA^3_\mu I^3 - ig' B_\mu Y$$

$$= -i\sqrt{g^2 + g'^2}[c_w(c_w Z_\mu + s_w A_\mu) I^3 + s_w(-s_w Z_\mu + c_w A_\mu)]$$

$$= -i\sqrt{g^2 + g'^2}[s_w c_w A_\mu (I^3 + Y) + Z_\mu (c_w^2 I^3 - s_w^2 Y)]$$

$$= -i\sqrt{g^2 + g'^2}[s_w c_w A_\mu (I^3 + Y) + Z_\mu (I^3 - s_w^2 (I^3 + Y))]$$

From these relations, we find the following simple prescriptions:

A couples to $Q = (I^3 + Y)$; the coupling strength is

$$e = \sqrt{g^2 + g'^2}\; s_w c_w = g g' / \sqrt{g^2 + g'^2}$$

This is the photon field, and we can identify e with the electron charge and Q with the electric charge of f.

W couples only to SU(2) doublets, with the universal strength $g/\sqrt{2}\ ,\qquad g = e/s_w$

Z couples with strength $g/c_w = e/(c_w s_w)$ to the quantum number

$$Q_Z = I^3 - s_w^2 Q$$

To complete the specification of the Standard Model, we assign the fermions in each generation of quarks and leptons the quantum numbers

$$\nu_{eL} \ : \quad I^3 = +\frac{1}{2} \ , \ Y = -\frac{1}{2} \ , \ Q = 0 \qquad \nu_{eR} \ : \quad I^3 = 0 \ , \ Y = 0 \ , \ Q = 0$$

$$e^-_L \ : \quad I^3 = -\frac{1}{2} \ , \ Y = -\frac{1}{2} \ , \ Q = -1 \qquad e^-_R \ : \quad I^3 = 0 \ , \ Y = -1 \ , \ Q = -1$$

$$u_L \ : \quad I^3 = +\frac{1}{2} \ , \ Y = \frac{1}{6} \ , \ Q = \frac{2}{3} \qquad u_R \ : \quad I^3 = 0 \ , \ Y = \frac{2}{3} \ , \ Q = \frac{2}{3}$$

$$d_L \ : \quad I^3 = -\frac{1}{2} \ , \ Y = \frac{1}{6} \ , Q = -\frac{1}{3} \qquad d_R \ : \quad I^3 = 0 \ , \ Y = -\frac{1}{3} \ , \ Q = -\frac{1}{3}$$

This gives the correct electric charge assignments for all species.

The other important feature is that the left-handed fermions are assigned to SU(2) doublets, while the right-handed fermions are assigned to SU(2) singlets.

The fact that the W couples only to left-handed species is a crucial property that shapes the Standard Model, both positively and negatively.   It is therefore important to understand that this feature is extremely well supported experimentally.   In the next part of this lecture, I will review some surprisingly strong pieces of evidence for this structure.

For these applications, I will go to energies $E \ll m_W$ and approximate

$$\frac{1}{q^2 - m_W^2} \to -\frac{1}{m_W^2}$$

In this limit, the W exchange can be written as the dimension-6 operator

$$\delta\mathcal{L} = \frac{g^2}{2m_W^2}\; J^+_\mu J^{-\mu}$$

where

$$\begin{aligned} J^+_\mu &= \nu_L^\dagger \overline{\sigma}_\mu e_L + u_L^\dagger \overline{\sigma}_\mu d_L + \cdots \\ J^-_\mu &= e_L^\dagger \overline{\sigma}_\mu \nu_L + d_L^\dagger \overline{\sigma}_\mu u_L + \cdots \end{aligned}$$

and the coefficient is conventionally defined as

$$\frac{g^2}{2m_W^2} = \frac{4G_F}{\sqrt{2}}$$

This theory is called the V-A theory, since

$$u_L^\dagger \overline{\sigma}^\mu d_L = \overline{u}\gamma^\mu \frac{1-\gamma^5}{2} d$$

It reflects maximal parity violation for the charge-changing weak interactions.

To discuss the consequences of V-A theory, I should first explain my conventions for fermions. For a Dirac fermion, I set

$$\Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \qquad \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix}$$

with

$$\sigma^\mu = (1, \vec{\sigma})^\mu \qquad \overline{\sigma}^\mu = (1, -\vec{\sigma})^\mu$$

Then, for example, a vector current takes the form

$$j^\mu = \overline{\psi}\gamma^\mu\psi = \psi_L^\dagger \overline{\sigma}^\mu \psi_L + \psi_R^\dagger \overline{\sigma}^\mu \psi_R$$

and divides neatly into L and R pieces. The L and R fields are linked by the fermion mass term. If we can ignore masses, the L and R fermion numbers are separately conserved.

The labels L,R here is called chirality. For a massless fermion, this is identical to the fermion helicity; for a massive fermion, there is a change of basis.

Some properties of these fermions are

For massive fermions moving in the 3 direction

$$p = (E, 0, 0, p)$$

$$U_L = \begin{pmatrix} \sqrt{E+p}\,\xi_L \\ \sqrt{E-p}\,\xi_L \end{pmatrix} \qquad V_L = \begin{pmatrix} \sqrt{E-p}\,\xi_R \\ -\sqrt{E+p}\,\xi_R \end{pmatrix}$$

$$U_R = \begin{pmatrix} \sqrt{E-p}\,\xi_R \\ \sqrt{E+p}\,\xi_R \end{pmatrix} \qquad V_R = \begin{pmatrix} \sqrt{E+p}\,\xi_L \\ -\sqrt{E-p}\,\xi_L \end{pmatrix}$$

with

$$\xi_L = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \xi_R = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Here U_L , for example, is the L helicity spinor, written in the chirality basis. For massless fermions, we use only the top (L) or the bottom (R) two components, which I call u, v.

The matrix element $\langle 0 | \, j^\mu \left| e^-_L e^+_R \right\rangle$ is given by

$$v^\dagger_R \overline{\sigma}^\mu u_L = \sqrt{2E}(-1 \ 0) \ (1, -\sigma^1, -\sigma^2, -\sigma^3)^\mu \begin{pmatrix} 0 \\ 1 \end{pmatrix} \sqrt{2E}$$
$$= 2E \ (0, 1, -i, 0)^\mu \quad = 2E \ \sqrt{2}\epsilon^\mu_-$$

the polarization vector for the spin 1 virtual photon.

So for a current-current annihilation process such as

$$e^-_L e^+_R \to \mu^-_L \mu^+_R$$

we find $(u^\dagger_L \overline{\sigma}_\mu v_R)(v^\dagger_R \overline{\sigma}^\mu u_L) \quad = (2E)^2 \ 2 \ \epsilon'_- \cdot \epsilon_-$

$$= s(1 + \cos\theta) = -2u$$

Another way to write this is

$$|(u^\dagger_L \overline{\sigma}_\mu v_R)(v^\dagger_R \overline{\sigma}^\mu u_L)|^2 = 4(2p_{e-} \cdot p_{\mu+})(2p_{e^+} \cdot p_{\mu-})$$

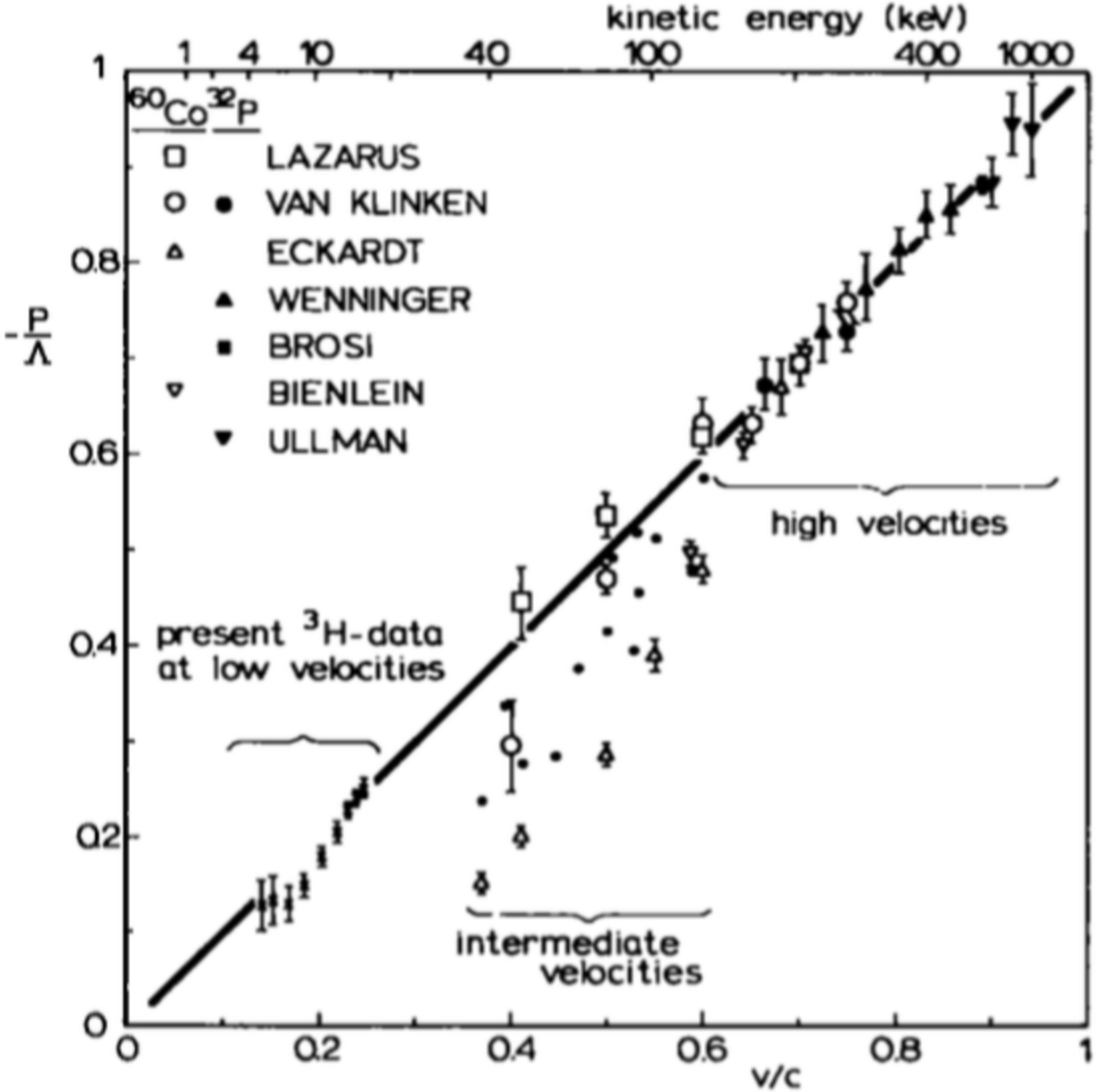
It is a nice exercise to check this answer using the usual trace theorems.

Now we can look into the consequences of the V-A theory.

1. The V-A theory implies that electrons emitted in β decay are left-handed. More precisely, for an electron that is not completely relativistic,

$$Pol(e^-) = \frac{(\sqrt{E+p})^2 - (\sqrt{E-p})}{(\sqrt{E+p})^2 + (\sqrt{E-p})} = \frac{p}{E} = v$$

By looking at a variety of β transitions, we can test the dependence on v.



Koks and van Klinken

2. The V-A structure of the weak coupling leads to a matrix element for muon decay

$$|\mathcal{M}(\mu^- \to \nu_\mu e^- \overline{\nu}_e)|^2 \sim (2p_\mu \cdot p_{\overline{\nu}})(2p_e \cdot p_\nu)$$

The neutrinos emitted in muon decay are not visible, but still this expression leads to a characteristic shape.

Recall formulae for 3-body phase space:

$$p_\mu = (m_\mu, \vec{0}) = p_e + p_\nu + p_{\overline{n}u}$$

$$x_i = \frac{2p_i \cdot p_\mu}{m_\mu^2} \qquad x_e + x_\nu + x_{\overline{\nu}} = 2$$

$$2p_e \cdot p_\nu = (p_e + p_\nu)^2 = (p_\mu - p_{\overline{\nu}})^2 = m_\mu^2(1 - x_{\overline{\nu}})$$

and (Dalitz!)

$$\int \Pi_3 = \frac{m_\mu^2}{128\pi^2} \int dx_e dx_{\overline{\nu}}$$

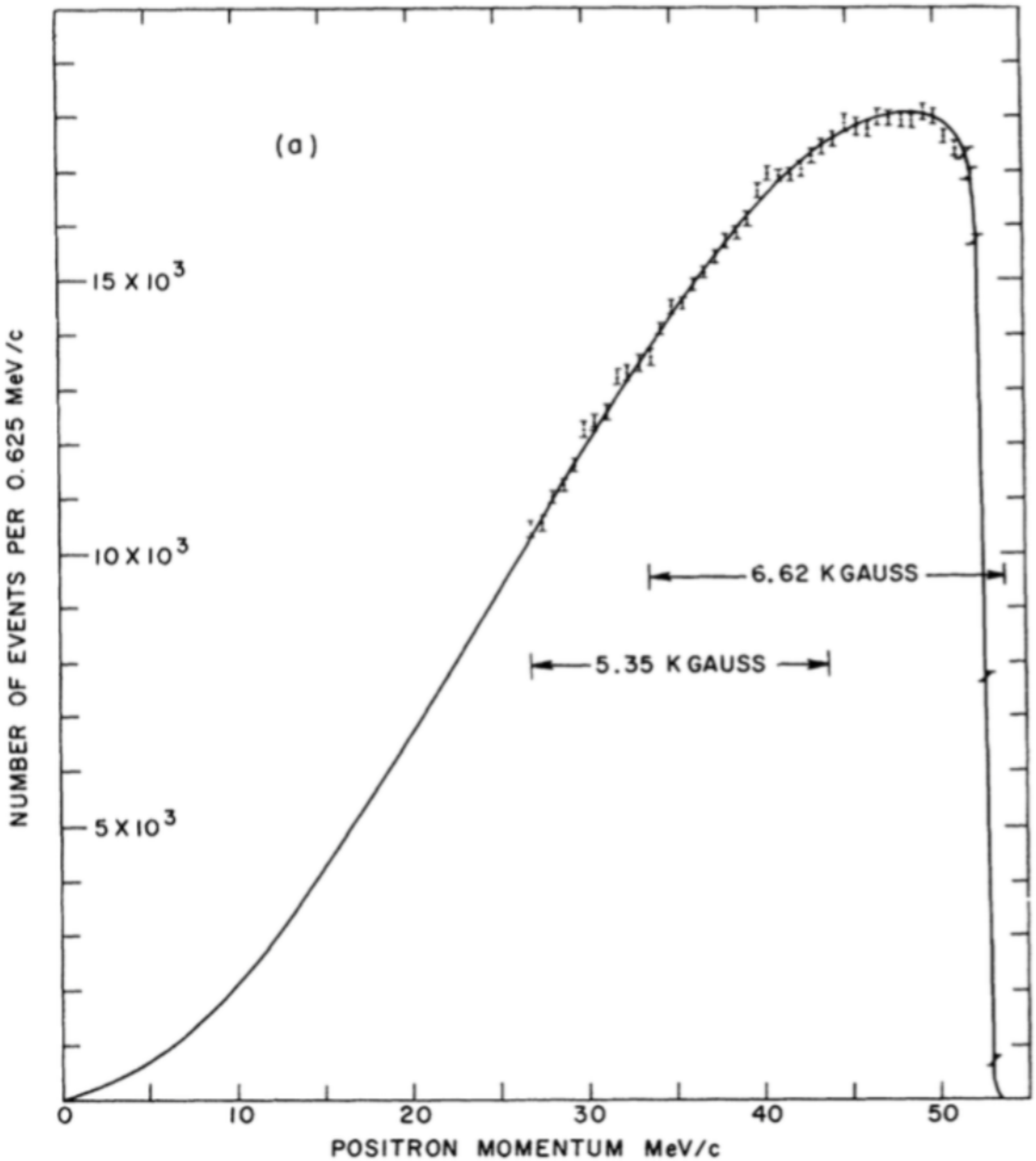
Then the muon decay rate is proportional to

$$\int_0^1 dx_e \int_{1-x_e}^1 dx_{\overline{\nu}}\ x_{\overline{\nu}}(1 - x_{\overline{\nu}})$$

that is,

$$\frac{d\Gamma}{dx_e} \sim \int_{1-x_e}^1 dx_{\overline{\nu}}\ x_{\overline{\nu}}(1 - x_{\overline{\nu}}) = (\frac{x_e^2}{2} - \frac{x_e^3}{3})$$

This shape, with a double zero at x = 0 and zero slope at the endpoint, is seen in the data.



(a)
15 X 10^3
10 X 10^3
5 X 10^3
6.62 K GAUSS
5.35 K GAUSS
NUMBER OF EVENTS PER 0.625 MeV/c
0
10
20
30
40
50
POSITRON MOMENTUM MeV/c

Bardon et al.

3. Charged pion decay is mediated by the V-A operator

$$\frac{4G_F}{\sqrt{2}}\ (d_L^\dagger \overline{\sigma}^\mu u_L)[\nu_{eL}^\dagger \overline{\sigma}_\mu e_L + \nu_{\mu L}^\dagger \overline{\sigma}_\mu \mu_L]$$

At first sight, it might seem that the pion must decay equally often to e and μ. This would contradict experiment, which says that almost all decays are to μ. But, what is the real prediction of V-A ?

The pion matrix element is

$$\langle 0 | \, (d_L^\dagger \overline{\sigma}^\mu u_L) \left| \pi^+(p) \right\rangle = -i \frac{1}{2} F_\pi p^\mu$$

where F_π is the pion decay constant, equal to 135 MeV. Then the complete matrix element involves

$$p_\mu \ u_{\nu L}^\dagger(p_\nu) \overline{\sigma}^\mu v(p_{\ell+})$$

The neutrino must be left-handed, by V-A. But, the pion is spin 0, so the lepton must also be left-handed. The neutrino and lepton spinors are

$$U_L = \begin{pmatrix} \sqrt{2E_\nu}\ \xi_L \\ 0 \end{pmatrix} \qquad V_L = \begin{pmatrix} \sqrt{E_\ell - p_\ell}\xi_R \\ \times \end{pmatrix}$$

Then the matrix element is proportional to

$$|\mathcal{M}|^2 \sim (E_\ell - p_\ell)E_\nu = \frac{m_\ell^2}{m_\pi^2}(\frac{m_\pi^2 - m_\ell^2}{2m_\pi^2})$$

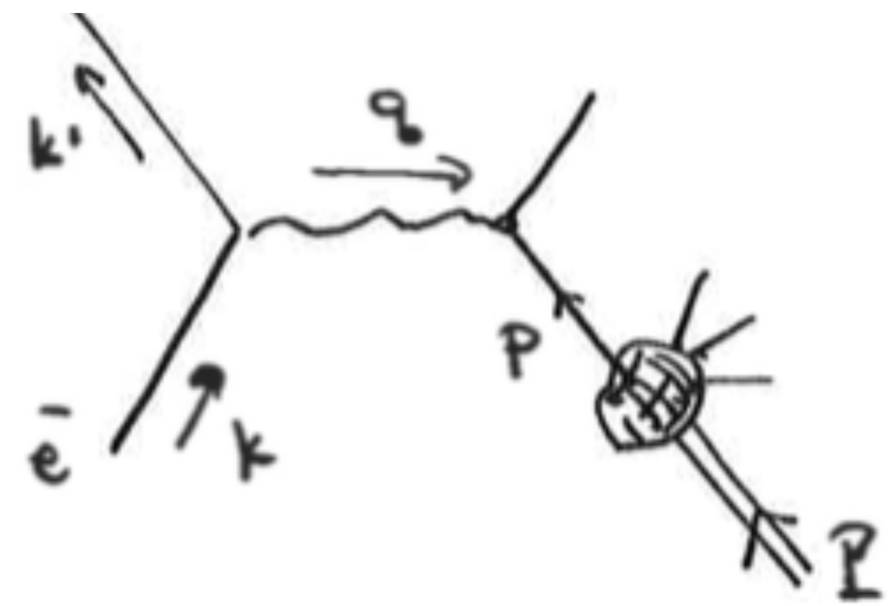
There is another factor of E_ν from phase space. Then V-A predicts

$$\frac{BR(\pi^- \to e^-\overline{\nu})}{BR(\pi^- \to \mu^-\overline{\nu})} = \frac{m_e^2}{m_\mu^2}\left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right)^2 = 1.28 \times 10^{-4}$$

comppared to experiment: 1.23×10^{-4}

4. The helicity structure of the V-A interaction between leptons and quarks is also seen in neutrino deep inelastic scattering. Electron deep inelastic scattering has the kinematics:

in leading order in QCD:



In neutrino deep inelastic scattering, we create this kinematic situation by producing neutrinos from pion decay, using an absorber (iron from a battleship) to remove muons, and then impinging the beam on a large target.

The kinematic variables of deep inelastic scattering are

$$s = (k+p)^2 \qquad\qquad Q^2 = -q^2$$

$$y = \frac{q^0}{k^0} = \frac{2P\cdot q}{2P\cdot k} \qquad\qquad x = \frac{Q^2}{2P\cdot q}$$

so, $Q^2 = xys$. The quark is a parton with momentum fraction ξ , $0 < \xi < 1$. Then in the lepton-parton reaction

$$\hat{s} = 2p\cdot k = 2\xi P\cdot k$$

$$\hat{t} = q^2 = -Q^2$$

$$\hat{u} = -2p\cdot k' = -2\xi P\cdot(k-q) = -\hat{s}(1-y)$$

The final quark is on shell, so

$$0 = (p+q)^2 = 2p\cdot q - Q^2 = 2\xi P\cdot q - Q^2$$

and ξ is equal to the observable x !

What concerns us here is the distribution in y. For the reaction

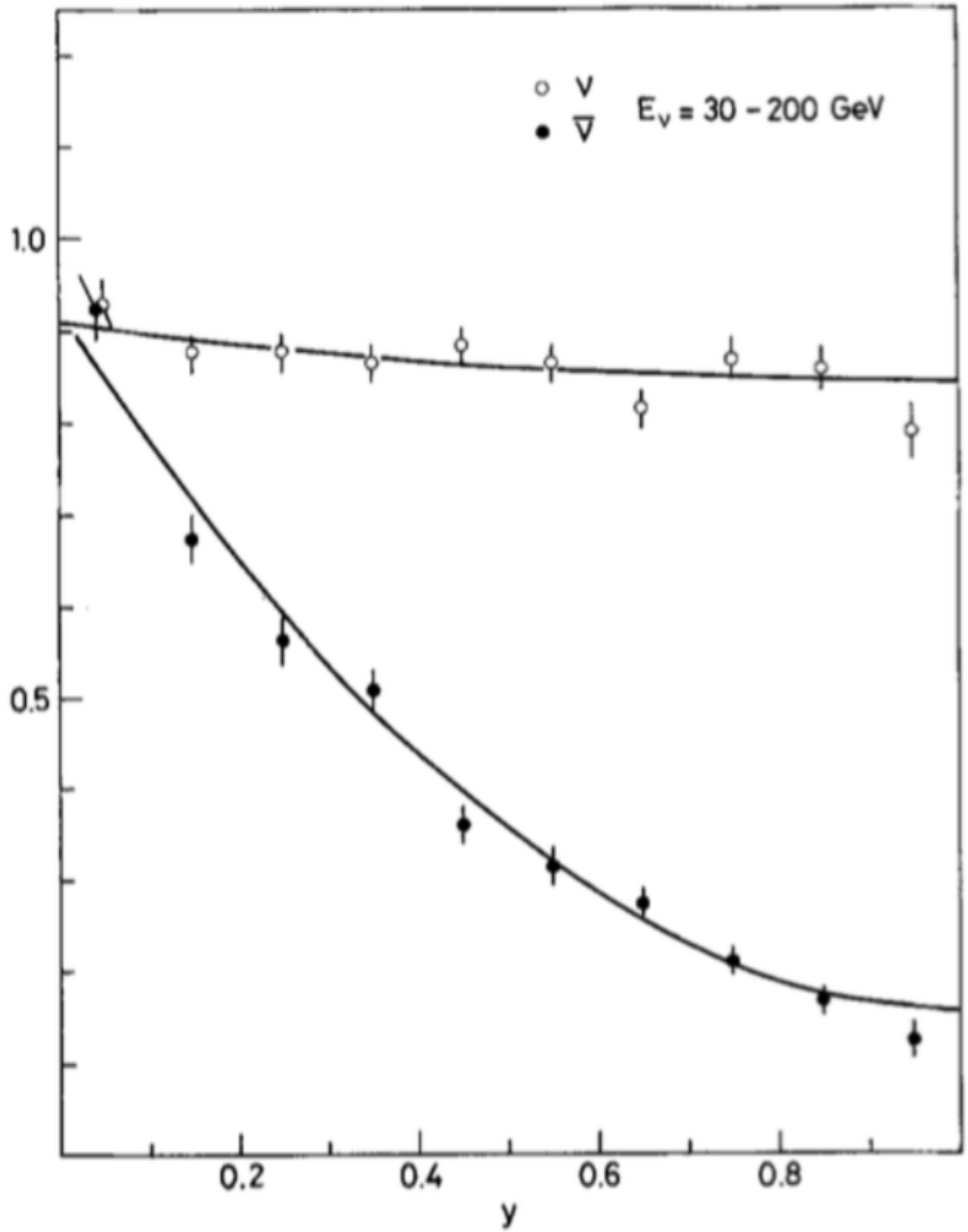
$$\nu + d \to \mu^- + u$$

the basic current-current amplitudes would be

$$|(u_L^\dagger(p_\mu)\overline{\sigma}^\mu u_L(\nu))(u_L^\dagger(u)\overline{\sigma}_\mu u_L(d))|^2$$
$$= 4(2p_\mu \cdot p_u)(2p_\nu \cdot p_d) = 4s^2$$

$$|(u_L^\dagger(p_\mu)\overline{\sigma}^\mu u_L(\nu))(u_R^\dagger(u)\overline{\sigma}_\mu u_R(d))|^2$$
$$= 4(2p_\mu \cdot p_d)(2p_\nu \cdot p_u) = 4u^2 = 4s^2(1-y)^2$$

Neutrinos from π^+decay are L. V-A says that they have no charge-changing weak coupling to R quarks. Then the $(1-y)^2$ term should be absent. Conversely, antineutrinos are R, so the deep inelastic cross section should be proportional to $(1-y)^2$.



CDHS experiment

In the more modern era, we test these predictions in collider physics. For example, the Standard Model predicts that

$$\frac{d\sigma}{d\cos\theta_*}(d\overline{u} \to W^- \to \mu^-\overline{\nu}) \sim u^2 \sim (1+\cos\theta_*)^2$$

$$\frac{d\sigma}{d\cos\theta_*}(u\overline{d} \to W^+ \to \mu^+\nu) \sim t^2 \sim (1-\cos\theta_*)^2$$

These angular distributions are well verified at the LHC.

The neutral current amplitudes are more complex, because the photon and Z couple to both L and R fermions. In e^+e^- annihilation (for example, at LEP), the angular distributions are

$$\frac{d\sigma}{d\cos\theta}(e^-_L e^+_R \to f_L \overline{f}_R) = \frac{\pi\alpha^2}{2s} F_{LL}(s)(1+\cos\theta)^2$$

$$\frac{d\sigma}{d\cos\theta}(e^-_R e^+_L \to f_L \overline{f}_R) = \frac{\pi\alpha^2}{2s} F_{RL}(s)(1-\cos\theta)^2$$

$$\frac{d\sigma}{d\cos\theta}(e^-_L e^+_R \to f_R \overline{f}_L) = \frac{\pi\alpha^2}{2s} F_{LR}(s)(1-\cos\theta)^2$$

$$\frac{d\sigma}{d\cos\theta}(e^-_R e^+_L \to f_R \overline{f}_L) = \frac{\pi\alpha^2}{2s} F_{RR}(s)(1+\cos\theta)^2$$

where

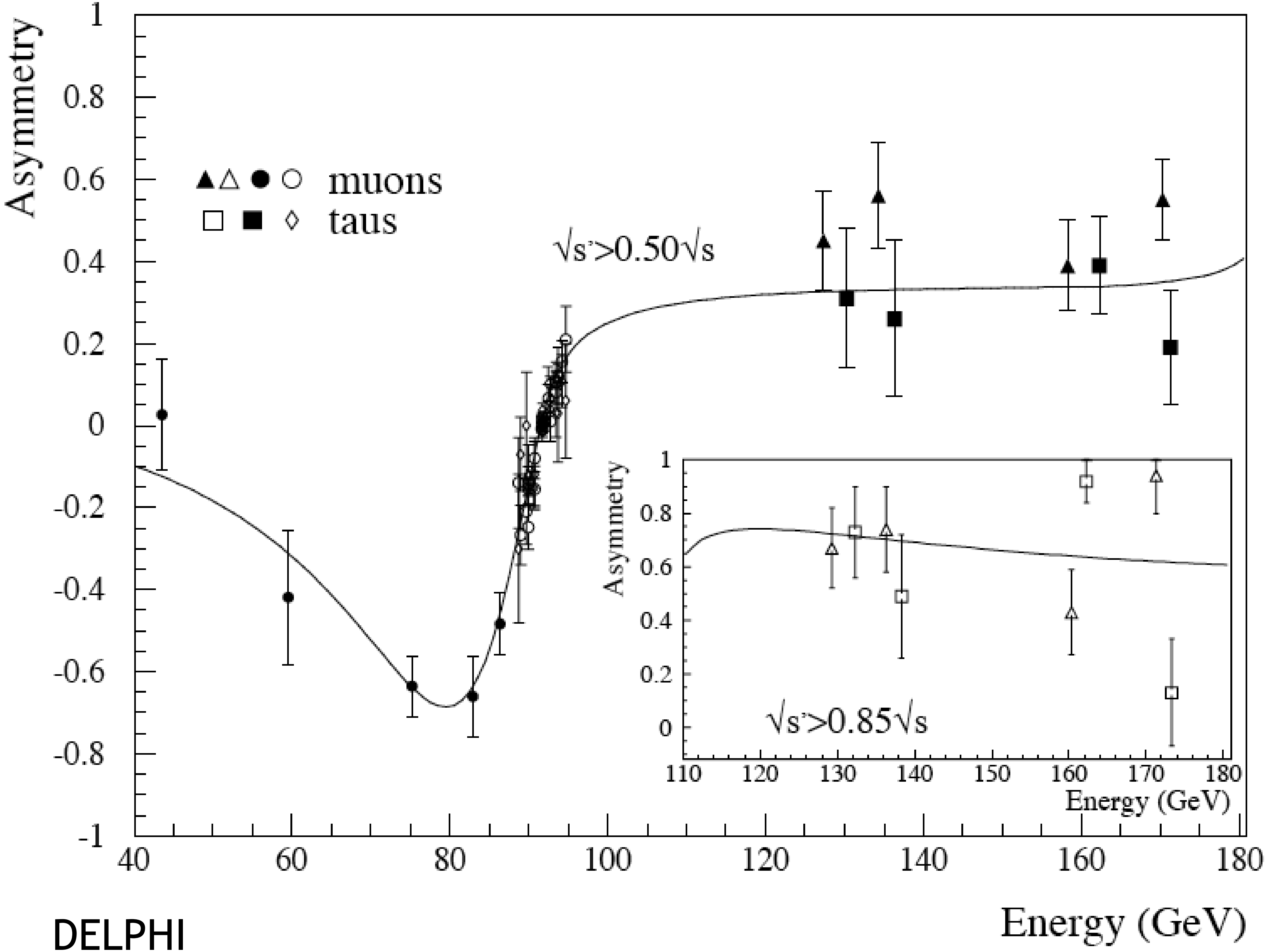
$$F_{LL} = \left| Q_f \; + \; \frac{(1/2 - s_w^2)(I_f^3 - s_w^2 Q_f)}{s_w^2 c_w^2} \frac{s}{s - m_Z^2} \right|^2$$

$$F_{RL} = \left| Q_f \; + \; \frac{(-s_w^2)(I_f^3 - s_w^2 Q_f)}{s_w^2 c_w^2} \frac{s}{s - m_Z^2} \right|^2$$

$$F_{LR} = \left| Q_f \; + \; \frac{(1/2 - s_w^2)(-s_w^2 Q_f)}{s_w^2 c_w^2} \frac{s}{s - m_Z^2} \right|^2$$

$$F_{RR} = \left| Q_f \; + \; \frac{(-s_w^2)(-s_w^2 Q_f)}{s_w^2 c_w^2} \frac{s}{s - m_Z^2} \right|^2$$

Note, for $s > m_Z^2$, constructive interference for LL and RR, destructive interference for RL and LR. Then, with unpolarized beams (as at LEP), we expect a positive forward-backward asymmetry.



DELPHI

It is interesting to explore the high energy limits of the expressions $F_{IJ}(s)$. Begin with $F_{RL}(s)$. In the limit $s \gg m_Z^2$, this becomes

$$F_{RL} \to \left| \frac{s_w^2 c_w^2 (I_f^3 + Y_f) - s_w^2 I_f^3 + s_w^4 (I_f^3 + Y_f)}{s_w^2 c_w^2} \right|^2$$
$$= \left| \frac{s_w^2 Y_f}{s_w^2 c_w^2} \right|^2 = \left| \frac{(-1) Y_f}{c_w^2} \right|^2 = \left| \frac{g'^2}{e^2} Y_{eR} Y_f \right|^2$$

This is exactly the amplitude for s-channel B boson exchange, in the situation where the original SU(2)xU(1) symmetry of the model is not broken.

The simplicity of this expression tells us that it is useful to analyze the high-energy limit of the weak interactions from the viewpoint that broken symmetry is restored at high energy.

Here is the same analysis for $F_{LL}(s)$:

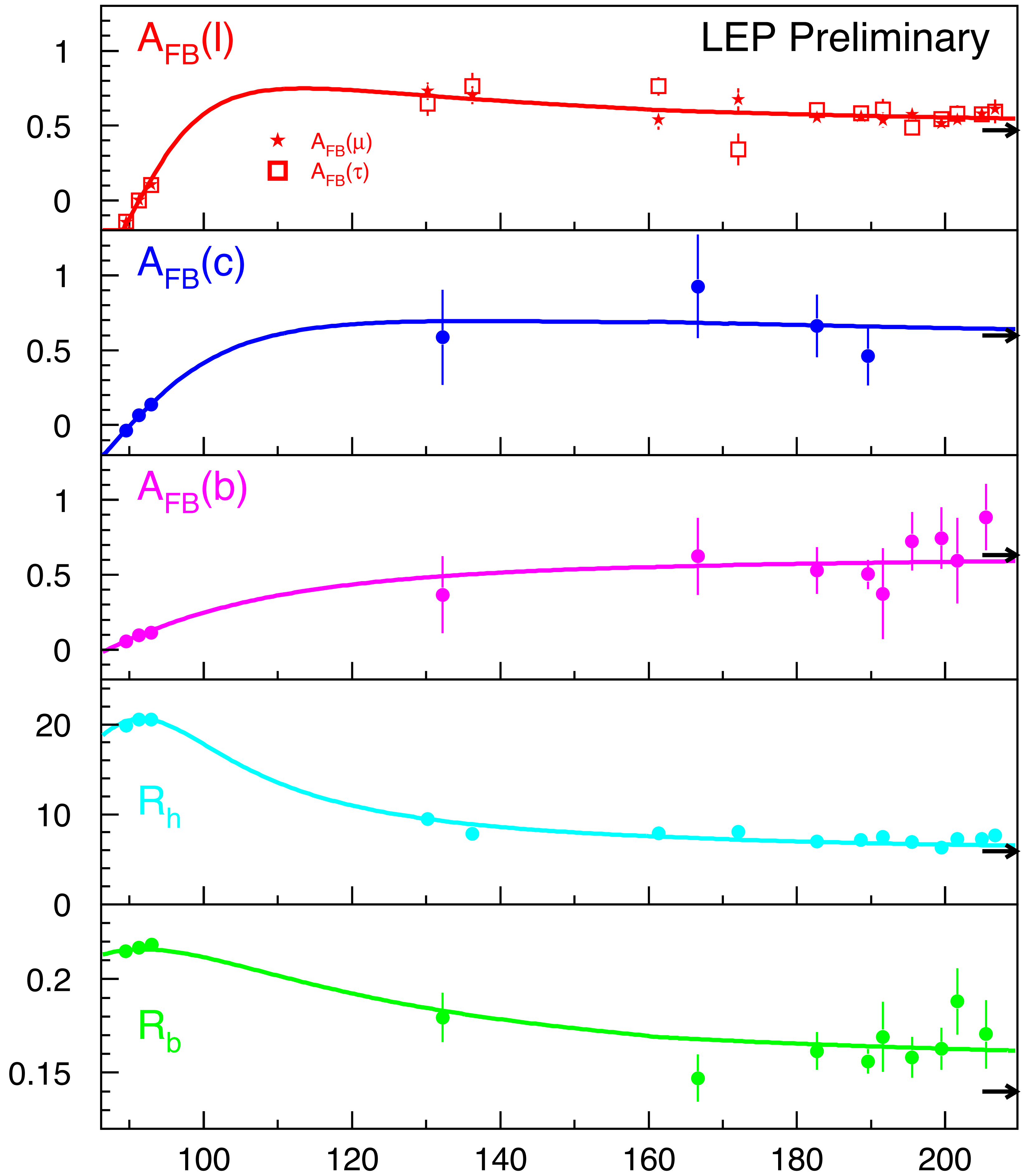
$$F_{LL} \to \left| \frac{s_w^2 c_w^2 (I_f^3 + Y_f) + (1/2 - s_w^2)(I_f^3 - s_w^2(I_f^3 + Y_f))}{s_w^2 c_w^2} \right|^2$$

$$= \left| \frac{(1/2) c_w^2 I_f^3 + (1/2) s_w^2 Y_f}{s_w^2 c_w^2} \right|^2$$

$$= \left| \frac{g^2}{e^2} I_{eL}^3 I_f^3 + \frac{g'^2}{e^2} Y_{eL} Y_f \right|^2$$

so the result is a coherent sum of A^3 and B exchanges as expected in the theory with unbroken symmetry.

Here is the approach to the limit of the symmetric theory as measured at LEP:



data compilation
by Hildreth