

# The Electroweak Standard Model

## 4. The Standard Model Higgs Boson

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So far in these lectures, we have discussed all of the heavy particles of the Standard Model except one. In this lecture, I will discuss the predictions of the Standard Model for the properties of the Higgs boson.

The basics of the theory are extremely simple. A general Higgs field configuration can be simplified by a gauge transformation to the form

$$\varphi(x) = \exp[-i\alpha^a(x)\sigma^a/2] \begin{pmatrix} 0 \\ (v + h(x))/\sqrt{2} \end{pmatrix}$$

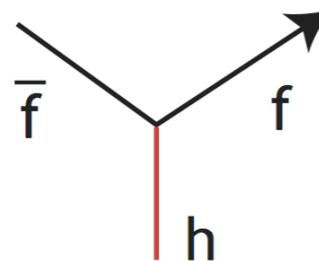
Here  $v$  is the vacuum expectation value of the field.

From  $m_W$  and  $g$ , we extract

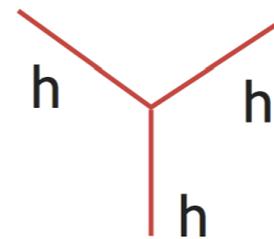
$$v = 250 \text{ GeV}$$

The dynamical part of the field is a single scalar field  $h(x)$ . The vertices of  $h(x)$  are given by shifting  $v$ . Thus, the vertices of  $h(x)$  are completely determined by known information from the Standard Model.

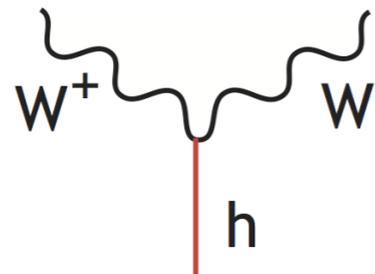
So,



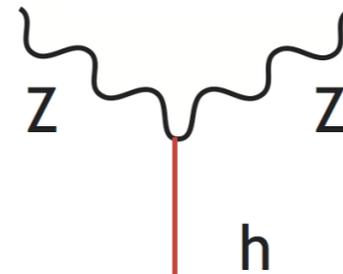
$$= -i \frac{m_f}{v}$$



$$= -3i \frac{m_h^2}{v}$$



$$= 2i \frac{m_W^2}{v} g_{\mu\nu}$$



$$= 2i \frac{m_Z^2}{v} g_{\mu\nu}$$

within the Standard Model, there is no freedom. The decay widths of the Higgs boson will depend on the Higgs boson mass, but, once this is known, these widths can be computed precisely.

These couplings imply that a heavy Higgs boson will decay dominantly by

$$h \rightarrow W^+ W^- , \quad h \rightarrow Z Z , \quad h \rightarrow t \bar{t}$$

The theory of these Higgs boson decays is very simple.

However, by now you all know that the LHC experiments exclude a Standard Model Higgs boson in the mass range where decay to these particles would be permitted. The Higgs resonance found at the LHC has a mass of 125 GeV.

Therefore, all of the actual decays of the Higgs boson are suppressed in some way, by factors

$$\frac{m_f^2}{m_W^2} , \quad \frac{\alpha_w}{4\pi} , \quad \left(\frac{\alpha_s}{4\pi}\right)^2$$

However, this means that the theory of Higgs boson decays is very rich, with a large number of decay modes accessible.

Begin with the decays to fermions. The matrix element for Higgs decay to a light fermion is

$$i\mathcal{M}(h \rightarrow f_R \bar{f}_R) = -i \frac{m_f}{v} u_R^\dagger v_R = -i \frac{m_f}{v} (2E)$$

Summing over final fermion helicities and integrating over phase space

$$\Gamma(h \rightarrow f \bar{f}) = \frac{1}{2m_h} \frac{1}{8\pi} \frac{m_f^2 m_h^2}{v^2} \cdot 2$$

or, using  $v^2 = 4m_W^2/g^2$

$$\Gamma(h \rightarrow f \bar{f}) = \frac{\alpha_w}{8} m_h \frac{m_f^2}{m_W^2}$$

For final leptons, we can immediately evaluate this:

$$\Gamma(h \rightarrow \tau^+ \tau^-) = 260 \text{ keV} \quad \Gamma(h \rightarrow \mu^+ \mu^-) = 9 \text{ keV}$$

for  $m_h = 125 \text{ GeV}$ .

For quarks, a few more details must be added.

The mass in this formula should be the  $\overline{MS}$  mass evaluated at  $Q = m_h$ . This is related to the quark mass as usually quoted by

$$m_f(m_h) = m_f(m_f) \left[ \frac{\alpha_s(m_h)}{\alpha_s(m_f)} \right]^{4/b_0} (1 + \mathcal{O}(\alpha_s))$$

The appropriate values of quark masses (in MeV) are

$m_u$	$m_d$	$m_s$	$m_c$	$m_b$
1.5	3	60	700	2800

Also, there is a QCD correction that is larger than the one for  $e^+e^-$  annihilation:

$$3\left(1 + \frac{17}{3\pi}\alpha_s(m_h) + \dots\right) = 3 \cdot 1.24$$

Then, for example,

$$\Gamma(h \rightarrow b\bar{b}) = \frac{\alpha_w m_h}{8} \left( \frac{2.8}{m_W} \right)^2 \cdot 3 \cdot (1.24) = 2.4 \text{ MeV}$$

This will turn out to correspond to a BR of 58%. So the total width of the Higgs is about 4.1 MeV, and the other fermion BRs are

$\tau^+ \tau^-$	$c\bar{c}$	$s\bar{s}$	$\mu^+ \mu^-$
6.3%	3%	0.03%	0.02%

Did you expect that  $BR(\tau^+ \tau^-) > BR(c\bar{c})$  despite the color factor 3?

For a heavy Higgs that can decay to W and Z bosons on shell, the decay amplitudes would be

$$i\mathcal{M}(h \rightarrow W^+ W^-) = i \frac{2m_W^2}{v} \epsilon_+^* \cdot \epsilon_-^*$$
$$i\mathcal{M}(h \rightarrow ZZ) = i \frac{2m_Z^2}{v} \epsilon_1^* \cdot \epsilon_2^*$$

For a very heavy Higgs, there is a further enhancement for the longitudinal polarization states

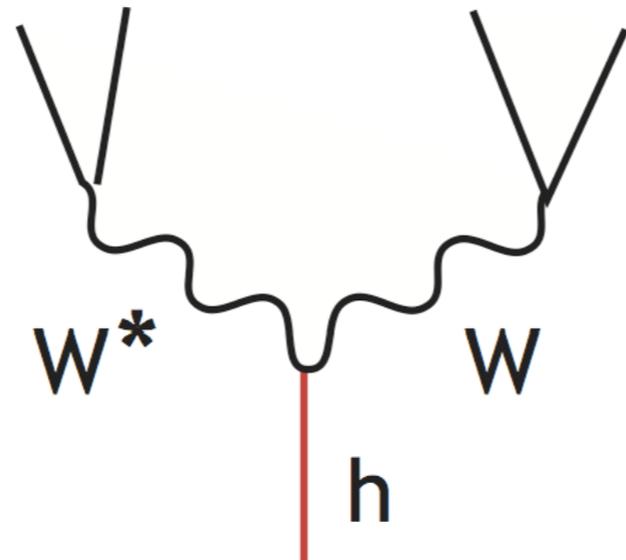
$$\epsilon_1^* \cdot \epsilon_2^* = \frac{k_1 \cdot k_2}{m_Z^2} = \frac{m_h^2}{2m_Z^2}$$

This factor is just

$$\lambda / (g^2 + g'^2)$$

so that the longitudinal Z and W couple like (heavy) Higgs bosons rather than gauge bosons, as predicted by the GBET.

For the actual situation of a 125 GeV Higgs boson, one or both of the Ws or Zs must be off shell. Then the decay is best described as a  $h \rightarrow 4$  fermion process



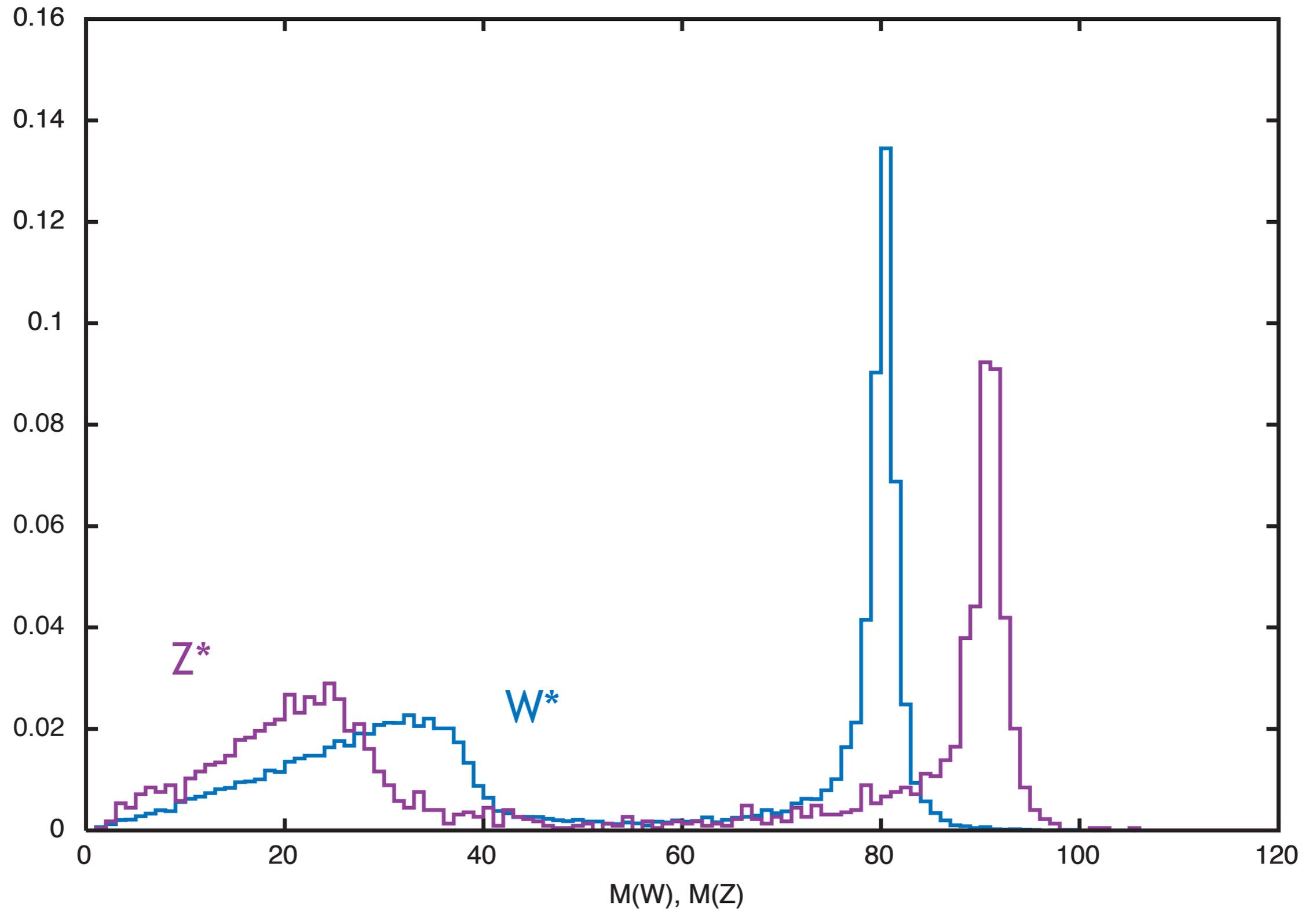
The rate is suppressed by a factor of  $\alpha_w$  and by the off-shell W or Z propagator. The result is that the rate is competitive with  $b\bar{b}$  for W and a factor 10 smaller for Z.

The Standard Model branching fractions are

$$BR(h \rightarrow WW^*) = 22\% \quad BR(h \rightarrow ZZ^*) = 2.7\%$$

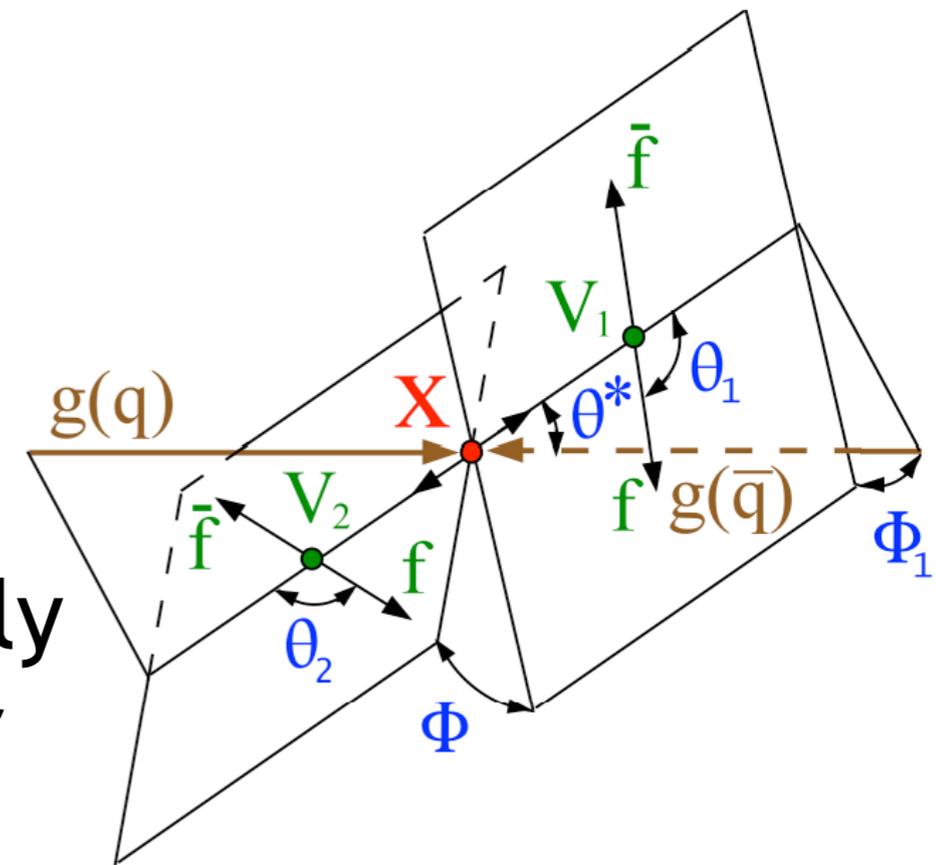
# 2-jet mass distributions in $h \rightarrow WW^*, ZZ^*$ decays

$m(h) = 120 \text{ GeV}$



The Higgs decay to  $ZZ^*$  is exceptionally interesting because it is completely reconstructable when both Zs decay to charged leptons. The angular distribution of the leptons permits a spin analysis.

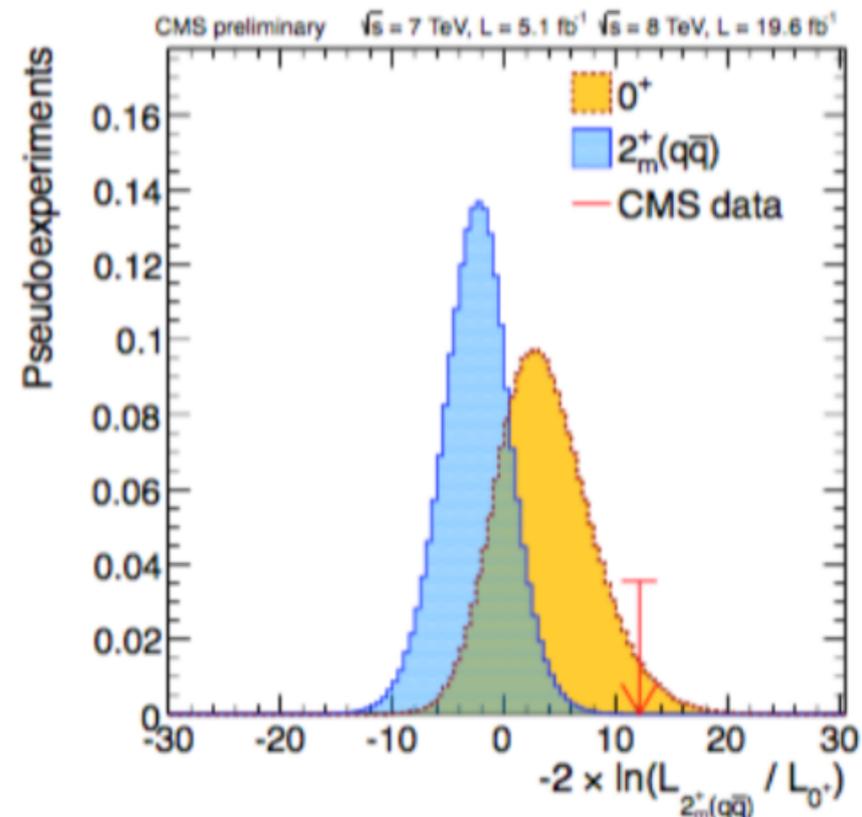
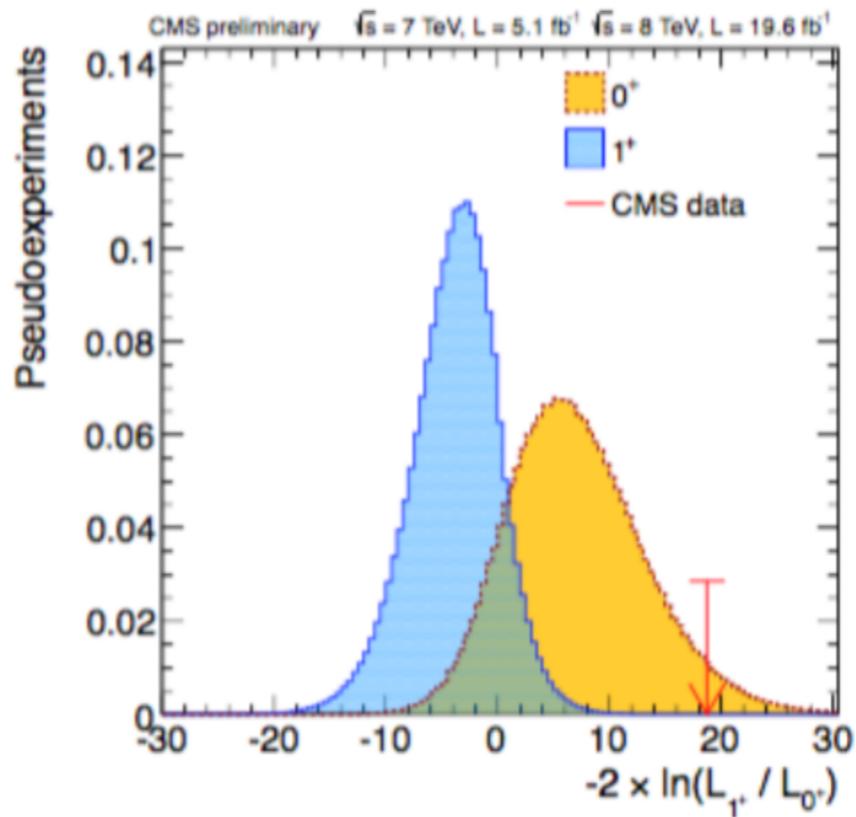
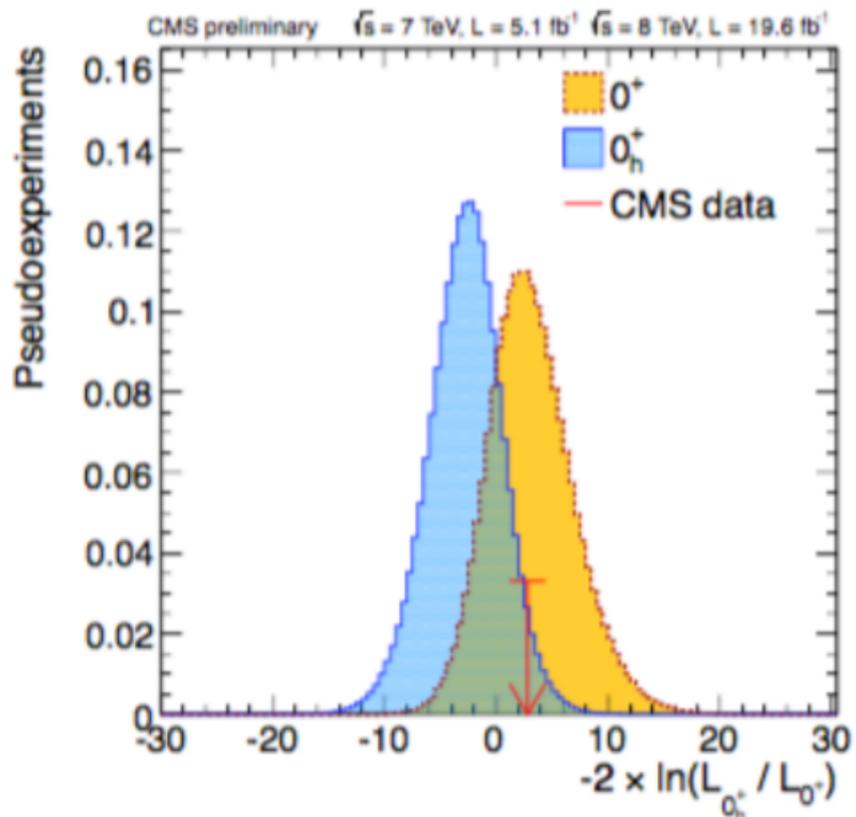
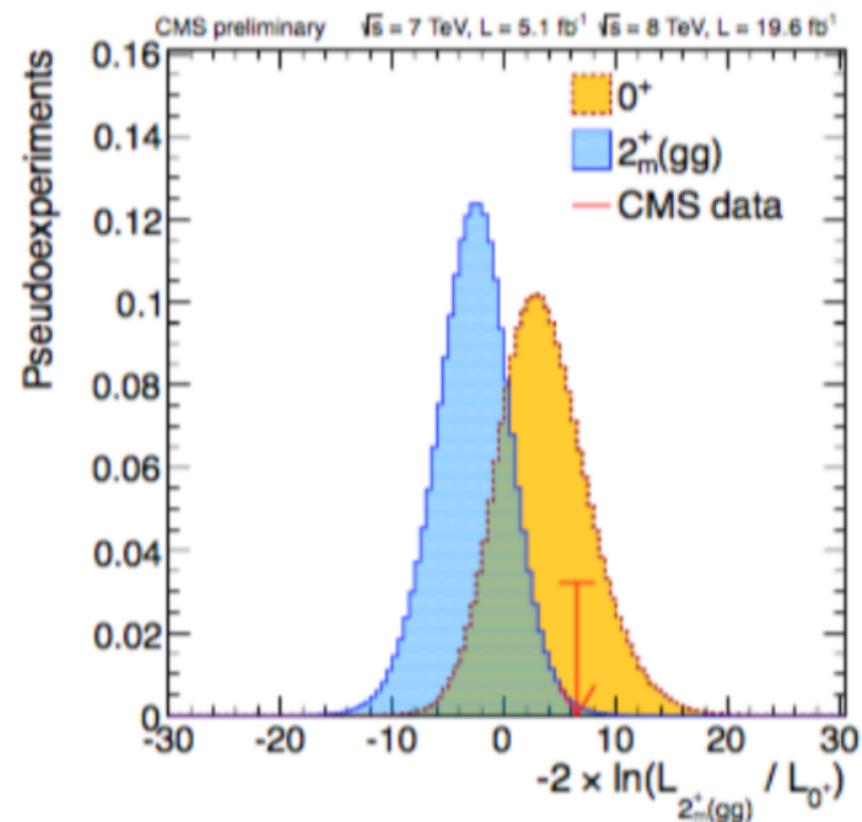
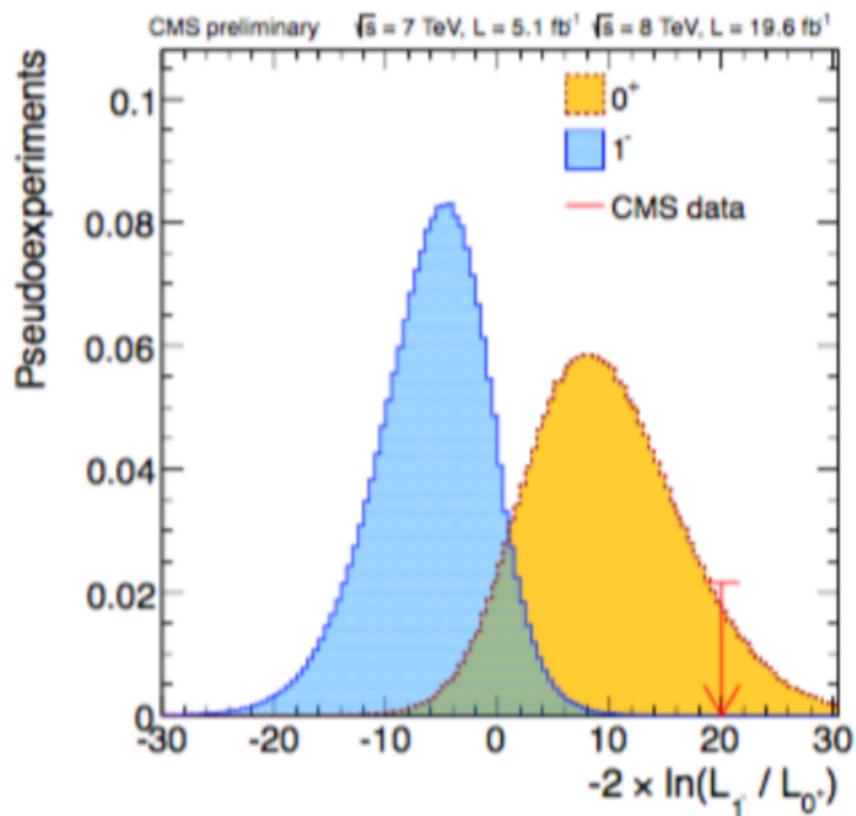
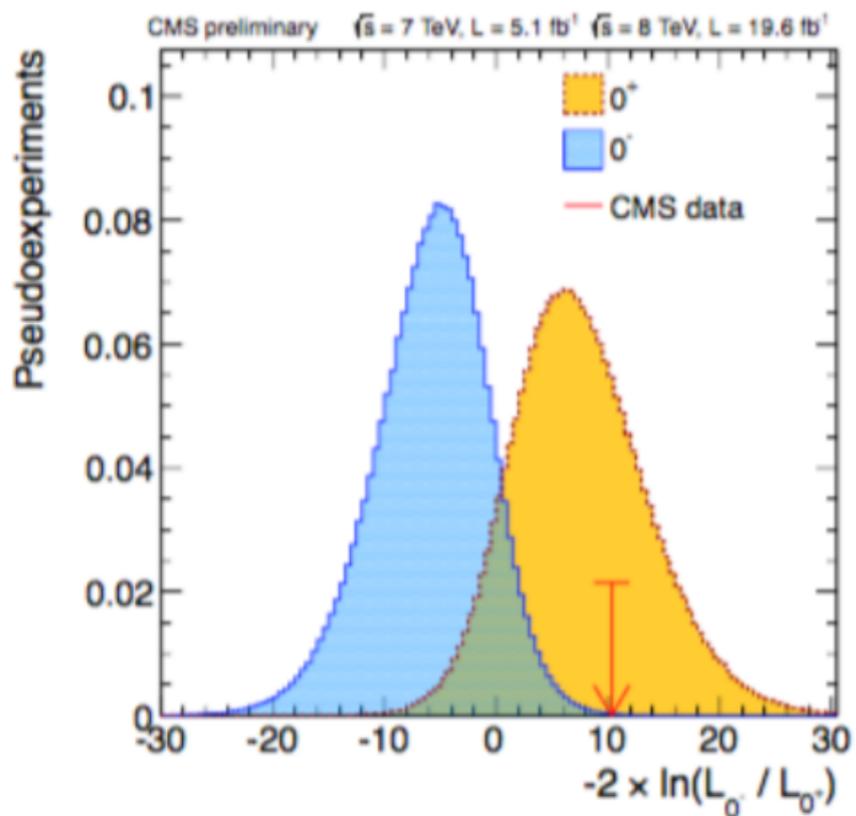
For the Standard Model amplitude, the two Zs are preferentially longitudinally polarized, and their decay planes are preferentially parallel. This contrasts with other possible assignments



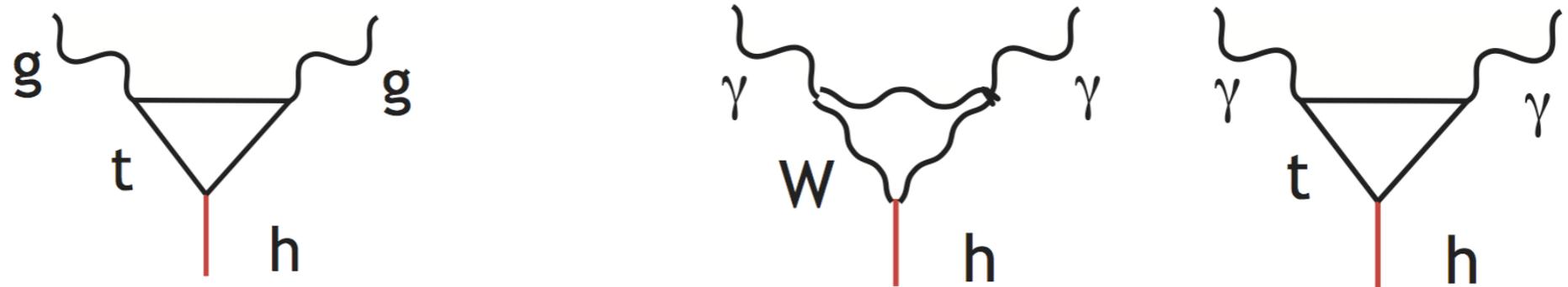
$$0^- \quad h \epsilon^{\mu\nu\lambda\sigma} Z_{\mu\nu} Z^{\lambda\sigma}$$

$$0^+ \quad h Z_{\mu\nu} Z^{\mu\nu}$$

or assignments to spin 2.



Finally, there are loop processes that allow the Higgs to decay to massless vector boson states  $gg$  and  $\gamma\gamma$ , and to  $Z\gamma$ .



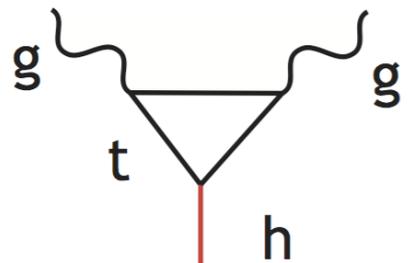
Begin with the  $hgg$  vertex. Integrating out the top quark loop gives an effective operator

$$\delta\mathcal{L} = \frac{1}{4} A h F_{\mu\nu}^a F^{\mu\nu a}$$

where  $F_{\mu\nu}^a$  is the QCD field strength and  $A$  has dimension  $(\text{GeV})^{-1}$ . This operator yields the vertex

$$-iA\delta^{ab}(k_1 \cdot k_2 g^{\mu\nu} - k_1^\nu k_2^\mu)$$

For a quark of mass  $m_f$ , we might estimate the size of the diagram as



The diagram shows a triangular loop of top quarks (t) with two external gluon lines (g) and one external Higgs boson line (h). The Higgs line is red, and the top quark lines are black. The diagram is followed by an approximation symbol and the expression  $\frac{\alpha_s m_f}{v} \cdot \frac{1}{M}$ .

$$\sim \frac{\alpha_s m_f}{v} \cdot \frac{1}{M}$$

where  $M$  is the momentum that flows in the loop

$$M \sim \max(m_h, 2m_f)$$

There are two cases: For  $2m_f < m_h$ , the diagram is suppressed by a factor  $2m_f/m_h$ . For  $2m_f > m_h$ , the factors of  $m_f$  cancel, and the diagram is at full strength no matter how large  $m_f$  is.

So, this diagrams gets large contributions only from those quarks that are too heavy to be decay products of the Higgs. In the Standard Model, this is uniquely the top quark.

To compute the diagram for the top quark, we can start from the top quark QCD vacuum polarization, which has the value

$$i(k^2 g^{\mu\nu} - k^\mu k^\nu) \text{tr}[t^a t^b] \frac{\alpha_s}{3\pi} \log \frac{\Lambda^2}{m_t^2}$$

$$= i(k^2 g^{\mu\nu} - k^\mu k^\nu) \delta^{ab} \frac{\alpha_s}{6\pi} \log \frac{\Lambda^2}{m_t^2}$$

Now introduce a zero momentum Higgs boson by shifting  $v \rightarrow (v + h)$  where  $v$  appears in this expression through

$$m_t^2 = y_t^2 v^2 / 2$$

The hgg vertex is then

$$i(k^2 g^{\mu\nu} - k^\mu k^\nu) \delta^{ab} \frac{\alpha_s}{3\pi} \frac{1}{v}$$

Comparing to our previous expression, we find

$$A = \frac{\alpha_s}{3\pi v} = \frac{g\alpha_s}{6\pi m_W}$$

From this expression, we can compute the partial width  $\Gamma(h \rightarrow gg)$  in the limit  $m_h \ll 4m_t^2$

$$\Gamma(h \rightarrow gg) = \frac{\alpha_w \alpha_s^2}{72\pi^2} \frac{m_h^3}{m_W^2}$$

The full expression is

$$\Gamma(h \rightarrow gg) = \frac{\alpha_w \alpha_s^2}{72\pi^2} \frac{m_h^3}{m_W^2} \cdot \left| \frac{3}{2} \tau (1 - (\tau - 1) (\sin^{-1} \frac{1}{\sqrt{\tau}})^2) \right|^2$$

where  $\tau = 4m_t^2/m_h^2$ .

An interesting feature of the argument I have given is that we have related the Higgs coupling to  $gg$  to the top quark contribution to the QCD  $\beta$  function. We can use a similar idea to obtain the Higgs coupling to  $\gamma\gamma$ , from the  $t$  and  $W$  contributions to the QED coupling constant renormalization.

Write the photon vacuum polarization amplitude due to W bosons and top quarks

$$\begin{aligned}
 & i(k^2 g^{\mu\nu} - k^\mu k^\nu) \frac{\alpha}{4\pi} \left[ -\frac{22}{3} + \frac{1}{3} + \frac{4}{3} \cdot 3 \cdot \left(\frac{2}{3}\right)^2 \right] \log \frac{\Lambda^2}{v^2} \\
 & = -i(k^2 g^{\mu\nu} - k^\mu k^\nu) \frac{\alpha}{3\pi} \left[ \frac{21}{4} - \frac{4}{3} \right] \log \frac{\Lambda^2}{v^2}
 \end{aligned}$$

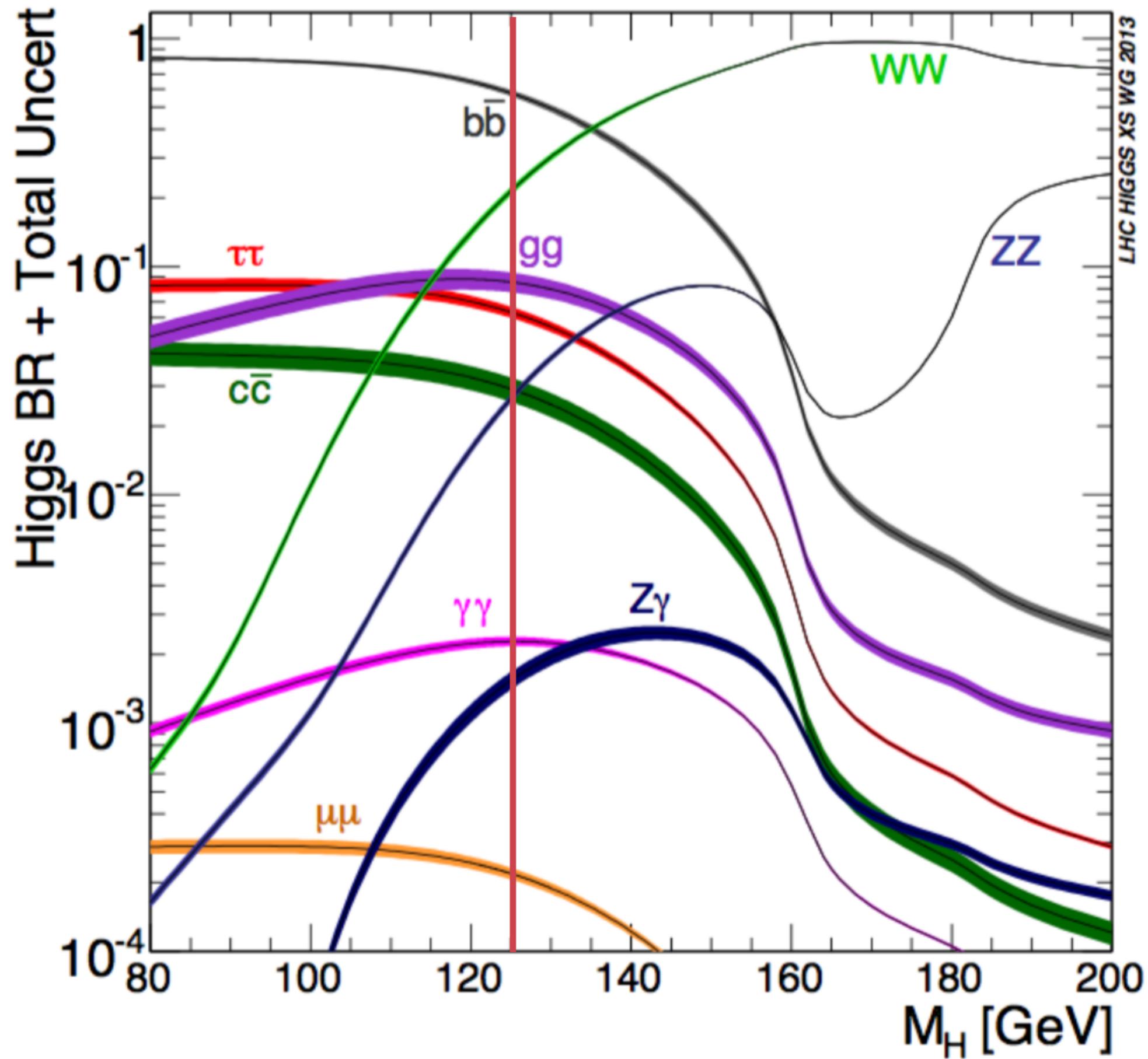
Then, following the same logic, we find in the limit  $m_h \ll (2m_W, 2m_t)$

$$\Gamma(h \rightarrow \gamma\gamma) = \frac{\alpha_w \alpha^2}{144\pi^2} \frac{m_h^3}{m_W^2} \left| \frac{21}{4} - \frac{4}{3} \right|^2$$

Careful evaluation, including QCD corrections to the gluon width, gives the branching ratios

$$BR(h \rightarrow gg) = 8.6\% \qquad BR(h \rightarrow \gamma\gamma) = 0.23\%$$

We can now put all of the pieces together and graph the Standard Model predictions for the various branching ratios of the Higgs as a function of the Higgs mass.

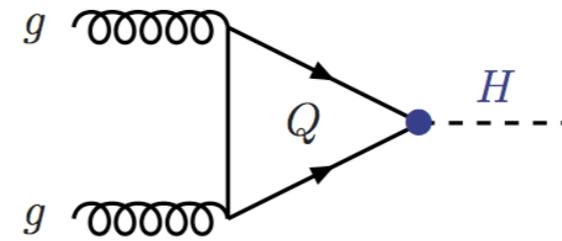


LHC Higgs  
XS WG

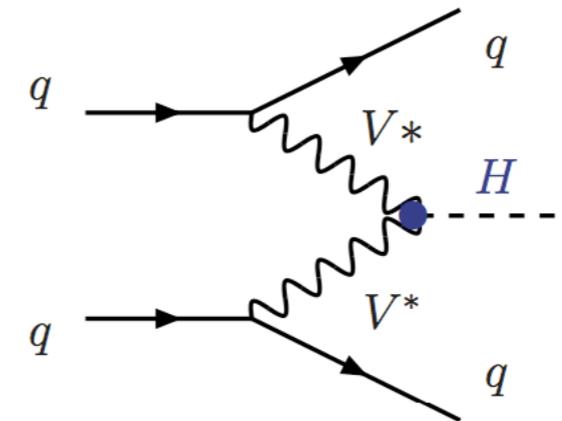
With this introduction to the Standard Model Higgs properties, I can very briefly discuss the study of the Higgs boson at the LHC.

The important production modes for the Higgs boson at hadron colliders are:

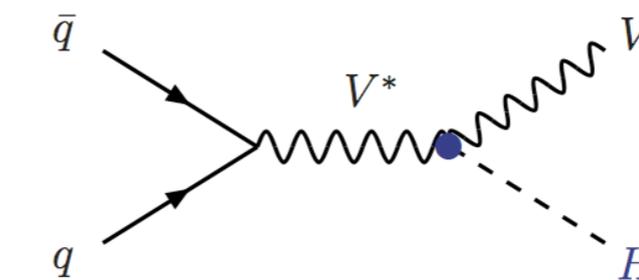
gluon-gluon fusion



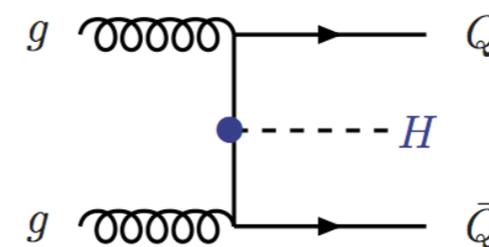
vector boson fusion

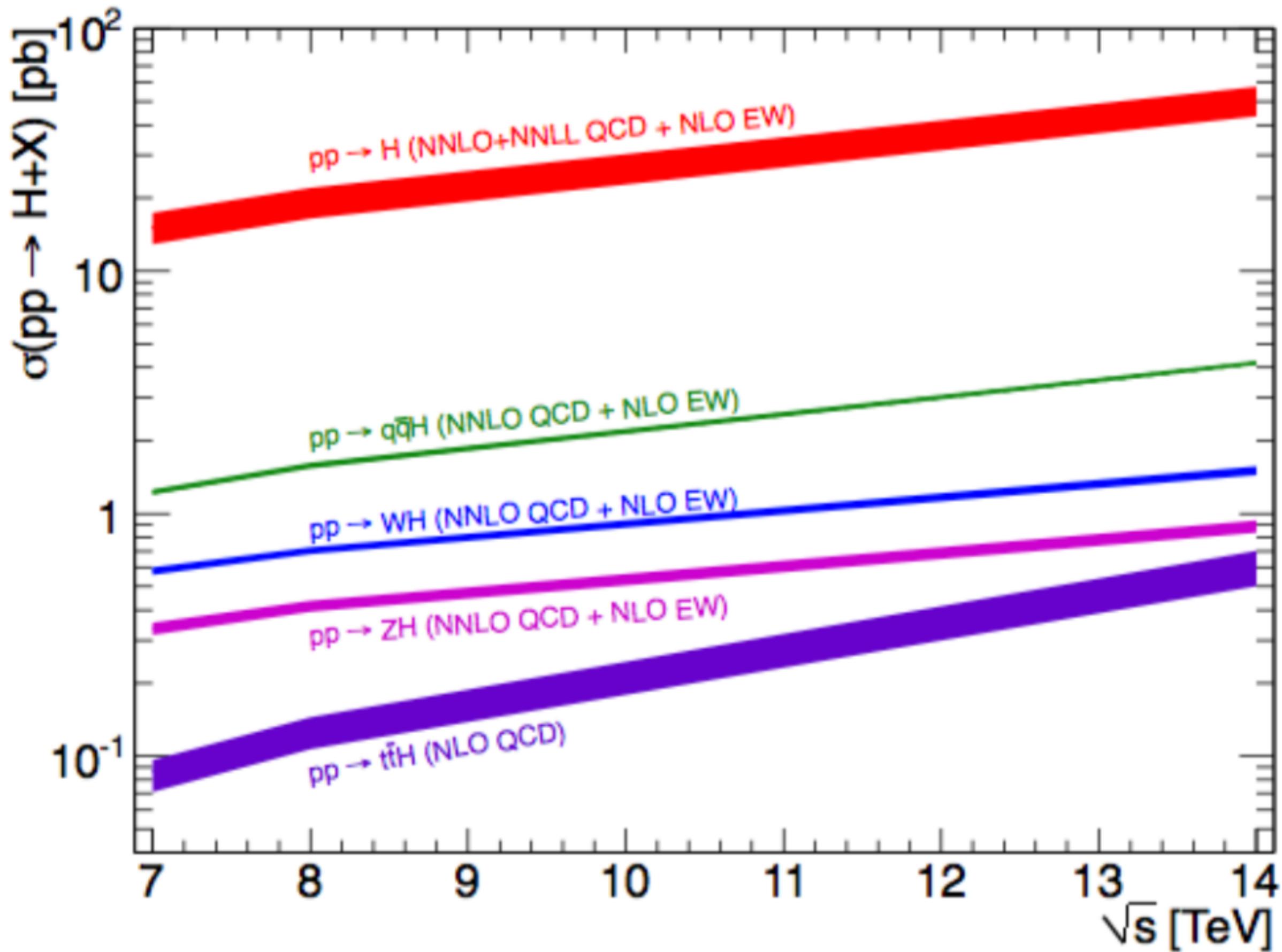


“Higgsstrahlung”  
associated production  
w.  $W, Z$



associated production  
with top





These four reactions have different advantages for the precision study of Higgs decays:

gluon-gluon fusion:

highest cross section, access to rare decays

WW fusion:

tagged Higgs decays, access to invisible and exotic modes  
smallest theoretical error on production cross section

Higgsstrahlung:

tagged Higgs decays

boosted Higgs, for the study of  $b\bar{b}$  decay

associated production with top:

access to the Higgs coupling to top

The original strategy for observing the Higgs boson at the LHC used the characteristic decay modes in which the Higgs could be reconstructed as a resonance,

$$h \rightarrow \gamma\gamma \qquad h \rightarrow ZZ^* \rightarrow \ell^+ \ell^- \ell'^+ \ell'^-$$

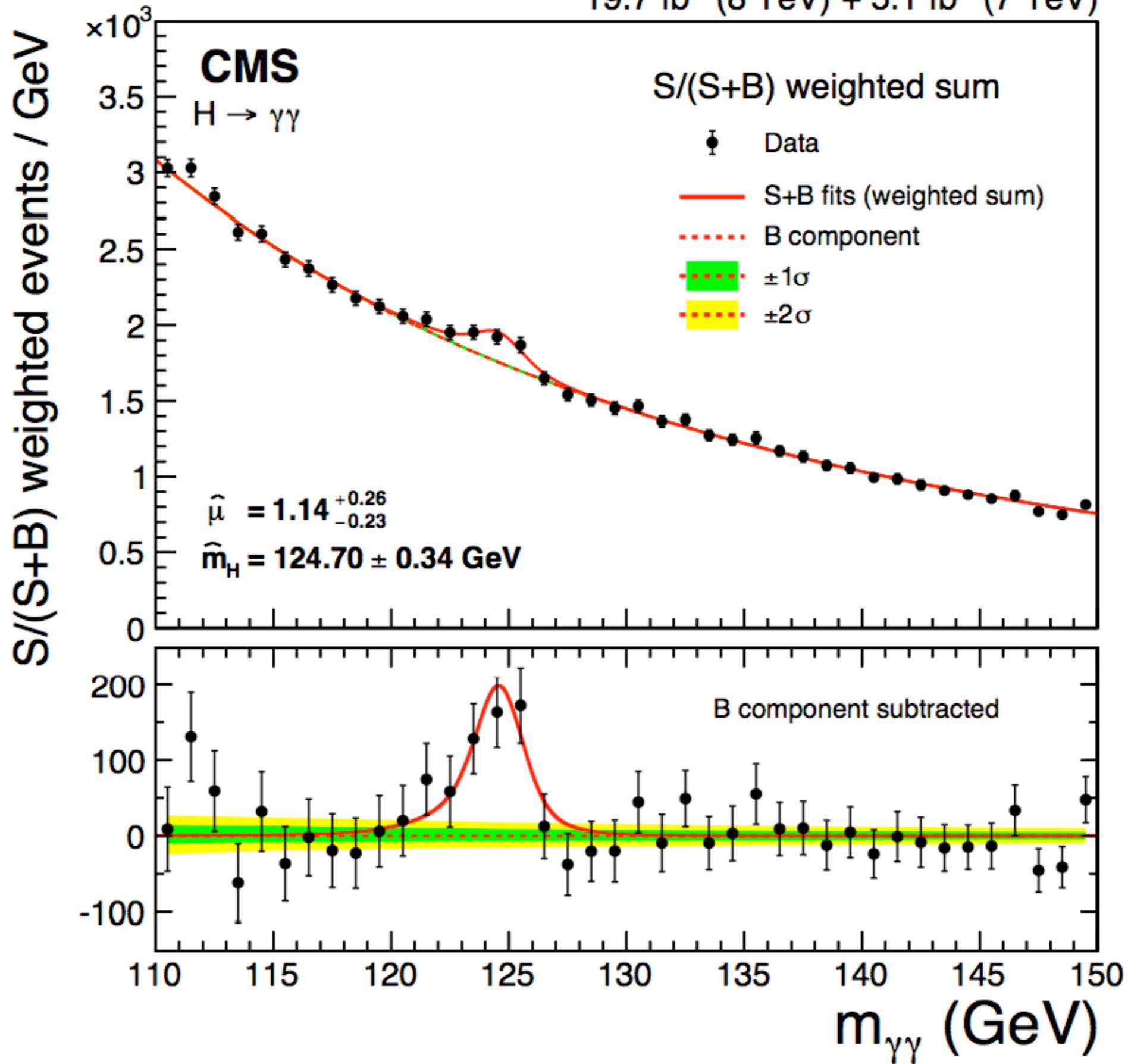
Note that these modes correspond to branching ratios of  
0.23%                      and                      0.012%

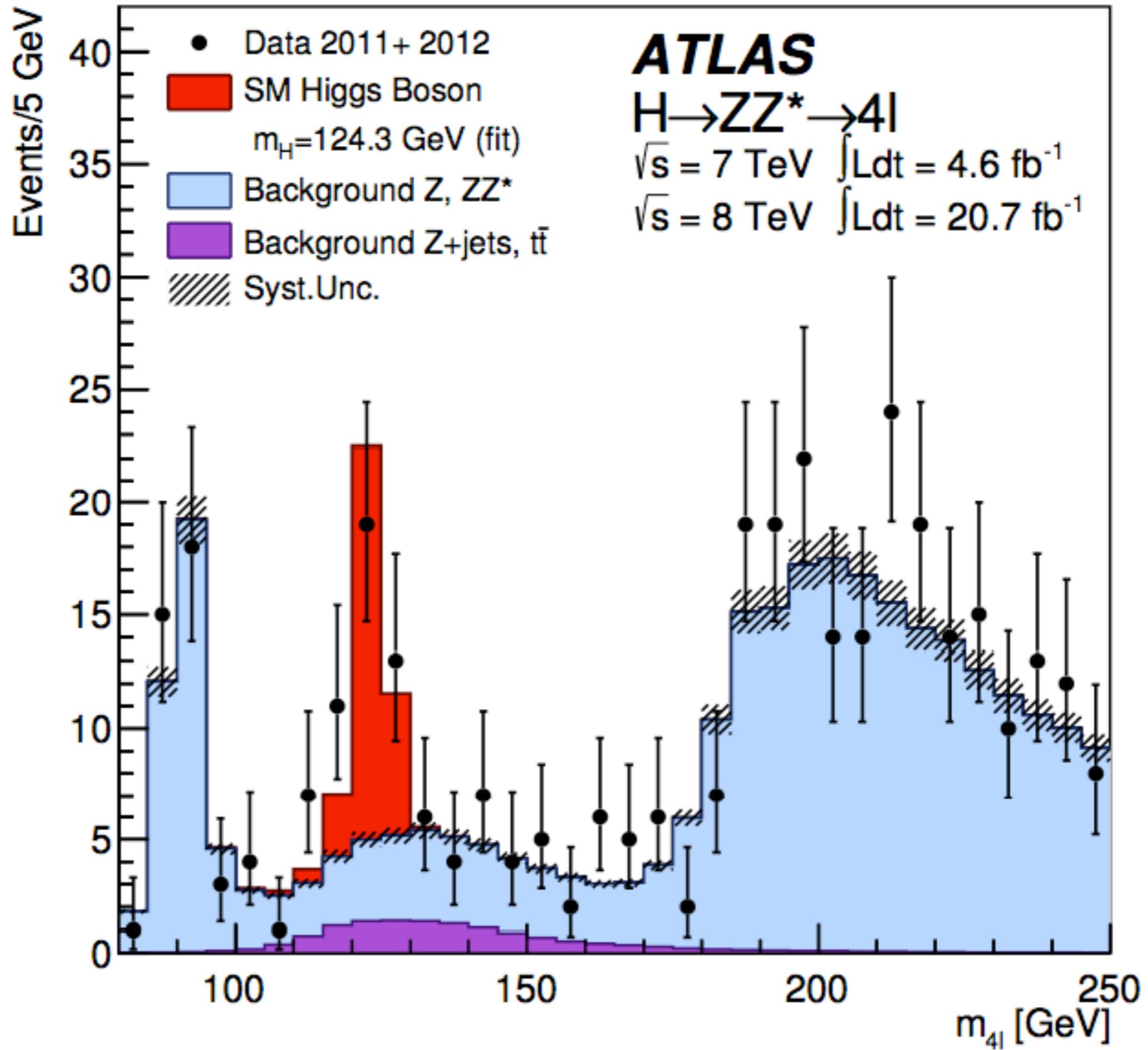
respectively. With a production cross section of about 20 pb, these processes have rates

$$4 \times 10^{-13} \qquad \text{and} \qquad 2 \times 10^{-14}$$

of the pp total cross section.

19.7 fb<sup>-1</sup> (8 TeV) + 5.1 fb<sup>-1</sup> (7 TeV)





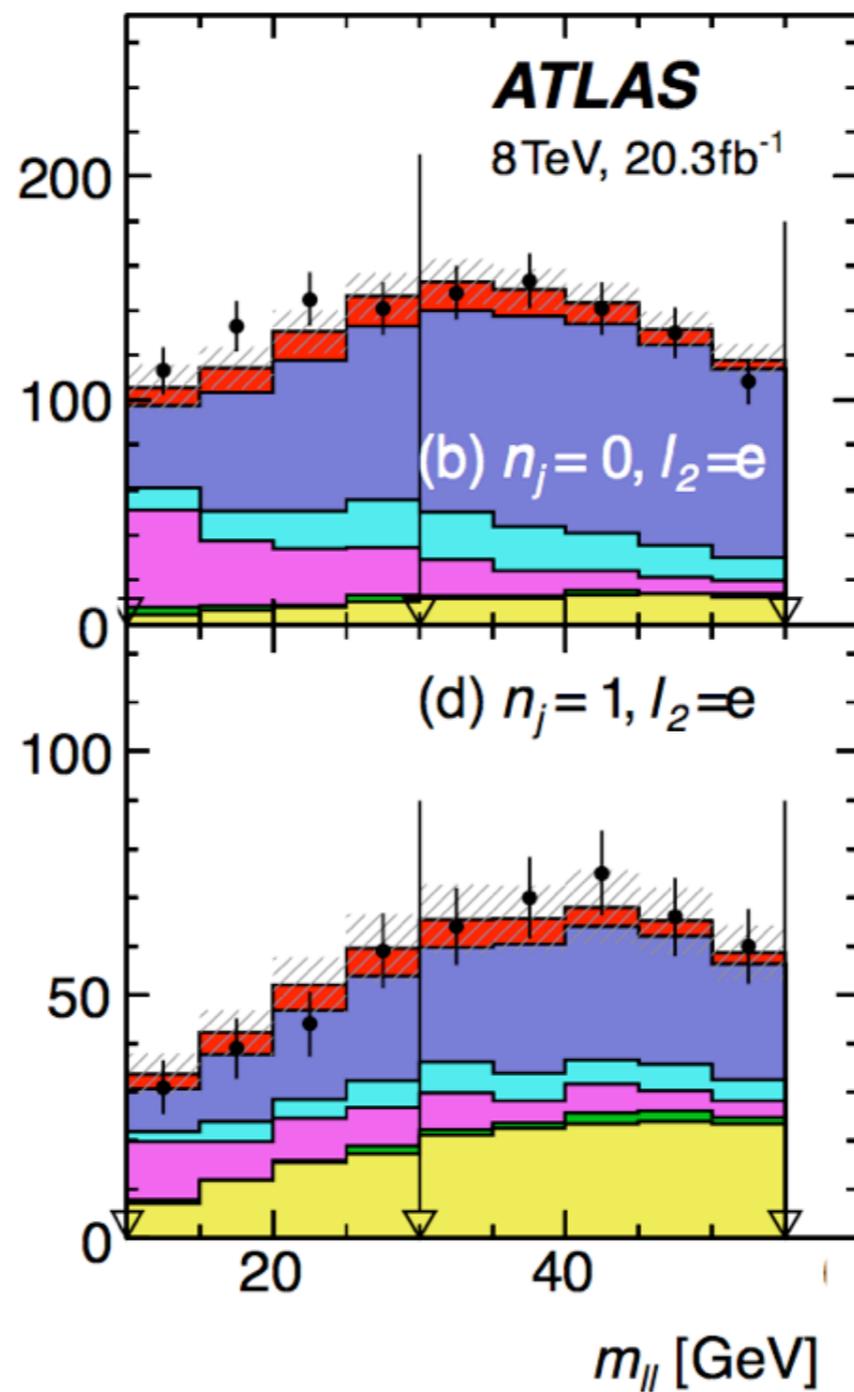
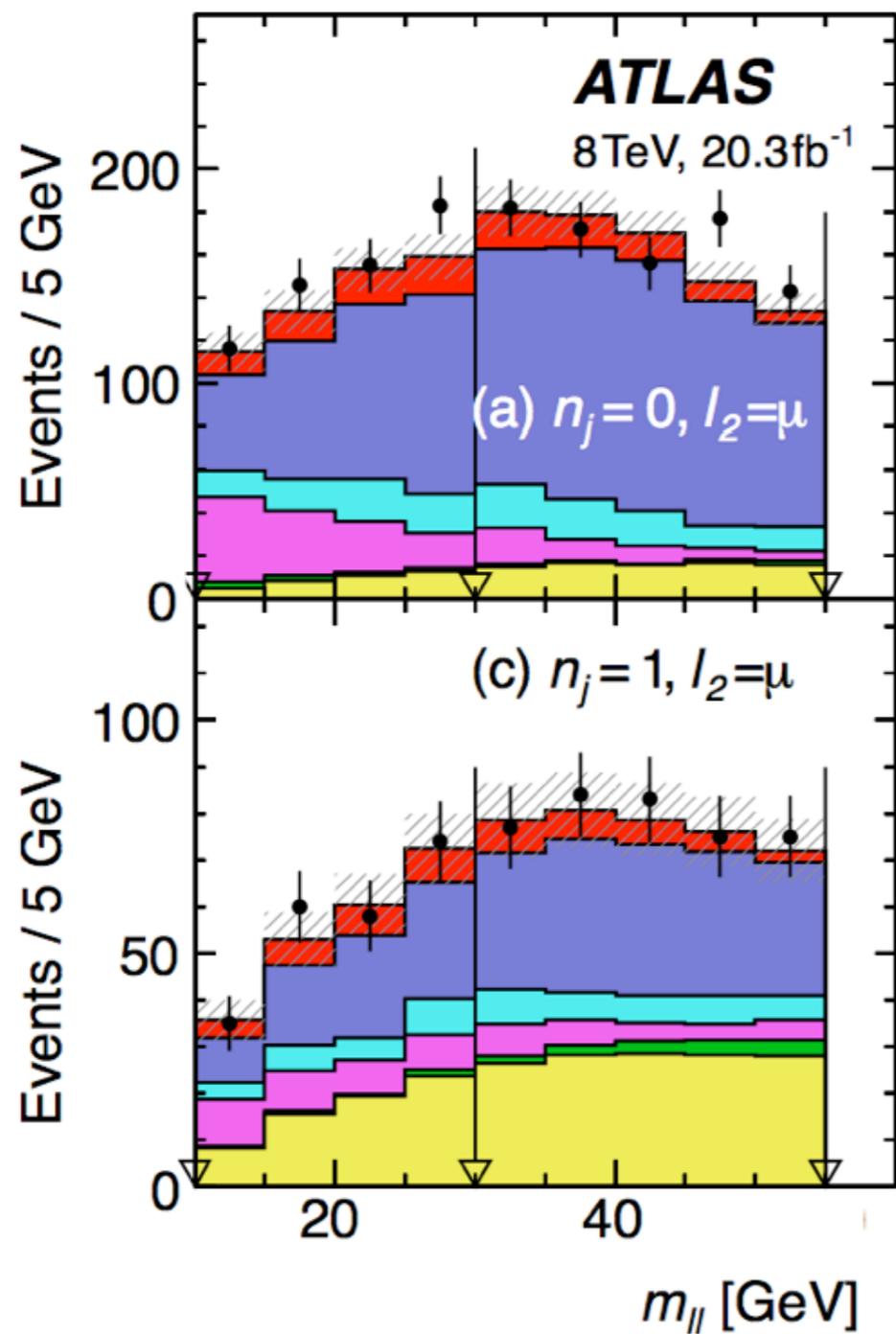
Once we are convinced that the Higgs resonance is actually present at a mass of 125 GeV, we can look for its signatures in other decay modes. These have larger rates, but they produce events that are not obviously distinguishable from other Standard Model reactions.

An example is  $pp \rightarrow h \rightarrow W^+ W^- \rightarrow \ell^+ \ell^- \nu \bar{\nu}$ . This is not obviously distinguishable from

$$pp \rightarrow W^+ W^- \rightarrow \ell^+ \ell^- \nu \bar{\nu}$$

The signal to background can be enhanced by going to a region where  $m(\ell^+ \ell^-)$  and the angle between the two leptons are relatively small. It is also necessary to apply a jet veto ( $n_j = 0, 1$ ) to avoid background from

$$pp \rightarrow t\bar{t} \rightarrow b\bar{b}\ell^+ \ell^- \nu \bar{\nu}$$



**ATLAS  $H \rightarrow WW^*$**

$\sqrt{s} = 8 \text{ TeV}, 20.3 \text{ fb}^{-1}$

Obs  $\pm$  stat

Exp  $\pm$  syst

Higgs

WW

Misid

VV

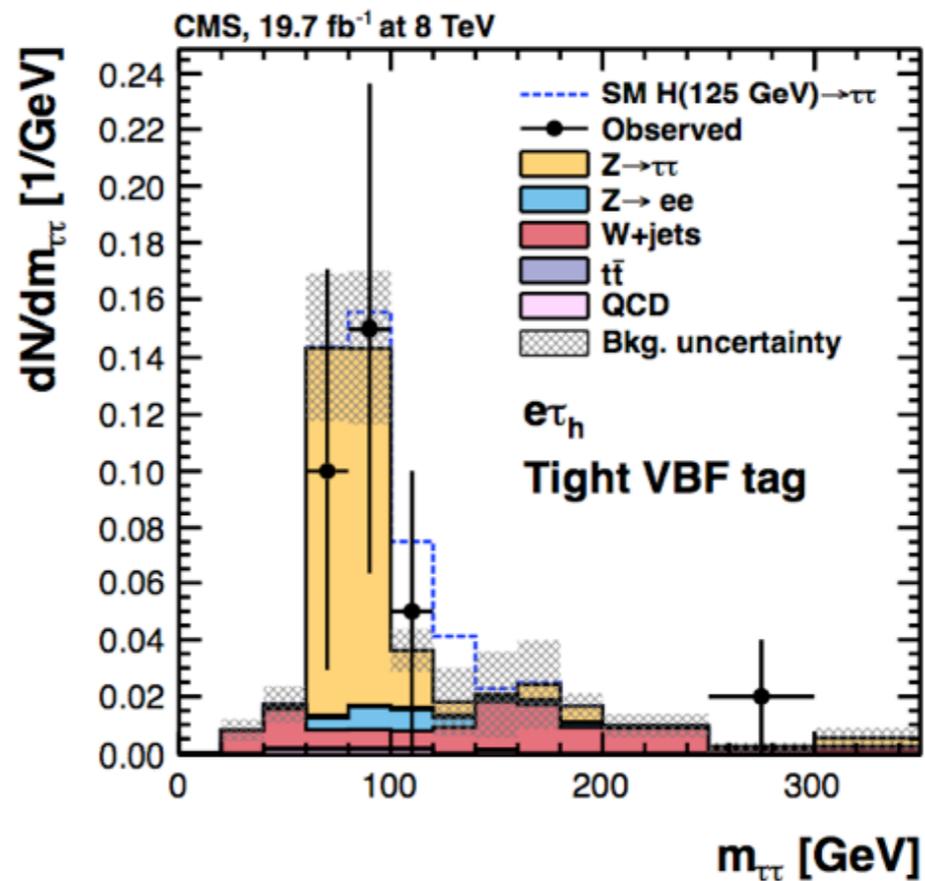
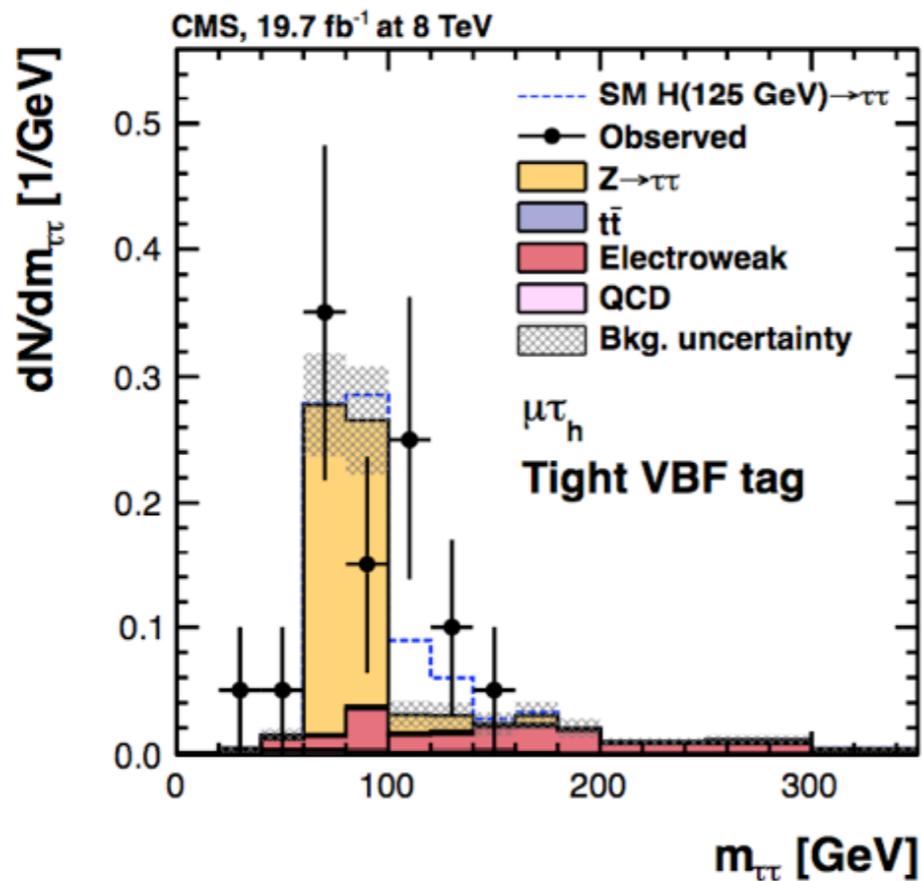
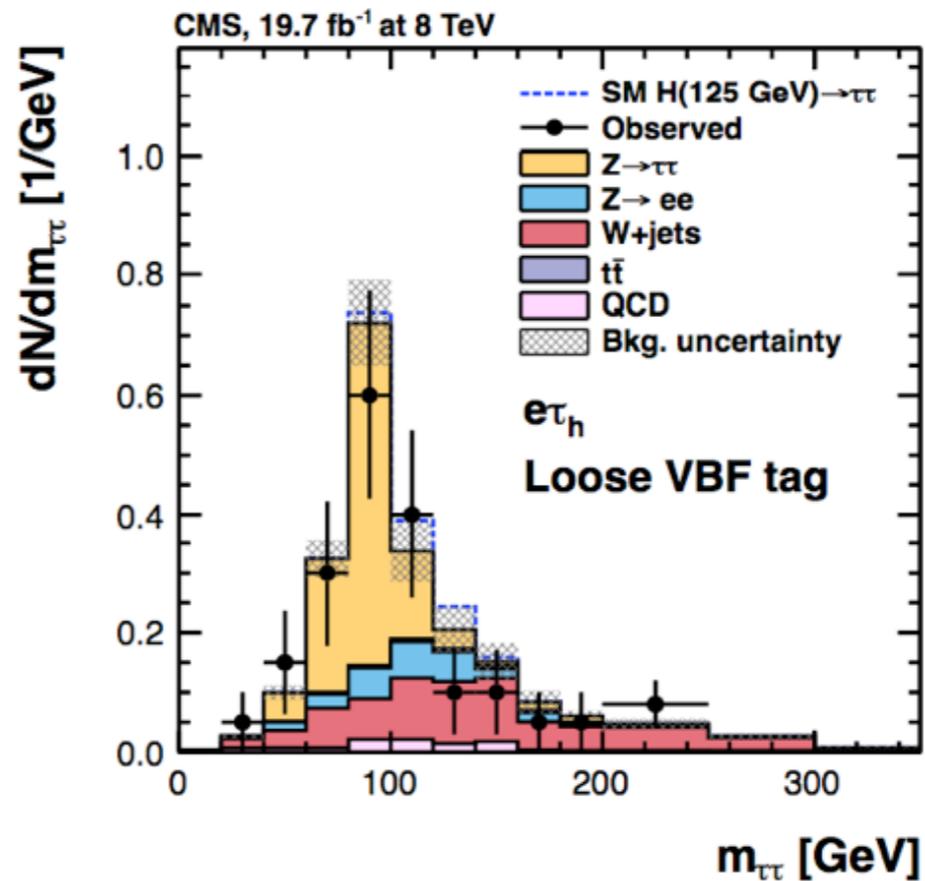
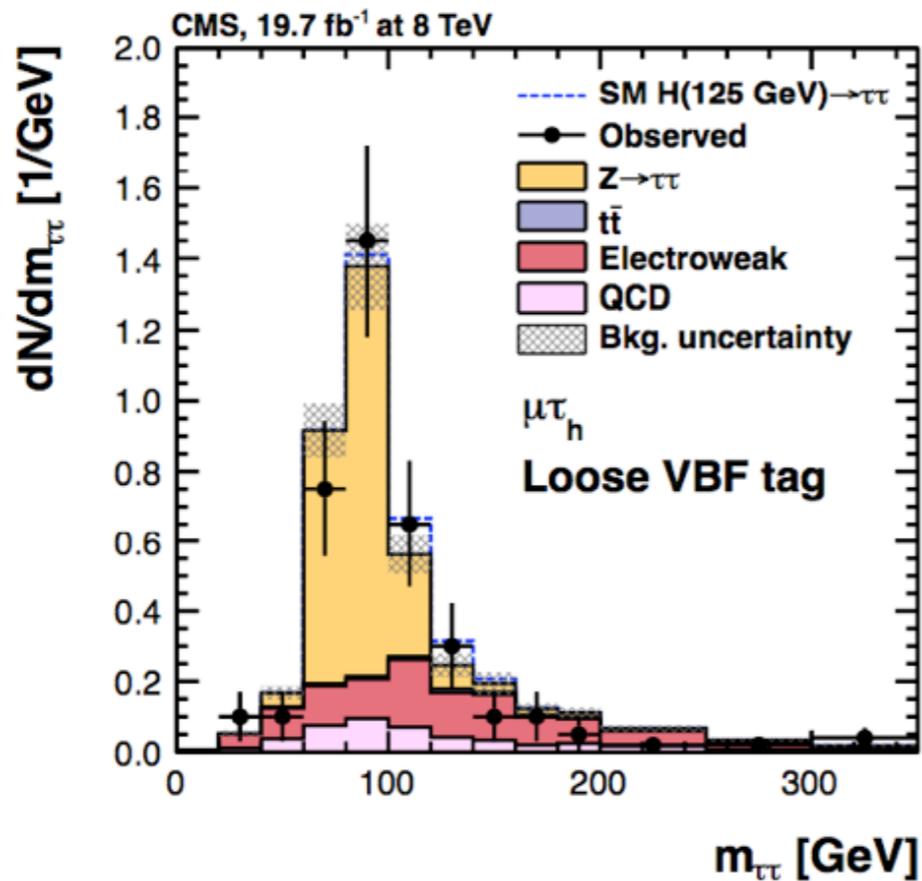
DY

Top

For  $pp \rightarrow h \rightarrow \tau^+ \tau^-$  , important backgrounds are

$$pp \rightarrow Z \rightarrow \tau^+ \tau^- \quad pp \rightarrow W^+ W^-$$

and QCD reactions where jets fake the  $\tau$  signature. The strongest analyses use the vector boson fusion signature, with forward jets, to minimize the QCD background.



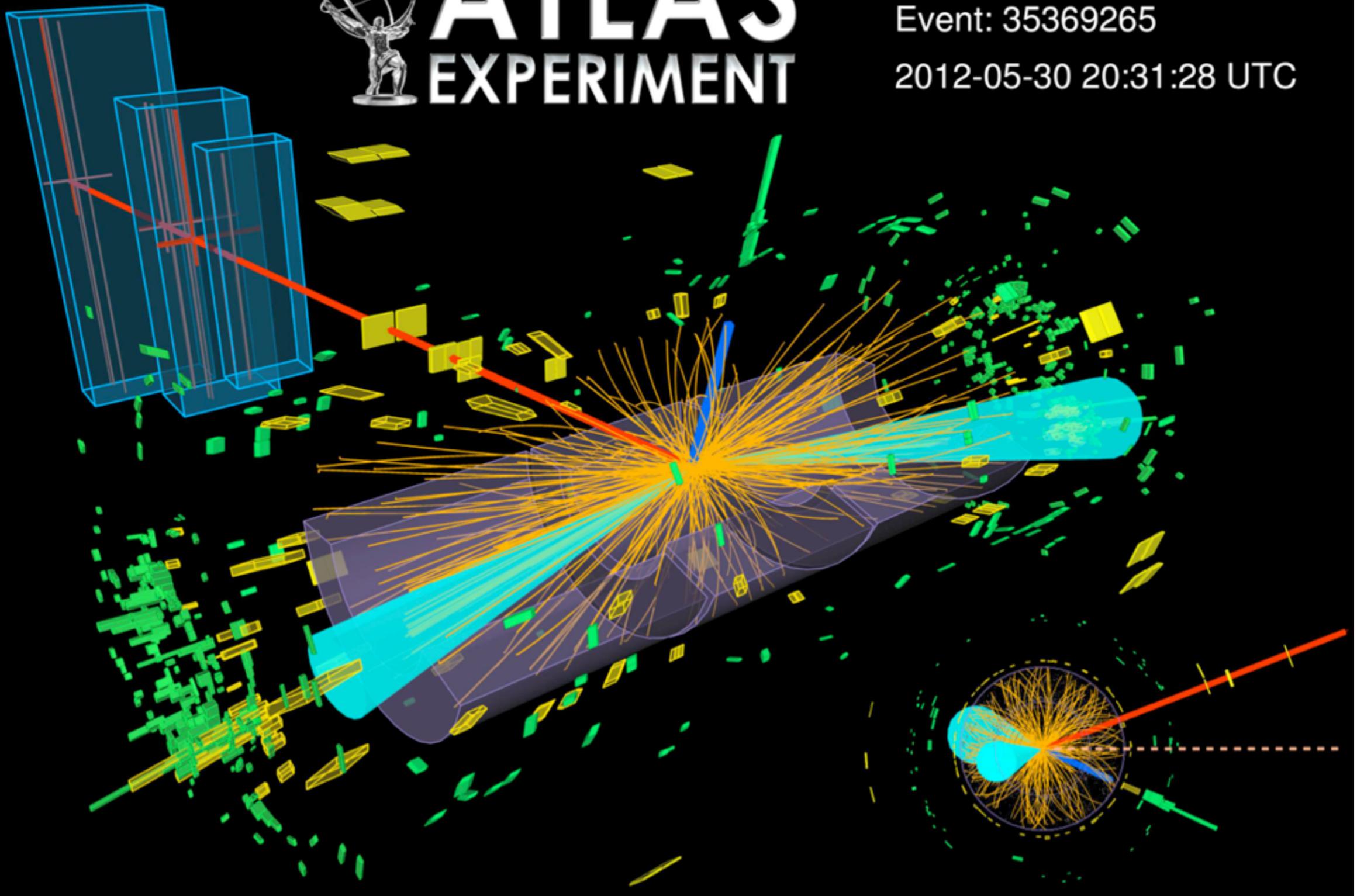


# ATLAS EXPERIMENT

Run: 204153

Event: 35369265

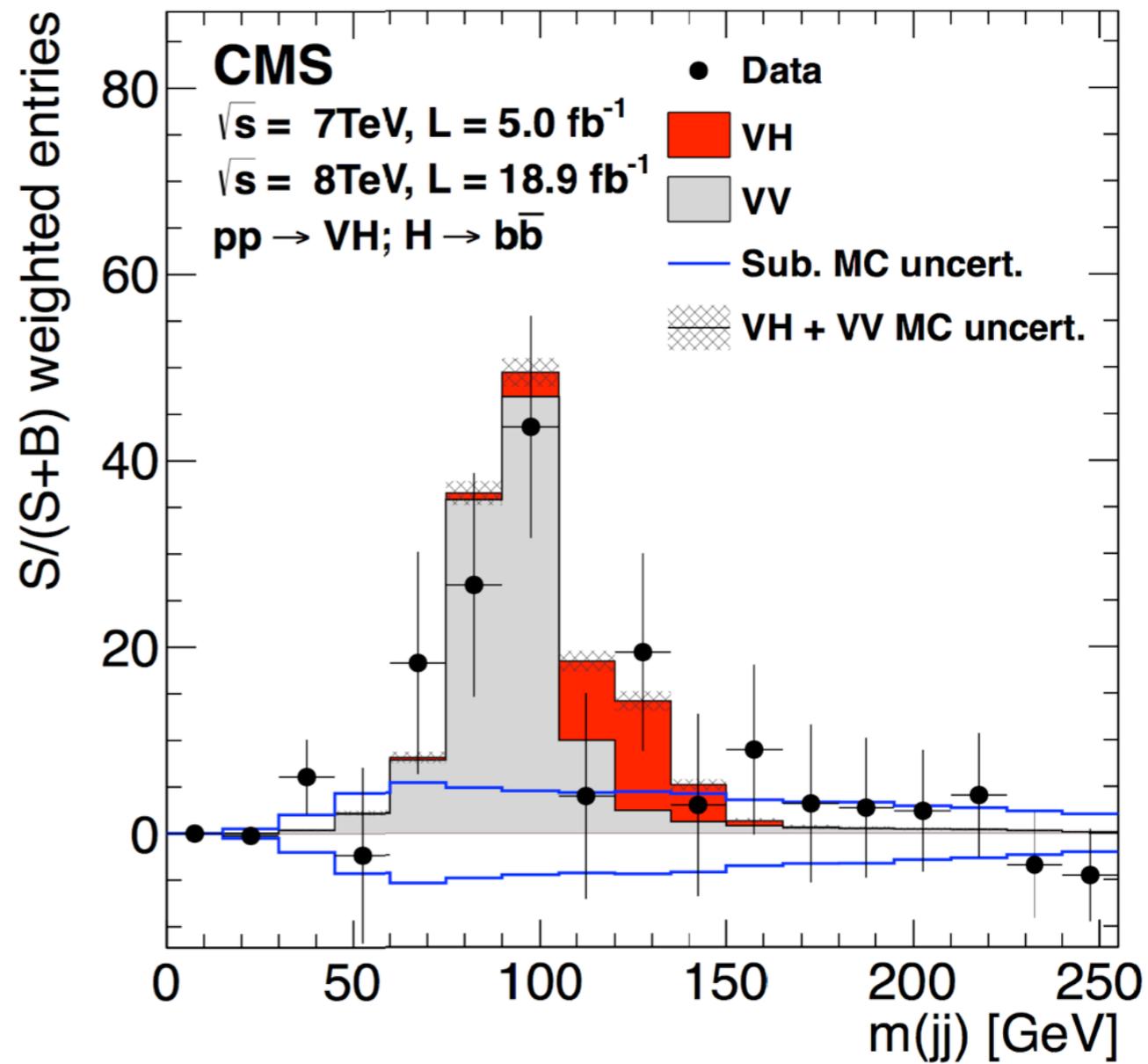
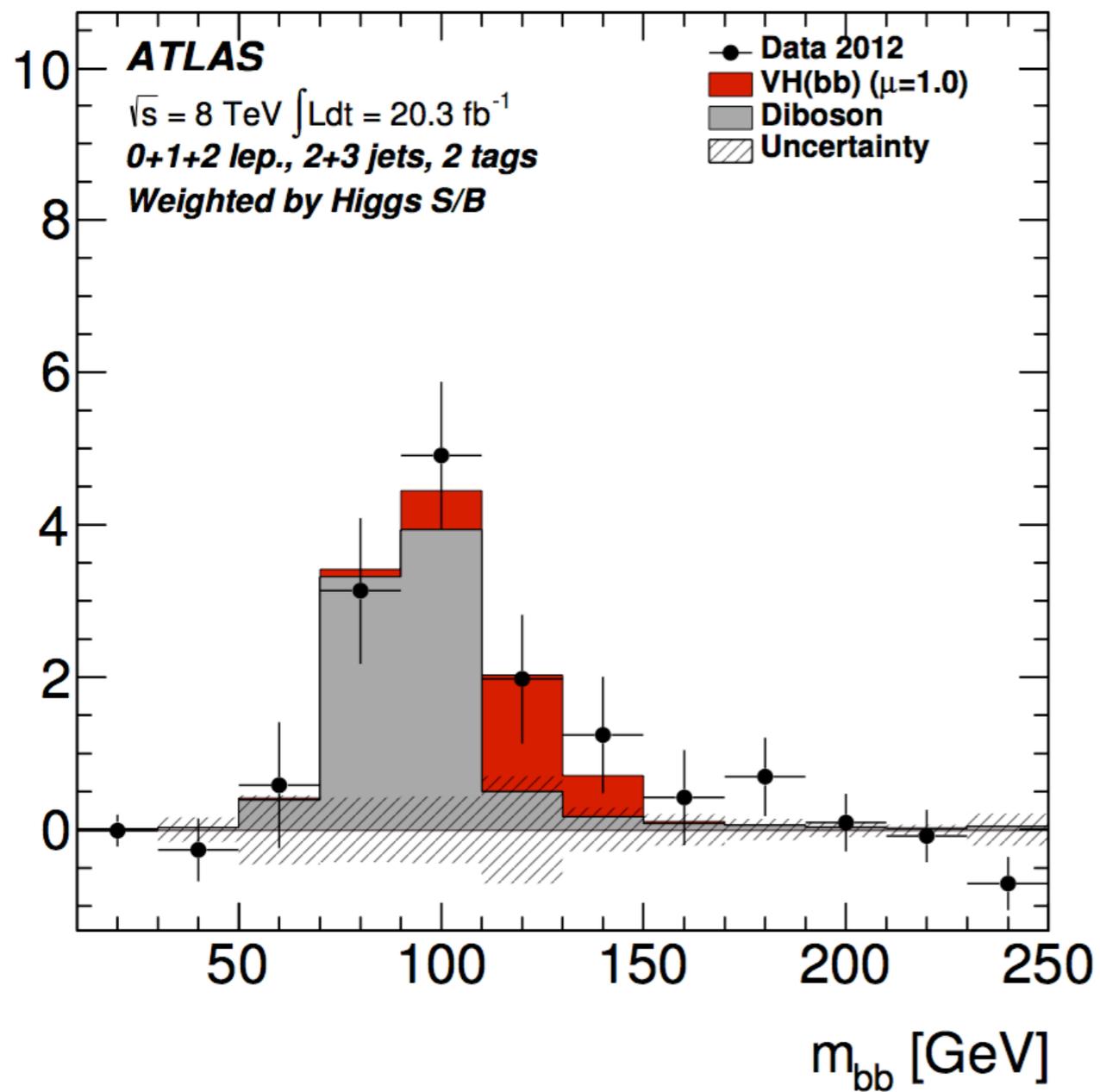
2012-05-30 20:31:28 UTC



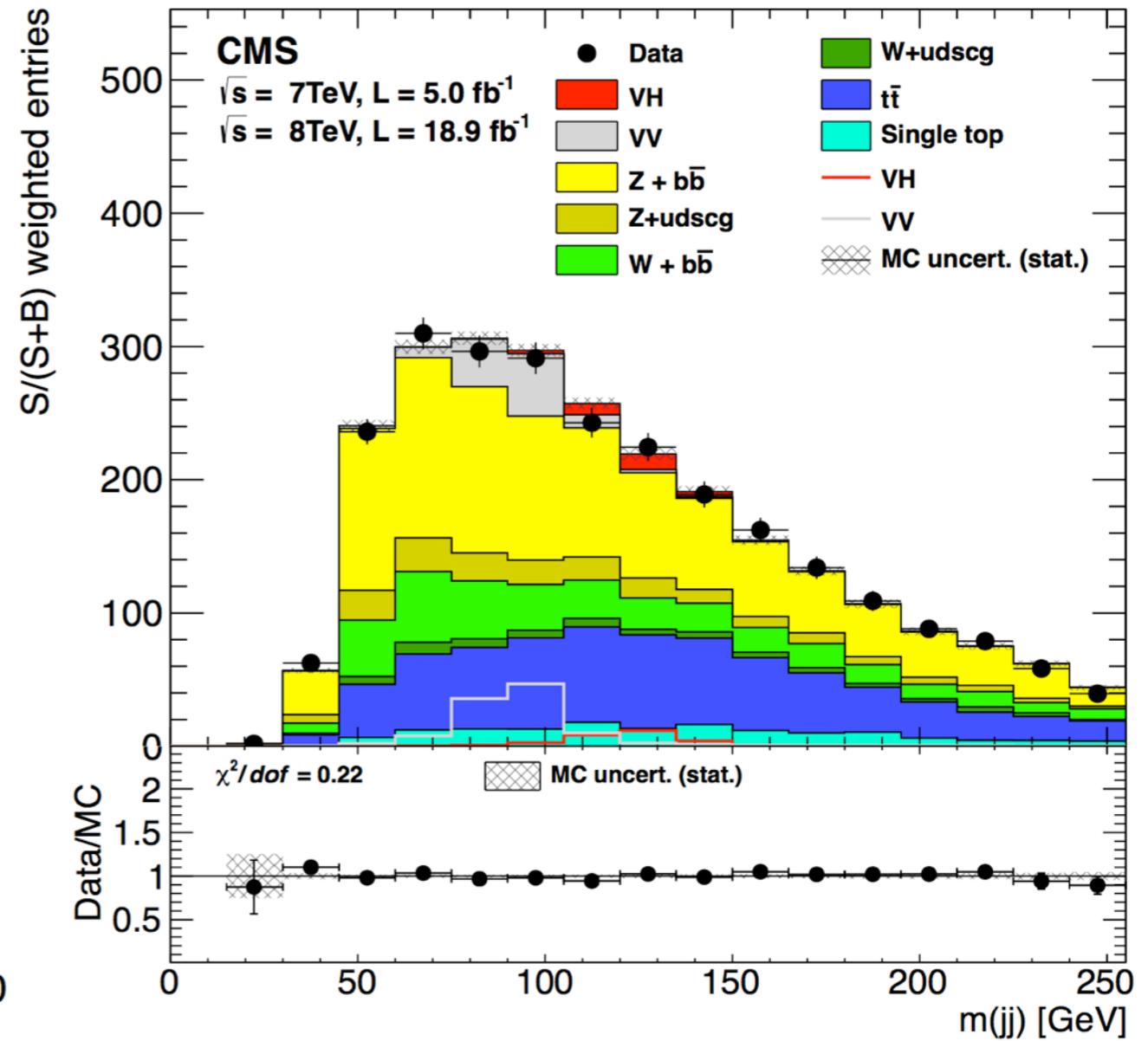
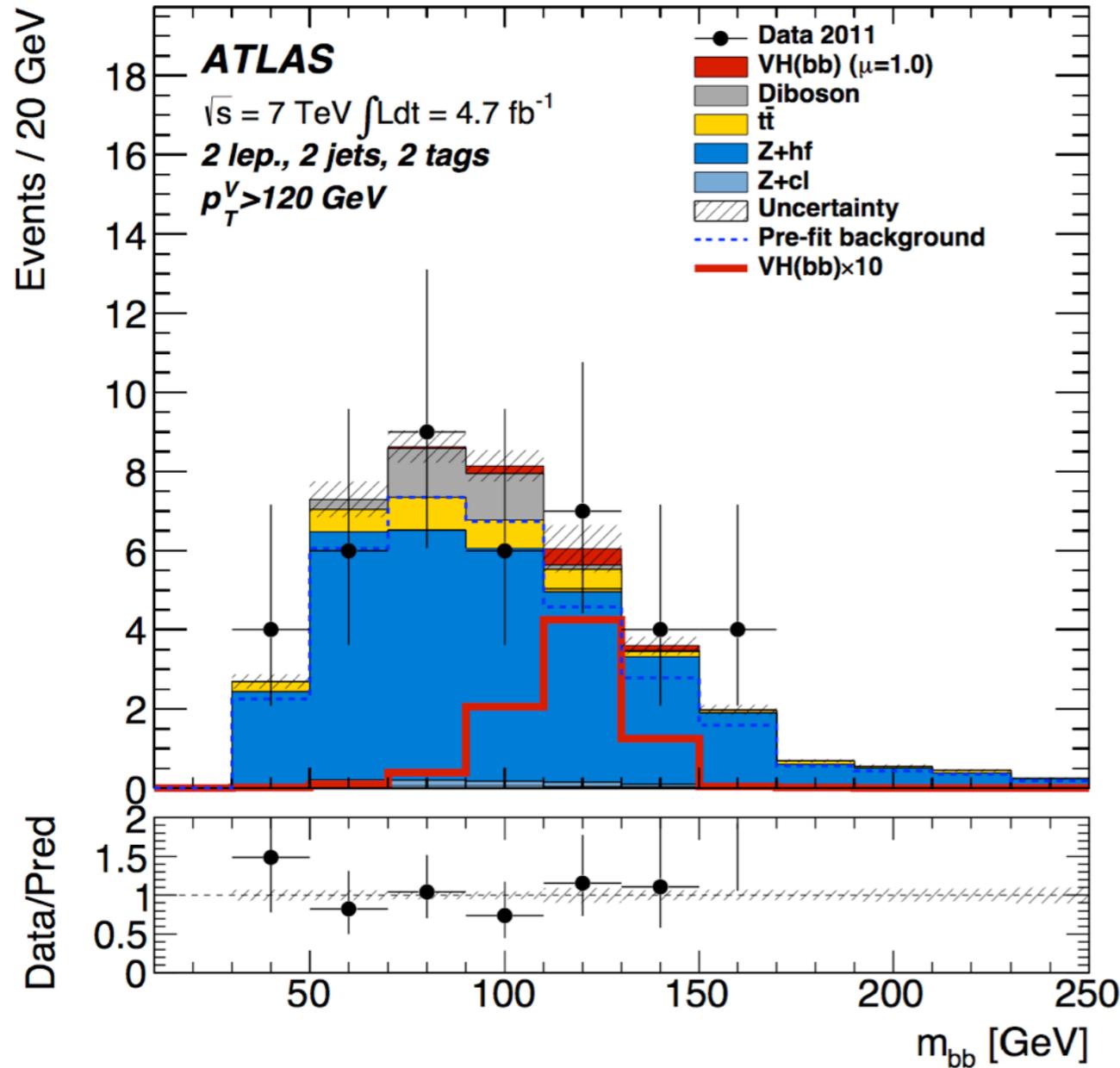
The most challenging of the major modes is the largest one,  $h \rightarrow b\bar{b}$ . Observing this mode in gg production is probably hopeless, since  $gg \rightarrow b\bar{b}$  with 125 GeV mass jets is about a million times larger. Current analyses use associated production with W or Z. However, the reactions

$$\begin{aligned} pp &\rightarrow Vh, \quad h \rightarrow b\bar{b} \\ pp &\rightarrow VZ, \quad Z \rightarrow b\bar{b} \\ pp &\rightarrow Vg, \quad g \rightarrow b\bar{b} \end{aligned}$$

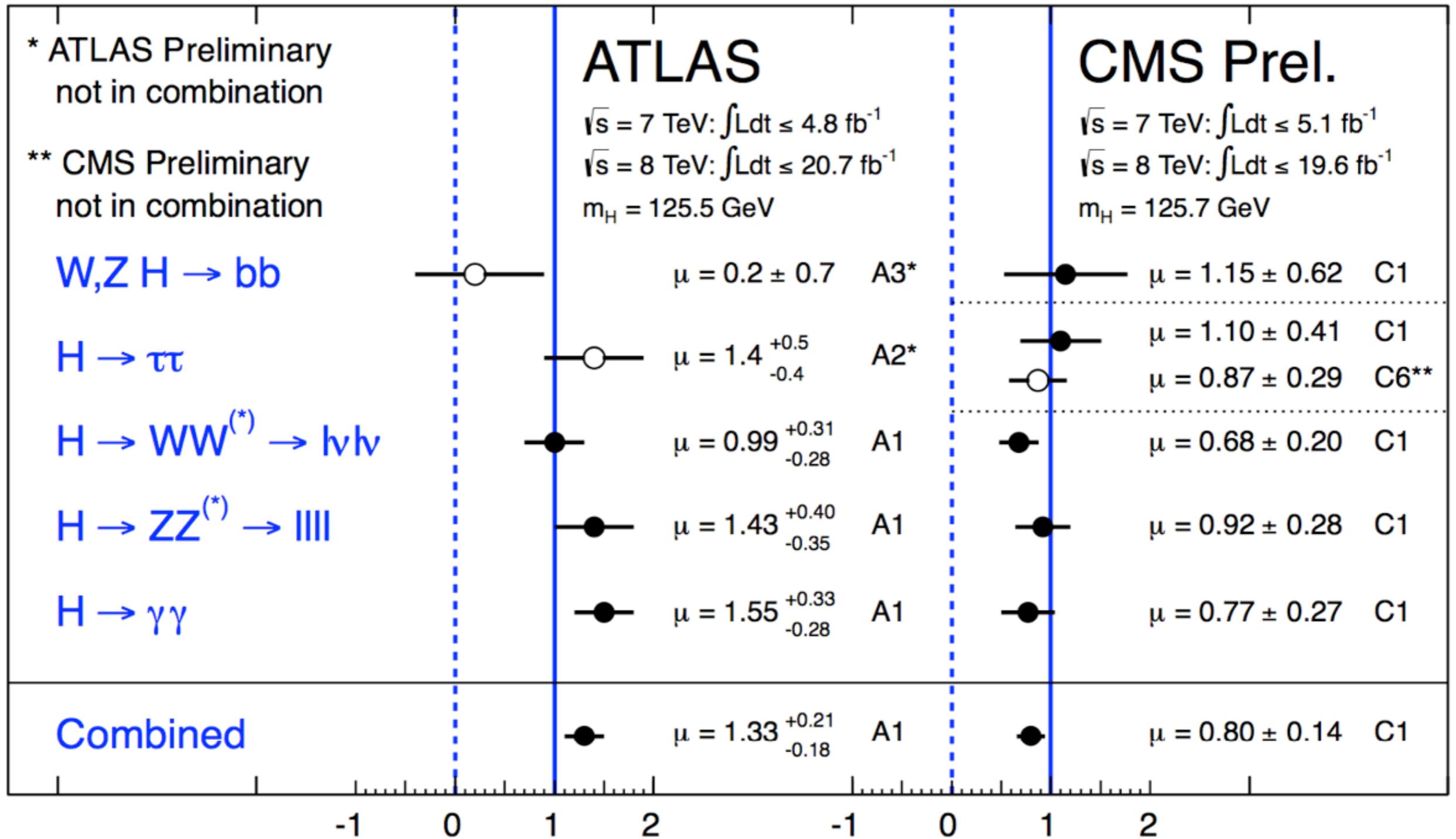
are difficult to distinguish. It is thought that this can be done using properties of boosted  $h, Z, g$  systems including the jet mass and color flow.



Here are sample plots from some signal regions that are not background-subtracted.



# Here is a summary of Higgs observations from LHC Run 1



PDG summary 2014

Best fit signal strength ( $\mu$ )