The Electroweak Standard Model

2. Precision tests at the Z resonance

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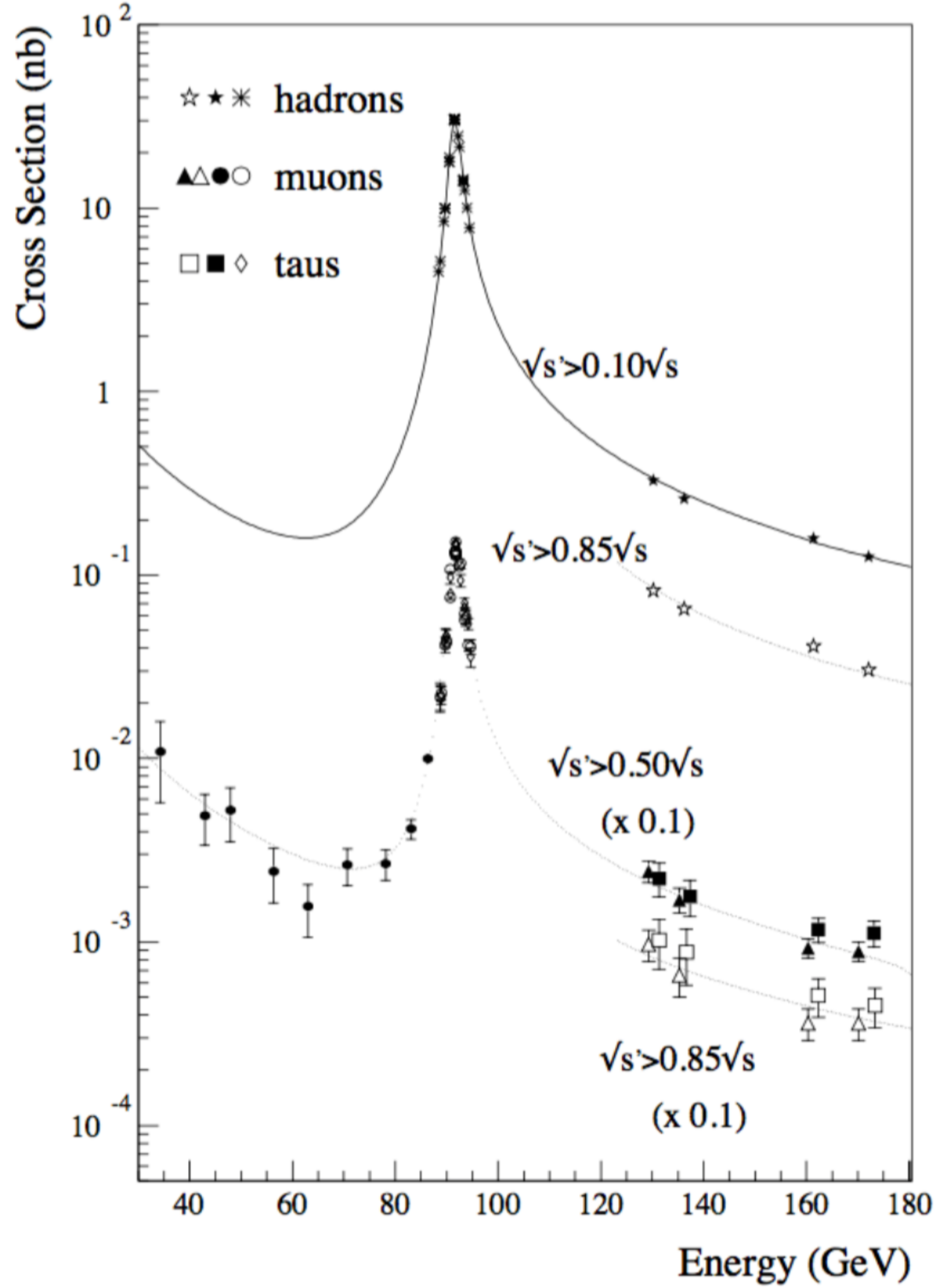
We saw in the previous lecture that the Z boson appears as a resonance in e^+e^- annihilation. In the 1990's, the accelerators LEP and SLC tuned their energy to the Z mass to produce large numbers of Z bosons at rest in the lab, in an appropriate setting for precision measurements.

LEP also operated above 200 GeV, to study the electroweak pair production of W and Z bosons. I will discuss that program in the next lecture.

In this lecture, I will review the precision weak interaction experiments at the Z, which continue to provide important constraints on the Standard Model and its generalizations.

The e^+e^- cross section in the vicinity of the Z resonance.



DELPHI

To begin, we should work out the Z width and branching fractions at leading order.

The leading order matrix element for Z decay to $f_L \overline{f}_R$ is

$$\mathcal{M}(Z \to f_L \overline{f}_R) = i\frac{g}{c_w}\ Q_{Zf}\ u_L^\dagger \overline{\sigma}^\mu v_R\ \epsilon_{Z\mu}$$

with

$$Q_Z = I^3 - s_w^2 Q$$

Recall from the previous lecture that

$$u_L^\dagger \overline{\sigma}^\mu v_R = (2E)\ \sqrt{2}\ (\epsilon_-^\mu)^*$$

We can integrate over the fermion direction, but it is simpler, and equivalent, to average over the direction of the Z polarization. Then

$$\langle |\mathcal{M}|^2 \rangle = \frac{2}{3}\frac{g^2}{c_w^2} Q_Z^2 m_Z^2$$

Then

$$\Gamma(Z \to f_L \overline{f}_R) = \frac{1}{2m_Z} \frac{1}{8\pi} \left\langle |M|^2 \right\rangle$$

So, finally,

$$\Gamma(Z \to f_L \overline{f}_R) = \frac{\alpha_w m_Z}{6c_w^2} \; Q_Z^2 \; N_f$$

where

$$\alpha_w = \frac{g^2}{4\pi} \qquad\qquad N_f = \begin{cases} 1 & \text{lepton} \\ 3(1 + \alpha_s/\pi + \cdots) & \text{quark} \end{cases}$$

The widths to right-handed species $f_R \overline{f}_L$ obey the same formulae. Now we only need to evaluate these formulae and sum over all Standard Model species that can appear in Z decays.

It is worth pausing to ask what values of coupling constants we should use to evaluate this formula.

Begin with α . You all know that $\alpha = 1/137$. However, α is a running coupling constant that takes larger values as the length scale on which it is considered decreases. At $Q = 91.\ \mathrm{GeV}$, $\alpha(Q) = 1/128$. Later in the lecture, I will defend a value of s_w^2

$$s_w^2 = 0.23$$

For this value, we find

$$\alpha_w = \frac{1}{29.6} \qquad \alpha' = \frac{1}{98.}$$

It is interesting to compare these to other fundamental Standard Model couplings at the same scale:

$$\alpha_s = \frac{1}{8.5} \qquad \alpha_t = \frac{y_t^2}{4\pi} = \frac{1}{12.7}$$

We combine with these values the values of the Q_Z .
It is useful to tabulate these for one Standard Model generation:

species	Q_{ZL}	Q_{ZR}	S_f	A_f
ν	$+\frac{1}{2}$	$-$	0.250	1.00
e	$-\frac{1}{2}+s_w^2$	$+s_w^2$	0.126	0.15
u	$+\frac{1}{2}-\frac{2}{3}s_w^2$	$-\frac{2}{3}s_w^2$	0.144	0.67
d	$-\frac{1}{2}+\frac{1}{3}s_w^2$	$+\frac{1}{3}s_w^2$	0.185	0.94

In this table, the quantities evaluated numerically are

$$S_f = Q_{ZL}^2 + Q_{ZR}^2 \qquad A_f = \frac{Q_{ZL}^2 - Q_{ZR}^2}{Q_{ZL}^2 + Q_{ZR}^2}$$

The first of these gives the total decay rate for the species f . The second gives the polarization asymmetry, the preponderance of f_L over f_R in Z decays.

It is possible to measure both the rates and the asymmetries in Z resonance experiments.

The S_f are tested by the Z total width and branching ratios. At the level of our leading-order theory, the width is

$$\Gamma_Z = \frac{\alpha_w m_Z}{6c_w^2}\Big[\underset{\nu}{3 \cdot 0.25} + \underset{e}{3 \cdot 0.126} + \underset{u}{2 \cdot (3.1) \cdot 0.144} + \underset{d}{3 \cdot (3.1) \cdot 0.185}\Big]$$

The separate terms in this formula give the branching ratios

$$BR(\nu_e \overline{\nu}_e) = 6.7\% \qquad BR(e^+e^-) = 3.3\%$$

$$BR(u\overline{u}) = 11.9\% \qquad BR(d\overline{d}) = 15.3\%$$

The numerical value of the total is $\Gamma_Z = 2.49\ \text{GeV}$

This can be compared to the value obtained from the Z resonance lineshape

$$\Gamma_Z = 2.4952 \pm 0.0023\ \text{GeV}$$

The precision of the Z resonance measurements is quite remarkable, reaching parts per mil for many variables. To discuss the rapport between theory and experiment at this level, we need to include electroweak radiative corrections, which typically are of order 1%.

As I continue to discuss the experimental results, I will make reference to radiative corrections that are particularly important.

To give a complete accounting of radiative corrections, I should give a precise account of the renormalization conventions used. Please let me postpone that discussion to later in the lecture (where, in any event, I will still not treat it completely).

To begin the review of experiments, I should discuss the measurement of the Z mass and width in more detail.

Ideally, the Z is a Breit-Wigner resonance,

$$\sigma \sim \left| \frac{1}{s - m_Z^2 + i m_Z \Gamma_Z} \right|^2$$

however, the line shape is distorted by initial state radiation. The magnitude of collinear photon radiation is given by the parameter

$$\beta = \frac{2\alpha}{\pi}(\log \frac{s}{m_e^2} - 1) = 0.108 \text{ at the Z}$$

In addition, since the Z is narrow, the effect is magnified, since relatively soft radiation can push the CM energy off resonance. The size of the correction on the Z peak can be roughly estimated as $-\beta \cdot \log \frac{m_Z}{\Gamma_Z} = -40\%$

To make a proper accounting, we need to resum collinear photon radiation just as we resum collinear gluon radiation in parton distributions.

Fadin and Kuraev computed the parton distribution of an electron in the electron and computed this in QED

$$f_e(z,s) = \frac{\beta}{2}(1-z)^{\beta/2-1}(1+\frac{3}{8}\beta) - \frac{1}{4}\beta(1+z) + \cdots$$

This function, for each electron, would be convolved with the Breit-Wigner. The theory was extended to include 2 orders of subleading logs and finite corrections of order α^2.

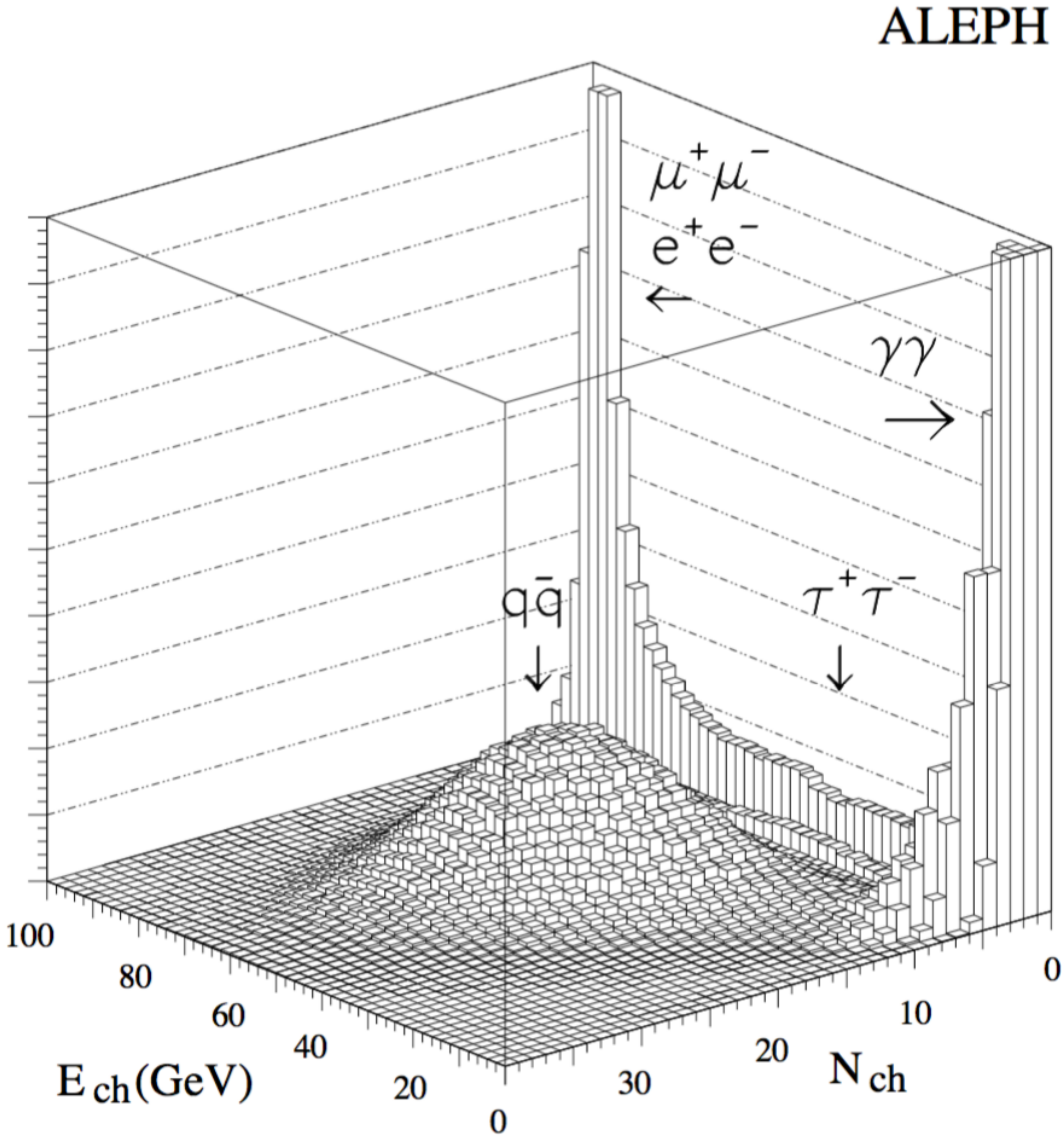
The experimental aspects of the measurement were also very challenging. The energy of the LEP ring was calibrated using resonant depolarization of a single beam and then corrected for 2-beam effects.

However, this calibration was found to depend on the season and the time of day. Some contributing effects were the changes in the size of the LEP/LHC tunnel due to the annual change in the water level of Lake Geneva and current surges in the magnets due to the passage of the TGV.

To measure the branching ratios, we need only collect Z events and sort them into categories.

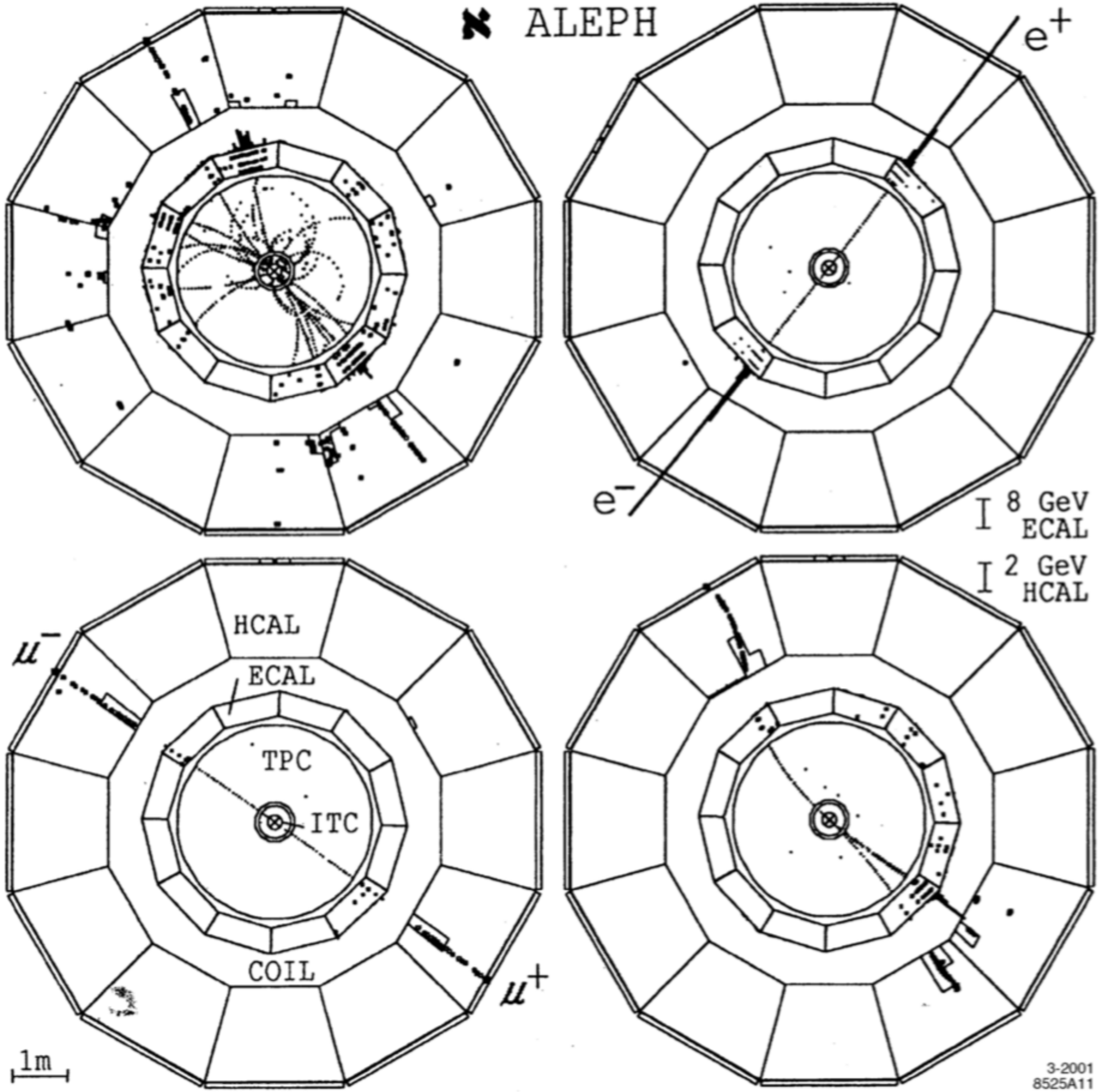
The major backgrounds are from Bhabha and 2-gamma events; these do not resemble Z events (unlike the situation at LHC !). Nonresonant annihilations are at the level of parts per mil (except for tau - few %).

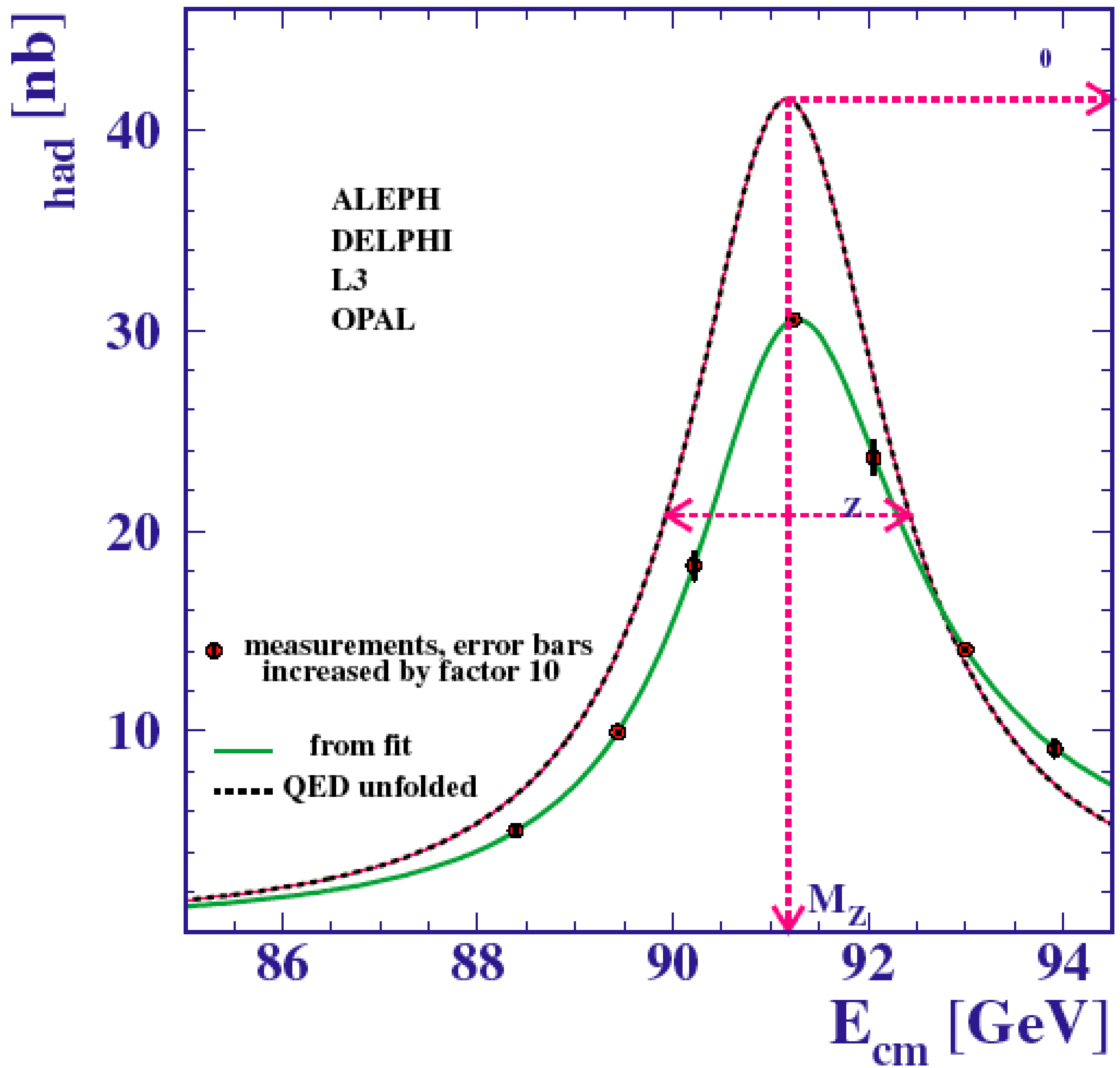
The various leptonic and hadronic decay modes have different, characteristic, forms.


ALEPH
$\mu^+\mu^-$
e^+e^-
$\gamma\gamma$
$q\bar{q}$
$\tau^+\tau^-$
100
80
60
40
20
0
E_{ch}(GeV)
0
10
20
30
N_{ch}

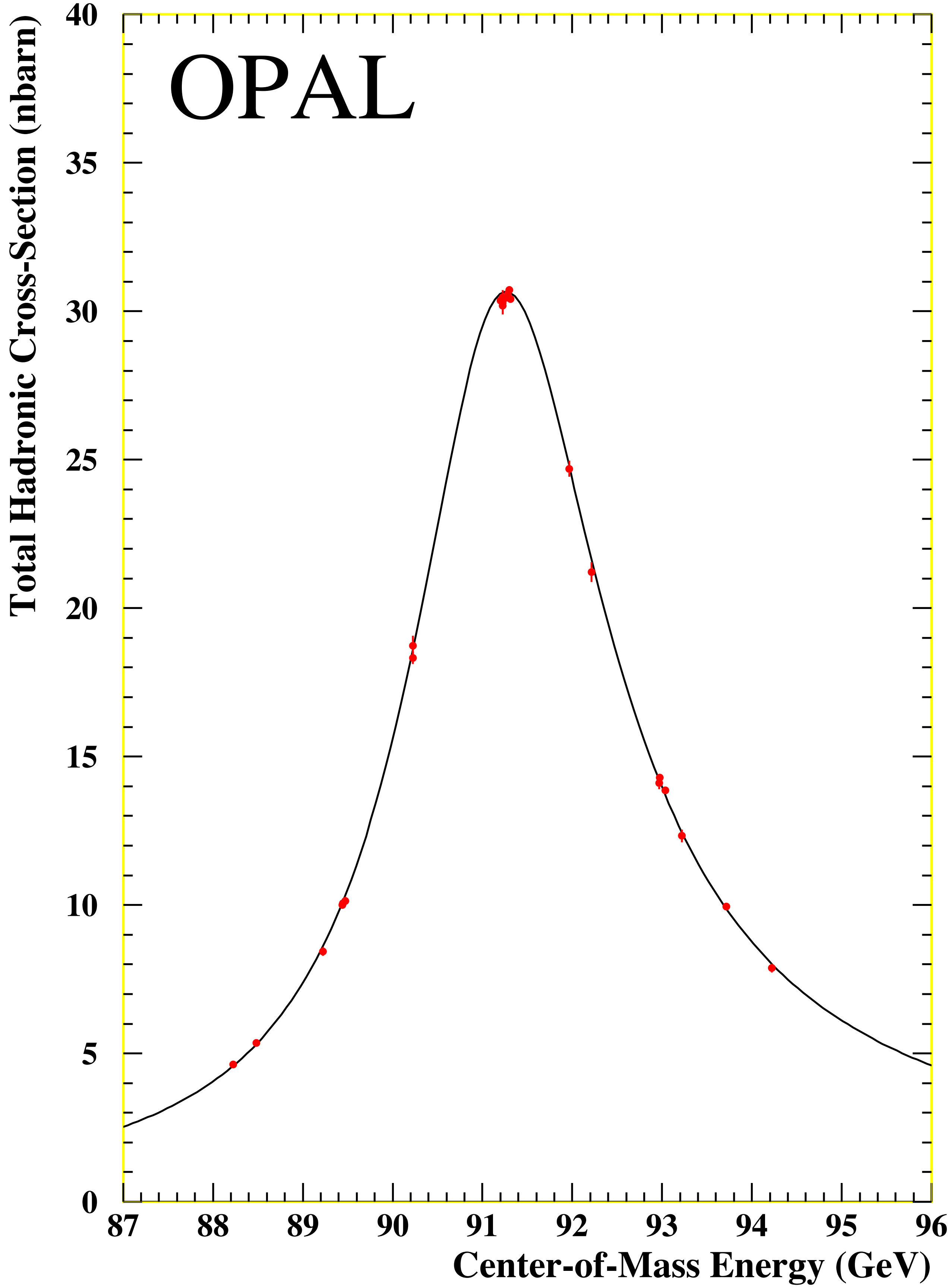
	ALEPH	DELPHI	L3	OPAL
$q\bar{q}$ final state				
acceptance	$s'/s > 0.01$	$s'/s > 0.01$	$s'/s > 0.01$	$s'/s > 0.01$
efficiency [%]	99.1	94.8	99.3	99.5
background [%]	0.7	0.5	0.3	0.3
e^+e^- final state				
acceptance	$-0.9 < \cos\theta < 0.7$ $s' > 4m_\tau^2$	$\lvert\cos\theta\rvert < 0.72$ $\eta < 10°$	$\lvert\cos\theta\rvert < 0.72$ $\eta < 25°$	$\lvert\cos\theta\rvert < 0.7$ $\eta < 10°$
efficiency [%]	97.4	97.0	98.0	99.0
background [%]	1.0	1.1	1.1	0.3
$\mu^+\mu^-$ final state				
acceptance	$\lvert\cos\theta\rvert < 0.9$ $s' > 4m_\tau^2$	$\lvert\cos\theta\rvert < 0.94$ $\eta < 20°$	$\lvert\cos\theta\rvert < 0.8$ $\eta < 90°$	$\lvert\cos\theta\rvert < 0.95$ $m_{f\bar{f}}^2/s > 0.01$
efficiency [%]	98.2	95.0	92.8	97.9
background [%]	0.2	1.2	1.5	1.0
$\tau^+\tau^-$ final state				
acceptance	$\lvert\cos\theta\rvert < 0.9$ $s' > 4m_\tau^2$	$0.035 < \lvert\cos\theta\rvert < 0.94$ $s' > 4m_\tau^2$	$\lvert\cos\theta\rvert < 0.92$ $\eta < 10°$	$\lvert\cos\theta\rvert < 0.9$ $m_{f\bar{f}}^2/s > 0.01$
efficiency [%]	92.1	72.0	70.9	86.2
background [%]	1.7	3.1	2.3	2.7

final LEPEWWG Phys. Rept. 2005


ALEPH
e+
e−
8 GeV ECAL
2 GeV HCAL
HCAL
ECAL
TPC
ITC
COIL
μ−
μ+
1m
3-2001
8525A11



composite of the four LEP experiments, showing the effect of ISR


OPAL
Total Hadronic Cross-Section (nbarn)
40
35
30
25
20
15
10
5
0
87
88
89
90
91
92
93
94
95
96
Center-of-Mass Energy (GeV)

Two particular branching ratios merit special attention.

First, the Z decays invisibly, to neutrinos, 20% of the time. This decay affects the cross section

$$\sigma(e^+e^- \to Z \to \text{ hadrons})$$

by decreasing the Z peak height and increasing the width. Measurement of these parameters and comparison to Standard Model predictions gives

$$n_\nu = 2.9840 \pm 0.0082$$

Second, the Z branching ratio to b quarks is of special interest, particularly because the b belongs to the same SU(2)xU(1) multiplet as the t_L .

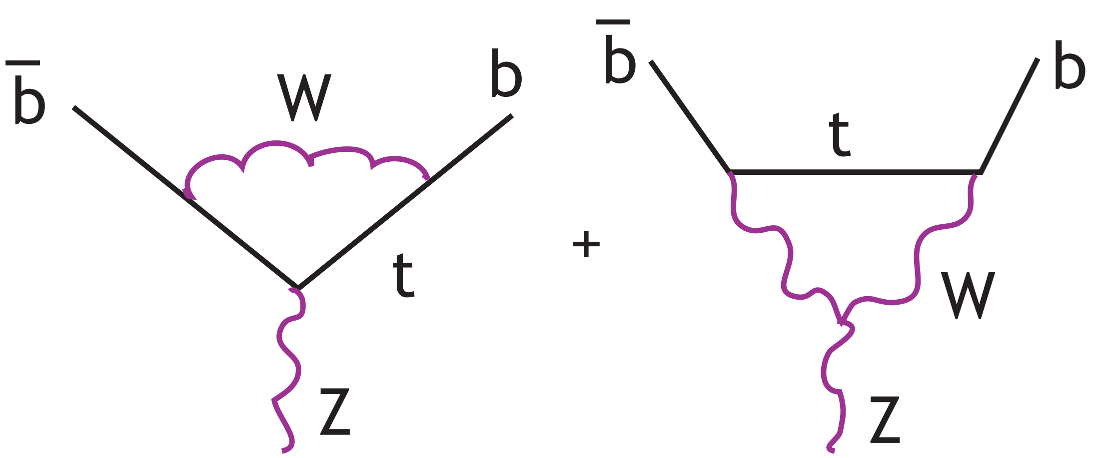
An observable that specifically tracks this effect is

$$R_b = \frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to \text{ hadrons})}$$

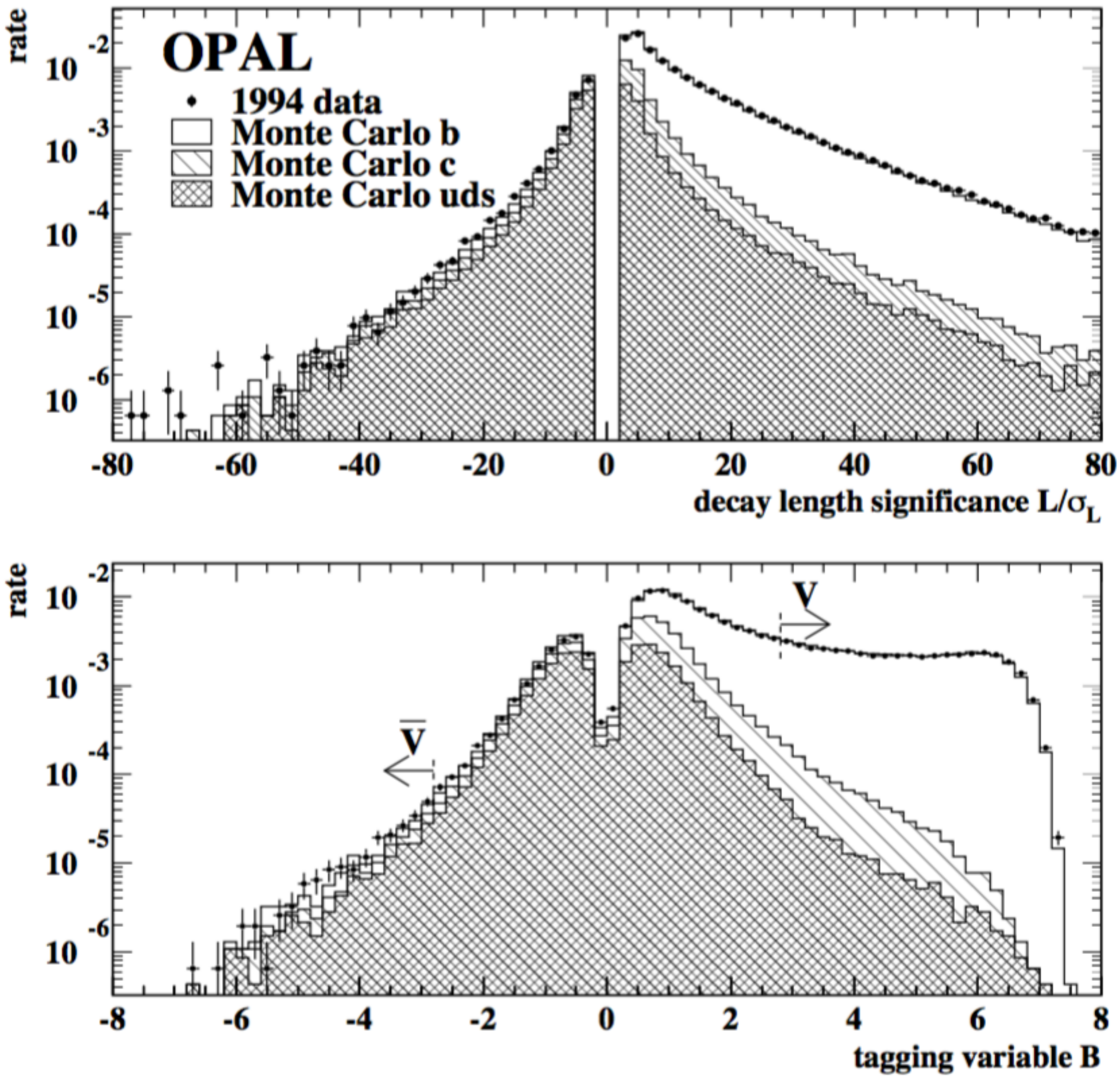
In the leading-order model, this quantity has the value

$$R_b = 0.22$$

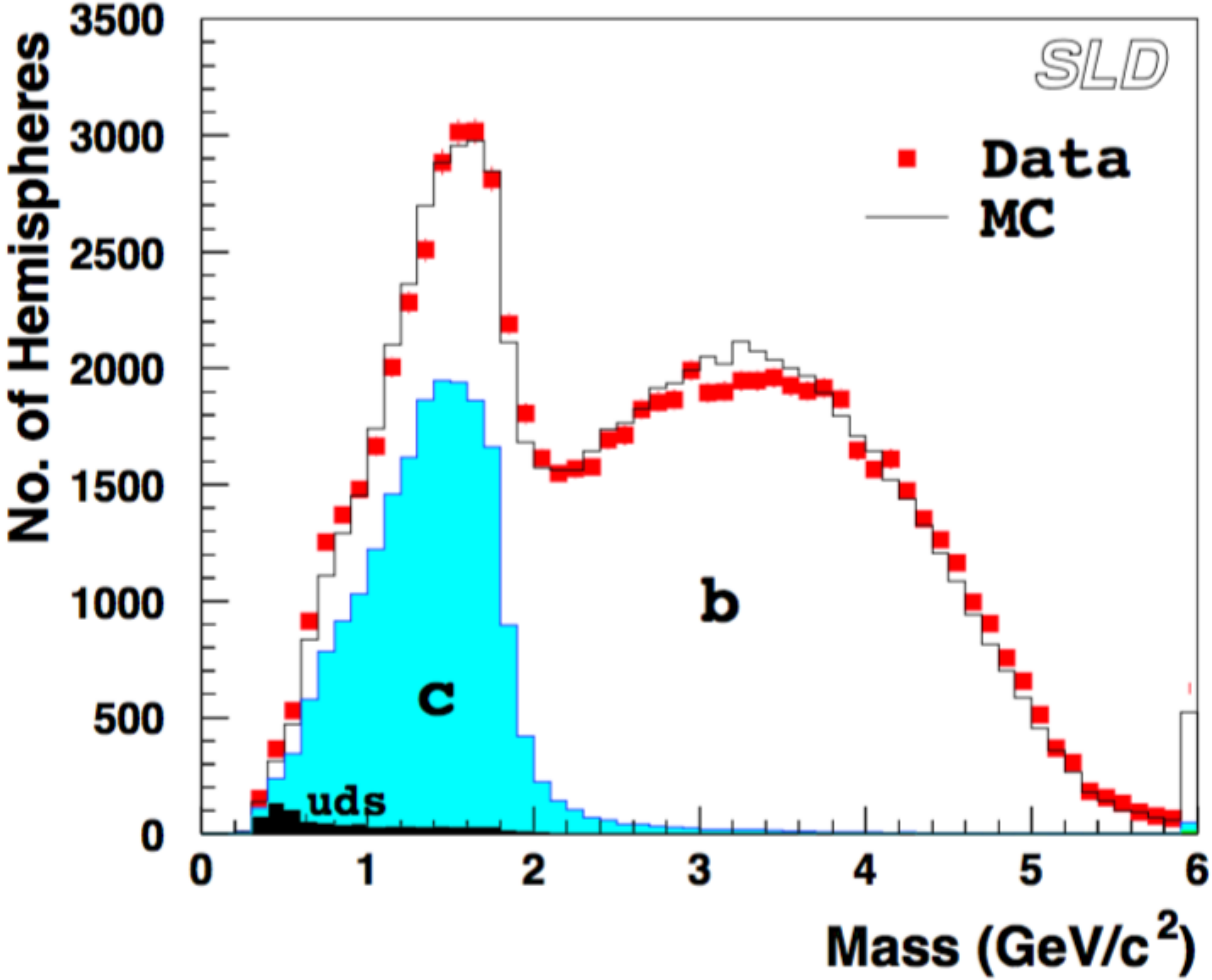
However there is a large radiative correction from diagrams involving the top quark



$$Q_{ZbL} = -(\frac{1}{2} - \frac{1}{3}s_w^2 - \frac{\alpha}{16\pi s_w^2}\frac{m_t^2}{m_W^2})$$



OPAL
1994 data
Monte Carlo b
Monte Carlo c
Monte Carlo uds
rate
decay length significance L/σ_L
tagging variable B
V
$\overline{V}$


SLD
Data
MC
b
c
uds
No. of Hemispheres
Mass (GeV/c^2)
0
500
1000
1500
2000
2500
3000
3500
0
1
2
3
4
5
6

b-tag working points used in these studies.

	ALEPH	DELPHI	L3	OPAL	SLD
b Purity [%]	97.8	98.6	84.3	96.7	98.3
b Efficiency [%]	22.7	29.6	23.7	25.5	61.8

The performance of SLD was much better due to its pixel vertex detector at 2 cm; however, the SLD statistics was 10 times smaller.

Final LEP/SLC results:

$$R_b = 0.21629 \pm 0.00066$$ (-2% from LO)

$$R_c = 0.1721 \pm 0.0030$$

Now turn to the Z asymmetries. These take very different values for l, c, b — all predicted by a common value of s_w^2 .

There are three very different methods to measure the lepton asymmetries:

from forward-backward asymmetries, esp. to quarks

from direct measurement using beam polarization

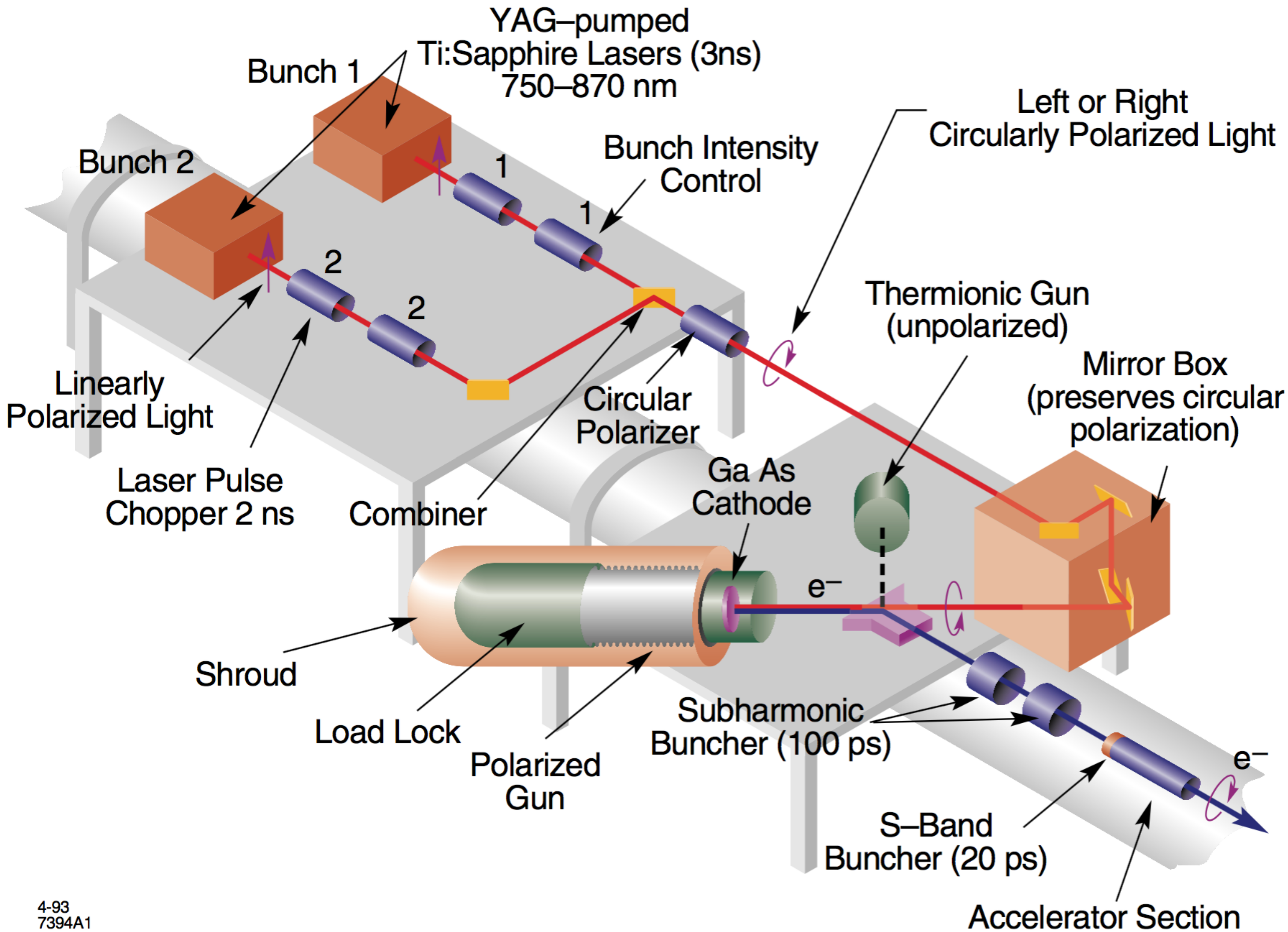
from tau lepton polarimetry

For unpolarized beams, the angular distribution for $e^+e^- \to f\overline{f}$ is :

$$\frac{d\sigma}{d\cos\theta}(e^+e^- \to f\overline{f}) \sim \left(\frac{1+A_e}{2}\right)\left(\frac{1+A_f}{2}\right)(1+\cos\theta)^2$$
$$+\left(\frac{1-A_e}{2}\right)\left(\frac{1+A_f}{2}\right)(1-\cos\theta)^2$$
$$+\left(\frac{1+A_e}{2}\right)\left(\frac{1-A_f}{2}\right)(1-\cos\theta)^2$$
$$+\left(\frac{1-A_e}{2}\right)\left(\frac{1-A_f}{2}\right)(1+\cos\theta)^2$$

This leads to

$$A_{FB} = \frac{3}{4}\ A_e\ A_f$$

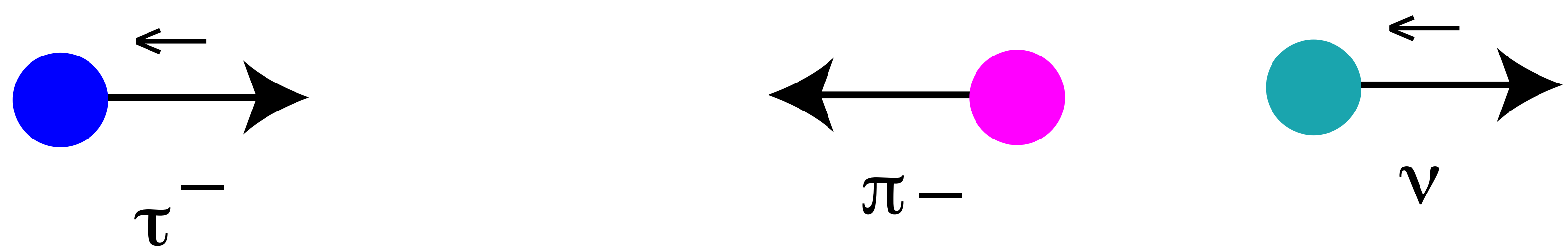


4 km to the right, measure a cross section asymmetry.

$$A_\ell = 0.1513 \pm 0.0021$$

Since τ leptons decay through V-A weak interactions, their decays are sensitive to the τ polarization.

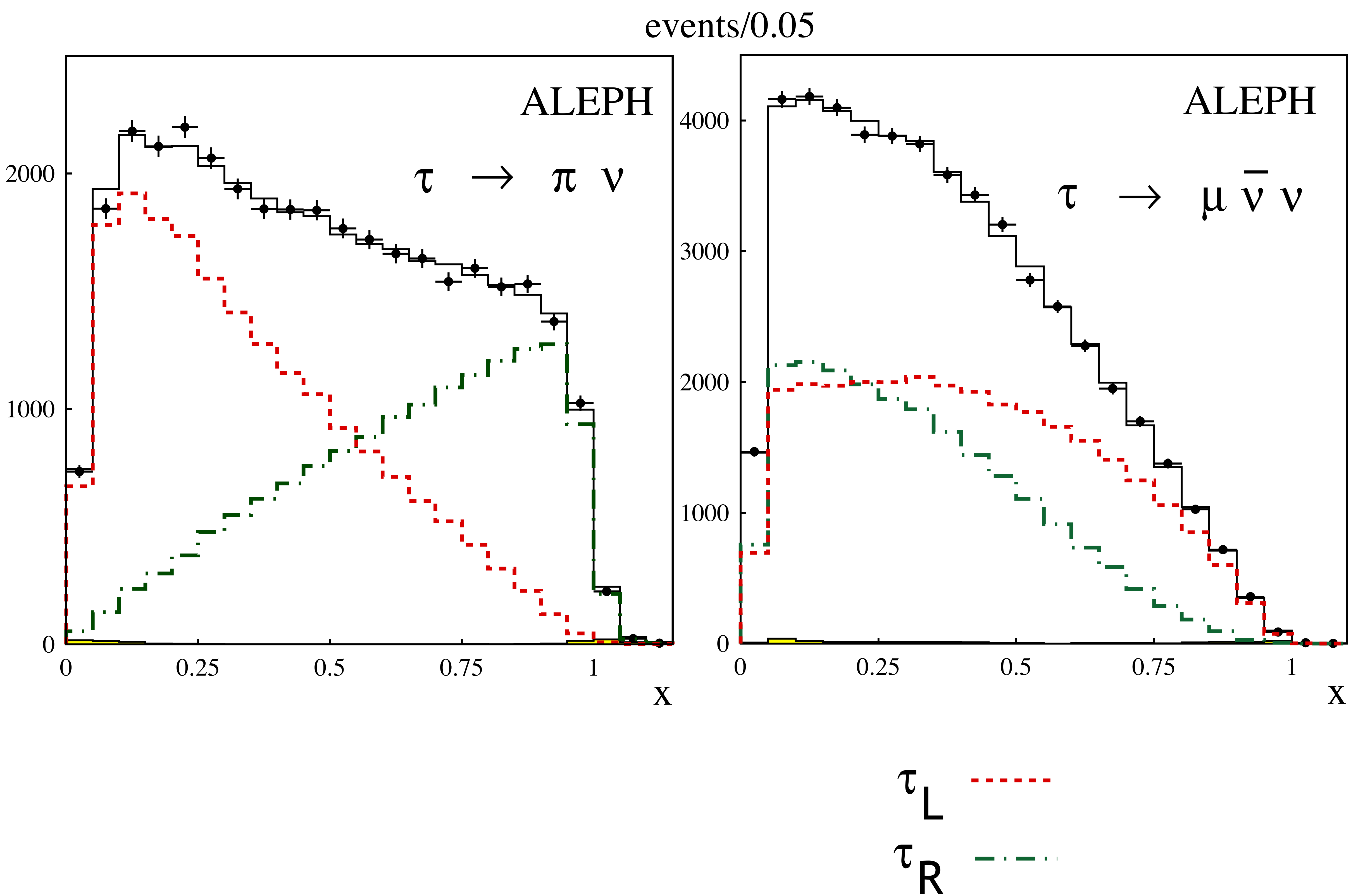
The easiest case to understand is $\tau^- \to \nu_\tau \pi^-$. A τ at rest with $S^3 = -\frac{1}{2}$ decays to a forward ν_τ and a backward π^-.



A highly boosted τ has then has

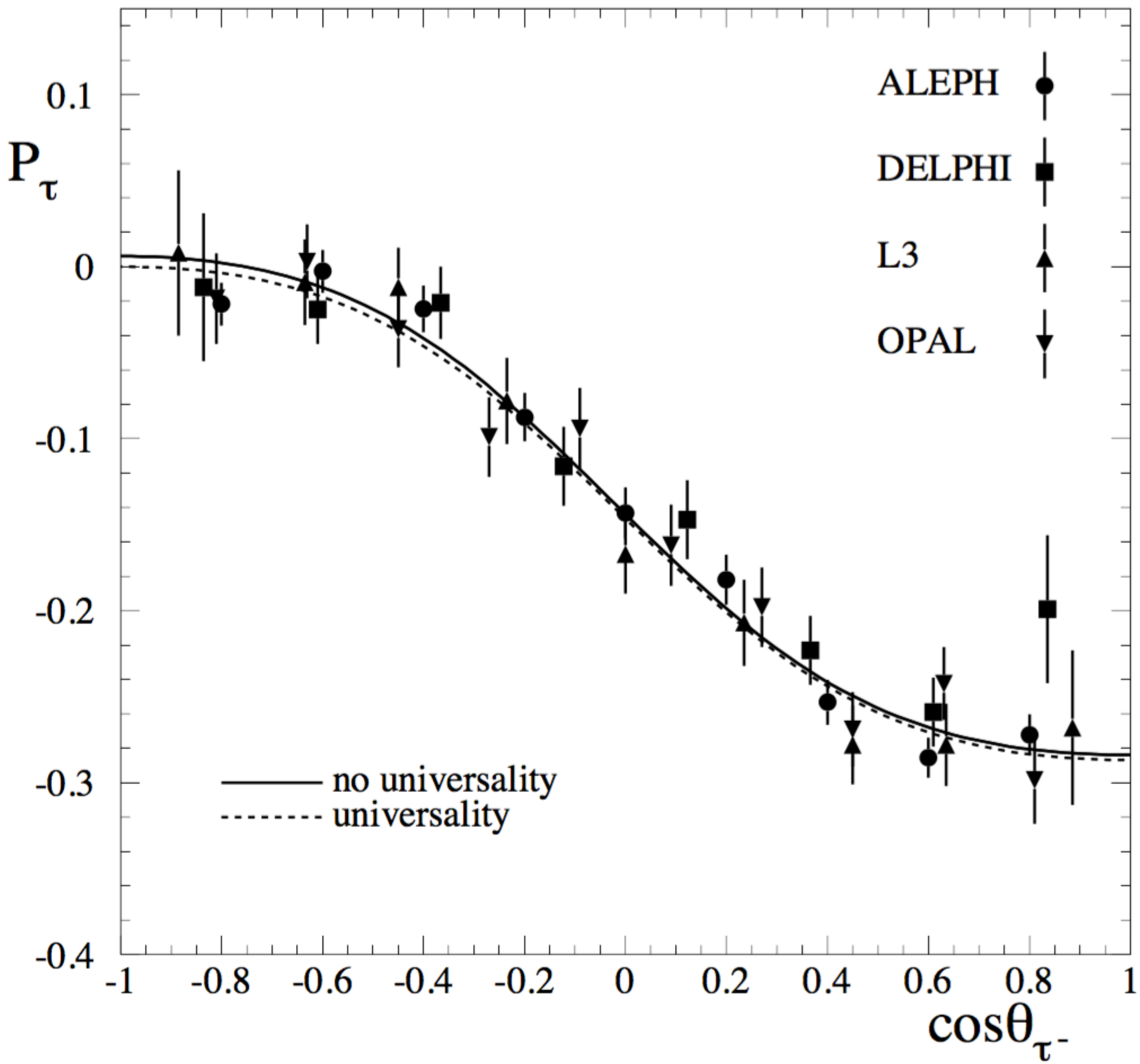
$$\tau_L \ : \quad \frac{d\Gamma}{dx} \sim (1-x) \qquad \tau_R \ : \quad \frac{d\Gamma}{dx} \sim x$$

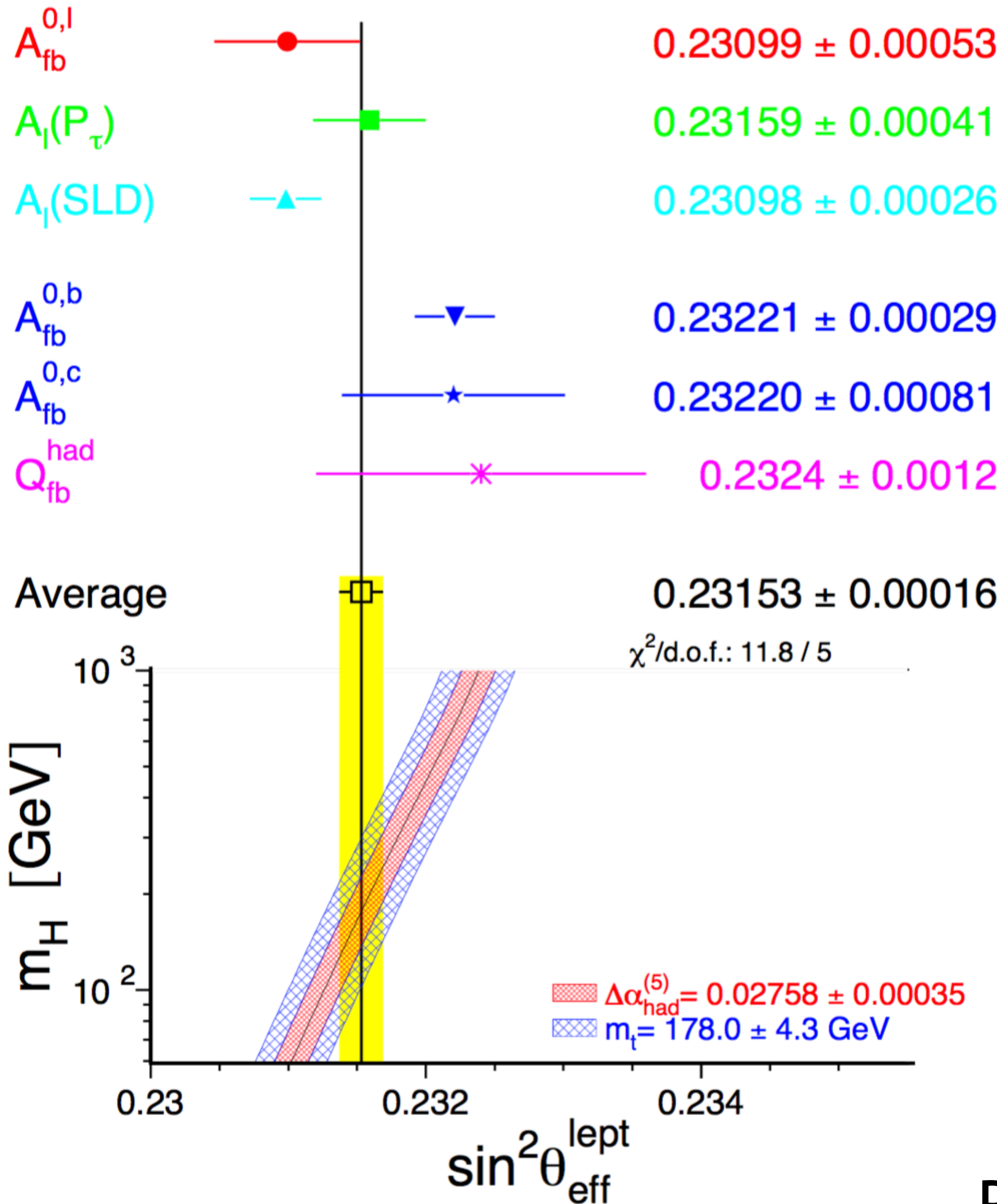
where $x = E_\pi / E_\tau$. Similar asymmetries appear in the other prominent τ decay modes.



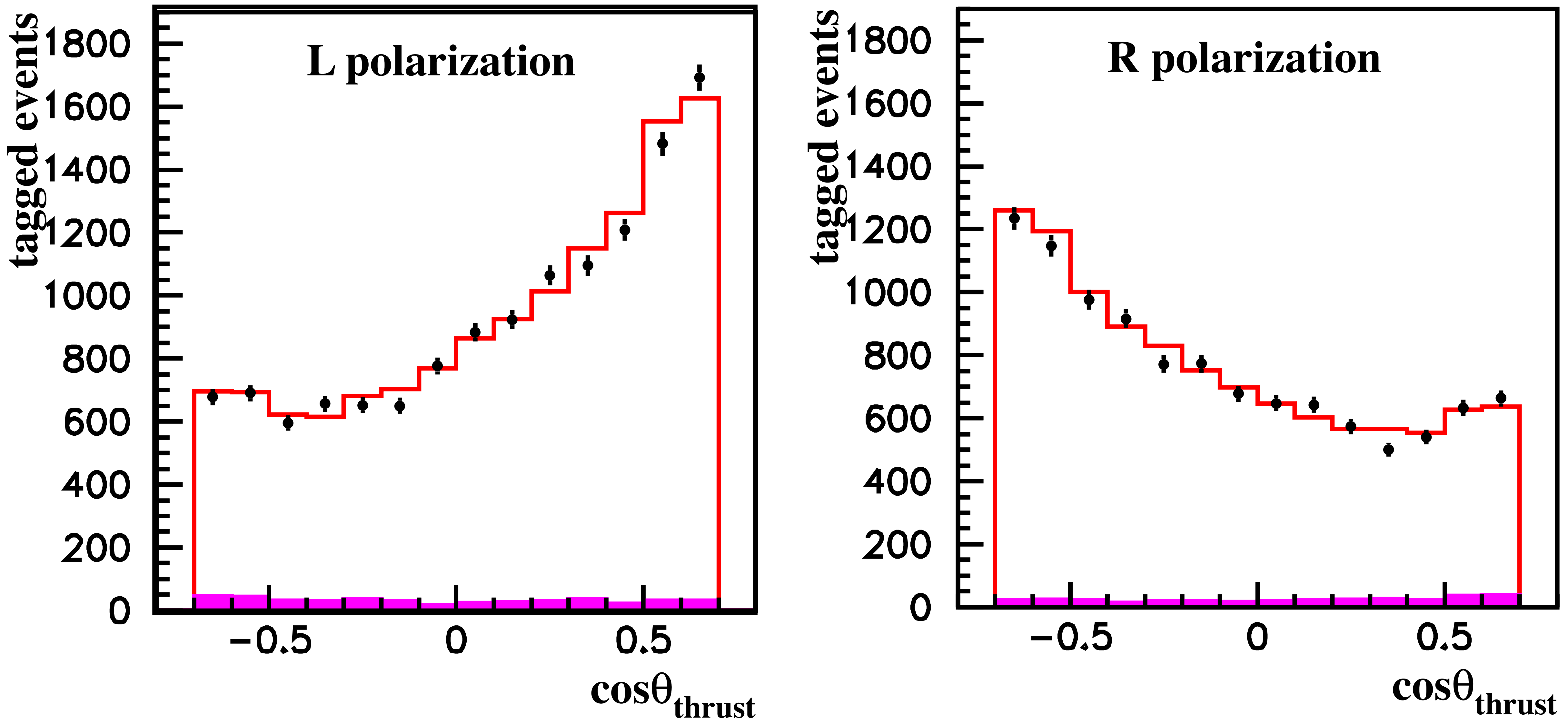
events/0.05
ALEPH
$\tau \rightarrow \pi \nu$
ALEPH
$\tau \rightarrow \mu \bar{\nu} \nu$
2000
1000
0
4000
3000
2000
1000
0
0
0.25
0.5
0.75
1
x
0
0.25
0.5
0.75
1
x
τ_L
τ_R

There is also a correlation between τ polarization and $\cos\theta$ that can be used to improve the measurement.

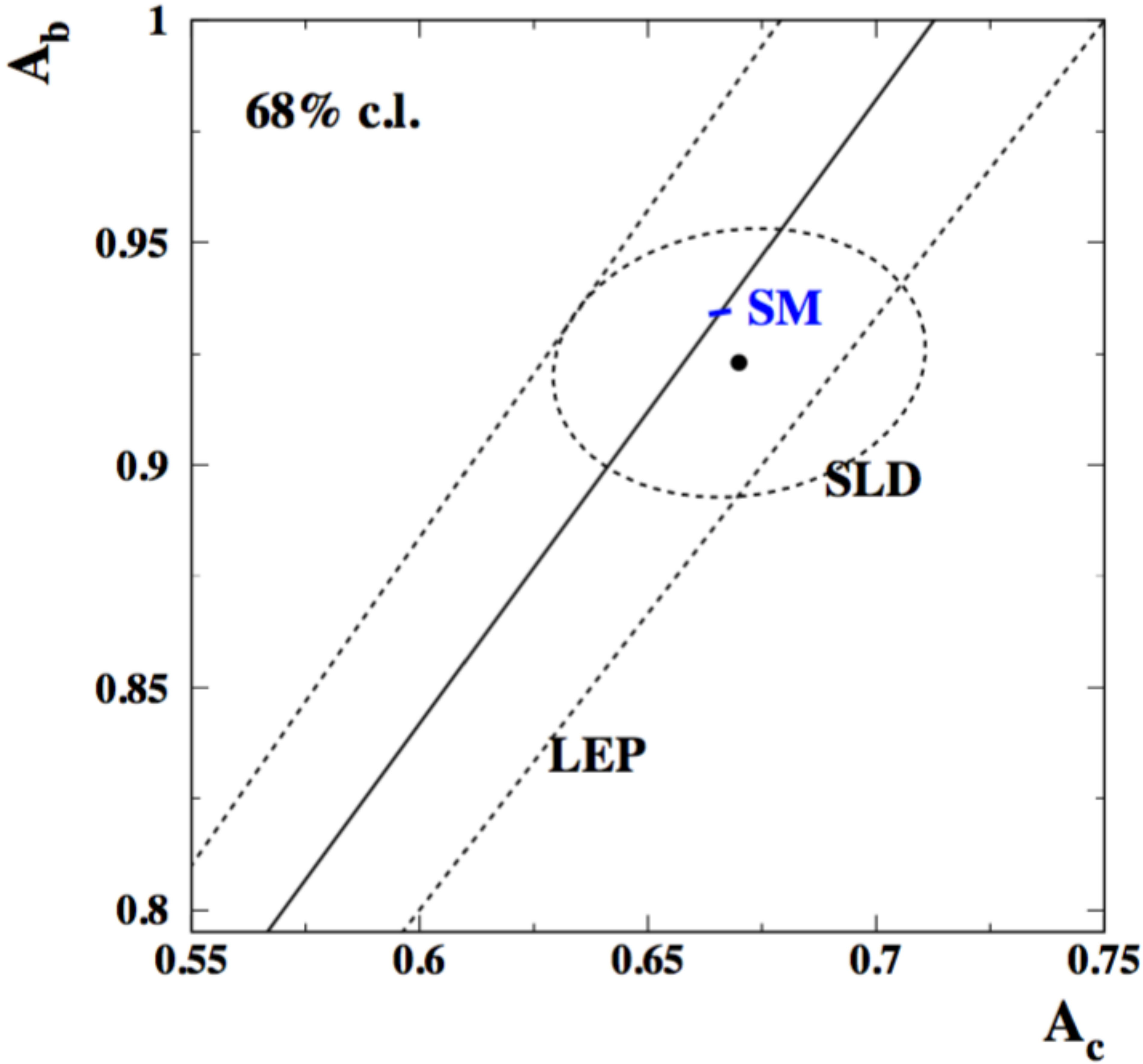




final LEPEWWG
Phys. Rept. 2006


L polarization
R polarization
tagged events
0
200
400
600
800
1000
1200
1400
1600
1800
−0.5
0
0.5
cosθ thrust

SLD

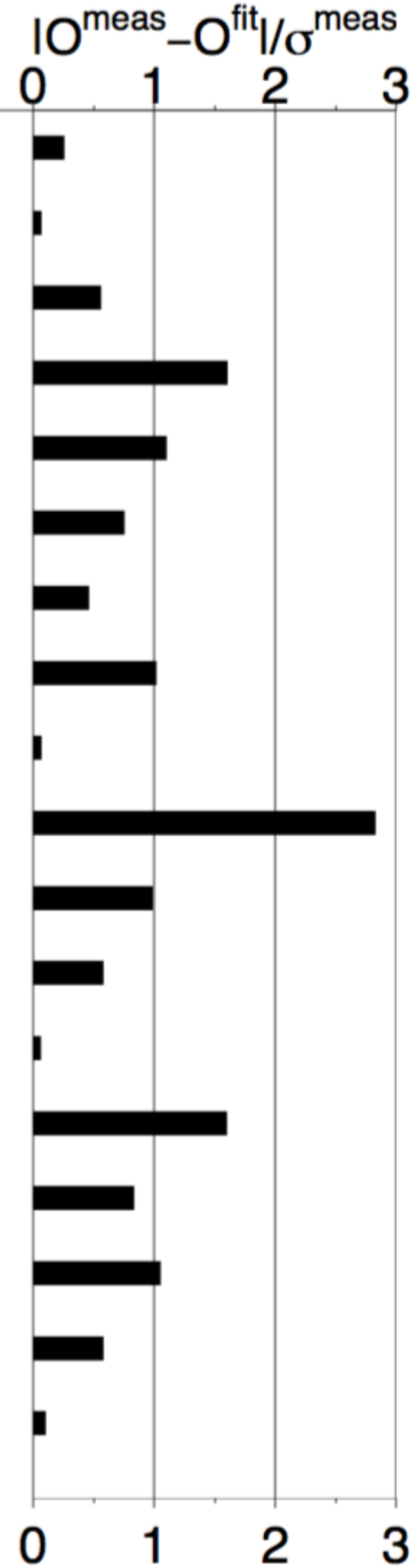


68% c.l.
SM
SLD
LEP
A_b
A_c
1
0.95
0.9
0.85
0.8
0.55
0.6
0.65
0.7
0.75

Here is a summary of the LEP and SLC precision measurements, compiled in the LEP EWWG summary report: Phys.Rept. 427, 257 (2006).

Measurements are shown in terms of the pull (in σ) with respect to the best-fit Standard Model parameters.

	Measurement	Fit
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02758 ± 0.00035	0.02767
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4965
σ^0_{had} [nb]	41.540 ± 0.037	41.481
R_l	20.767 ± 0.025	20.739
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01642
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1480
R_b	0.21629 ± 0.00066	0.21562
R_c	0.1721 ± 0.0030	0.1723
$A^{0,b}_{fb}$	0.0992 ± 0.0016	0.1037
$A^{0,c}_{fb}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1480
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.425 ± 0.034	80.389
Γ_W [GeV]	2.133 ± 0.069	2.093
m_t [GeV]	178.0 ± 4.3	178.5



final LEPEWWG
Phys. Rept. 2006

Now we must discuss the renormalization prescription for the computation of 1-loop radiative corrections.

The Standard Model has a large number of parameters. However, for the specific processes that I have discussed in this lecture, the tree-level predictions depend only on 3 parameters

$$g \ , \ g' \ , \ v$$

The 1-loop corrections will include divergent corrections, included quadratically divergent corrections from v^2 . However, when the corrections to these three parameters are fixed, all 1-loop corrections are made finite. Each specific reaction will obtain a finite correction, which is a prediction of the Standard Model.

Different schemes are used to fix the three underlying divergent amplitudes. Each gives different expressions for the cross sections. These expressions become identical when observables are related to other observables. Three common schemes are

Marciano-Sirlin: fix $\alpha(m_Z), m_Z, m_W$ to their experimental values

on-shell Z: fix $\alpha(m_Z), G_F, m_Z$ to their experimental values

$\overline{MS}$ subtraction

In most analyses today, the 3 unknown constants in each scheme are varied to give the best global fit to the corpus of precision data.

There are many possible definitions of θ_w .

Marciano-Sirlin scheme: define θ_w by $c_w = m_W / m_Z$

this leads to: $s_w^2 = 0.22290 \pm 0.00008$

on-shell Z scheme: define θ_w by

$$\sin^2 2\theta_w = (2 c_w s_w)^2 = \frac{4\pi\alpha(m_Z)}{\sqrt{2} G_F m_Z^2}$$

this leads to

$$s_w^2 = 0.231079 \pm 0.000036$$

Both definitions lead to the same expressions relating observables to observables, but only when finite 1-loop corrections are included.

One particular class of radiative corrections is very simple to analyze. This is the case in which new particles have no direct coupling to light fermions but appear in Z processes only through vector boson vacuum polarization amplitudes.

These are called oblique radiative corrections .

They are most simply discussed as a power series in

$$m_Z^2/M^2$$

where M is the mass of a new particle from beyond the Standard Model.

Define the vacuum polarization amplitudes

$$= ie^2 \Pi_{QQ} g^{\mu\nu}$$

$$= i \frac{e^2}{s_w c_w} (\Pi_{3Q} - s_w^2 \Pi_{QQ}) g^{\mu\nu}$$

$$= i \frac{e^2}{s_w^2 c_w^2} (\Pi_{33} - 2 s_w^2 \Pi_{3Q} + s_w^2 \Pi_{QQ}) g^{\mu\nu}$$

$$= i \frac{e^2}{s_w^2} \Pi_{11} g^{\mu\nu}$$

Each amplitude has a Taylor expansion in q^2/M^2 :

$$\Pi_{QQ}(q^2) = A q^2 + \cdots$$

$$\Pi_{3Q}(q^2) = B q^2 + \cdots$$

$$\Pi_{33}(q^2) = C + D q^2 + \cdots$$

$$\Pi_{11}(q^2) = E + F q^2 + \cdots$$

Of the 6 constants on the previous slide, 3 contribute to the renormalizations of g , g' , v . This leaves 3 combinations that are finite at 1 loop. These are

$$S = \frac{16\pi}{m_Z^2}[\Pi_{33}(m_Z^2) - \Pi_{33}(0) - \Pi_{3Q}(m_Z^2)]$$

$$T = \frac{4\pi}{s_w^2 m_W^2}[\Pi_{11}(0) - \Pi_{33}(0)]$$

$$U = \frac{16\pi}{m_Z^2}[\Pi_{11}(m_Z^2) - \Pi_{11}(0) - \Pi_{33}(m_Z^2) + \Pi_{33}(0)]$$

Roughly, T parametrizes the correction to $m_W/m_Z c_w$, S parametrizes the q^2/M^2 correction, and U, with both suppressions, is very small in most BSM models.

The leading oblique corrections to electroweak observables can then be expressed as, for example,

$$\frac{m_W^2}{m_Z^2} - c_0^2 = \frac{\alpha c^2}{c^2 - s^2}\left(-\frac{1}{2}S + c^2 T\right)$$

$$s_*^2 - s_0^2 = \frac{\alpha}{c^2 - s^2}\left(\frac{1}{4}S - s^2 c^2 T\right)$$

This allows experiment to place constraints that can then be applied to a large class of models.

Some guidance about the expected sizes of S and T is given by the result for one new electroweak doublet:

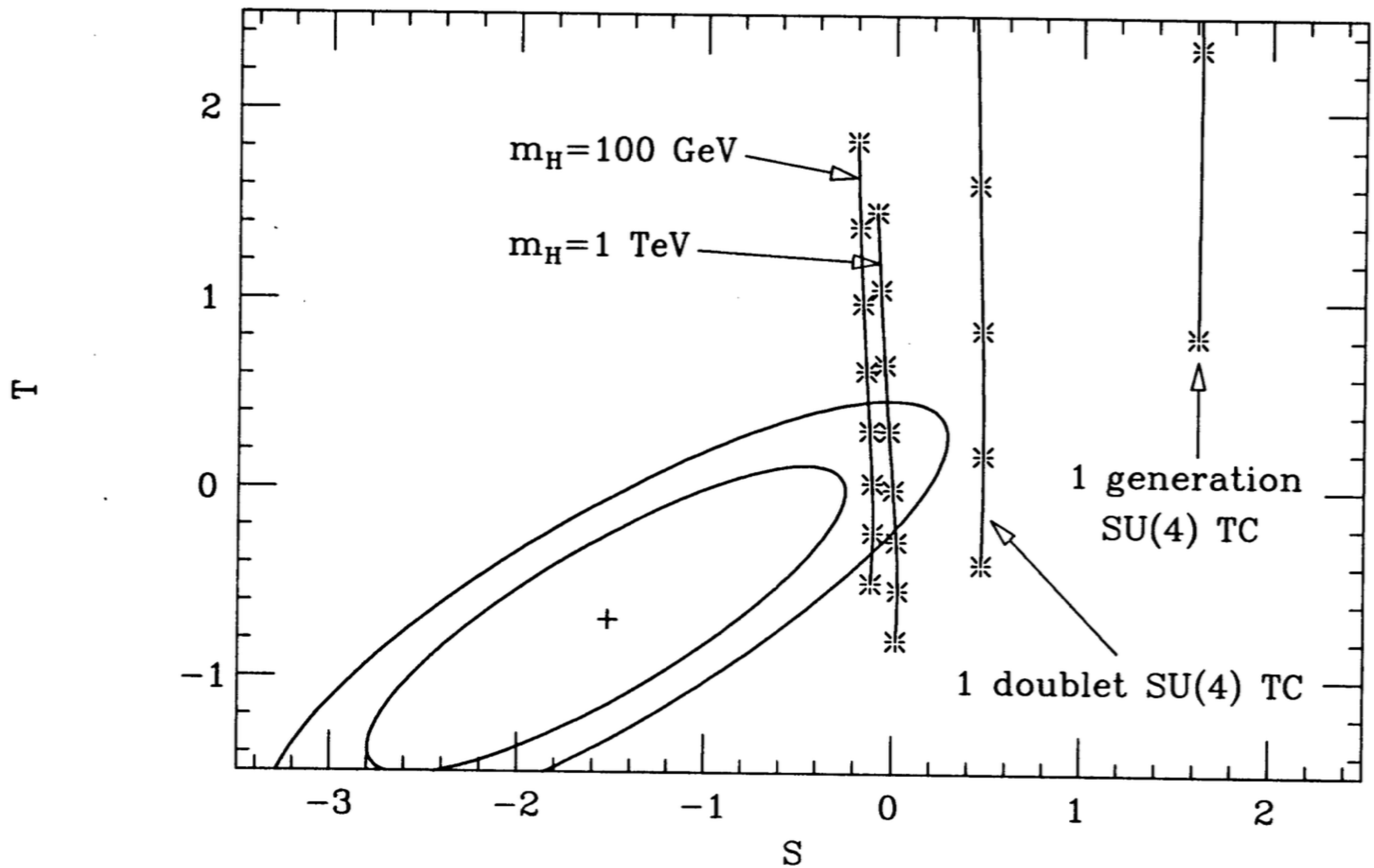
$$S = \frac{1}{6\pi} \qquad\qquad T = \frac{|m_U^2 - m_D^2|}{m_Z^2}$$

The effects of the SM top quark and Higgs boson can also be expressed (approximately) in the S,T framework

top: $$S = \frac{1}{6\pi}\log\frac{m_t^2}{m_Z^2} \qquad T = \frac{3}{16\pi s^2 c^2}\frac{m_t^2}{m_Z^2}$$

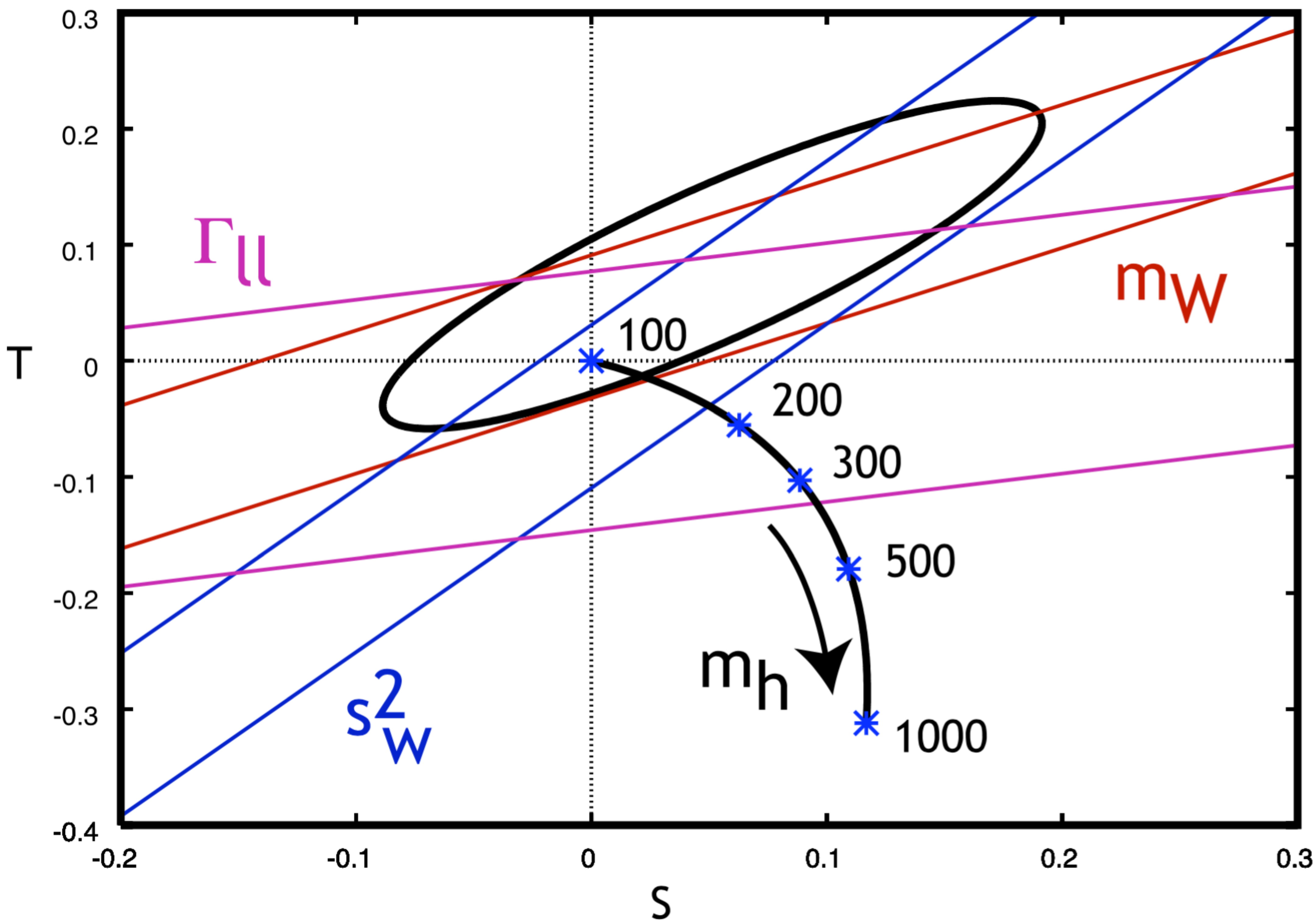
Higgs: $$S = \frac{1}{12\pi}\log\frac{m_h^2}{m_Z^2} \qquad T = -\frac{3}{16\pi c^2}\log\frac{m_h^2}{m_Z^2}$$

S,T fit c. 1991



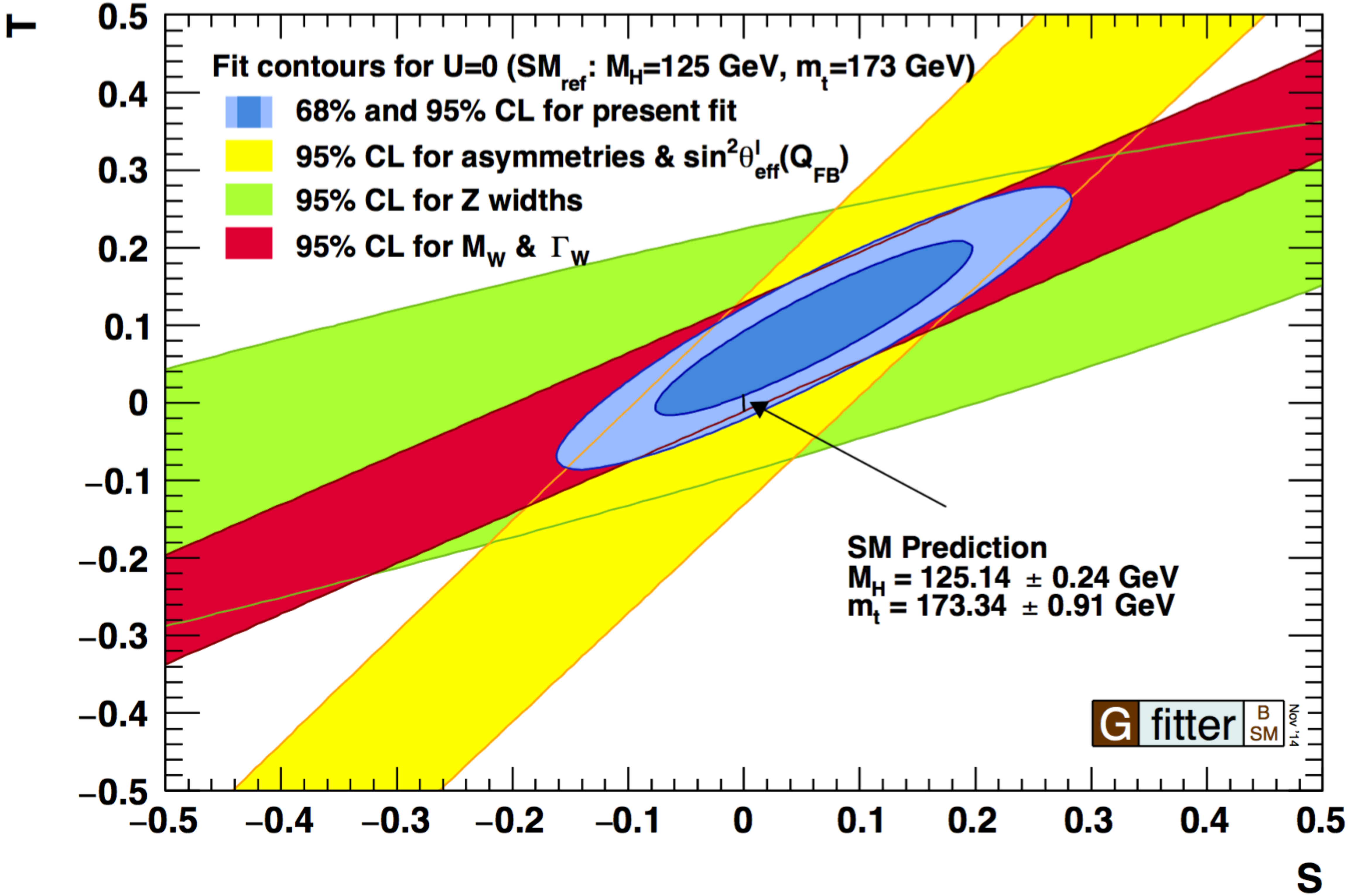
2
1
0
−1
−3
−2
−1
0
1
2
S
T
m_H=100 GeV
m_H=1 TeV
1 generation
SU(4) TC
1 doublet SU(4) TC

S,T fit c. 2008



LEP EWWG: within the MSM $m_h < 144\ (182)$ GeV (95% CL)

S,T fit c. 2014



T
S
Fit contours for U=0 (SM$_{ref}$: M$_H$=125 GeV, m$_t$=173 GeV)
68% and 95% CL for present fit
95% CL for asymmetries & $\sin^2\theta^l_{eff}(Q_{FB})$
95% CL for Z widths
95% CL for M_W & Γ_W
SM Prediction
M_H = 125.14 ± 0.24 GeV
m_t = 173.34 ± 0.91 GeV
G fitter B SM
Nov '14