Precision QCD

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Perturbative calculations

Perturbative calculations rely on the idea of an order-by-order expansion in the small coupling

$$\sigma \sim \underbrace{A}_{\text{LO}} + \underbrace{B\alpha_s}_{\text{NLO}} + \underbrace{C\alpha_s^2}_{\text{NNLO}} + \underbrace{D\alpha_s^3}_{\text{NNNLO}} + \ldots$$

- Perturbative calculations are possible because the coupling is small at high energy
- In QCD (or in a generic QFT) the coupling depends on the energy (renormalization scale)
- So changing scale the result changes. By how much? What does this dependence mean?
- In the following will discuss these issues through examples

Hard cross section

Born level cross section straightforward in principle

$$\sigma_{LO} = \int_m d\Phi_m |\mathcal{M}^{(0)}(\{p_i\})|^2 S(\{p_i\})$$

m-particle phase space
(e.g. Vegas)

Matrix element

measurement function
(constraint on phase space)

Leading order with Feynman diagrams

Get *any* LO cross-section from the Lagrangian

1. draw all Feynman diagrams
2. put in the explicit Feynman rules and get the amplitude
3. do some algebra, simplifications
4. square the amplitude
5. integrate over phase space + flux factor + sum/average over outgoing/incoming states

Automated tools for (1-3): FeynArts/Qgraf, Mathematica/Form etc.

Bottlenecks

a) number of Feynman diagrams diverges factorially

b) algebra becomes more cumbersome with more particles

But given enough computer power everything can be computed at LO

Diagrams for gluon amplitudes

Number of diagrams for gg $\rightarrow$ n gluons

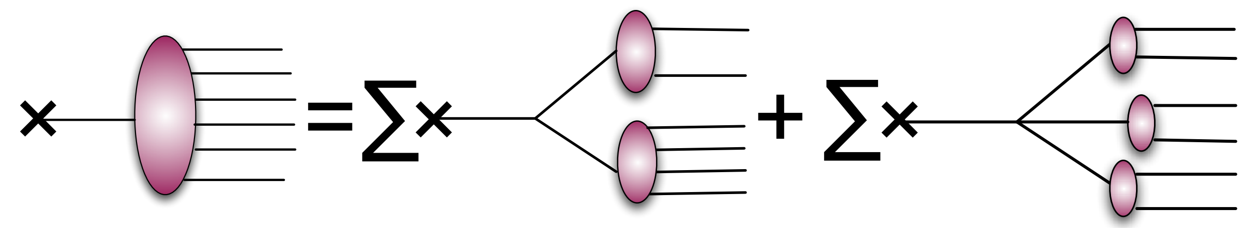
n	2	3	4	5	6	7	8
diag.	4	25	220	2485	34300	559405	10525900

- number of diagrams grows very fast
- complexity of each diagrams grows with n

Alternative methods?

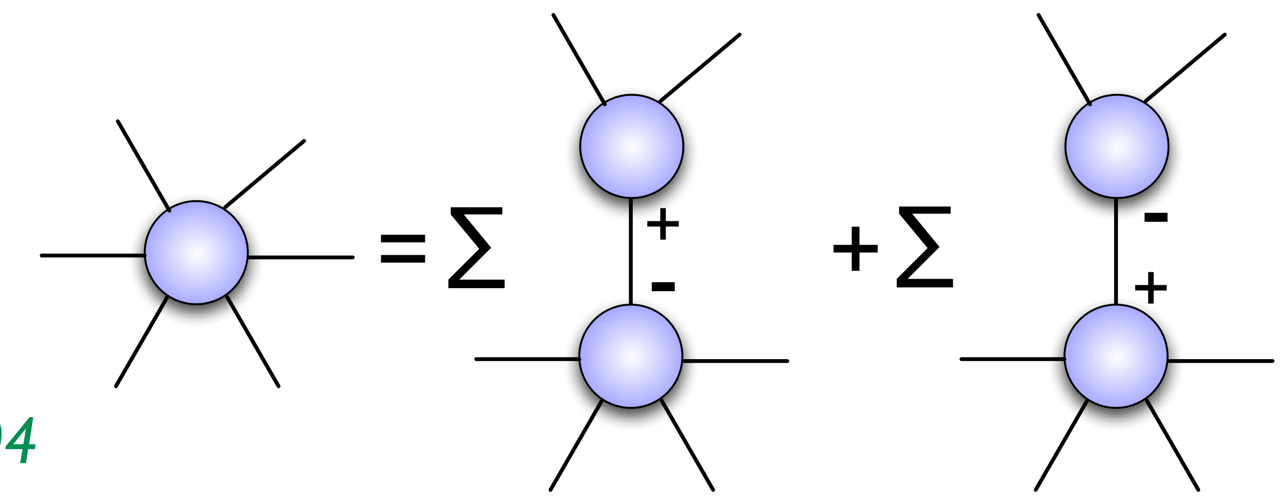
Techniques beyond Feynman diagrams

✓ Berends-Giele relations: compute helicity amplitudes recursively using off-shell currents



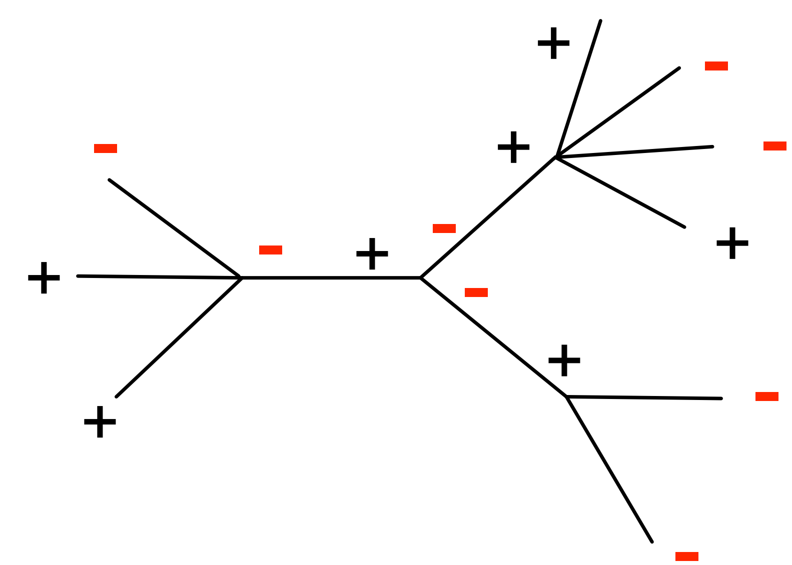
Berends, Giele ’88

✓ BCF relations: compute helicity amplitudes via on-shell recursions (use complex momentum shifts)



Britto, Cachazo, Feng ’04

✓ CSW relations: compute helicity amplitudes by sewing together MHV amplitudes [- - + + ... +]



Cachazo, Svrcek, Witten ’04

Benefits and drawbacks of LO

Benefits of LO:

- fastest option; often the only one
- test quickly new ideas with fully exclusive description
- many working, well-tested approaches
- highly automated, crucial to explore new ground, but no precision

Drawbacks of LO:

- large scale dependences, reflecting large theory uncertainty
- no control on normalization
- poor control on shapes
- poor modeling of jets

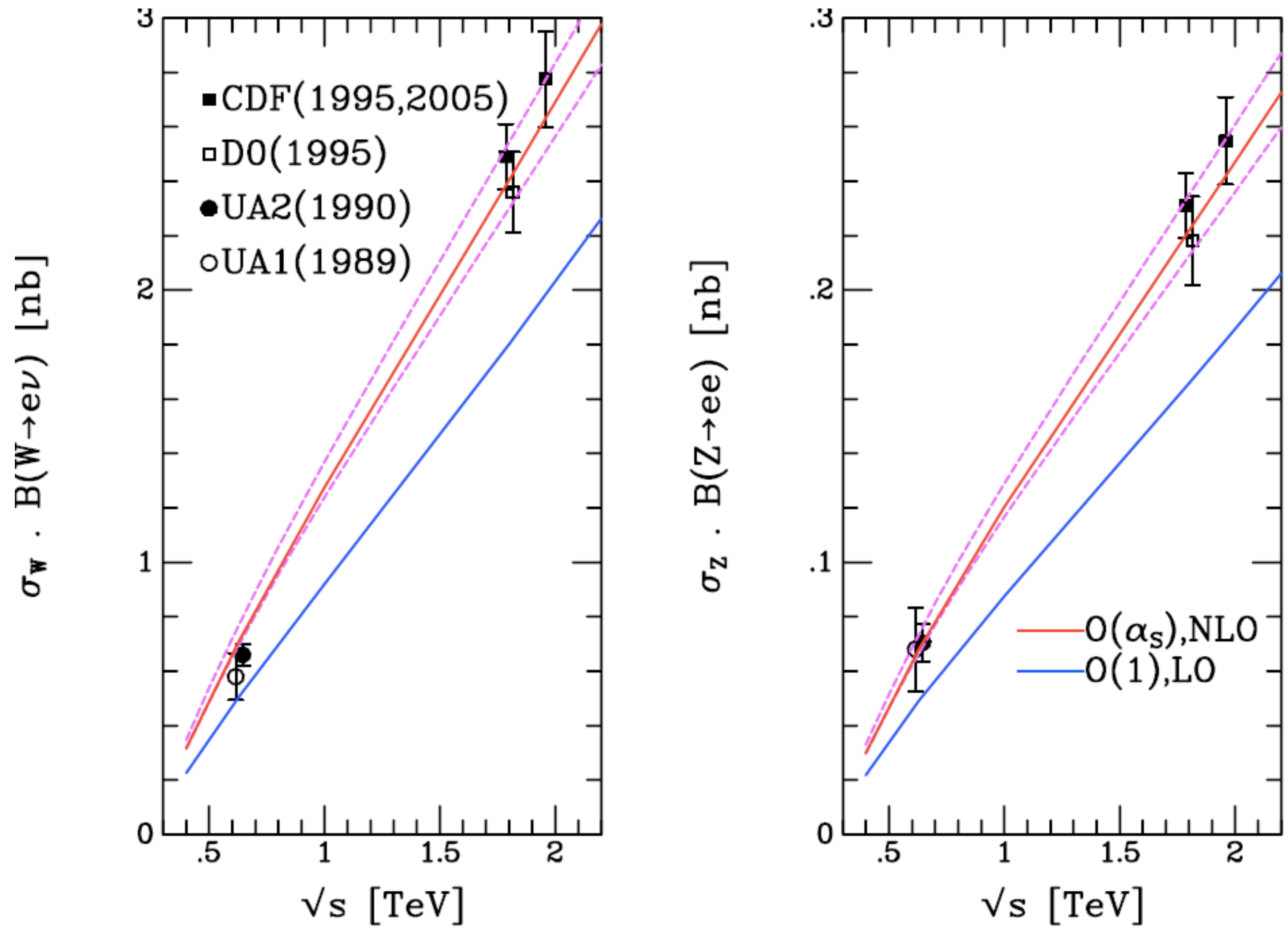
Example: W+4 jet cross-section $\propto \alpha_s(Q)^4$

Vary $\alpha_s(Q)$ by ±10% via change of Q ⇒ cross-section varies by ±40%

Is it necessary to go beyond LO?

Very early observation:

at least NLO corrections are needed to describe data



Drell Yan production is one of the first processes for which NLO corrections have been computed

Leading order n-jet cross-section

- Consider the cross-section to produce n jets. The leading order result at scale μ result will be

$$\sigma^{\rm LO}_{\rm njets}(\mu) = \alpha_s(\mu)^n A(p_i, \epsilon_i, \ldots)$$

- Instead, choosing a scale μ' one gets

$$\sigma^{\rm LO}_{\rm njets}(\mu') = \alpha_s(\mu')^n A(p_i, \epsilon_i, \ldots) = \alpha_s(\mu)^n \left(1 + n\, b_0\, \alpha_s(\mu) \ln \frac{\mu^2}{\mu'^2} + \ldots\right) A(p_i, \epsilon_i, \ldots)$$

So the change of scale is a NLO effect ($\propto \alpha_s$), but this becomes more important when the number of jets increases ($\propto$n)

- Notice that at Leading Order the normalization is not under control:

$$\frac{\sigma^{\rm LO}_{\rm njets}(\mu)}{\sigma^{\rm LO}_{\rm njets}(\mu')} = \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu')}\right)^n$$

NLO n-jet cross-section

Now consider n-jet cross-section at NLO. At scale μ the result reads

$$\sigma^{\rm NLO}_{\rm njets}(\mu) = \alpha_s(\mu)^n A(p_i, \epsilon_i, \dots) + \alpha_s(\mu)^{n+1}\left(B(p_i, \epsilon_i, \dots) - n b_0 \ln\frac{\mu^2}{Q_0^2}\right) + \dots$$

- So the NLO result compensates the LO scale dependence. The residual dependence is NNLO
- Scale dependence and normalization start being under control only at NLO, since a compensation mechanism kicks in
- Notice also that a good scale choice automatically resums large logarithms to all orders, while a bad one spuriously introduces large logs and ruins the PT expansion
- Scale variation is conventionally used to estimate the theory uncertainty, but the validity of this procedure should not be overrated

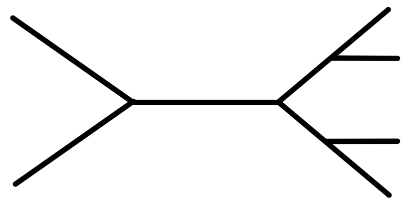
Next-to-leading order

Benefits of next-to-leading order (NLO)

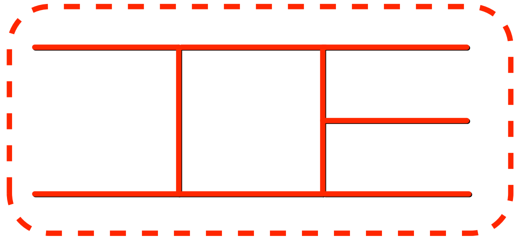
- reduce dependence on unphysical scales
- establish normalization and shape of cross-sections
- small scale dependence at LO can be very misleading, small dependence at NLO robust sign that PT is under control
- large NLO correction or large dependence at NLO robust sign that neglected other higher order are important
- through loop effects get indirect information about sectors not directly accessible

Ingredients at NLO

A full N-particle NLO calculation requires (e.g. for N=3):



- ☑ tree graph rates with N+1 partons
  ➔ soft/collinear divergences



Bottleneck for a long time. Now understood how to compute this automatically

- ☑ virtual correction to N-leg process
  ➔ divergence from loop integration,
  use e.g. dimensional regularization

- ☑ **set of subtraction terms to cancel divergences**

We won't have time to do detailed NLO calculations, but let's look a bit more in detail at the issue of divergences/subtraction

Regularization in QCD

<u>Regularization</u>: a way to make intermediate divergent quantities meaningful

- In QCD dimensional regularization is today the standard procedure, based on the fact that d-dimensional integrals are more convergent if one reduces the number of dimensions.

$$\int \frac{d^4 l}{(2\pi)^4} \to \mu^{2\epsilon} \int \frac{d^d l}{(2\pi)^d}\,, \quad d = 4 - 2\epsilon < 4$$

- N.B. to preserve the correct dimensions a mass scale μ is needed
- Divergences show up as intermediate poles 1/ε $\int_0^1 \frac{dx}{x} \to \int_0^1 \frac{dx}{x^{1-\epsilon}} = \frac{1}{\epsilon}$
- This procedure works both for UV divergences and IR divergences

Alternative regularization schemes: photon mass (EW), cut-offs, Pauli-Villard ...
Compared to those methods, dimensional regularizatiom has the big virtue that it leaves the regularized theory Lorentz invariant, gauge invariant, unitary etc.

Subtraction and slicing methods

- Consider e.g. an n-jet cross-section with some arbitrary infrared safe jet definition. At NLO, two divergent integrals, but the sum is finite

$$\sigma^J_{\rm NLO} = \int_{n+1} d\sigma^J_{\rm R} + \int_n d\sigma^J_{\rm V}$$

- Since one integrates over a different number of particles in the final state, real and virtual need to be evaluated first, and combined then

- This means that one needs to find a way of removing divergences before evaluating the phase space integrals

- Two main techniques to do this
  - *phase space slicing* ⇒ obsolete because of practical/numerical issues
  - *subtraction method* ⇒ most used in recent applications

Subtraction method

- The real cross-section can be written schematically as

$$d\sigma_R^J = d\phi_{n+1}|\mathcal{M}_{n+1}|^2 F_{n+1}^J(p_1,\ldots,p_{n+1})$$

where F^J is the arbitrary jet-definition

- The matrix element has a non-integrable divergence

$$|\mathcal{M}_{n+1}|^2 = \frac{1}{x}\mathcal{M}(x)$$

where x vanishes in the soft/collinear divergent region

- IR divergences in the loop integration regularized by taking D = 4-2ε

$$2\,\mathrm{Re}\{\mathcal{M}_V \cdot \mathcal{M}_0^*\} = \frac{1}{\epsilon}\mathcal{V}$$

Subtraction method

- The n-jet cross-section becomes

$$\sigma^J_{\rm NLO} = \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{M}(x) F^J_{n+1}(x) + \frac{1}{\epsilon}\mathcal{V}F^J_n$$

- Infrared safety of the jet definition implies

$$\lim_{x\to 0} F^J_{n+1}(x) = F^J_n$$

- KLN cancelation guarantees that

$$\lim_{x\to 0} \mathcal{M}(x) = \mathcal{V}$$

- One can then add and subtract the analytically computed divergent part

$$\sigma^J_{\rm NLO} = \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{M}(x) F^J_{n+1}(x) - \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{V}F^J_n + \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{V}F^J_n + \frac{1}{\epsilon}\mathcal{V}F^J_n$$

Subtraction method

- This can be rewritten exactly as

$$\sigma^J_{\mathrm{NLO}} = \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{M}(x) \left(F^J_1(x) - \mathcal{V} F^J_0 \right) + \mathcal{O}(1) \mathcal{V} F^J_0$$

$\Rightarrow$ Now both terms are finite and can be evaluated numerically

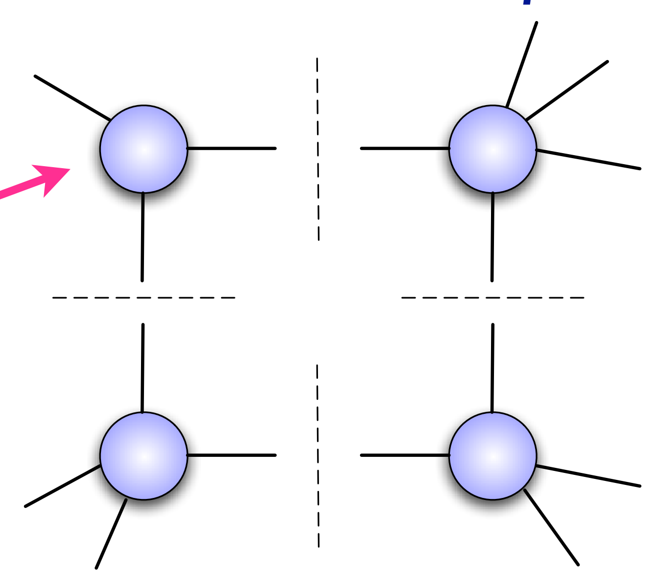
- Subtracted cross-section must be calculated separately for each process (but mostly automated now). It must be valid everywhere in phase space
- Systematized in the seminal papers of Catani-Seymour (dipole subtraction, '96) and Frixione-Kunszt-Signer (FKS method, '96)
- Subtraction used in all recent NLO applications and public codes (Event2, Disent, MCFM, NLOjet++, MC@NLO, POWHEG ...)

Two breakthrough ideas for one-loop

A number of breakthrough ideas developed in the last 10 years, most notably

1) "... we show how to use generalized unitarity to read off the (box) coefficients. The generalized cuts we use are quadrupole cuts ..."

NB: non-zero because cut gives complex momenta



Britto, Cachazo, Feng '04

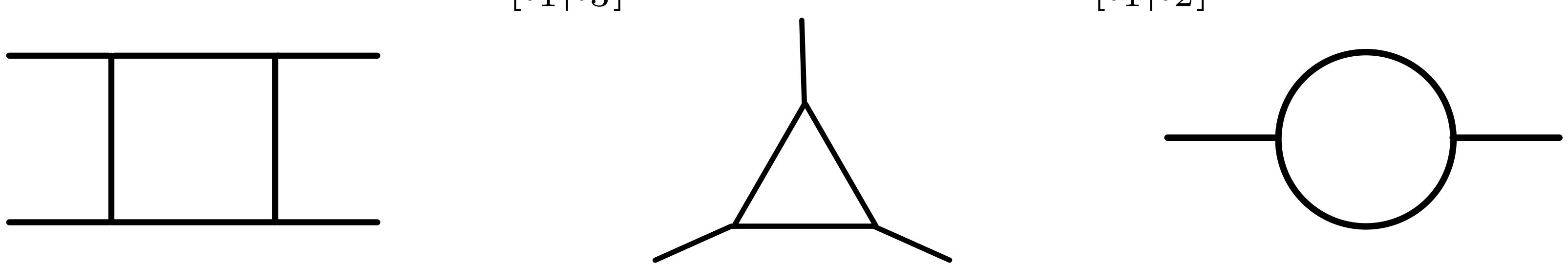
Quadrupole cuts: 4 on-shell conditions on 4 dimensional loop momentum) freezes the integration. But rational part of the amplitude, coming from $D=4-2\varepsilon$ not 4, computed separately

Two breakthrough ideas for one-loop

A number of breakthrough ideas developed in the last 10 years, most notably

2) The OPP method: "We show how to extract the coefficients of 4-, 3-, 2- and 1-point one-loop scalar integrals...."

$$\mathcal{A}_N = \sum_{[i_1|i_4]} \left(d_{i_1 i_2 i_3 i_4} \; I^{(D)}_{i_1 i_2 i_3 i_4} \right) + \sum_{[i_1|i_3]} \left(c_{i_1 i_2 i_3} \; I^{(D)}_{i_1 i_2 i_3} \right) + \sum_{[i_1|i_2]} \left(b_{i_1 i_2} \; I^{(D)}_{i_1 i_2} \right)$$



Ossola, Pittau, Papadopolous '06

Coefficients can be determined by solving system of equations: no loops, no twistors, just algebra!

NLO automation

Revolution in NLO techniques lead to automation of NLO calculations, considered now a solved problem

Various tools developed: Blackhat+Sherpa, GoSam+Sherpa, Helac-NLO, Madgraph5_aMC@NLO, NJet, OpenLoops+Sherpa, Samurai, Recola ...

- high-multiplicity processes still difficult (long run-time on clusters to obtain stable distributions, numerical instabilities).
  Edge: 4 to 6 particles in the final state, depends on the process

- also loop-induced processes automated (enhanced by gluon PDF)

- comparison to NLO is now the standard in most physics analysis

NLO automation: example

Hirschi, Frederix, Garzelli, Maltoni, Pittau 1103.0621

<u>Example</u>: heavy quarks and jets at NLO

Process		Syntax	Cross section (pb)	
Heavy quarks+vector bosons			LO 13 TeV	NLO 13 TeV
e.1	$pp \to W^{\pm} b\bar{b}$ (4f)	`p p > wpm b b~`	$3.074 \pm 0.002 \cdot 10^{2}\ ^{+42.3\%}_{-29.2\%}\ ^{+2.0\%}_{-1.6\%}$	$8.162 \pm 0.034 \cdot 10^{2}\ ^{+29.8\%}_{-23.6\%}\ ^{+1.5\%}_{-1.2\%}$
e.2	$pp \to Z\, b\bar{b}$ (4f)	`p p > z b b~`	$6.993 \pm 0.003 \cdot 10^{2}\ ^{+33.5\%}_{-24.4\%}\ ^{+1.0\%}_{-1.4\%}$	$1.235 \pm 0.004 \cdot 10^{3}\ ^{+19.9\%}_{-17.4\%}\ ^{+1.0\%}_{-1.4\%}$
e.3	$pp \to \gamma\, b\bar{b}$ (4f)	`p p > a b b~`	$1.731 \pm 0.001 \cdot 10^{3}\ ^{+51.9\%}_{-34.8\%}\ ^{+1.6\%}_{-2.1\%}$	$4.171 \pm 0.015 \cdot 10^{3}\ ^{+33.7\%}_{-27.1\%}\ ^{+1.4\%}_{-1.9\%}$
e.4*	$pp \to W^{\pm} b\bar{b}\, j$ (4f)	`p p > wpm b b~ j`	$1.861 \pm 0.003 \cdot 10^{2}\ ^{+42.5\%}_{-27.7\%}\ ^{+0.7\%}_{-0.7\%}$	$3.957 \pm 0.013 \cdot 10^{2}\ ^{+27.0\%}_{-21.0\%}\ ^{+0.7\%}_{-0.6\%}$
e.5*	$pp \to Z\, b\bar{b}\, j$ (4f)	`p p > z b b~ j`	$1.604 \pm 0.001 \cdot 10^{2}\ ^{+42.4\%}_{-27.6\%}\ ^{+0.9\%}_{-1.1\%}$	$2.805 \pm 0.009 \cdot 10^{2}\ ^{+21.0\%}_{-17.6\%}\ ^{+0.8\%}_{-1.0\%}$
e.6*	$pp \to \gamma\, b\bar{b}\, j$ (4f)	`p p > a b b~ j`	$7.812 \pm 0.017 \cdot 10^{2}\ ^{+51.2\%}_{-32.0\%}\ ^{+1.0\%}_{-1.5\%}$	$1.233 \pm 0.004 \cdot 10^{3}\ ^{+18.9\%}_{-19.9\%}\ ^{+1.0\%}_{-1.5\%}$
e.7	$pp \to t\bar{t}\, W^{\pm}$	`p p > t t~ wpm`	$3.777 \pm 0.003 \cdot 10^{-1}\ ^{+23.9\%}_{-18.0\%}\ ^{+2.1\%}_{-1.6\%}$	$5.662 \pm 0.021 \cdot 10^{-1}\ ^{+11.2\%}_{-10.6\%}\ ^{+1.7\%}_{-1.3\%}$
e.8	$pp \to t\bar{t}\, Z$	`p p > t t~ z`	$5.273 \pm 0.004 \cdot 10^{-1}\ ^{+30.5\%}_{-21.8\%}\ ^{+1.8\%}_{-2.1\%}$	$7.598 \pm 0.026 \cdot 10^{-1}\ ^{+9.7\%}_{-11.1\%}\ ^{+1.9\%}_{-2.2\%}$
e.9	$pp \to t\bar{t}\, \gamma$	`p p > t t~ a`	$1.204 \pm 0.001 \cdot 10^{0}\ ^{+29.6\%}_{-21.3\%}\ ^{+1.6\%}_{-1.8\%}$	$1.744 \pm 0.005 \cdot 10^{0}\ ^{+9.8\%}_{-11.0\%}\ ^{+1.7\%}_{-2.0\%}$
e.10*	$pp \to t\bar{t}\, W^{\pm} j$	`p p > t t~ wpm j`	$2.352 \pm 0.002 \cdot 10^{-1}\ ^{+40.9\%}_{-27.1\%}\ ^{+1.3\%}_{-1.0\%}$	$3.404 \pm 0.011 \cdot 10^{-1}\ ^{+11.2\%}_{-14.0\%}\ ^{+1.2\%}_{-0.9\%}$
e.11*	$pp \to t\bar{t}\, Z j$	`p p > t t~ z j`	$3.953 \pm 0.004 \cdot 10^{-1}\ ^{+46.2\%}_{-29.5\%}\ ^{+2.7\%}_{-3.0\%}$	$5.074 \pm 0.016 \cdot 10^{-1}\ ^{+7.0\%}_{-12.3\%}\ ^{+2.5\%}_{-2.9\%}$
e.12*	$pp \to t\bar{t}\, \gamma j$	`p p > t t~ a j`	$8.726 \pm 0.010 \cdot 10^{-1}\ ^{+45.4\%}_{-29.1\%}\ ^{+2.3\%}_{-2.6\%}$	$1.135 \pm 0.004 \cdot 10^{0}\ ^{+7.5\%}_{-12.2\%}\ ^{+2.2\%}_{-2.5\%}$
e.13*	$pp \to t\bar{t}\, W^{-}W^{+}$ (4f)	`p p > t t~ w+ w-`	$6.675 \pm 0.006 \cdot 10^{-3}\ ^{+30.9\%}_{-21.9\%}\ ^{+2.1\%}_{-2.0\%}$	$9.904 \pm 0.026 \cdot 10^{-3}\ ^{+10.9\%}_{-11.8\%}\ ^{+2.1\%}_{-2.1\%}$
e.14*	$pp \to t\bar{t}\, W^{\pm} Z$	`p p > t t~ wpm z`	$2.404 \pm 0.002 \cdot 10^{-3}\ ^{+26.6\%}_{-19.6\%}\ ^{+2.5\%}_{-1.8\%}$	$3.525 \pm 0.010 \cdot 10^{-3}\ ^{+10.6\%}_{-10.8\%}\ ^{+2.3\%}_{-1.6\%}$
e.15*	$pp \to t\bar{t}\, W^{\pm} \gamma$	`p p > t t~ wpm a`	$2.718 \pm 0.003 \cdot 10^{-3}\ ^{+25.4\%}_{-18.9\%}\ ^{+2.3\%}_{-1.8\%}$	$3.927 \pm 0.013 \cdot 10^{-3}\ ^{+10.3\%}_{-10.4\%}\ ^{+2.0\%}_{-1.5\%}$
e.16*	$pp \to t\bar{t}\, ZZ$	`p p > t t~ z z`	$1.349 \pm 0.014 \cdot 10^{-3}\ ^{+29.3\%}_{-21.1\%}\ ^{+1.7\%}_{-1.5\%}$	$1.840 \pm 0.007 \cdot 10^{-3}\ ^{+7.9\%}_{-9.9\%}\ ^{+1.7\%}_{-1.5\%}$
e.17*	$pp \to t\bar{t}\, Z\gamma$	`p p > t t~ z a`	$2.548 \pm 0.003 \cdot 10^{-3}\ ^{+30.1\%}_{-21.5\%}\ ^{+1.7\%}_{-1.6\%}$	$3.656 \pm 0.012 \cdot 10^{-3}\ ^{+9.7\%}_{-11.0\%}\ ^{+1.8\%}_{-1.9\%}$
e.18*	$pp \to t\bar{t}\, \gamma\gamma$	`p p > t t~ a a`	$3.272 \pm 0.006 \cdot 10^{-3}\ ^{+28.4\%}_{-20.6\%}\ ^{+1.3\%}_{-1.1\%}$	$4.402 \pm 0.015 \cdot 10^{-3}\ ^{+7.8\%}_{-9.7\%}\ ^{+1.4\%}_{-1.4\%}$

Similar tables for
- boson+jets
- diboson+jets
- triboson+jets
- four bosons
- heavy quarks + jets
- heavy quarks + bosons
- single top
- single Higgs
- Higgs pair
- ...

The 2007 Les Houches wishlist

Process ($V \in \{Z, W, \gamma\}$)	Comments
Calculations completed since Les Houches 2005	
1. $pp \to VV$jet	WWjet completed by Dittmaier/Kallweit/Uwer [3]; Campbell/Ellis/Zanderighi [4] and Binoth/Karg/Kauer/Sanguinetti (in progress)
2. $pp \to$ Higgs+2jets	NLO QCD to the gg channel completed by Campbell/Ellis/Zanderighi [5]; NLO QCD+EW to the VBF channel completed by Ciccolini/Denner/Dittmaier [6,7]
3. $pp \to V\,V\,V$	ZZZ completed by Lazopoulos/Melnikov/Petriello [8] and WWZ by Hankele/Zeppenfeld [9]
Calculations remaining from Les Houches 2005	
4. $pp \to t\bar{t}\,b\bar{b}$	relevant for $t\bar{t}H$
5. $pp \to t\bar{t}$+2jets	relevant for $t\bar{t}H$
6. $pp \to VV\,b\bar{b}$,	relevant for VBF $\to H \to VV$, $t\bar{t}H$
7. $pp \to VV$+2jets	relevant for VBF $\to H \to VV$; VBF contributions calculated by (Bozzi/)Jäger/Oleari/Zeppenfeld [10–12]
8. $pp \to V$+3jets	various new physics signatures
NLO calculations added to list in 2007	
9. $pp \to b\bar{b}b\bar{b}$	Higgs and new physics signatures
Calculations beyond NLO added in 2007	
10. $gg \to W^*W^*$ $\mathcal{O}(\alpha^2\alpha_s^3)$	backgrounds to Higgs
11. NNLO $pp \to t\bar{t}$	normalization of a benchmark process
12. NNLO to VBF and Z/γ+jet	Higgs couplings and SM benchmark
Calculations including electroweak effects	
13. NNLO QCD+NLO EW for W/Z	precision calculation of a SM benchmark

Table 1: The updated experimenter's wishlist for LHC processes

CLOSED

with Feynman diagrams

with Feynman diagrams or unitarity/onshell methods

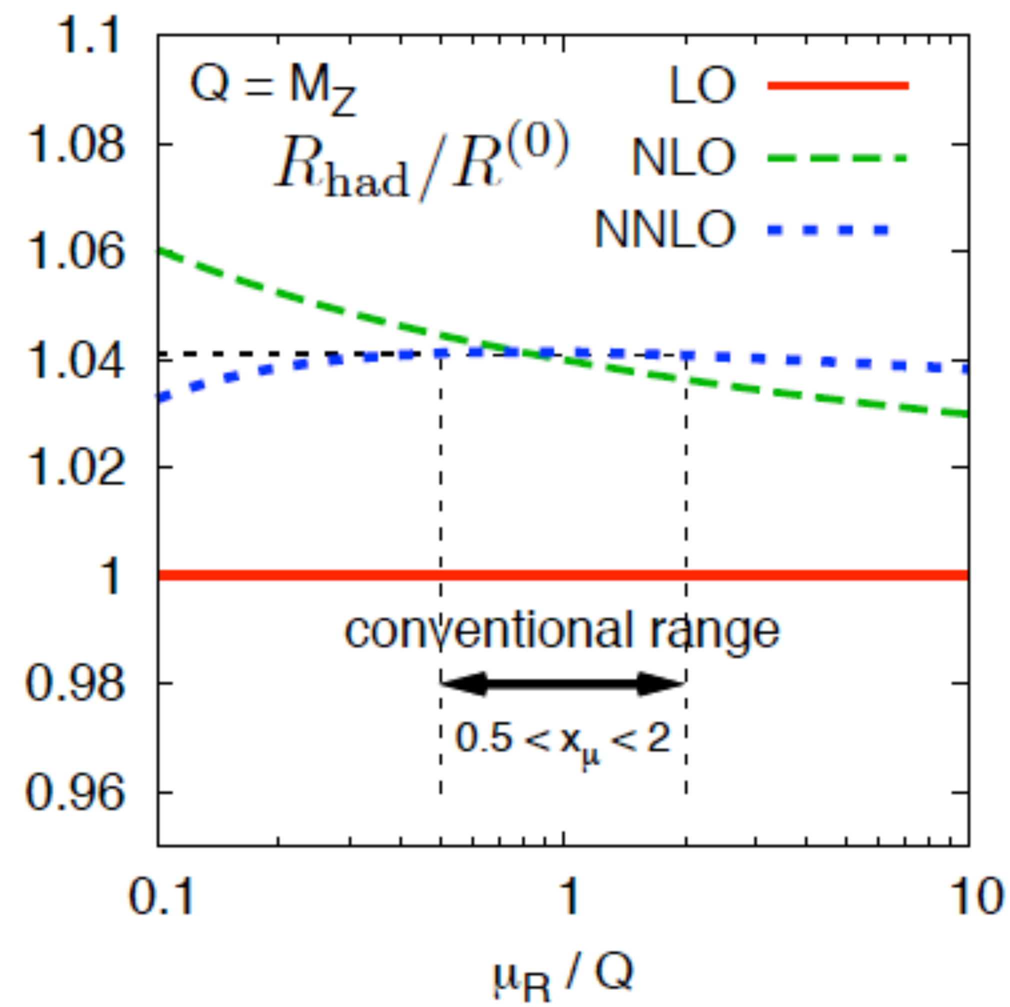
The NLO multi-leg Working group report 0803.0494

Uncertainties

The "unpleasant" feature that cross-sections depend on the choice of renormalization and factorization scale can be turned into something useful, i.e. a way to quantify the theoretical error

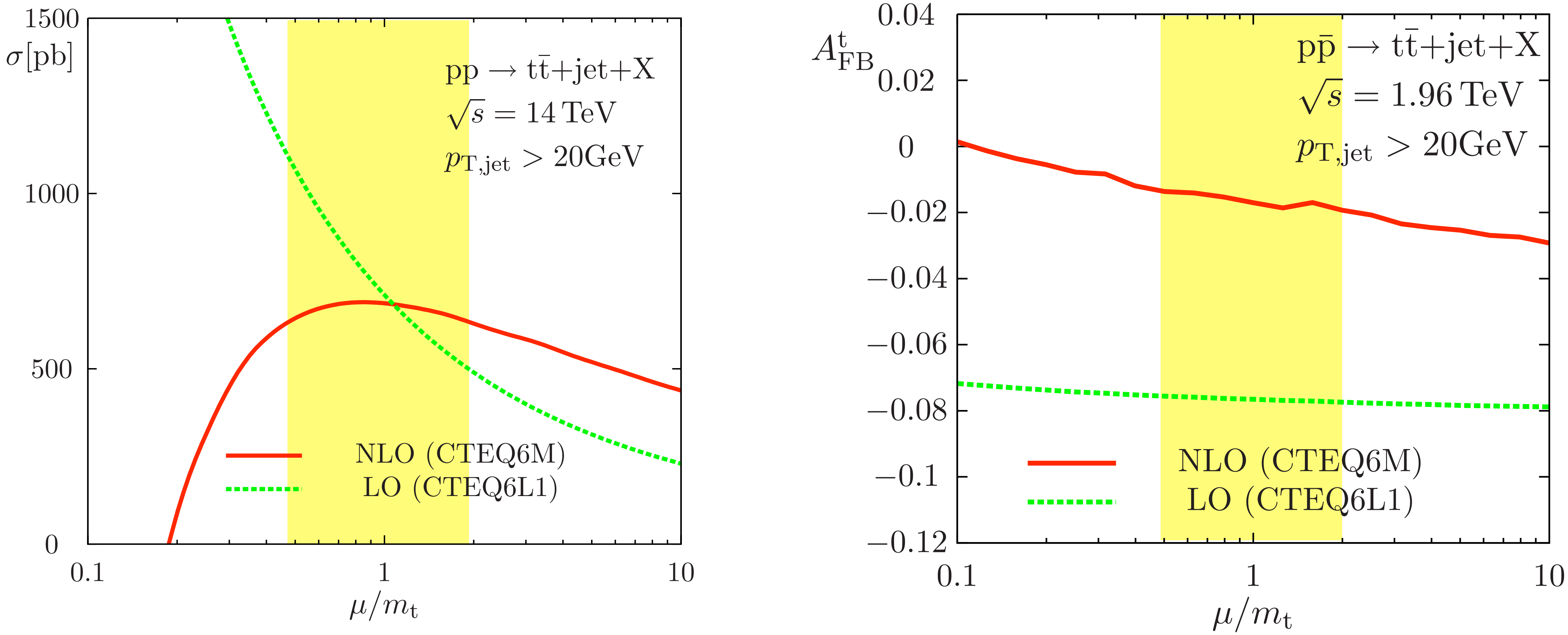
Example: R-ratio (again!)
Fix both scales to the scale at which the hard process occurs (Q) and vary them up and down by a factor 2

NB:
- the factor 2 is conventional
- it is a procedure that seemed to work well in practice
- in complicated processes large degree of freedom in the choice of the scale



1. Example of NLO: tt+1jet

Dittmaier, Kallweit, Uwer ’08

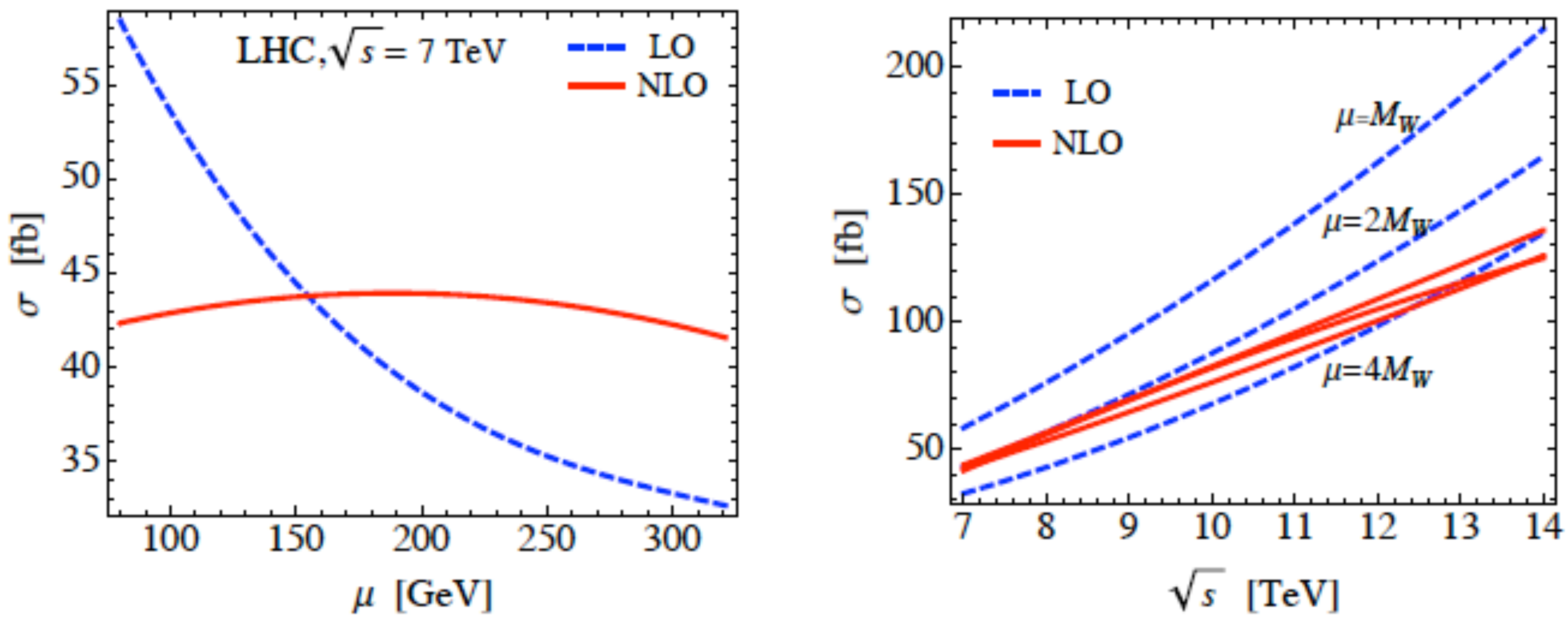


- improved stability of NLO result [but no decays]
- forward-backward asymmetry at the Tevatron compatible with zero
- LO scale uncertainty underestimates shift to NLO for the asymmetry

2. Example of NLO: WW+2jets

LO calculations: very large theoretical uncertainties

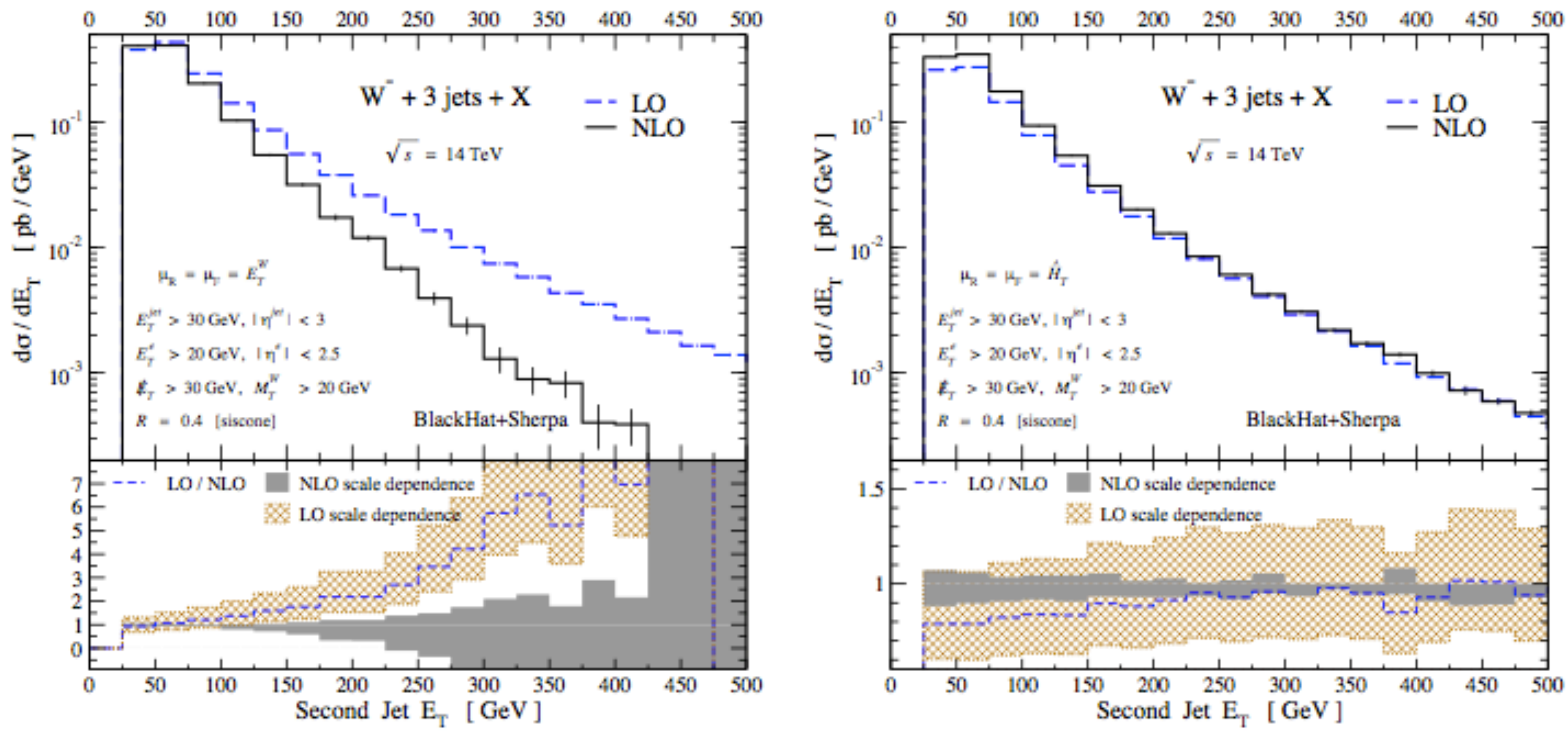
<u>Example</u>: cross-section for W^+W^- + 2 jet production at the LHC



Melia, et al. '11

3. Example of NLO: W+3jets

Scale choice: example of W+3 jets (problem more severe with more jets)



... large logarithms can appear in some distributions, invalidating even an NLO prediction.

Bern et al. '09

Automated NLO

- few years ago: each NLO calculation resulted in a paper. Now, as for leading order, just run a code and get the results (also for distributions)
- possibility to do precise studies of signal and backgrounds using the same tool (very practical + avoid errors)
- what lead to this remarkable progress? the fact that

1. leading order can be computed automatically and efficiently (e.g. via recursion relations)
2. one can reduce the one-loop to product of tree-level amplitudes
3. it was well understood how to subtract singularities
4. the basis of master integrals was known

But for item 2. everything was there since the time of Passarino-Veltman (even item 2. was understood, but no efficient/practical method exited).
We will now compare this to the current status of NNLO

NNLO

NNLO is one of the most active areas in QCD now

First let's ask the question: when do we need NNLO?

NNLO most important in three different situations

Benchmark processes (measured with highest accuracy)	Input to PDFs fits + backgrounds to Higgs studies	Very large NLO corrections (moderate precision needs NNLO)
- $Z \to l^+l^-$ - $W \to l\square$ - ...	- Diboson - Boson + jet - top-pairs - ...	- Higgs - Higgs + jet - ...

Plus more reliable estimate of theory uncertainty

Still early days, but in the few cases examined (e.g. Higgs and Drell Yan, WW, ZZ, top ...), better agreement with data at NNLO

Some history of NNLO

- first NNLO computation of a collider process was inclusive Drell-Yan production by Hamberg, van Neerven and Matsuura in '91
- second NNLO calculation: Higgs production in gluon-gluon fusion by Harlander and Kilgore in '02

  Both calculations refer to inclusive, total cross-sections that are not measurable

- first exclusive NNLO computation (for fiducial volume cross-sections) was Higgs $\to \gamma\gamma$ in '04 by Anastasiou, Melnikov and Petriello, followed by other exclusive calculations of Higgs and Drell-Yan processes
- only last year NNLO corrections to $2 \to 2$ processes also with QCD partons in the final state started to appear. This indicates a more complete understanding of NNLO

  Many things at NNLO are new and took a while to understand. Today's technology is likely not to be finalized yet

Ingredients for NNLO

Remember crucial steps for automated NLO were

1. leading order can be computed automatically and efficiently (e.g. via recursion relations)
2. one can reduce the one-loop to product of tree-level amplitudes
3. it was well understood how to subtract singularities
4. the basis of master integrals was known

At NNLO the situation is very different

1. leading order of very limited importance
2. no procedure to reduce two-loop to tree-level (unitarity approaches at two face still many outstanding issues)
3. subtraction of singularities far from trivial
4. basis set of master integrals not known, integrals not all/always known analytically

And all this for simple processes (no result exist, or has been attempted, for any $2 \to 3$ scattering process)

Ingredients for NNLO

What changed in the last years (and is undergoing more changes)

1. technology to compute integrals
2. extension of systematic FKS subtraction to NNLO

NNLO subtraction

While at NLO the bottleneck has been for a long time the calculation of virtual (one-loop) amplitudes, at NNLO the bottleneck comes mostly from finding a method to cancel divergences before numerical integration.

Two main approaches

Slicing:
partition the phase space with a (small) slicing parameter so that divergences are all below the slicing cut. In the divergent region use an approximate expression, neglecting finite terms, above use the exact (finite) integrand.

Subtraction:
since IR singularities of amplitudes are knows, add and subtract counterterms so as to make integrals finite. "Easy" at NLO, but complicated at NNLO due to the more intricate structure of (overlapping) singularities

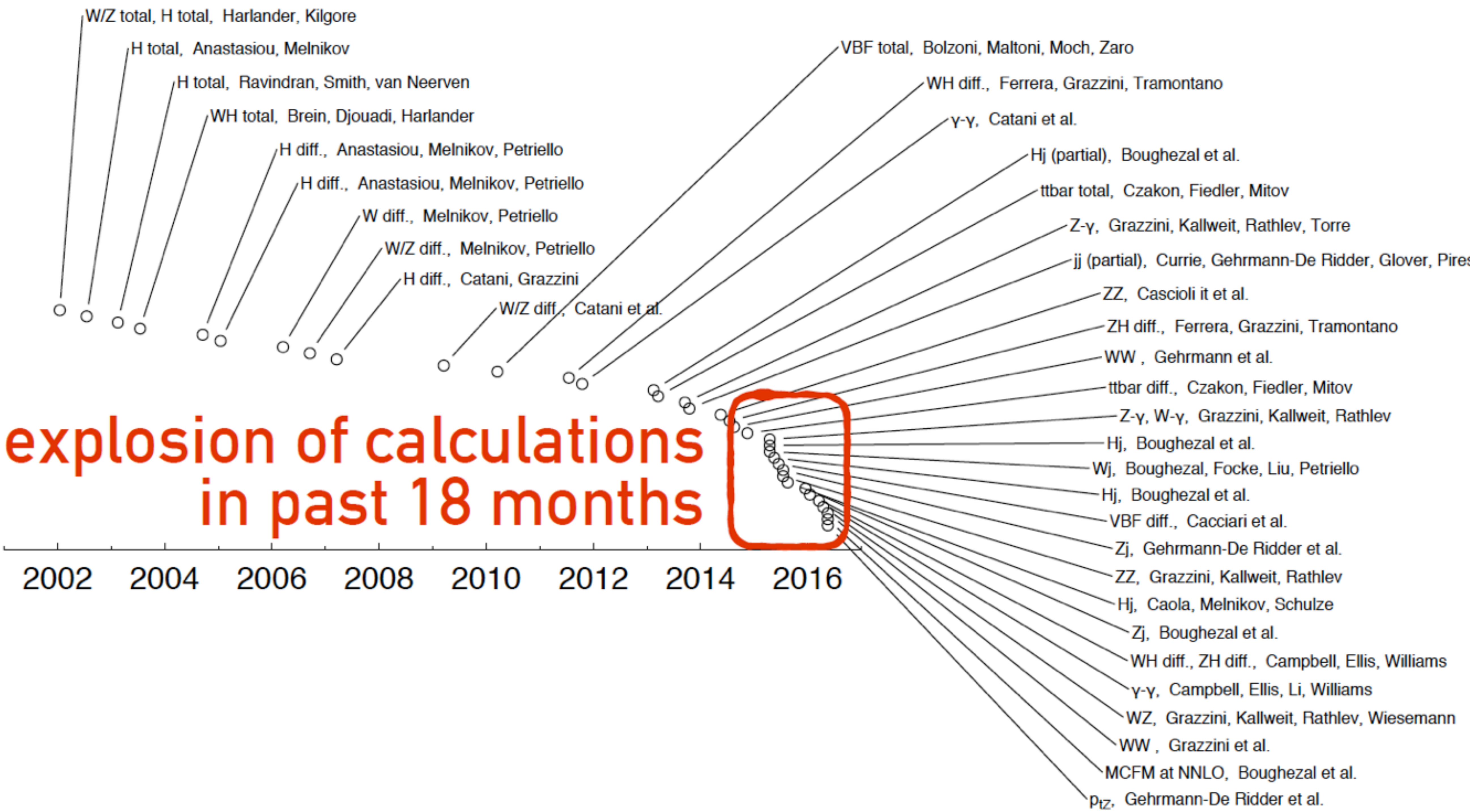
NNLO

Different practical realizations:

- antenna subtraction
- q_T subtraction (slicing)
- colorful subtraction
- sector improved residue subtraction scheme
- N-jettiness subtraction/slicing ☞ *new kid in town*

Obviously, two-loop integrals are also needed. Lots of progress here too. I will not discuss this here, only mention Henn's conjecture to compute integrals using differential equations

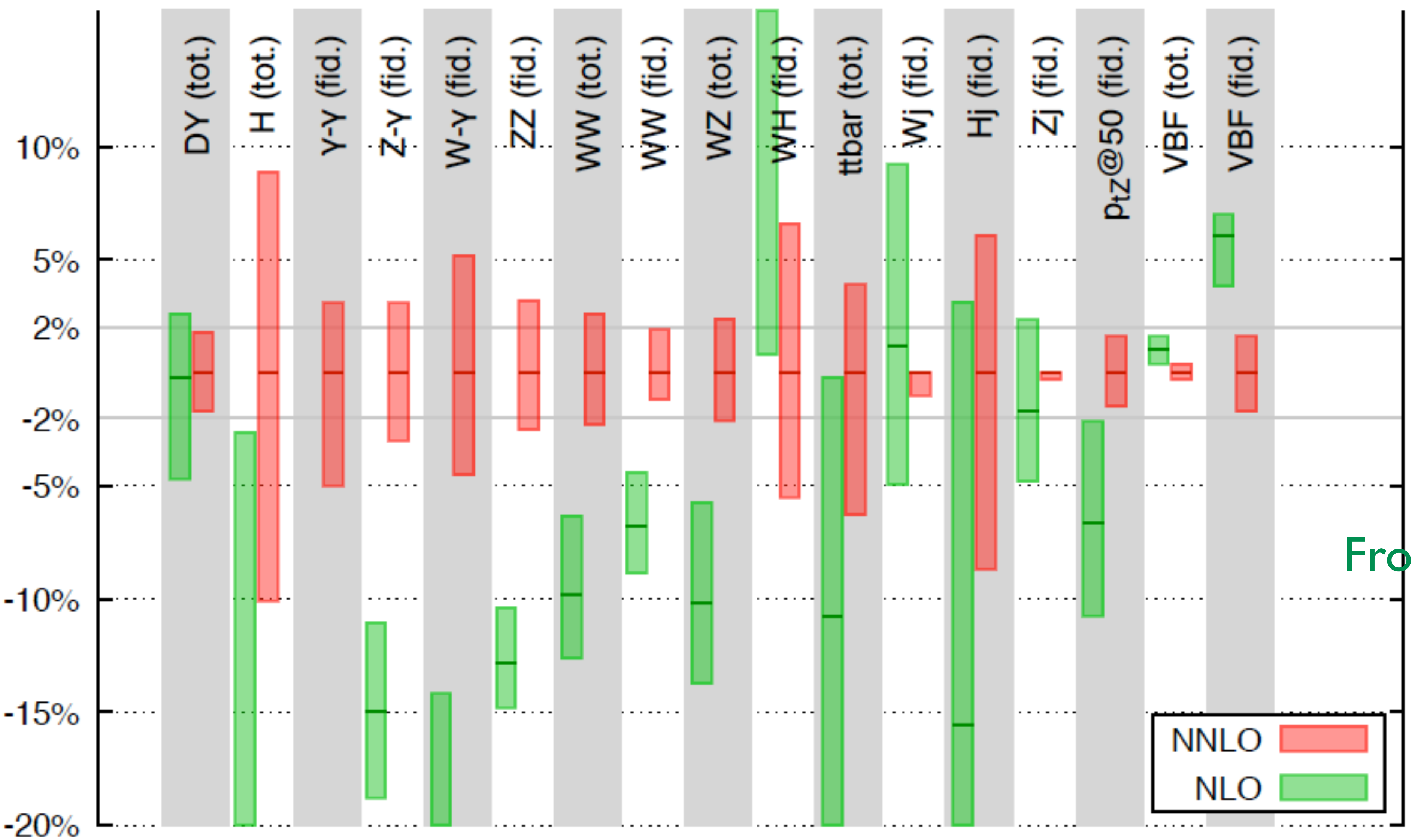
Collider processes known at NNLO



From talk given by G. Salam at LHCP2016

Is the NNLO in the NLO error band?

How well does the NLO scale uncertainty band account for the size of the NNLO correction?



From talk given by G. Salam at LHCP2016

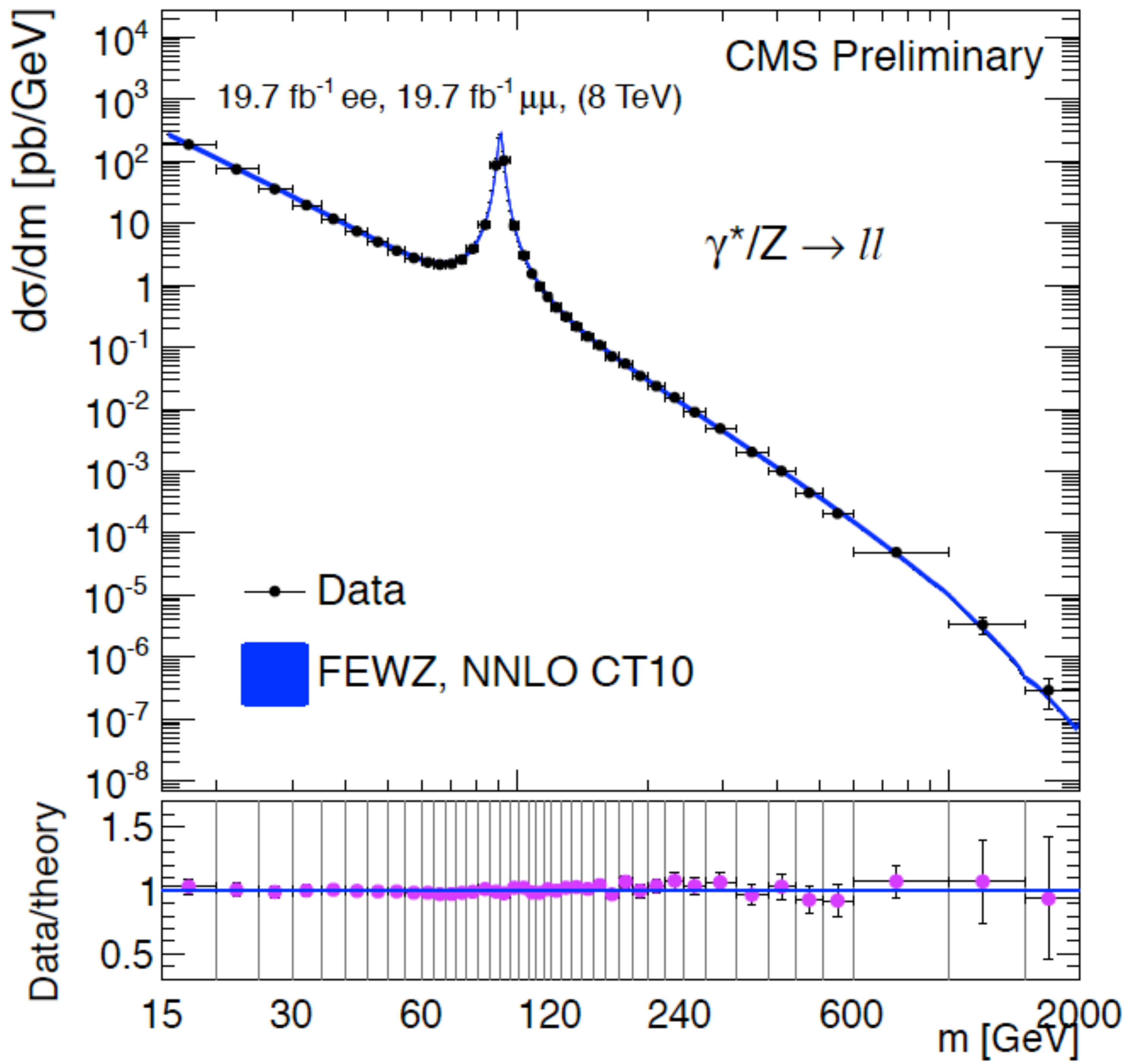
For many processes NNLO scale band is ~±2%

But only in 3/17 cases is NNLO (central) within NLO scale band...

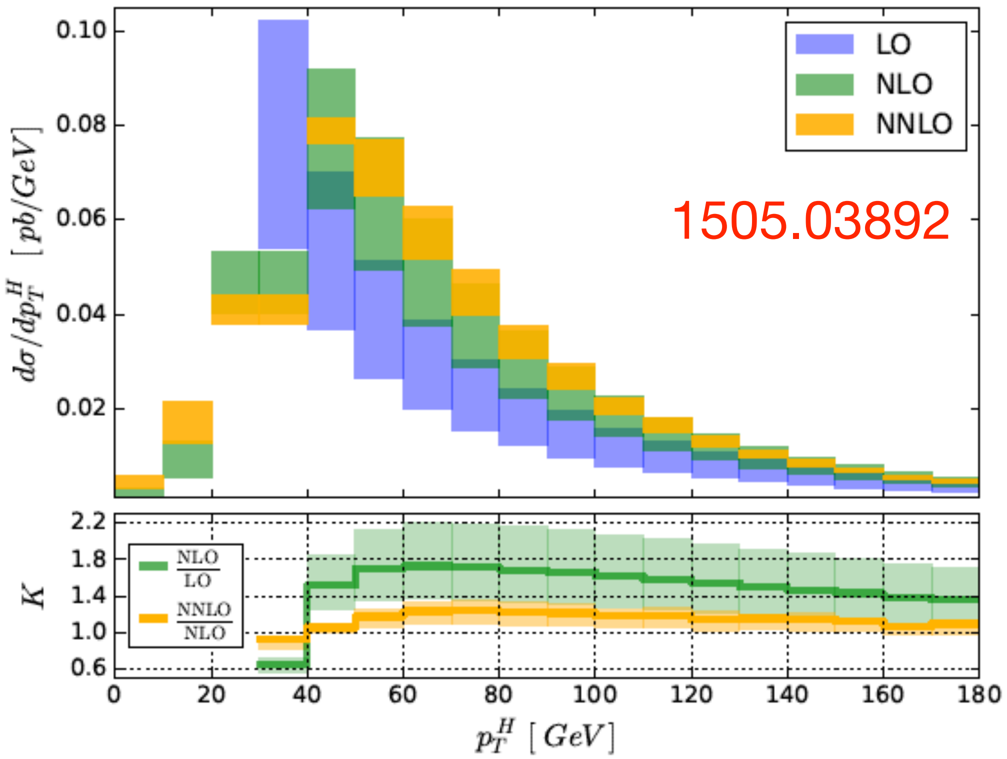
NNLO vs LHC data

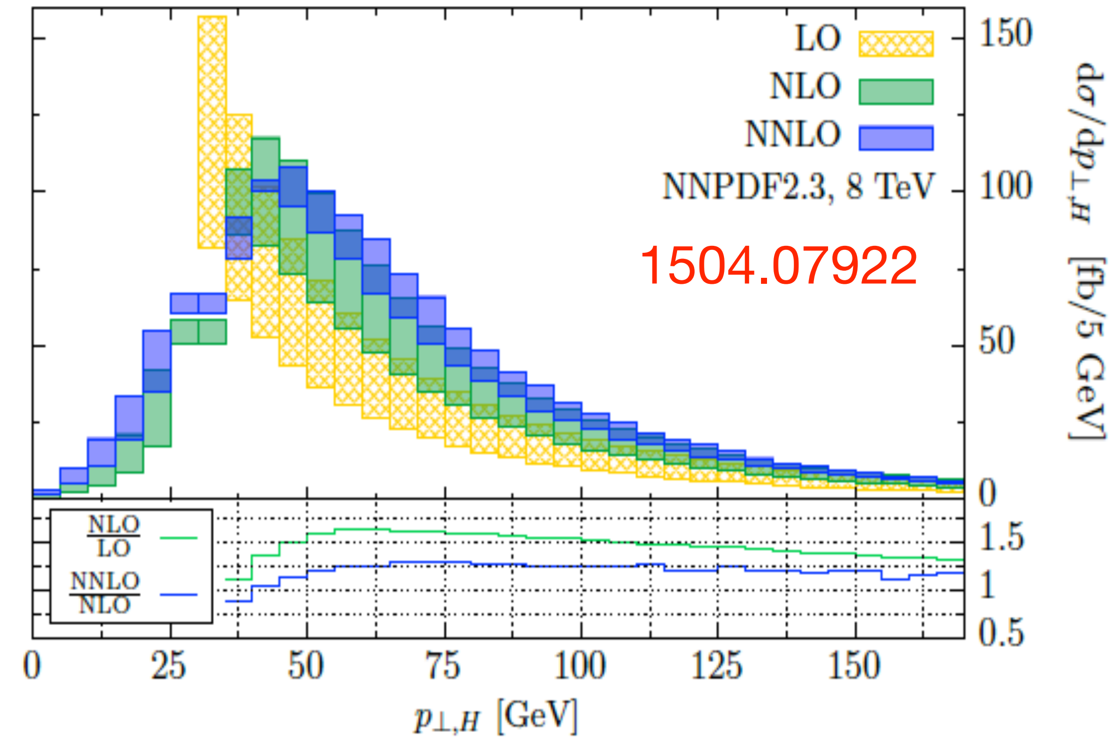
Impressive agreement between experiment and NNLO theory

CMS-PAS-SMP-14-003



NNLO: Higgs +1jet

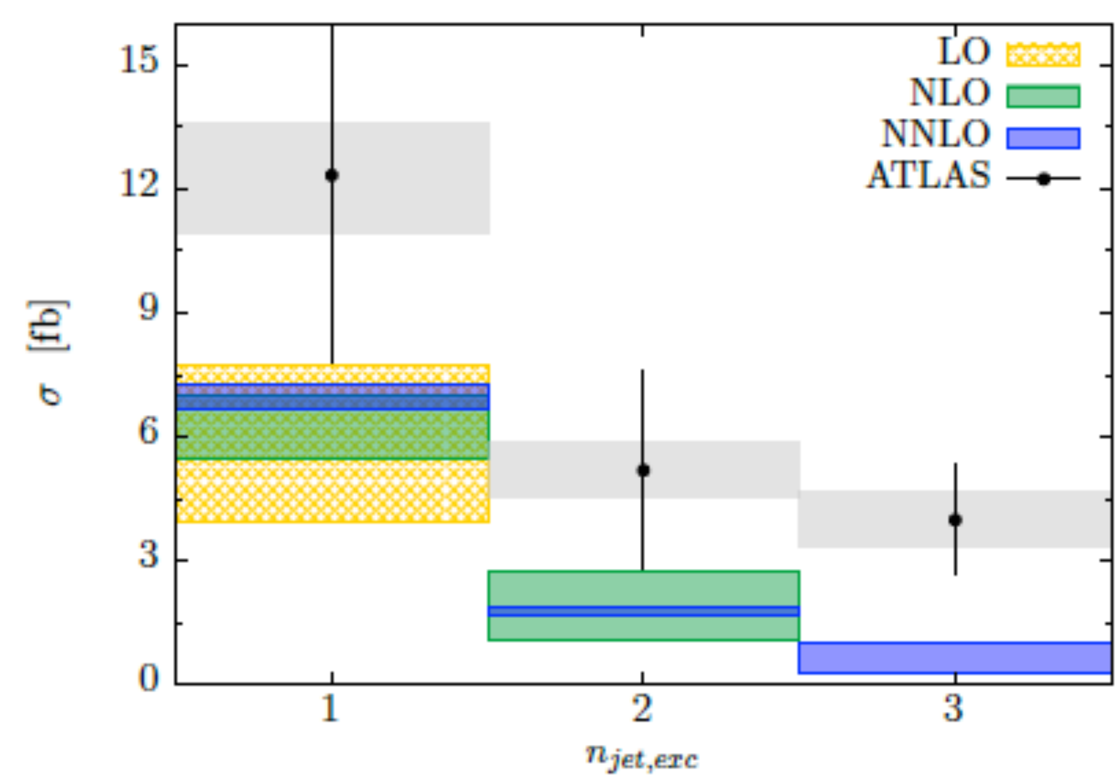


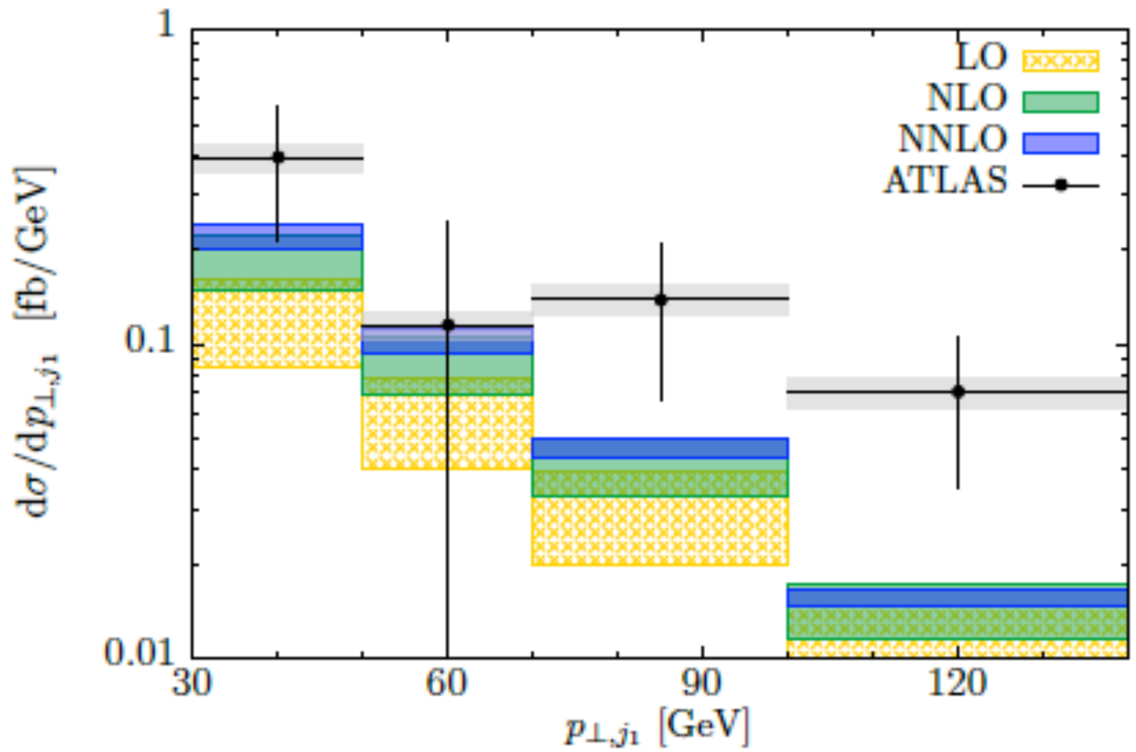


- first applications of new techniques to achieve cancellations of intermediate divergences
- large K-factor (≈1.15-1.20)
- useful comparison between independent calculations

NNLO: Higgs +1jet

Decays of Higgs to bosons also included. Fiducial cross-sections compared to ATLAS and CMS data



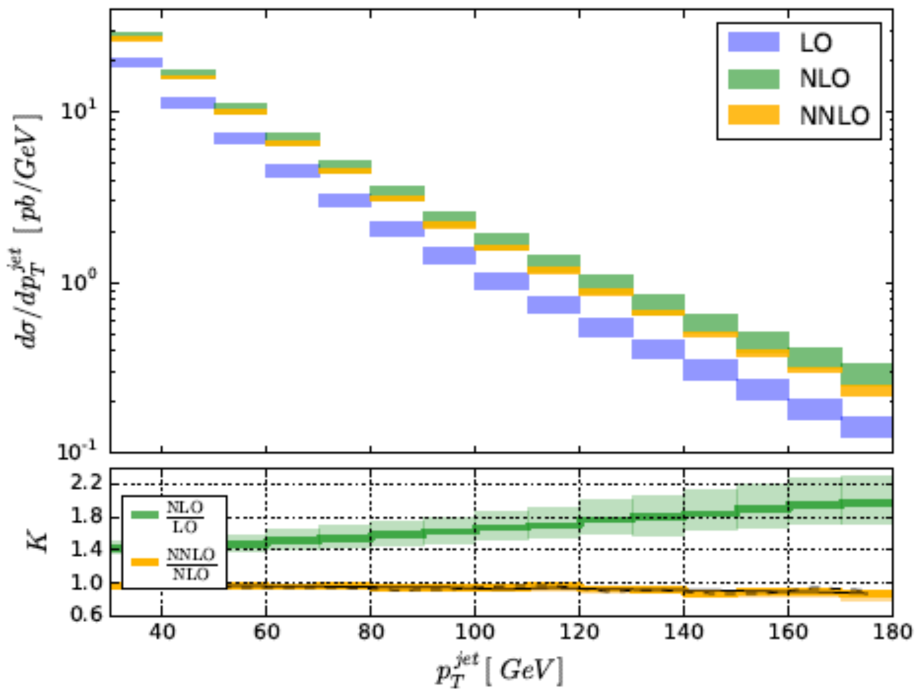


1508.02684

NNLO: V+1jet

W+1jet

1504.02131



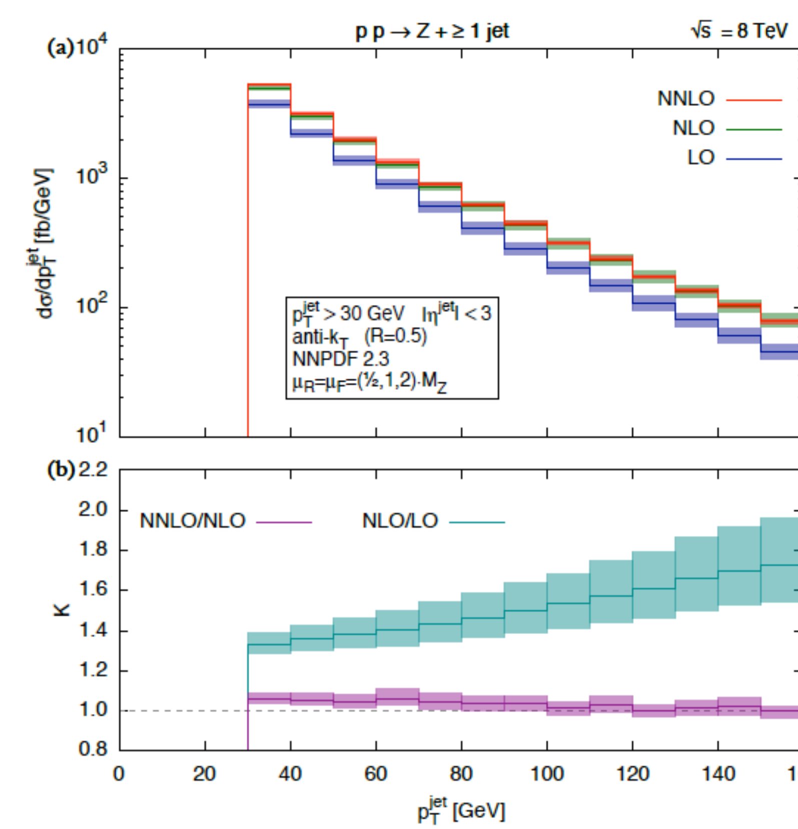
$p_T^{jet} > 30$ GeV, $\lvert\eta_{jet}\rvert < 2.4$	
Leading order:	533^{+39}_{-38} pb
Next-to-leading order:	797^{+63}_{-49} pb
Next-to-next-to-leading order:	787^{+0}_{-8} pb

- flat K-factor (≈1)
- huge reduction of theory error

Z+1jet

1507.02850

$$\sigma_{LO} = 103.6^{+7.7}_{-7.5} \text{ pb}$$
$$\sigma_{NLO} = 144.4^{+9.0}_{-7.2} \text{ pb}$$
$$\sigma_{NNLO} = 151.0^{+4.9}_{-3.6} \text{ pb}$$

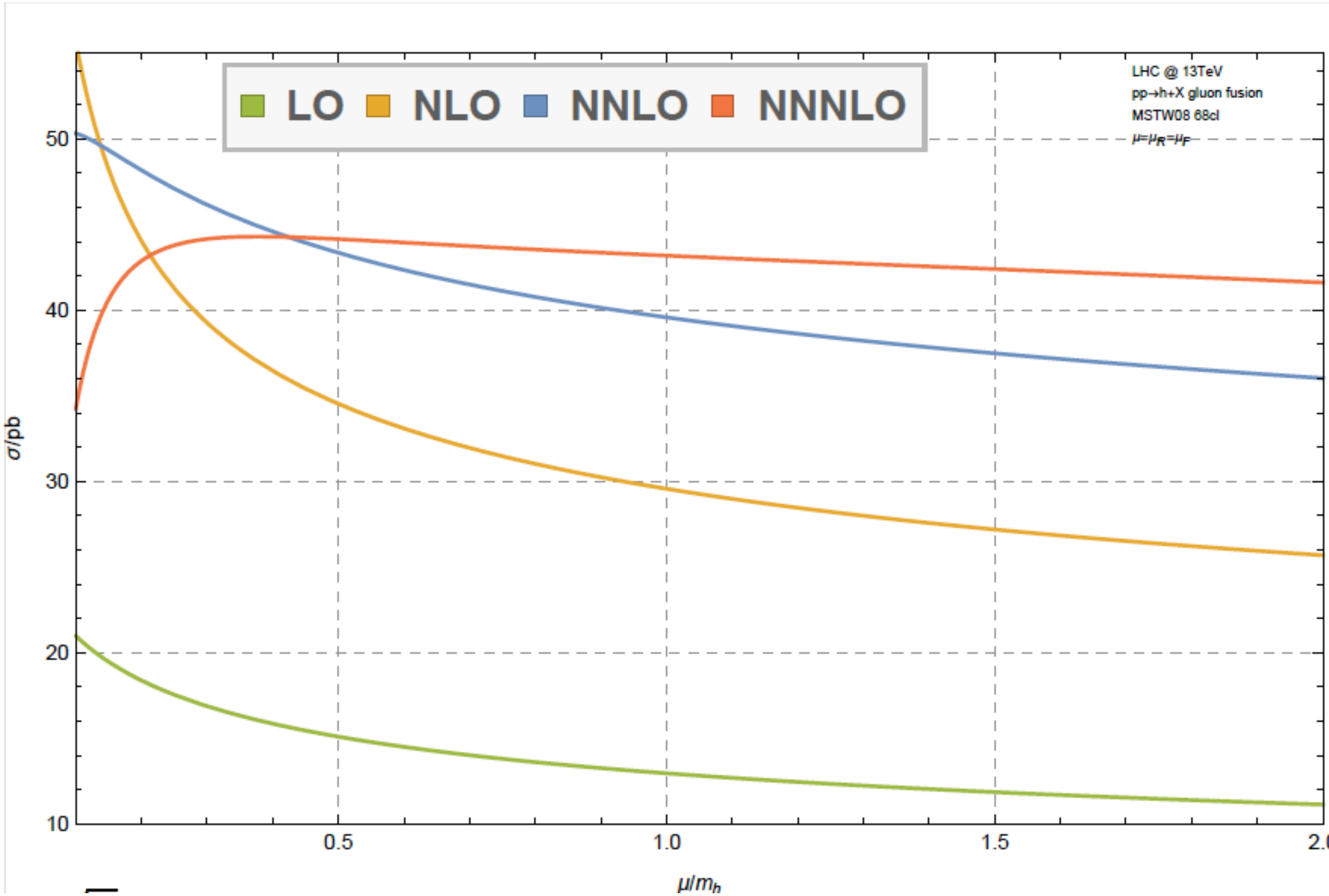


- similar features in Z+jet
- other observables ($p_{t,Z}$, y_Z, ...) non-trivial K-factor

Summary of perturbative calculations

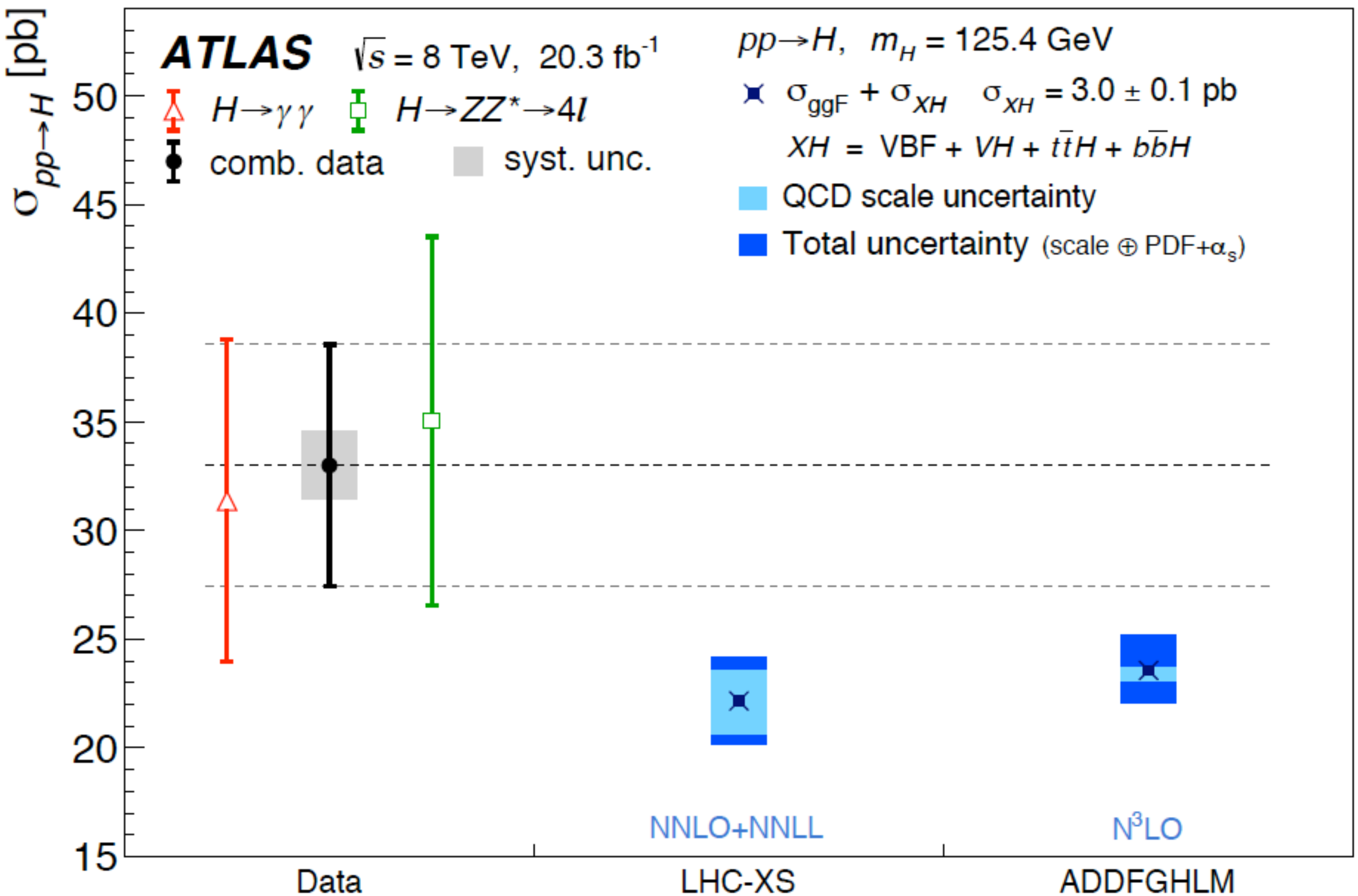
- LO: fully automated. Edge: 10-12 particles in the final state
- NLO: also automated. Edge: 4-6 particles in the final state
- NNLO: the new frontier. Lots of new $2 \to 2$ processes in the last year ($2 \to 1$ more than10 years old). Currently no $2 \to 3$ calculation for the LHC
- NNNLO: fully inclusive Higgs production *(new in 2015)*

Higgs production at N3LO



N^3LO finally stabilizes the slowly convergent perturbative expansion

Higgs production: theory vs data



Intermediate recap

QCD is a field very active

- NLO revolution belongs already to the past, NNLO the current hottest field.
  Only in the last few months: H+1jet, Z+1jet, W+1jet, VBF Higgs, VV, dijets at NNLO and even Higgs at N3LO
- many other important theoretical and phenomenological developments (NLO multi-jet merging, matching, inclusion of EW corrections, resummations ...)
- tools getting more and more refined. Drastic improvement in theory uncertainties and more attention paid towards a solid estimate

Very exciting to work on QCD as new ideas/calculations are promptly used in LHC analyses