Precision QCD

Giulia Zanderighi (CERN & Oxford)

4th Lecture

European School of High Energy Physics — June 2016

Outline

Jets

- jet definitions
- infrared safety
- applications (jet area, pile-up subtraction, quality measures, jet-substructure)

Jets: ten years ago

Cones are IR unsafe!

The Cone is too rigid!

IR unsafety affects jet cross-sections by less than 1%, so don't need to care!

kt collects too much soft radiation!

Cones have a well-defined circular area!

Jet area not well defined in kt: U.E. and pile-up subtraction too difficult!

What about dark towers??

After all, if D=1.35 R Cone and kt are practically the same thing....

Where do jets enter ?

Essentially everywhere at colliders!

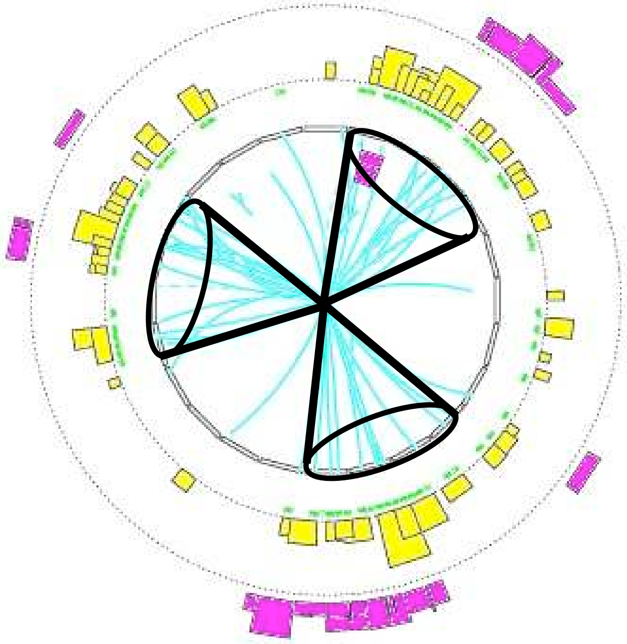
Jets are an essential tool for a variety of studies:

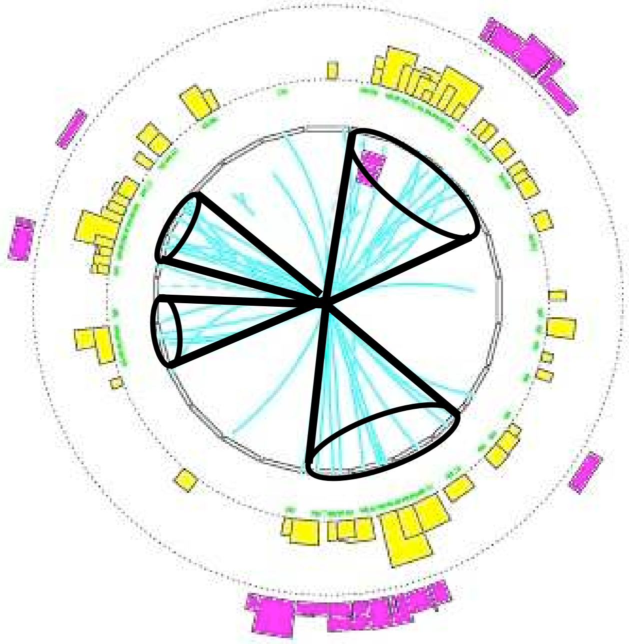
- top reconstruction
- mass measurements
- most Higgs and New Physics searches
- general tool to attribute structure to an event
- jet-veto cross-sections
- instrumental for QCD studies, e.g. inclusive-jet measurements
  ⇒ important input for PDF determinations

Jets

Jets provide a way of projecting away the multiparticle dynamics of an event ⇒ leave a simple quasi-partonic picture of the hard scattering

The projection is fundamentally ambiguous ⇒ jet physics is a rich subject



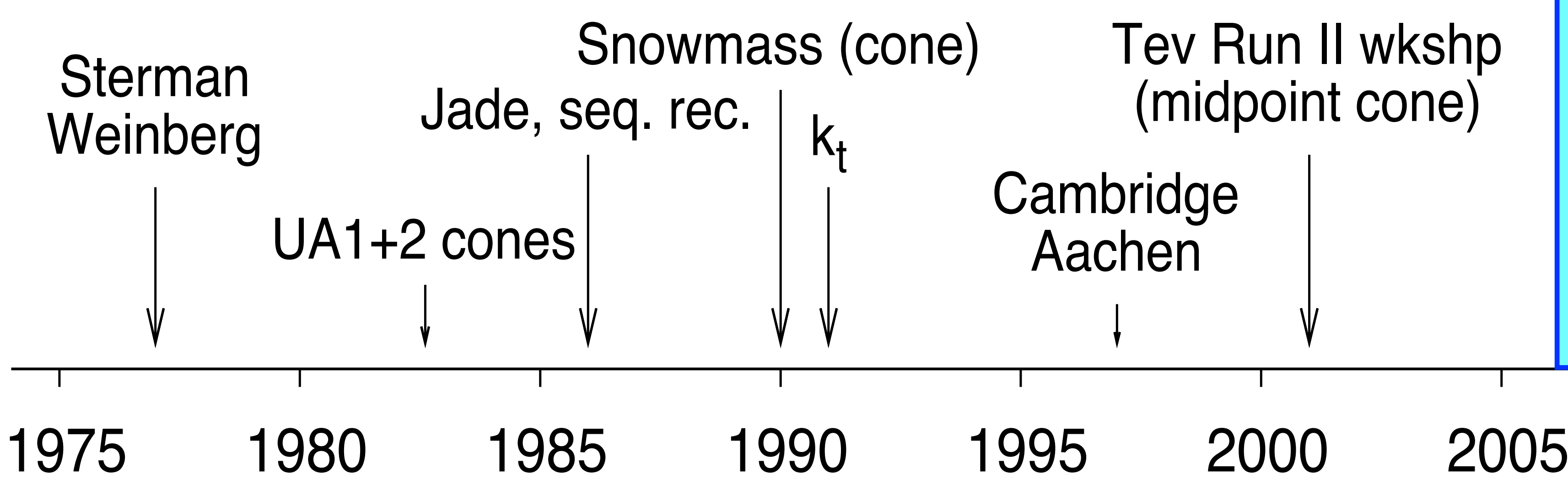


<u>Ambiguities:</u>

1) Which particles should belong to a same jet ?

2) How does recombine the particle momenta to give the jet-momentum?

Jet developments



fast-kt, SISCone, anti-kt, jet-areas, jet-flavour, non-perturbative effects, quality measures, jet-substructure as Higgs discovery channel ...

Two broad classes of jet algorithms

Today many extensions of the original Sterman-Weinberg jets.
Modern jet-algorithms divided into two broad classes

Jet algorithms

Cone type
(UA1,JetCLU, Midpoint, SISCone..)

top down approach: cluster particles according to distance in coordinate-space
Idea: put cones along dominant direction of energy flow

Sequential
(kt-type, Jade, Cambridge/Aachen...)

bottom up approach: cluster particles according to distance in momentum-space
Idea: undo branchings occurred in the PT evolution

Jet requirements

Snowmass accord

FERMILAB-Conf-90/249-E
[E-741/CDF]

Toward a Standardization of Jet Definitions

Several important properties that should be met by a jet definition are [3]:

1. Simple to implement in an experimental analysis;
2. Simple to implement in the theoretical calculation;
3. Defined at any order of perturbation theory;
4. Yields finite cross section at any order of perturbation theory;
5. Yields a cross section that is relatively insensitive to hadronization.

Other desirable properties:

- flexibility
- few parameters
- fast algorithms
- transparency
- ...

Inclusive k_t/Durham-algorithm

Catani et. al '92-'93; Ellis&Soper '93

Inclusive algorithm:

1. For any pair of final state particles i,j define the distance

$$d_{ij} = \frac{\Delta y_{ij}^2 + \Delta\phi_{ij}^2}{R^2} \min\{k_{ti}^2, k_{tj}^2\}$$

2. For each particle i define a distance with respect to the beam

$$d_{iB} = k_{ti}^2$$

3. Find the smallest distance. If it is a d_{ij} recombine i and j into a new particle ($\Longrightarrow$ recombination scheme); if it is d_{iB} declare i to be a jet and remove it from the list of particles

4. repeat the procedure until no particles are left

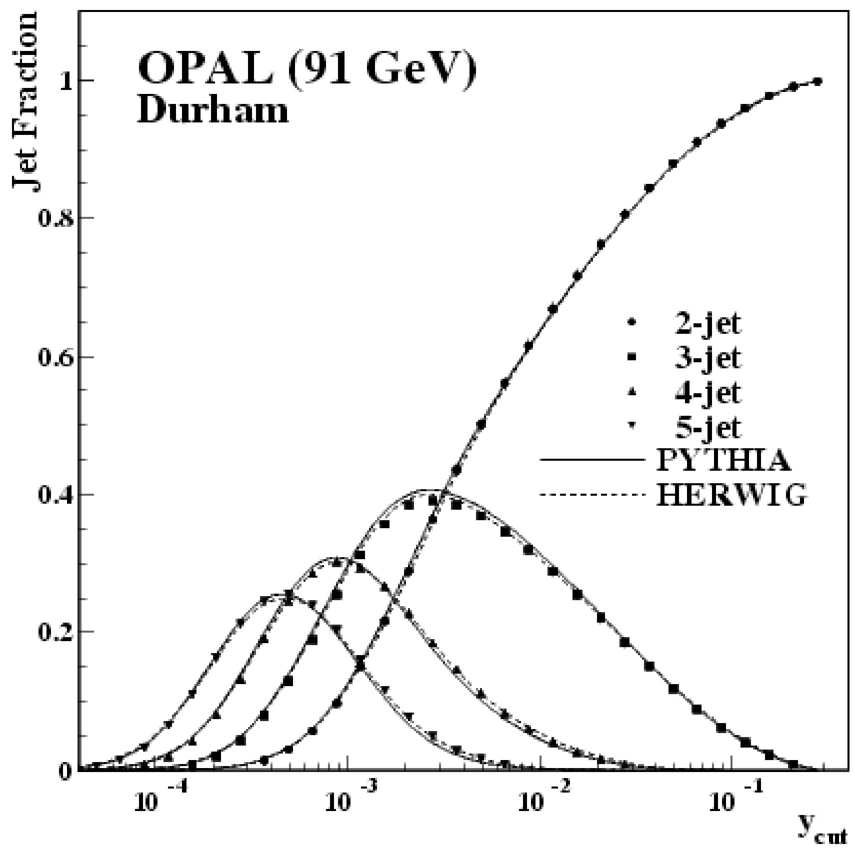
Exclusive version: stop when all d_{ij}, d_{iB} > d_{cut} or when reaching n-jets

k_t/Durham-algorithm in e^+e^-

k_t originaly designed in e^+e^- and most widely used algorithm in e^+e^- (LEP)

$$y_{ij} = 2\min\{E_i^2, E_j^2\}\,(1 - \cos\theta_{ij}^2)$$

- can specify events using y_{23}, y_{34}, y_{45}, y_{56} ...
- resolution parameter related to minimum transverse momentum between jets



Satisfies fundamental requirements:

1. Collinear safe: collinear particles recombine early on
2. Infrared safe: soft particles do not influence the clustering sequence

$\Rightarrow$ *collinear + infrared safety important: it means that cross-sections can be computed at higher order in pQCD (no divergences)!*

The CA and the anti-k_t algorithm

The Cambridge/Aachen: sequential algorithm like k_t, but uses only angular properties to define the distance parameters

$$d_{ij} = \frac{\Delta R_{ij}^2}{R^2} \qquad d_{iB} = 1 \qquad \Delta R_{ij}^2 = (\phi_i - \phi_j)^2 + (y_i - y_j)^2$$

Dotshitzer et. al '97; Wobisch & Wengler '99

The anti-kt algorithm: designed not to recombine soft particles together

$$d_{ij} = \min\{1/k_{ti}^2, 1/k_{tj}^2\}\Delta R_{ij}^2/R^2 \qquad d_{iB} = 1/k_{ti}^2$$

Cacciari, Salam, Soyez '08

Recombination schemes in e^+e^-

Given two massless momenta p_i and p_j how does one recombine them to build p_{ij} ? Several choices are possible.

Most common ones:

1. E-scheme $\quad p_{ij} = p_i + p_j$
2. E_0-scheme $\quad \vec{p}_{ij} = \vec{p}_i + \vec{p}_j \qquad E_{ij} = |\vec{p}_{ij}|$
3. P_0-scheme $\quad E_{ij} = E_i + E_j \qquad \vec{p}_{ij} = \frac{E_{ij}}{|\vec{p}_i + \vec{p}_j|}(\vec{p}_i + \vec{p}_j)$

E_0/P_0-schemes give massless jets, along with the idea that the hard parton underlying the jet is massless

E-scheme give massive jets. Most used in recent analysis also at the hadron-colliders (other possibilities there too)

Cone algorithms

1. A particle i at rapidity and azimuthal angle (yi, φi) ⊂ cone C iff

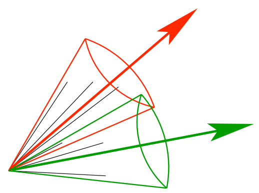
$$\sqrt{(y_i - y_C)^2 + (\phi_i - \phi_C)^2} \leq R_{\mathrm{cone}}$$

2. Define

$$\bar{y}_C \equiv \frac{\sum_{i\in C} y_i \cdot p_{T,i}}{\sum_{i\in C} p_{T,i}} \qquad \bar{\phi}_C \equiv \frac{\sum_{i\in C} \phi_i \cdot p_{T,i}}{\sum_{i\in C} p_{T,i}}$$

3. If weighted and geometrical averages coincide $(y_C, \phi_C) = (\bar{y}_C, \bar{\phi}_C)$ a stable cone (⇒ jet) is found, otherwise set $(y_C, \phi_C) = (\bar{y}_C, \bar{\phi}_C)$ & iterate

4. Split-merge on overlapping jets (2nd par: overlap parameter f)

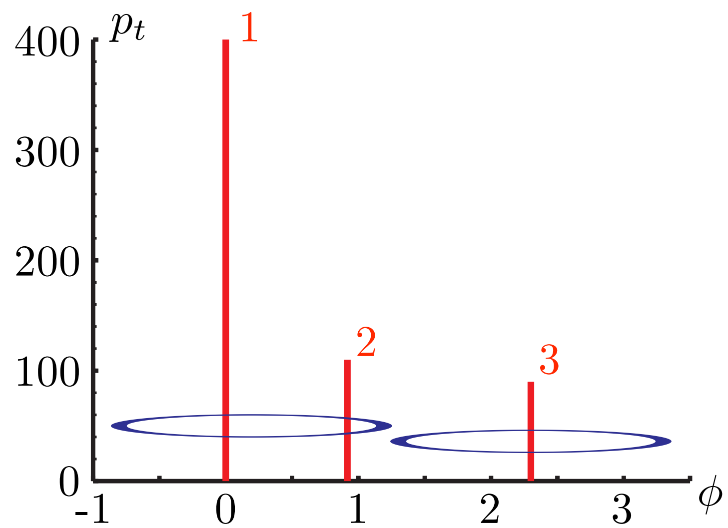


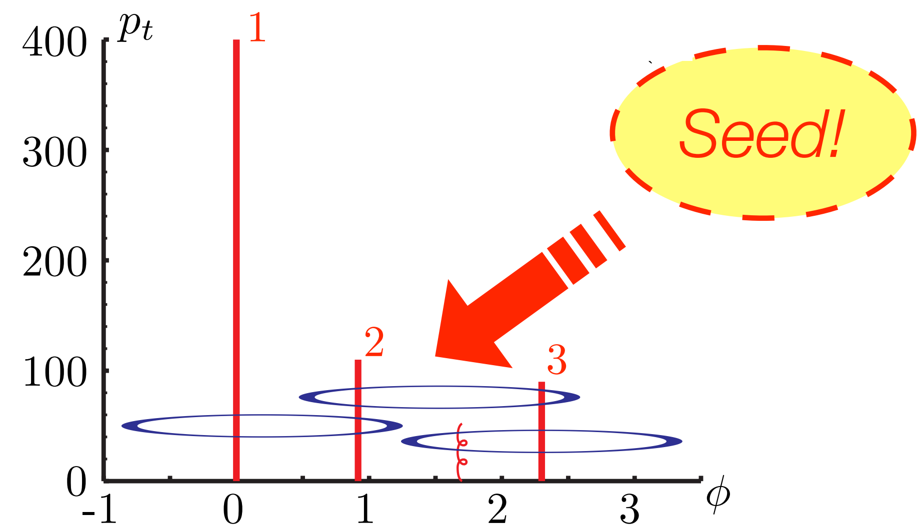
Ideally: place trial cones everywhere and find all stable cones

Practically (JetClu, MidPoint, PxCone..): introduce trial directions (seeds)

Seeds make cone algorithms infrared unsafe

Jets: infrared unsafety of cones





3 hard ⇒ 2 stable cones 3 hard + 1 soft ⇒ 3 stable cones

Soft emission changes the hard jets ⇒ algorithm is IR unsafe

<u>Midpoint algo:</u> take as seed position of emissions and midpoint between two emissions (postpones the infrared satefy problem)

Seedless cones

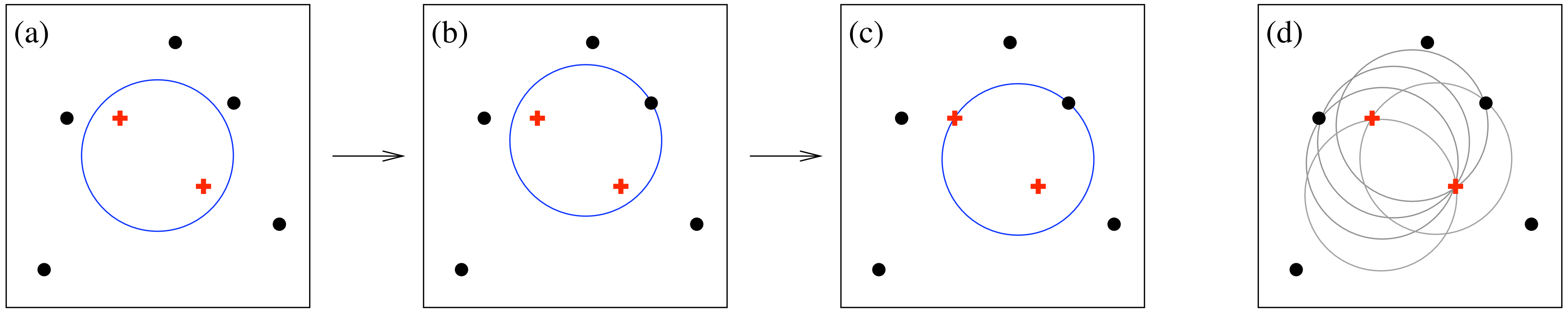
Solution:
use a seedless algorithm, i.e. consider all possible combinations of particles as candidate cones, so find all stable cones [⇒ jets]

Blazey '00

The problem:
clustering time growth as $N2^N$. So for an event with 100 particles need 10^{17} ys to cluster the event ⇒ prohibitive beyond PT (N=4,5)

Better solution:

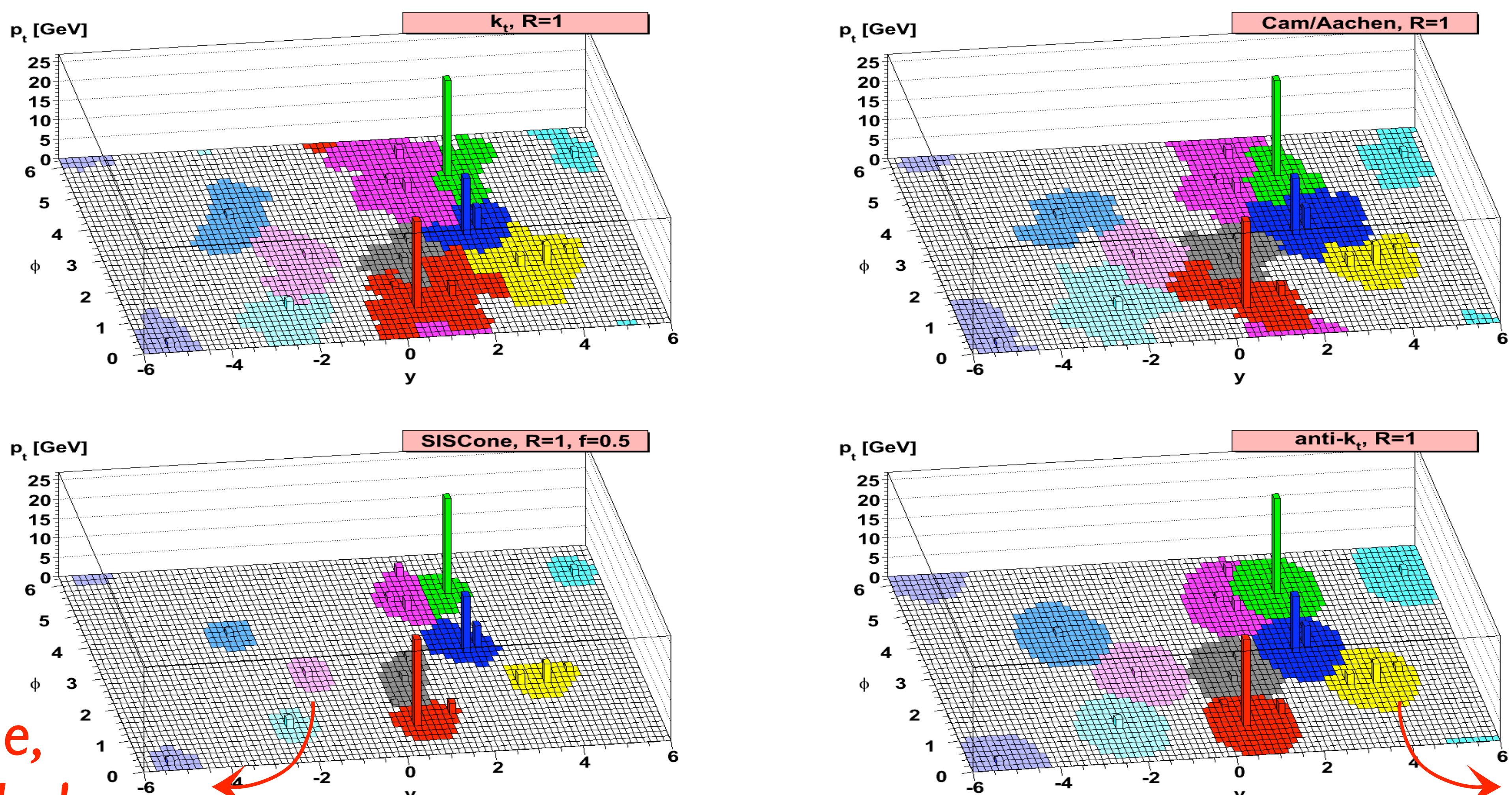
SISCone recasts the problem as a computational geometry problem, the identification of all distinct circular enclosures for points in 2D and finds a solution to that ⇒ $N^2 \ln N$ time IR safe algorithm



Salam, Soyez '07

Jet area

Given an infrared safe, fast jet-algorithm, can define the jet area A as follows: fill the event with an infinite number of infinitely soft emissions uniformly distributed in η-φ and make A proportional to the # of emissions clustered in the jet



NB: cone, not circular!

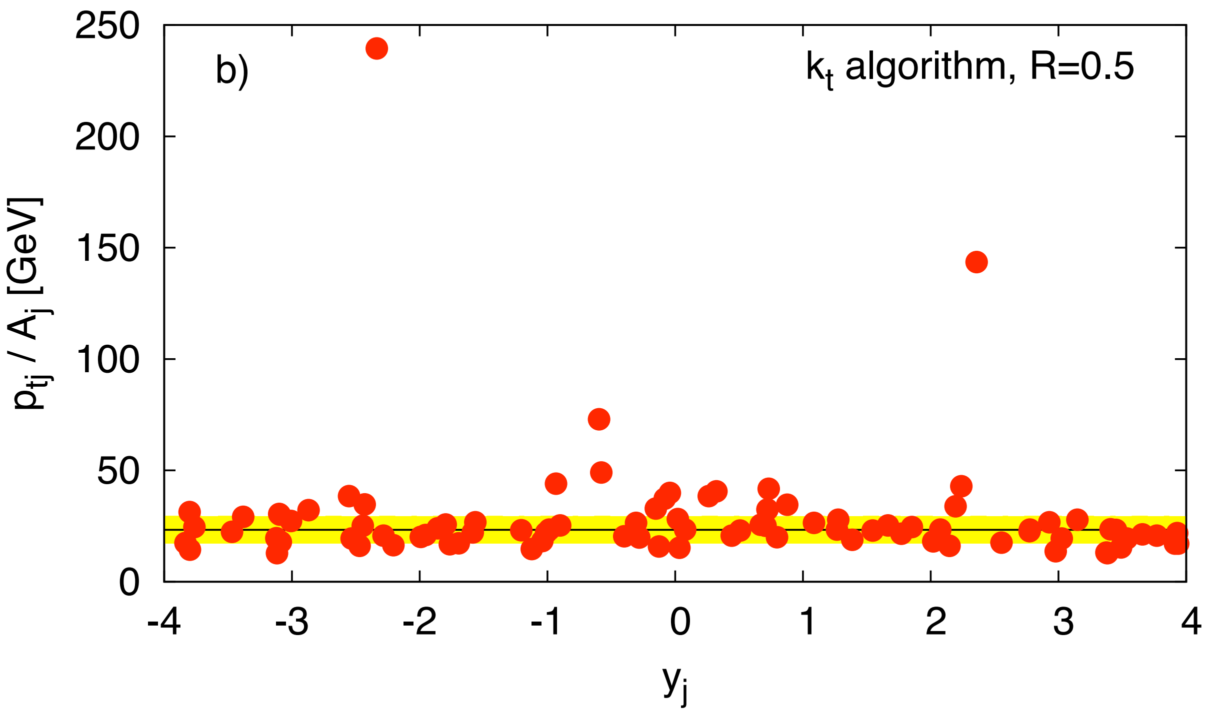
NB: new anti-kt

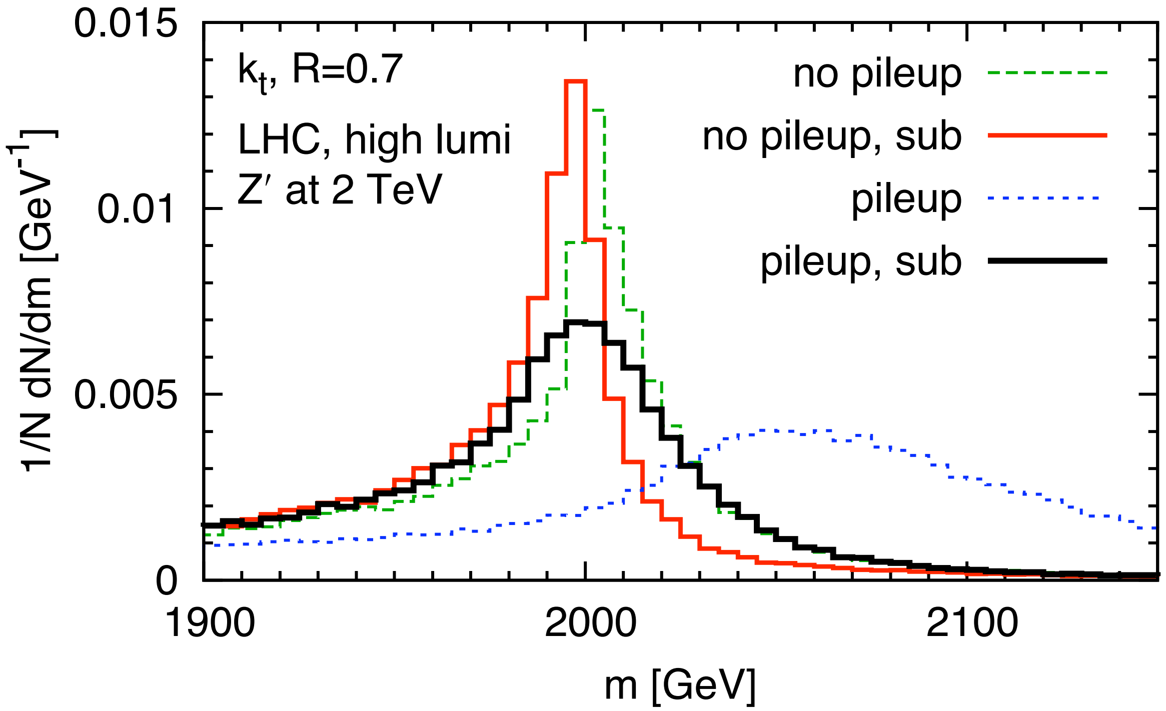
What jet areas are good for

jet-area ≡ catching area of the jet when adding soft emissions

⇒ simple area based subtraction for a variety of algorithms

Get $\rho = \frac{p_{t,j}}{A_j}$ from the majority of (pile-up) jets, define $p_j^{\rm sub} = p_j - A_j \rho$





Cacciari et al. '07

Remember: pileup = generic p-p interaction (hard, soft, single-diffractive...) overlapping with hard scattering

Quality measures of jets

Suppose you are searching for a heavy state (H→gg, Z'→qq, ...)

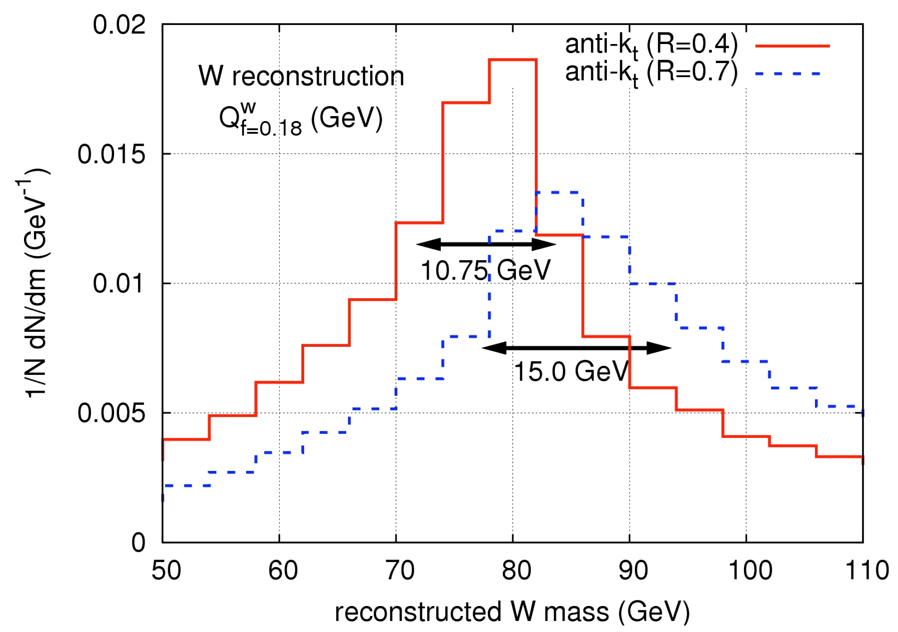
The object is reconstructed through its decay products

⇒ *Which jet algorithm (JA) is best ? Does the choice of R matter?*

<u>Define:</u> Q^w_f(JA, R) ≡ width of the smallest mass window that contains a fraction f of the generated massive objects

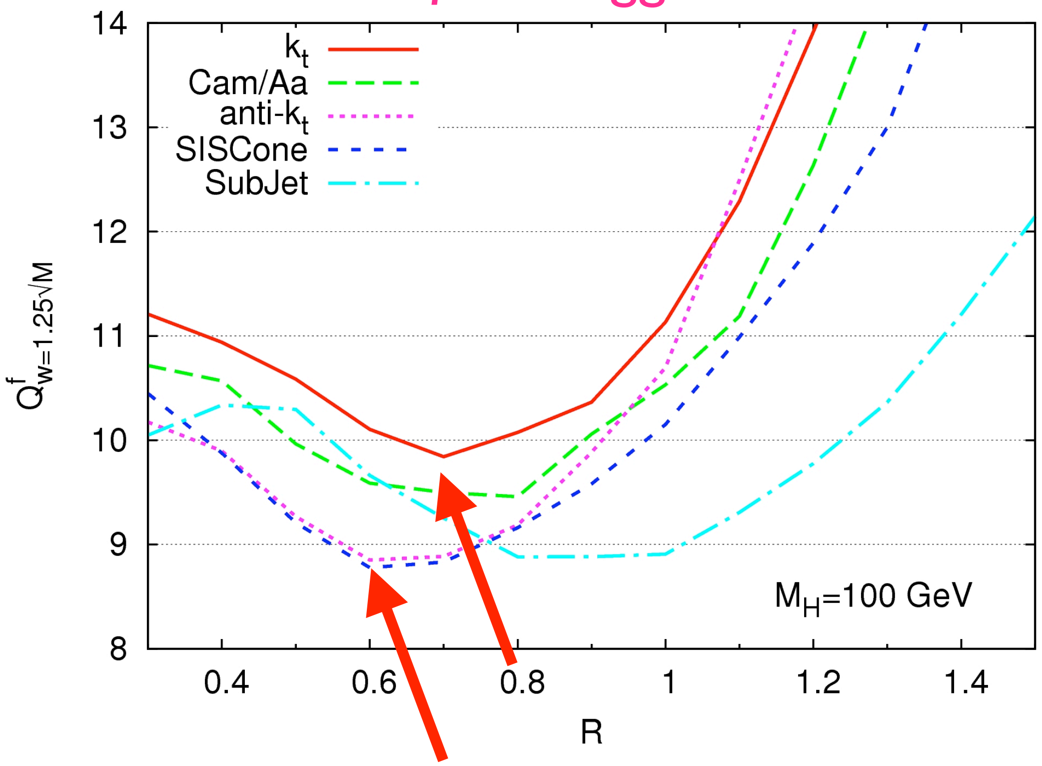
- good algo ⇔ small Q^w_f(JA, R)
- ratios of Q^w_f(JA,R): mapped to ratios of effective luminosity (with same $S/\sqrt{B}$)

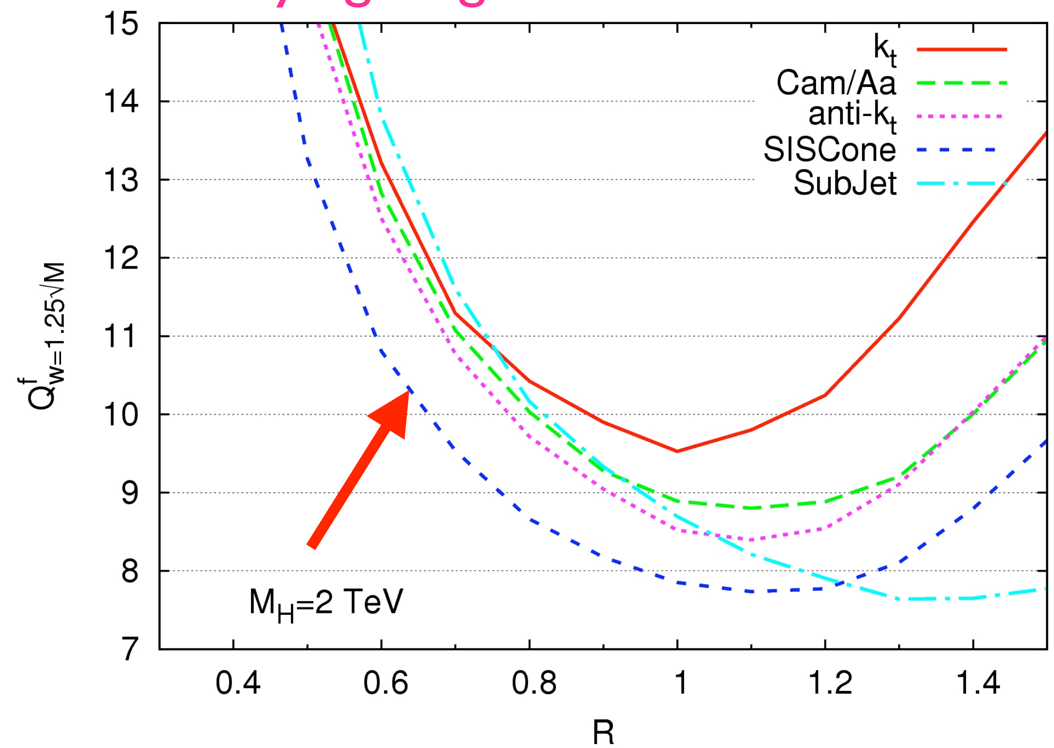
$$\mathcal{L}_2 = \rho_{\mathcal{L}} \mathcal{L}_1 \qquad \rho_{\mathcal{L}} = \frac{Q_z^f(JA_2, R_2)}{Q_z^f(JA_1, R_1)}$$



Quality measures: sample results

NB: Here "fake Higgs" = narrow resonance decaying to gluons

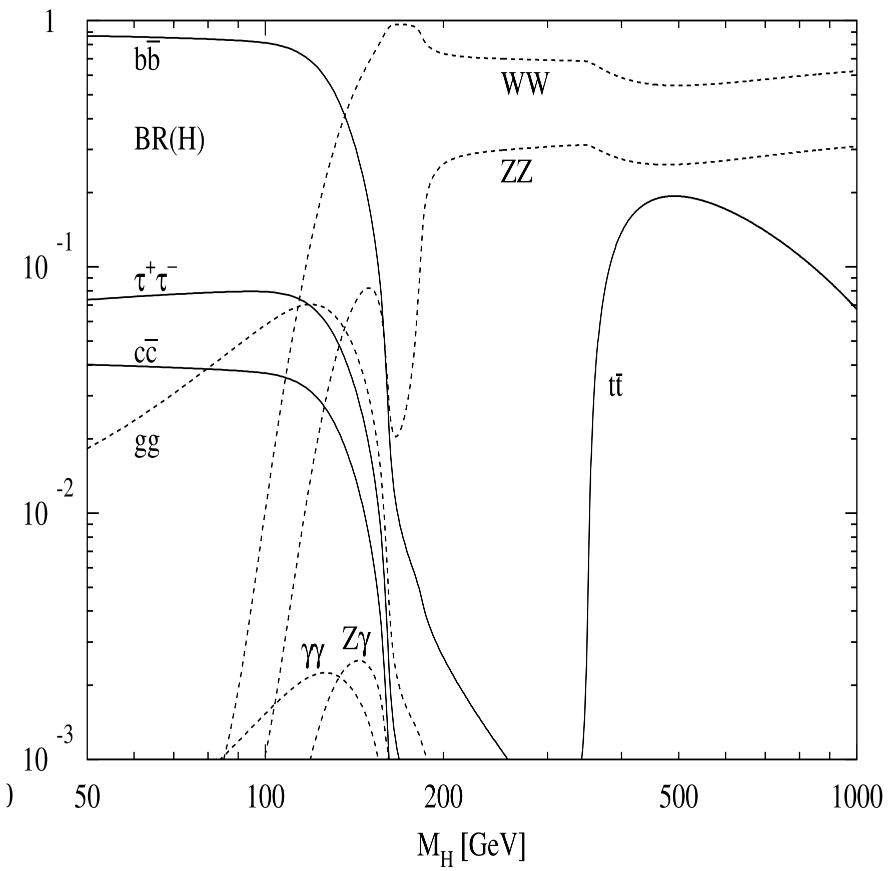


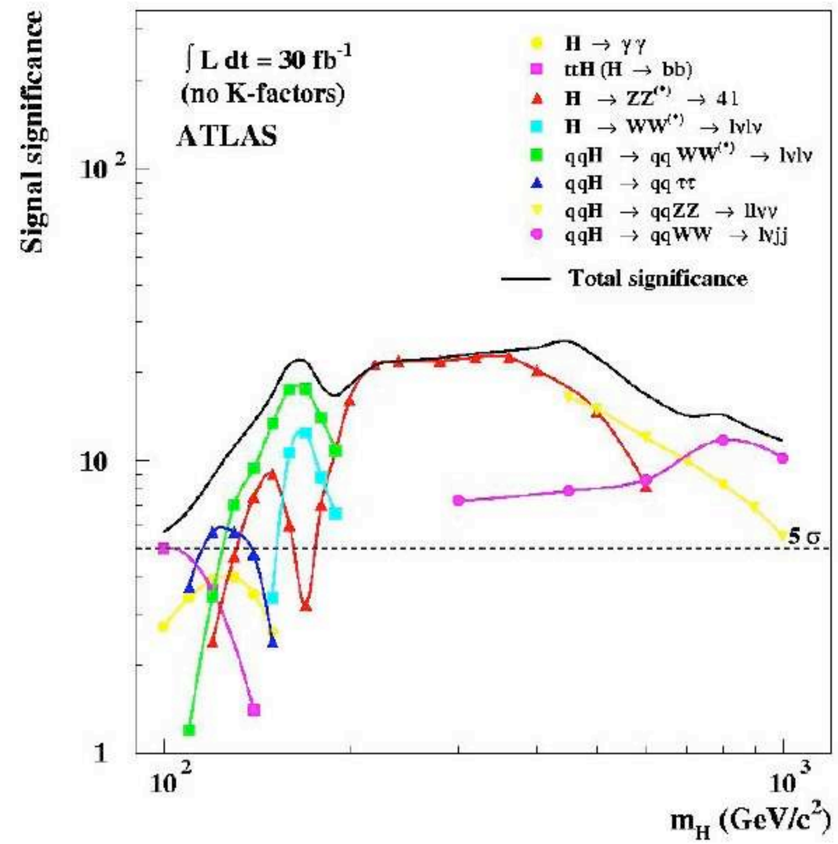


- At 100GeV: use a Tevatron standard algo (k_t, R=0.7) instead of best choice (SISCone, R=0.6 $\Rightarrow$ lose $\rho_{\mathcal{L}} = 0.8$ in effective luminosity
- At M_H=2 TeV: use M_H=100GeV best choice again loose in effective luminosity

A good choice of jet-algorithm does matter!
Bad choice of algorithm $\Leftrightarrow$ lost in discrimination power!

Z/W+ H (→bb) rescued ?





⇒ Light Higgs hard: H→bb dominant, but overwhelmed by background

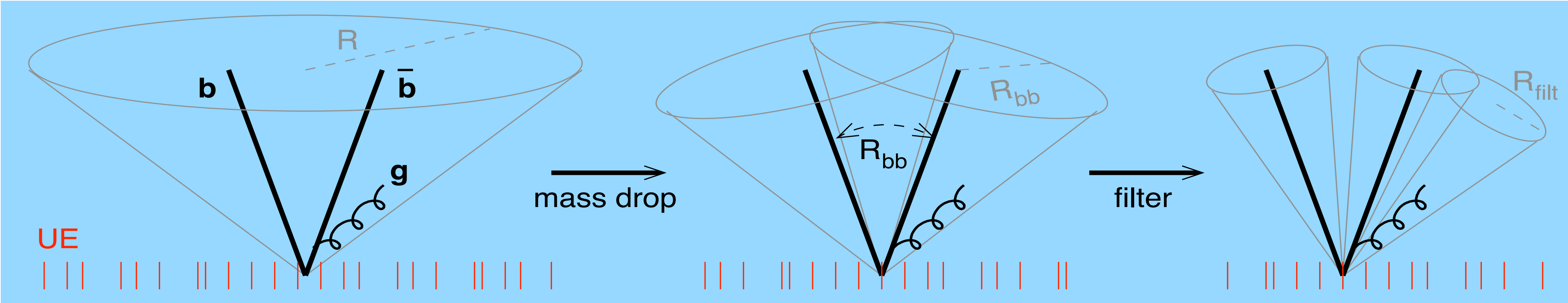
Conclusion [ATLAS TDR]:

The extraction of a signal from H → bb decays in the WH channel will be very difficult at the LHC even under the most optimistic assumptions [...]

Z/W + H (→bb) rescued ?

Boosted Higgs at high p_t: central decay products ⇒ single massive jet

Use jet-finding geared to identify the characteristic structure of fast-moving Higgs that decays into a bb-pair close in angle

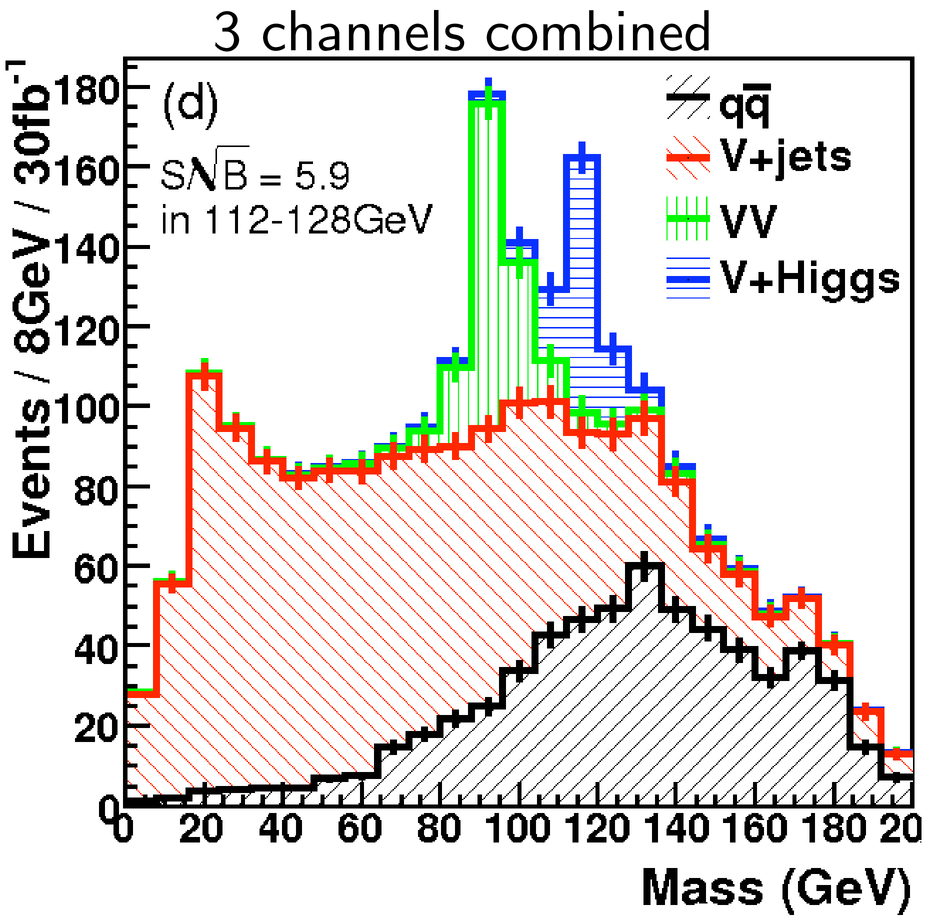


1. cluster the event with e.g. CA algo and large-ish R
2. undo last recomb: large mass drop + symmetric + b tags
3. filter away the UE: take only the 3 hardest sub-jets

<u>Related ideas for 2- and 3-body decays (boosted tops):</u> Butterworth, Cox & Forshaw; Butterworth, Ellis & Raklev; Skiba & Tucker-Smith; Hodom; Baur; Agashe et al; Lille, Randall & Wang; Contino & Servant; Brooijmans; Thaler & Wang; Kaplan et al.; Almeida et al. [...]

Z/W + H (→bb) rescued ?

Mass of the sub-jets:



- with common & channel specific cuts: p_{tV}, p_{tH} > 200GeV , ...
- real/fake b-tag rate: 0.7/0.01
- NB: very neat peak for WZ (Z →bb)
  Important for calibration

Butterworth, Davison, Rubin, Salam '08

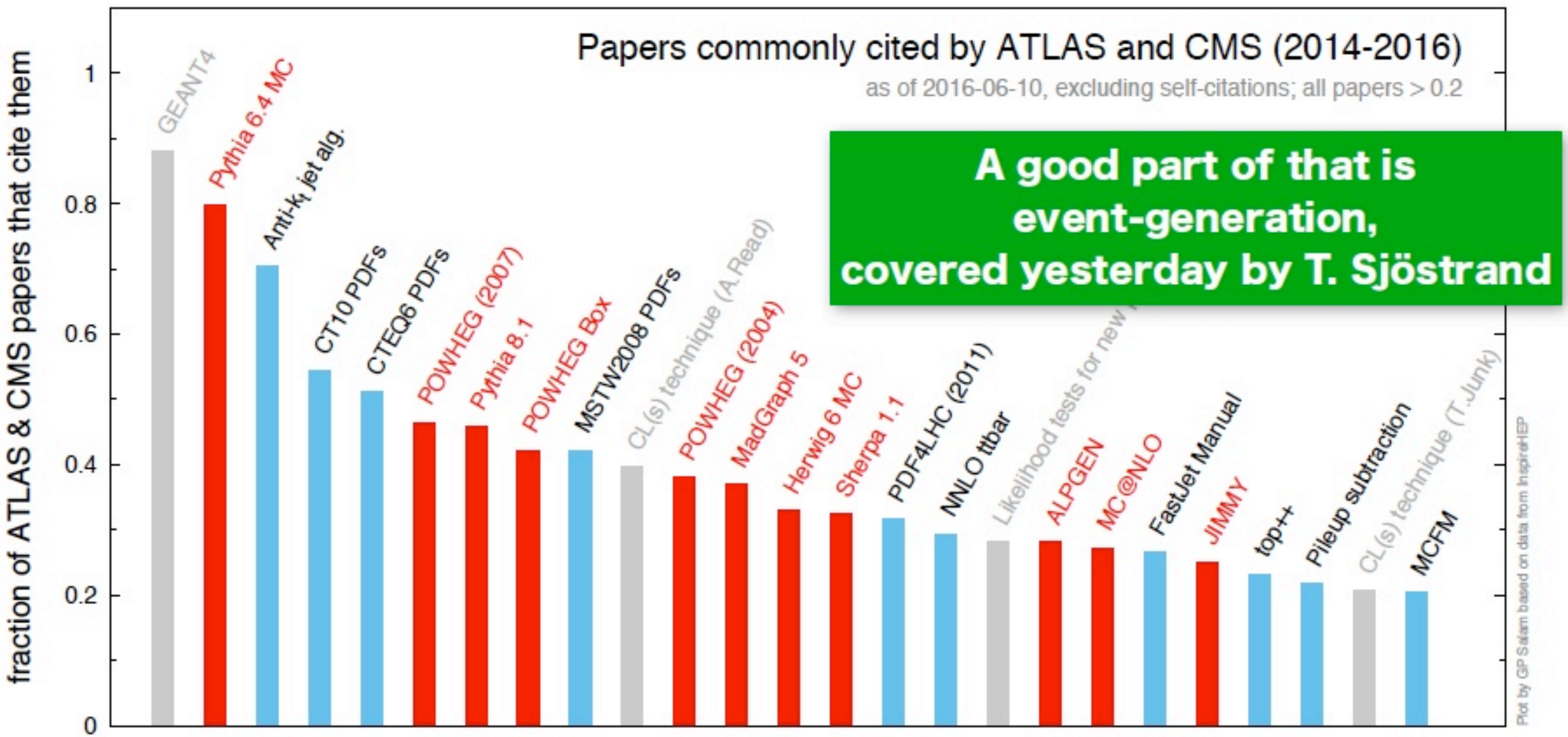
This result was the starting point of a new area of research in QCD that focuses on highly boosted events and analyses the sub-structure of jets

Recap on jets

- Two major jet classes: sequential (anti-k_t, CA, k_t,...) and cones (UA1, midpoint, ...)
- Jet algorithm is fully specified by: clustering + recombination + split merge or removal procedure + all parameters
- Standard cones based on seeds are IR unsafe
- SISCone is an IR safe cone algorithm (no seeds)
- Using IR unsafe algorithms you might not be able to use available higher order calculations
- Using IR safe algorithms: can do sophisticated studies e.g. jet-areas for pile-up subtraction and much more
- Not all algorithms fare the same for BSM searches: quality measures quantify this
- Many new ideas in jet substructure (Higgs example)

Parton showers and event generators

Are they used at the LHC ? Well, yes...



Parton shower & Monte Carlo methods

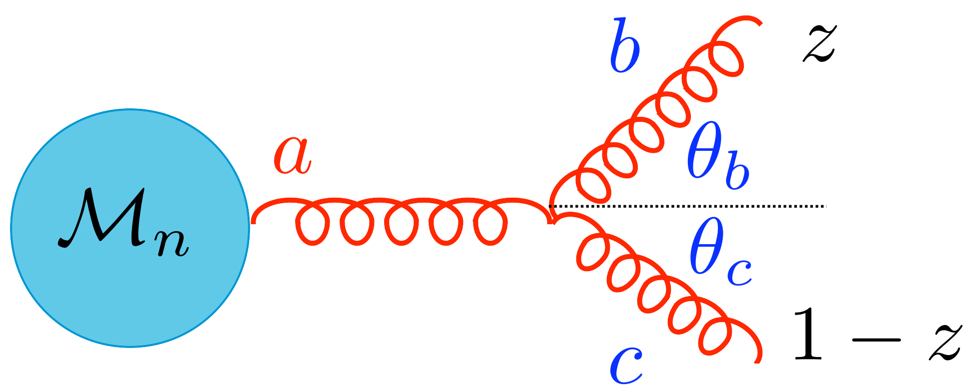
- today one can compute IR-safe quantities at NLO, few ones at NNLO, and very few at N^3LO. Difficult to expect much more in the coming years.
- we have also seen that sometimes large logs spoil the convergence of PT, NLO etc becomes useless
- now we adopt a different approach: we seek for an approximate result such that enhanced terms are taken into account to all orders
- this will lead to a 'parton shower' picture, which can be implemented in computer simulations, usually called Monte Carlo programs or event generators

Parton branching: the time-like case

<u>Assume:</u> $p_b^2, p_c^2 \ll p_a^2 \equiv t$ (scale of the branching)

$$
\begin{aligned}
p_a &= (E_a, 0, 0, p_{az}) \\
p_b &= (E_b, 0, E_b \sin\theta_b, E_b \cos\theta_b) \\
p_c &= (E_c, 0, -E_c \sin\theta_c, E_c \cos\theta_c)
\end{aligned}
$$

Time-like branching: t > 0



Kinematics: $z = \dfrac{E_b}{E_a} = 1 - \dfrac{E_c}{E_a}$

small angle approx.

$$t = (p_b + p_c)^2 = 2E_b E_c (1 - \cos\theta) \sim z(1-z) E_a^2 \theta^2$$

$$E_b \sin\theta_b = E_c \sin\theta_c \Rightarrow z\theta_b \sim (1-z)\theta_c$$

$$\theta = \theta_b + \theta_c = \frac{\theta_b}{1-z} = \frac{\theta_c}{z}$$

Parton branching: gluon case

Three-gluon vertex:

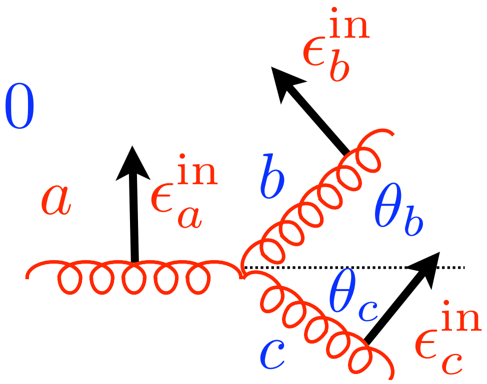
$$V_{ggg} = ig_s f_{ABC} \epsilon_a^\mu \epsilon_b^\nu \epsilon_c^\rho \left(g_{\mu\nu}(p_a - p_b)_\rho + g_{\nu\rho}(p_b - p_c)_\mu + g_{\rho\mu}(p_c - p_a)_\nu \right)$$

Use: $\epsilon_i \cdot p_i = 0$ and $p_a + p_b + p_c = 0$

$$V_{ggg} = -2ig_s f_{ABC} \left[(\epsilon_a \cdot \epsilon_b)(\epsilon_c \cdot p_b) - (\epsilon_b \cdot \epsilon_c)(\epsilon_a \cdot p_b) - (\epsilon_c \cdot \epsilon_a)(\epsilon_b \cdot p_c) \right]$$

Branching: in a plane. Natural to split polarization vectors in ϵ_i^{in} and ϵ_i^{out}

Properties: $\epsilon_i^{\text{in}} \cdot \epsilon_j^{\text{in}} = \epsilon_i^{\text{out}} \cdot \epsilon_j^{\text{out}} = -1 \qquad \epsilon_i^{\text{in}} \cdot \epsilon_j^{\text{out}} = \epsilon_i^{\text{out}} \cdot p_j = 0$

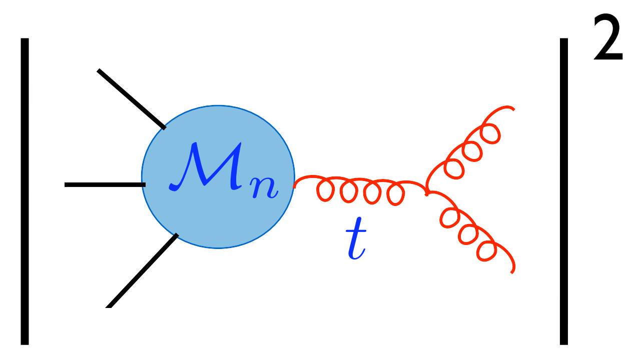


Explicitly:

$$\begin{aligned} \epsilon_a^{\text{in}} &= (0, 0, 1, 0) \\ \epsilon_b^{\text{in}} &= (0, 0, \cos\theta_b, -\sin\theta_b) \\ \epsilon_c^{\text{in}} &= (0, 0, \cos\theta_c, \sin\theta_c) \end{aligned} \quad \Rightarrow \quad \begin{aligned} \epsilon_a^{\text{in}} \cdot p_b &= -E_b\theta_b = -z(1-z)E_a\theta \\ \epsilon_b^{\text{in}} \cdot p_c &= E_c\theta = (1-z)E_a\theta \\ \epsilon_c^{\text{in}} \cdot p_b &= -E_b\theta = -zE_a\theta \end{aligned}$$

Parton branching: the gluon case

Squared matrix element for n+1 partons becomes:



$$|\mathcal{M}_{n+1}|^2 = \frac{4g_s^2}{t} C_A F(z;\epsilon_a,\epsilon_b,\epsilon_c)|\mathcal{M}_n|^2$$

NB: one "t" cancels completely

a	b	c	F(z; ε_a, ε_b, ε_c)
in	in	in	(1-z)/z + z/(1-z) + z(1-z)
in	out	out	z(1-z)
out	in	out	(1-z)/z
out	out	in	z/(1-z)

Averaging over incoming and summing over outgoing pol. we get

$$C_A\langle F\rangle = \hat{P}_{gg} = C_a\left[\frac{1-z}{z} + \frac{z}{1-z} + z(1-z)\right]$$

The gluon case: remarks

Soft singularities ($z \to 0,1$) are associated to soft gluon in the plane of the branching

Correlation between plane of branching and polarization of incoming gluon: take polarization of gluon at an angle φ to the plane then

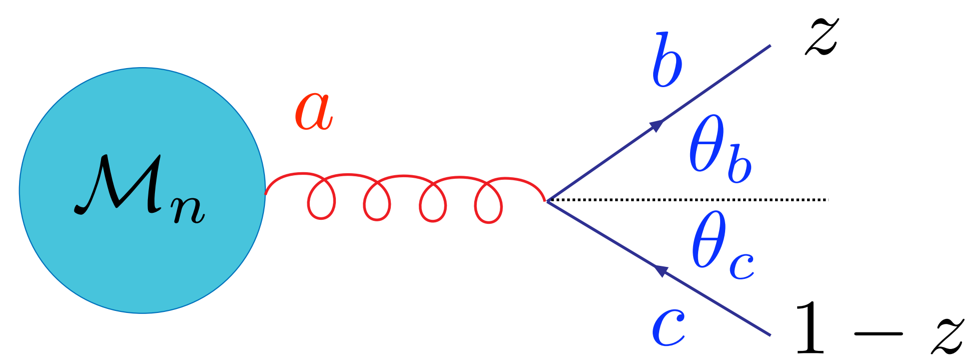
$$
\begin{aligned}
F_\phi &= \sum_{b,c} |\cos\phi\,\mathcal{M}(\epsilon_a^{\rm in},\epsilon_c,\epsilon_c) + \sin\phi\,\mathcal{M}(\epsilon_a^{\rm out},\epsilon_c,\epsilon_c)|^2 \\
&= \underbrace{\frac{1-z}{z} + \frac{z}{1-z} + z(1-z)}_{\text{unpolarized result}} + \underbrace{z(1-z)\cos 2\phi}_{\text{correction}}
\end{aligned}
$$

Correction favors polarization of branching gluon in the branching plane, but is weak (no soft enhancements)

Gluon splitting to quarks

Similarly start from 3-particle vertex:

$$V_{q\bar{q}g} = -ig_s\, t^A_{bc}\, \bar{u}(p_b)\, \gamma_\mu \epsilon^\mu_a\, v(p_c)$$



Fix a representation of the Dirac algebra (called Dirac rep.):

$$\gamma^0 = \begin{pmatrix} 1_{2\times 2} & 0_{2\times 2} \\ 0_{2\times 2} & -1_{2\times 2} \end{pmatrix} \qquad \gamma^i = \begin{pmatrix} 0_{2\times 2} & \sigma_i \\ -\sigma_i & 0_{2\times 2} \end{pmatrix}$$

To first order in the small angles the spinors are

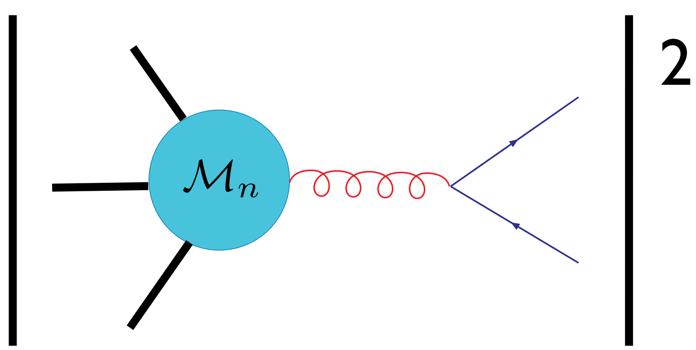
$$\frac{u_+(p_b)}{\sqrt{E_b}} = \begin{pmatrix} 1 \\ \theta_b/2 \\ 1 \\ \theta_b/2 \end{pmatrix} \quad \frac{u_-(p_b)}{\sqrt{E_b}} = \begin{pmatrix} \theta_b/2 \\ -1 \\ \theta_b/2 \\ -1 \end{pmatrix} \quad \frac{v_+(p_c)}{\sqrt{E_c}} = i\begin{pmatrix} -\theta_c/2 \\ -1 \\ \theta_c/2 \\ 1 \end{pmatrix} \quad \frac{v_-(p_c)}{\sqrt{E_c}} = i\begin{pmatrix} -1 \\ \theta_c/2 \\ -1 \\ \theta_c/2 \end{pmatrix}$$

Gluon splitting to quarks

Explicitly we find e.g.

$$-ig_s\bar{u}_+(p_b)\gamma_\mu\epsilon_a^{\text{in},\mu}v_-(p_c) = \sqrt{E_bE_c}(\theta_b-\theta_c) = \sqrt{z(1-z)}(1-2z)E_a\theta$$

Similarly to before define



a	b	c	F(z; ε_a, λ_b, λ_c)
in	$\pm$	$\mp$	$(1\text{-}2z)^2$
out	$\pm$	$\mp$	1

$$|\mathcal{M}_{n+1}|^2 = \frac{4g_s^2}{t}T_RF(z;\epsilon_a,\lambda_b,\lambda_c)|\mathcal{M}_n|^2$$

Averaged splitting function: $T_R\langle F\rangle \equiv \hat{P}_{qg}(z) = T_R\left[z^2+(1-z)^2\right]$

Angular correlation: $F_\phi = z^2+(1-z)^2-2z(1-z)\cos 2\phi$ (more important)

Last case: quark emitting gluon

Similarly to the previous cases for a quark emitting a gluon one obtains

$$|\mathcal{M}_{n+1}|^2 = \frac{4g_s^2}{t} C_F F(z;\lambda_a,\lambda_b,\epsilon_c)|\mathcal{M}_n|^2$$

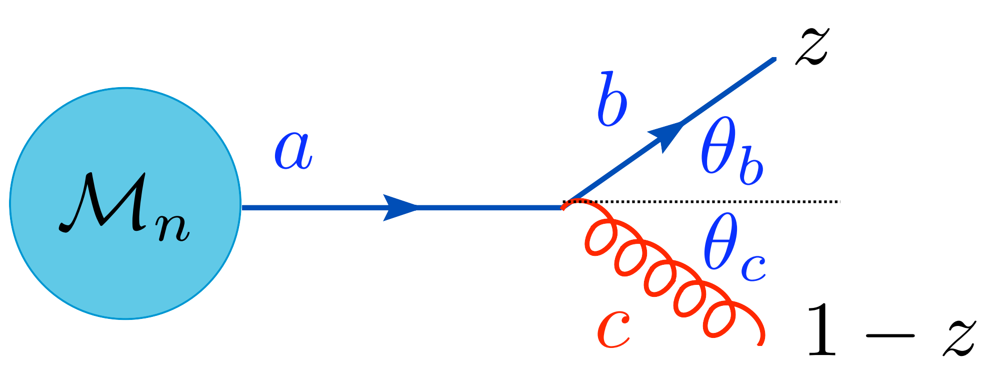
a	b	c	F(z; λ_a, λ_b, ε_c)
$\pm$	$\pm$	in	$(1+z)^2/(1-z)$
$\pm$	$\pm$	out	1-z

NB: helicity of the quark does not change during the branching

Averaged splitting function: $$C_F\langle F\rangle \equiv \hat{P}_{qq}(z) = C_F\frac{1+z^2}{1-z}$$

Angular correlation:

$$F_\phi = \frac{1+z^2}{1-z} + \frac{2z}{1-z}\cos 2\phi$$



Phase space

n-particle phase space (without branching): $d\Phi_n = d\Phi_{n-1} \frac{d^3 p_a}{(2\pi)^3 2E_a}$

(n+1)-particle phase space (with branching): $d\Phi_{n+1} = d\Phi_{n-1} \frac{d^3 p_b}{(2\pi)^3 2E_b} \frac{d^3 p_c}{(2\pi)^3 2E_c}$

At fixed p_b: $d^3 p_a = d^3 p_c \quad \Rightarrow \quad d\Phi_{n+1} = d\Phi_n \frac{d^3 p_b}{(2\pi)^3 2E_b} \frac{E_a}{E_c}$

$$t = p_a^2 \qquad z = \frac{E_b}{E_a}$$

$$p_b^2, p_c^2 \ll p_a^2 \equiv t$$

$$\begin{aligned} d^3 p_b &= p_b^2 dp_b \; \sin\theta d\theta \; d\phi \\ &\sim E_b^2 dE_b \; \theta d\theta \; d\phi \\ &= E_a^3 z^2 dz \; \frac{dt}{2z(1-z)E_a^2} \; d\phi \end{aligned}$$

$$\Rightarrow \quad d\Phi_{n+1} = d\Phi_n \frac{1}{4(2\pi)^3} dt\, dz\, d\phi$$

N-particle cross-section: $d\sigma_n = \mathcal{F} \, |\mathcal{M}_n|^2 \, d\Phi_n$ with $|\mathcal{M}_{n+1}|^2 = \frac{4g_s^2}{t} CF |\mathcal{M}_n|^2$

$$\boxed{d\sigma_{n+1} = d\sigma_n \frac{dt}{t} dz\, d\phi \frac{\alpha_s}{2\pi} C\, F}$$

Azimuthal averaged result

Averaging over azimuthal angles:

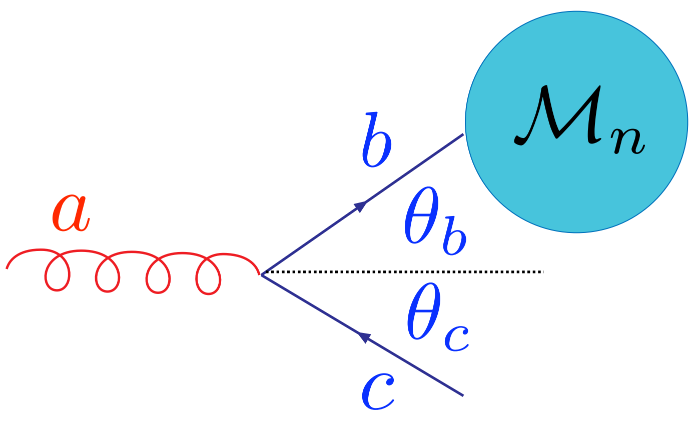
$$\int \frac{d\phi}{2\pi} C\, F = \hat{P}_{ba}(z)$$

The evolution equation becomes:

$$\boxed{d\sigma_{n+1} = d\sigma_n \frac{dt}{t} dz\, \frac{\alpha_s}{2\pi} \hat{P}_{ba}(z)}$$

Space-like branching

What are the modifications needed if an incoming parton splits?



The kinematics changes: $p_a^2, p_c^2 \ll |p_b^2| \equiv t$

Space-like branching: t < 0

Small angle approximation: $t = E_a E_c \theta_c^2$ (verify)

(n+1) particle phase space becomes: $d\Phi_{n+1} = d\Phi_n \frac{1}{4(2\pi)^3} dt \frac{dz}{z} d\phi$

The additional "z" is compensated by the different flux-factor, we find

Space-like or time-like braching: $d\sigma_{n+1} = d\sigma_n \frac{dt}{t} dz \frac{\alpha_s}{2\pi} \hat{P}_{ba}(z)$

Perturbative evolution

In exact analogy with what done for parton densities inside hadrons we want to write an evolution equation for the probability to have partons at the momentum scale Q^2 with momentum fraction z during PT branching

Start from DGLAP equation

$$Q^2 \frac{\partial f(x,Q^2)}{\partial Q^2} = \int_0^1 dz \frac{\alpha_s}{2\pi} \hat{P}(z) \left(\frac{1}{z} f\left(\frac{x}{z}, Q^2\right) - f(x,Q^2) \right)$$

Introduce a cut-off to regulate divergences

$$Q^2 \frac{\partial f(x,Q^2)}{\partial Q^2} = \int_0^{1-\epsilon} \frac{dz}{z} \frac{\alpha_s}{2\pi} \hat{P}(z) f\left(\frac{x}{z}, Q^2\right) - f(x,Q^2) \int_0^{1-\epsilon} dz \frac{\alpha_s}{2\pi} \hat{P}(z)$$

Introduce a Sudakov form factor (interpreted as the probability to evolve between two scales with no emission)

$$\Delta(Q^2) = exp\left\{ - \int_{Q_0}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \int_0^{1-\epsilon} dz \frac{\alpha_s}{2\pi} \hat{P}(z) \right\}$$

Perturbative evolution

The DGLAP equation becomes

$$Q^2 \frac{\partial}{\partial Q^2}\left(\frac{f(x,Q^2)}{\Delta(Q^2)}\right) = \frac{1}{\Delta(Q^2)}\int_0^{1-\epsilon} \frac{dz}{z}\frac{\alpha_s}{2\pi}\hat{P}(z) f\left(\frac{x}{z},Q^2\right)$$

Integrating the above equation one gets

$$f(x,Q^2) = f(x,Q_0^2)\frac{\Delta(Q^2)}{\Delta(Q_0^2)} + \int_{Q^0}^{Q^2}\frac{dk_\perp^2}{k_\perp^2}\frac{\Delta(Q^2)}{\Delta(k_\perp^2)}\int_0^{1-\epsilon}\frac{dz}{z}\frac{\alpha_s}{2\pi}\hat{P}(z) f\left(\frac{x}{z},k_\perp^2\right)$$

This equation has a probabilistic interpretation

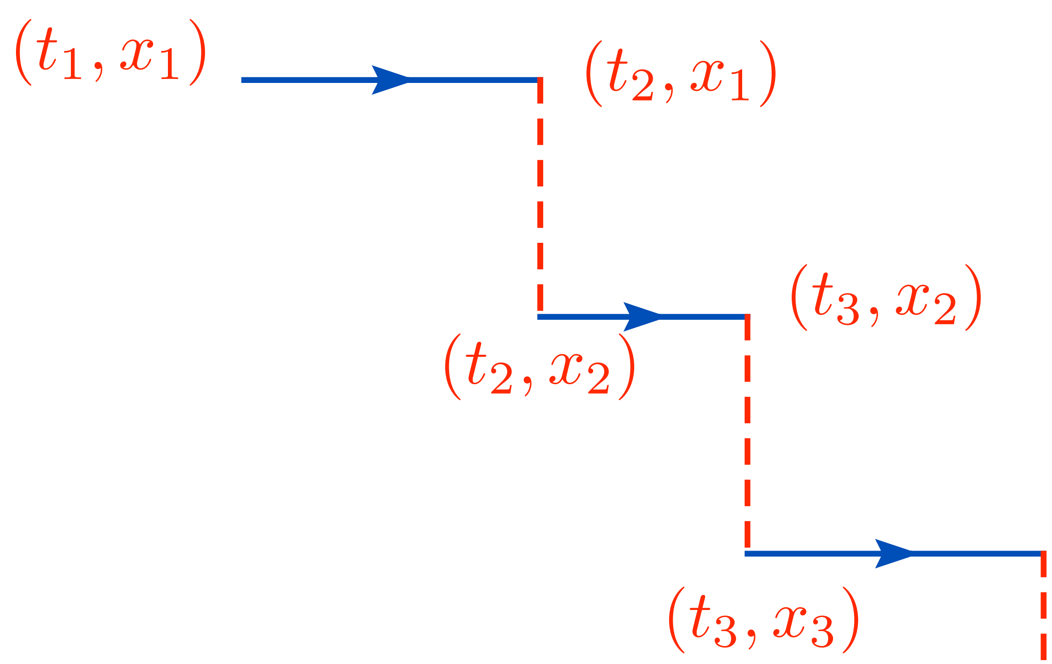
- First term: probability of evolving from Q_0^2 to Q^2 without emissions (ratio of Sudakovs $\Delta(Q^2)/\Delta(Q_0^2)$)
- Second term: emission at scale $k_\perp^2$ and evolution from $k_\perp^2$ to Q^2 without further emissions

Multiple branchings

Multiple branching can now be described using the above probabilistic equation

Denote by t the evolution variable (e.g $t = Q^2$)
Start from one parton at scale t_1 and momentum fraction x_1

The question is how to generate the values of t_2, x_2 and φ_2



Multiple branchings

1. t_2 generated with the correct probability by solving the equation (r = random number in [0,1])

$$\Delta(t_1)/\Delta(t_2) = r$$

   If t_2 smaller than cut-off evolution stops (no further branching)

2. Else, generate momentum fraction z = x_2/x_1 with $\text{Prob.} \sim \frac{\alpha_s}{2\pi} P(z)$

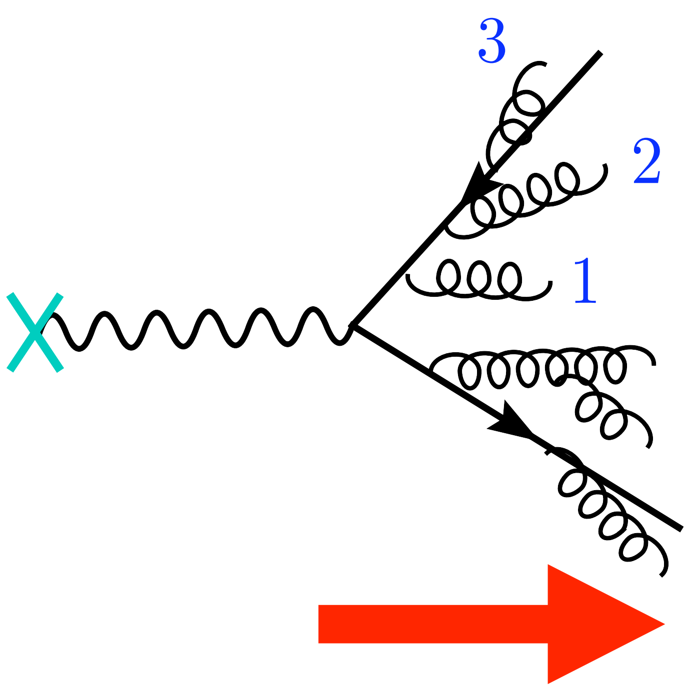
$$\int_\epsilon^{x_2/x_1} dz \frac{\alpha_s}{2\pi} P(z) = r' \int_\epsilon^{1-\epsilon} dz \frac{\alpha_s}{2\pi} P(z)$$

   ε: IR cut-off for resolvable branching

3. Azimuthal angles: generated uniformly in $(0,2\pi)$ (or taking into account polarization correlations)

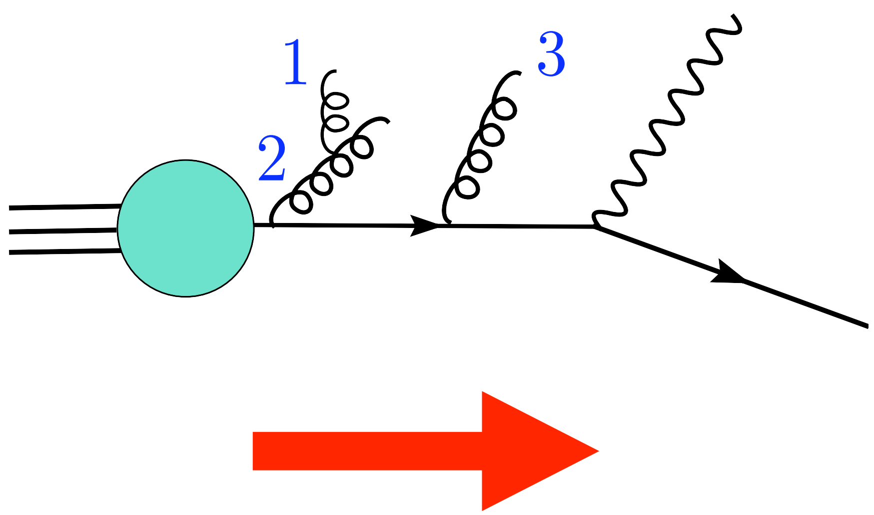
Space-like vs time-like evolution

Time-like: t evolves from a hard-scale downwards to an IR cut-off



$$Q > t_1 > t_2 > \cdots > Q_0$$

Space-like: t increases in the evolution up to the hard scale Q^2



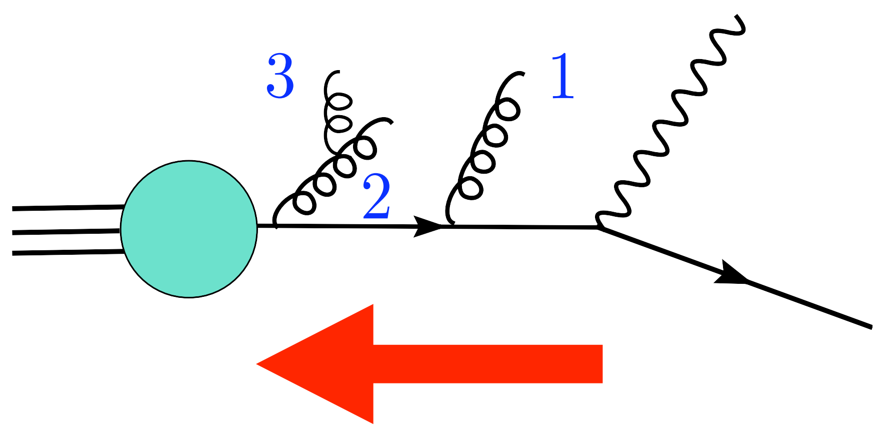
$$Q_0 < t_1 < t_2 < \ldots, Q$$

Each outgoing parton becomes a source of the new branching until the "no-branching" step is met (cut-off essential in parton shower)

⇒ a parton cascade develops, when all branchings are done partons are converted into hadrons via a hadronization model

Backward evolution

In space-like cases it is more convenient to start from the momentum fraction of the outgoing parton x_n and generate x_{n-1}, .. x_0 by backward evolution



$$Q > t_1 > t_2 > \cdots > Q_0$$

Essentially, the evolution proceeds as before but with a modified form factor which take the local parton density into account

We will not discuss backward evolution, despite its wide-spread use

Angular ordering

In the branching formalism discussed now we considered collinear enhancements to all orders in PT. But there are also soft enhancements.

When a soft gluon is radiated from a ($p_i p_j$) dipole one gets a universal eikonal factor

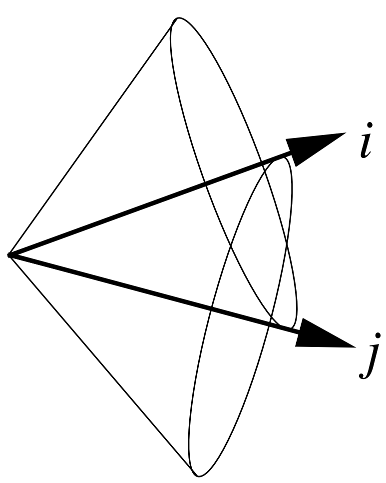
$$\omega_{ij} = \frac{p_i p_j}{p_i k \, p_j k} = \frac{1 - v_i v_j \cos\theta_{ij}}{\omega_k^2 (1 - v_i \cos\theta_{ik})(1 - v_j \cos\theta_{jk})}$$

Massless emitting lines $v_i = v_j = 1$, then

$$\omega_{ij} = \omega_{ij}^{[i]} + \omega_{ij}^{[j]} \qquad \omega_{ij}^{[i]} = \frac{1}{2}\left(\omega_{ij} + \frac{1}{1-\cos\theta_{ik}} - \frac{1}{1-\cos\theta_{jk}}\right)$$

Angular ordering

$$\int_0^{2\pi} \frac{d\phi}{2\pi} \omega_{ij}^{[i]} = \begin{cases} \frac{1}{\omega_k^2 (1-\cos\theta_{ik})} & \theta_{ik} < \theta_{ij} \\ 0 & \theta_{ik} > \theta_{ij} \end{cases}$$



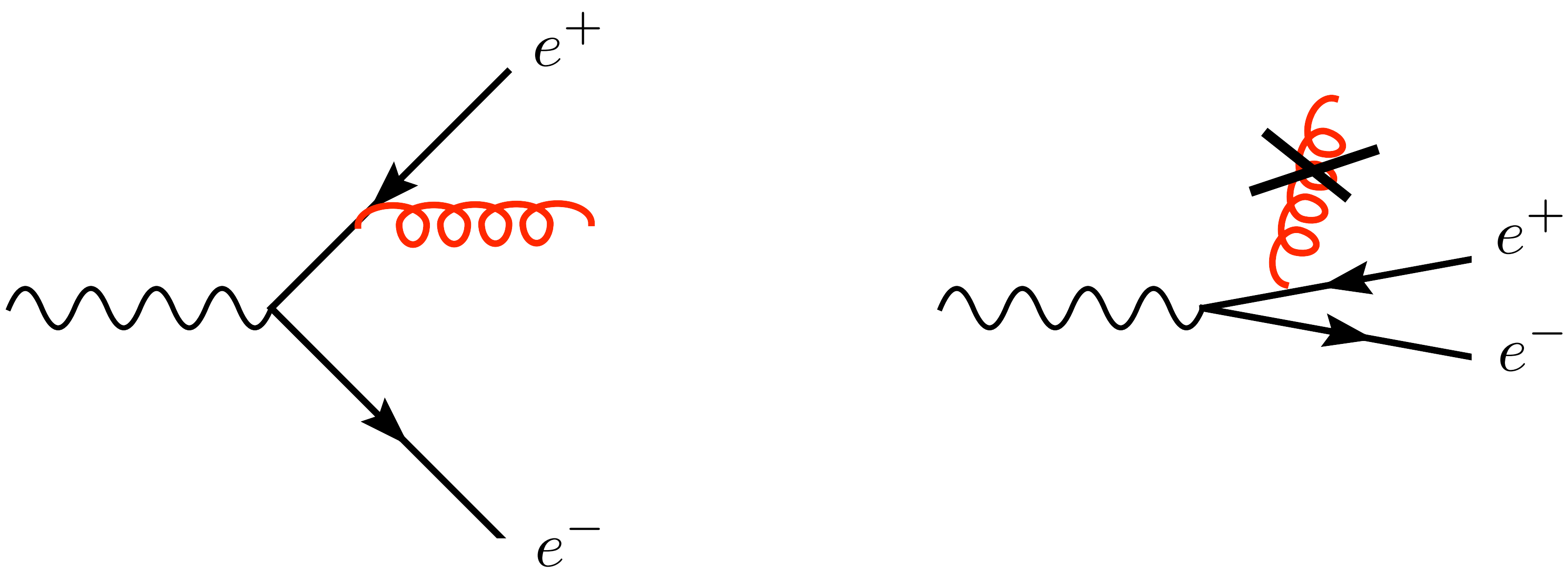
Proof: see e.g. QCD and collider physics, Ellis, Stirling, Webber

Angular ordering & coherence

A. O. is a manifestation of coherence of radiation in gauge theories

In QED

suppression of soft bremsstrahlung from an e+e- pair (Chudakov effect)
At large angles the e^+e^- pair is seen coherently as a system without total charge ⇒ radiation is suppressed



Angular ordering & coherence

Coherent a → b + c branching: replace the ordering variable $t = p_a^2$ with

$$\zeta = \frac{p_b p_c}{E_b E_c} \sim 1 - \cos\theta_{bc}$$

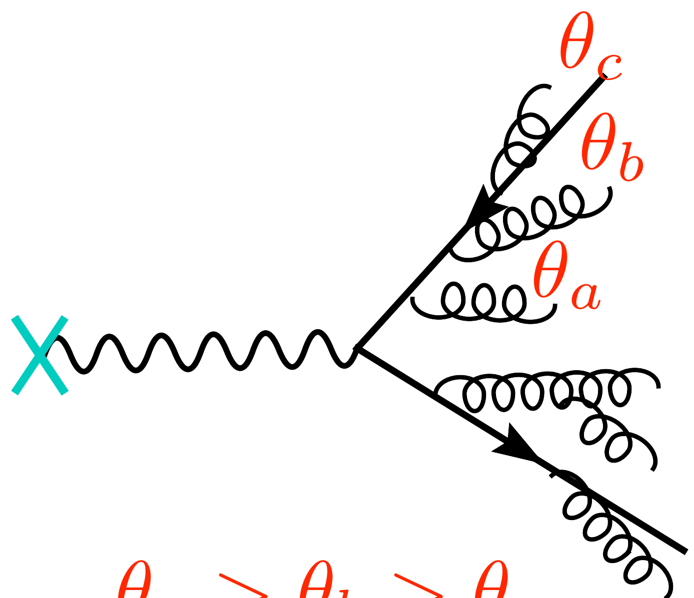
and require $\zeta' < \zeta$ at successive branchings

The basic formula for coherent branching

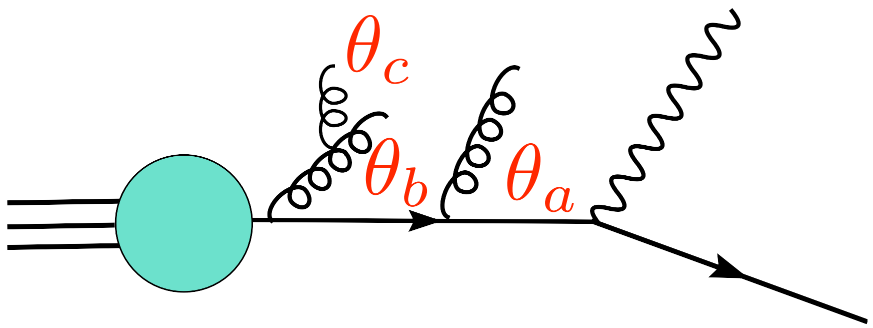
$$\boxed{d\sigma_{n+1} = d\sigma_n \frac{d\zeta}{\zeta} dz \frac{\alpha_s}{2\pi} \hat{P}_{ba}(z)}$$

NB: need collinear cut-off. Simplest choice: $\zeta_0 = \dfrac{t_0}{E^2}$

AO: time like vs space-like case



$\theta_a > \theta_b > \theta_c$



$\theta_a > \theta_b > \theta_c$

NB: angles decrease when moving away from the hard vertex, i.e. in the space-like case angles increase during the evolution

Accuracy issue

Formally, Monte Carlos are Leading Logs showers

- ✦ because they don't include any higher order corrections to the 1→2 splitting
- ✦ because they don't have any 1→3 splittings
- ✦ ....

However, they fare better than analytic Leading Log calculations

- because they have energy conservation (NLO effect) implemented
- because they have coherence
- because they have optimized choices for the coupling
- because they provide an exclusive description of the final state

So, despite not guaranteeing NLL accuracy, they fare usually better than Leading Log analytic calculations

The real issue is that we are not able to estimate the uncertainty

Warning

The above discussion is a simplification

- many details/subtleties not discussed enough, some not at all
- various MC differ in the choice of the ordering variable and in many details, but the basic idea remains the same
- purpose was to give an overall idea of how Monte Carlos and what they can/can't do

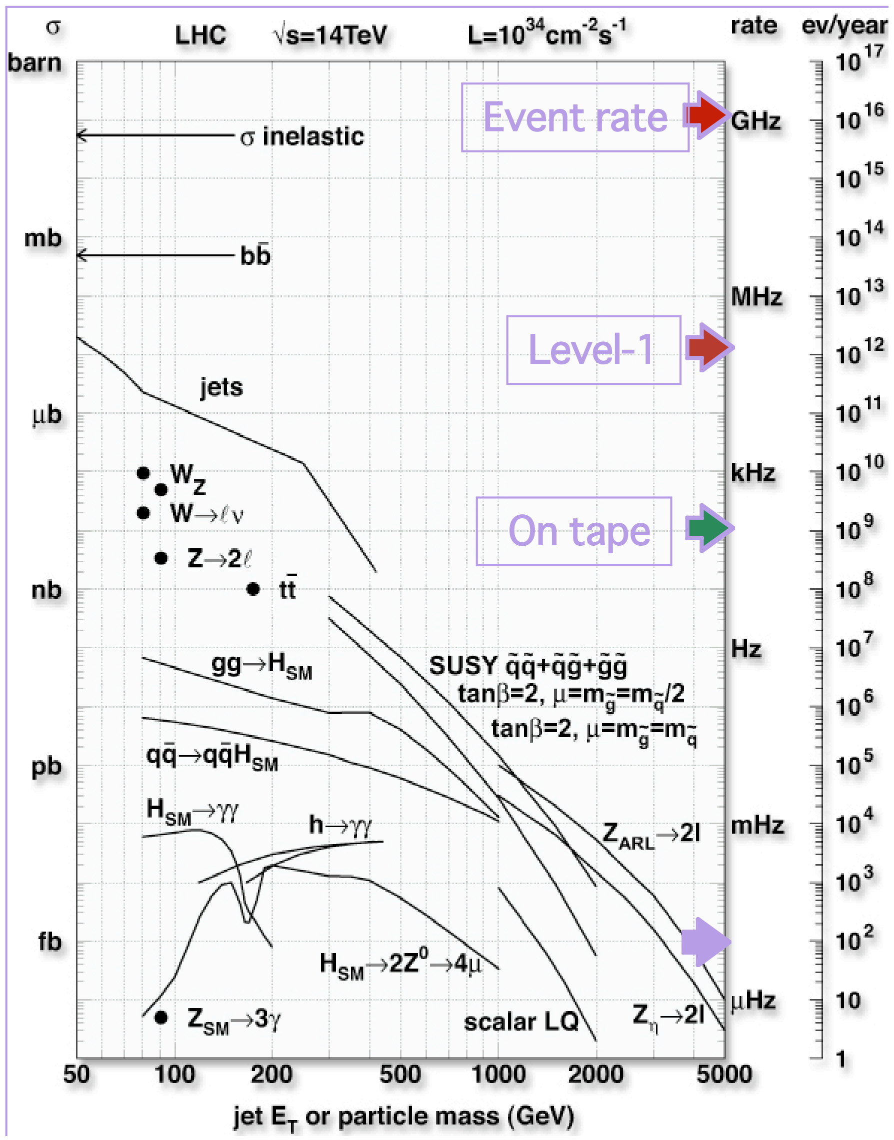
What I want to discuss next is

- hadronization/U.E. minimum bias and all that...
- improuvements to parton showers

Recap

- Monte Carlo as an approximation to higher orders
- parton evolution as branching process from higher to lower x
- parton shower based on Sudakov form factor (Prob. of evolving without branching) with corresponding evolution equation
- branching described by picking randomly 3 numbers (t, x, φ) with the right prob. distributions
- virtuality ordered shower: collinear emissions
- angular ordering needed for soft coherence effects

Cross sections



Final state	$\sim \sigma$
Total	$100\,\mathrm{mb}$
$W \to e\nu$	$20\,\mathrm{nb}$
$E \to e^+e^-$	$2\,\mathrm{nb}$
$b\bar{b}$	$0.8\,\mathrm{mb}$
$t\bar{t}$	$800\,\mathrm{pb}$
$H(m_H = 200\,\mathrm{GeV})$	$20\,\mathrm{pb}$

What is the bulk of the total cross-section made of?

Soft interactions

We talked a lot about high-energy scatterings, but what is the most likely thing which can happen when two protons collide at very high energy?

- most of the times, there will be only a low p_t momentum transfer between the partons in the protons

- only occasionally there will be a hard momentum transfer resulting in a hard interaction (outgoing jets at high p_t)

Perturbative QCD can describe hard interactions, but not the soft physics

What we can do is model (parametrize) soft effects, and fit them from data
$\Rightarrow$ Monte Carlo tuning

Some nomenclature

<u>Minimum bias:</u>

- event which one would see with a totally inclusive trigger
- a single inelastic particle-particle (proton-proton) interaction (predominantly dominantly soft)
- on average low transverse momentum, low multiplicity
- many minimum-bias events per bunch crossing at the LHC

<u>Pile-up:</u>

- many additional, generally soft proton-proton interactions

Nomenclature

The underlying event:

- all particle from a single particle collision, except the hard process of interest
  [beam remnant, initial state radiation, multi-parton interactions, minimum bias ...]
- an important area of physics, which will affect all LHC measurements of which we have still a very poor understanding and no first principle calculation

All this soft activity (additional energy) has nothing to do with the hard process $\Rightarrow$ needs to be subtracted

Soft underlying event in standard Herwig

<u>The UA5 model</u>: (herwig default for a long time)

Additional soft hadronic activity generated as a number of clusters distributed flat in rapidity and with exponential transverse momentum distribution

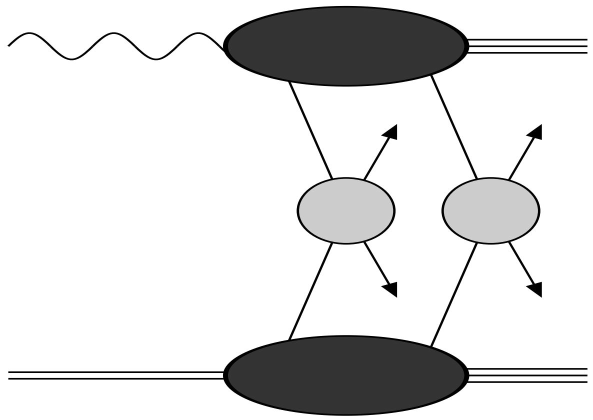
$$\mathrm{Prob}(p_t) \sim p_t e^{-b\sqrt{p_t^2+M^2}}$$

No matrix element, no physical model and practically too soft to fit data

UA5 soft underlying event obsolete: not recommended for serious use

Underlying event in Jimmy

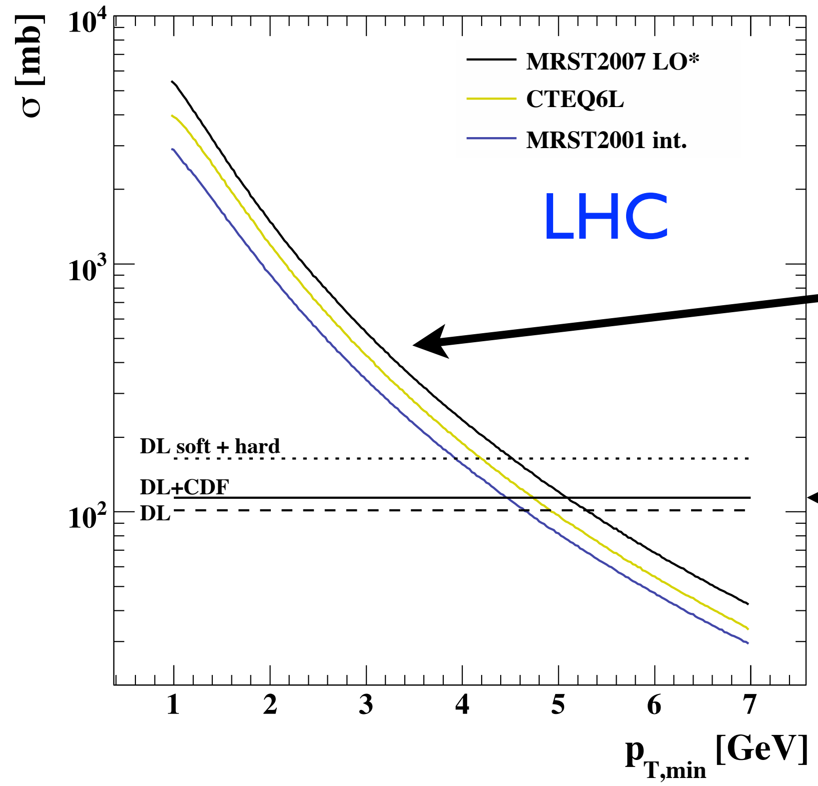
Jimmy is a plug-on to Herwig with a better treatment of the hard part of the U.E.



Issues:

- at high energies probe low-x PDFs
- the gluon PDF grows at small x
- if the parton density grows, it is reasonable to assume that more than one hard event per collision can take place: multi-particle interactions (MPI)
- this assumption is also necessary to unitarize the cross-section

Problems with unitarity



total "hard" cross-section assuming one parton-parton per pp collision

total cross section (few models)

⇒ assumption of one parton-parton per pp collision leads to inconsistency

- without MPI: cross-section for inclusive jet-production (computed in PT with steep PDFs) exceeds the total (γp, pp) cross-section
- with MPI: inclusive jet-cross section exceeds total cross-section by a factor corresponding to the mean multiplicity of MPI

Standard Jimmy model

<u>Assumption</u>:

- At fixed impact parameter b, scatters are independent and obey Poisson statistics (eikonal model)

$$\sigma_n = \int d^2b \frac{(A(b)\sigma_a)^n}{n!} e^{-A(b)\sigma_a}$$

normalized matter density

parton-parton cross-section

Inclusive cross-section is

$$\sigma_{\text{inc}} = \sum_{n=0}^{\infty} \int d^2b\, n\, \frac{(A(b)\sigma_a)^n}{n!} e^{-A(b)\sigma_a} = \sigma_a$$

Standard Jimmy model

Total cross-section with at least one scatter of type a is

$$\sigma_{\mathrm{tot,a}} = \sum_{n=1}^{\infty} \int d^2b \frac{(A(b)\sigma_a)^n}{n!} e^{-A(b)\sigma_a} = \int d^2b \left(1 - e^{-A(b)\sigma_a}\right)$$

Probability of n scatters given that there is at least one

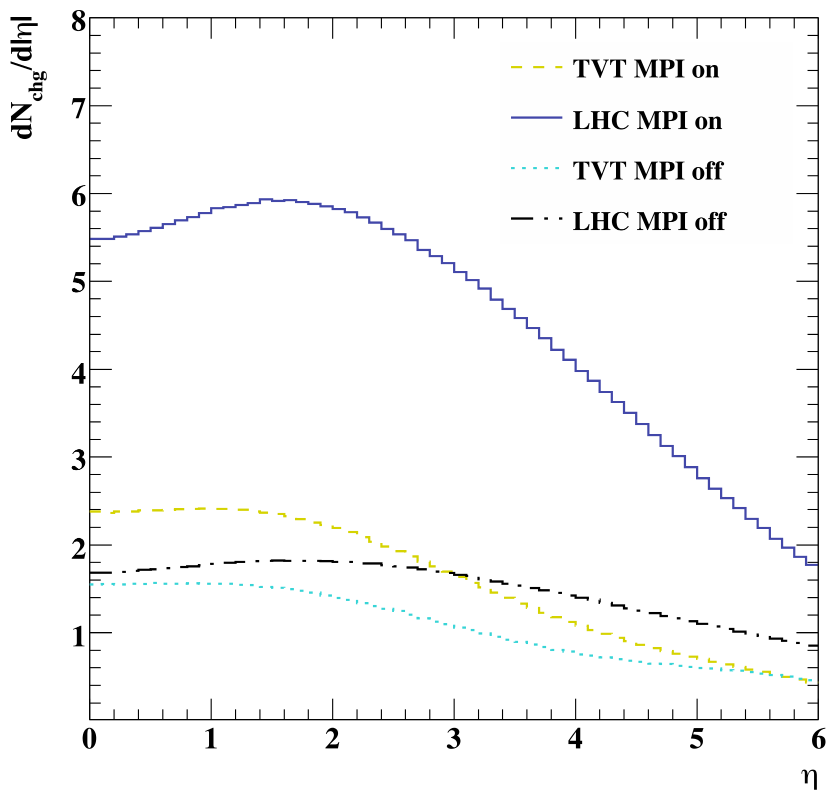
$$P_{n|1} = \frac{\int d^2b \frac{(A(b)\sigma_a)^n}{n!} e^{-A(b)\sigma_a}}{\int d^2b (1 - e^{-A(b)\sigma_a})}$$

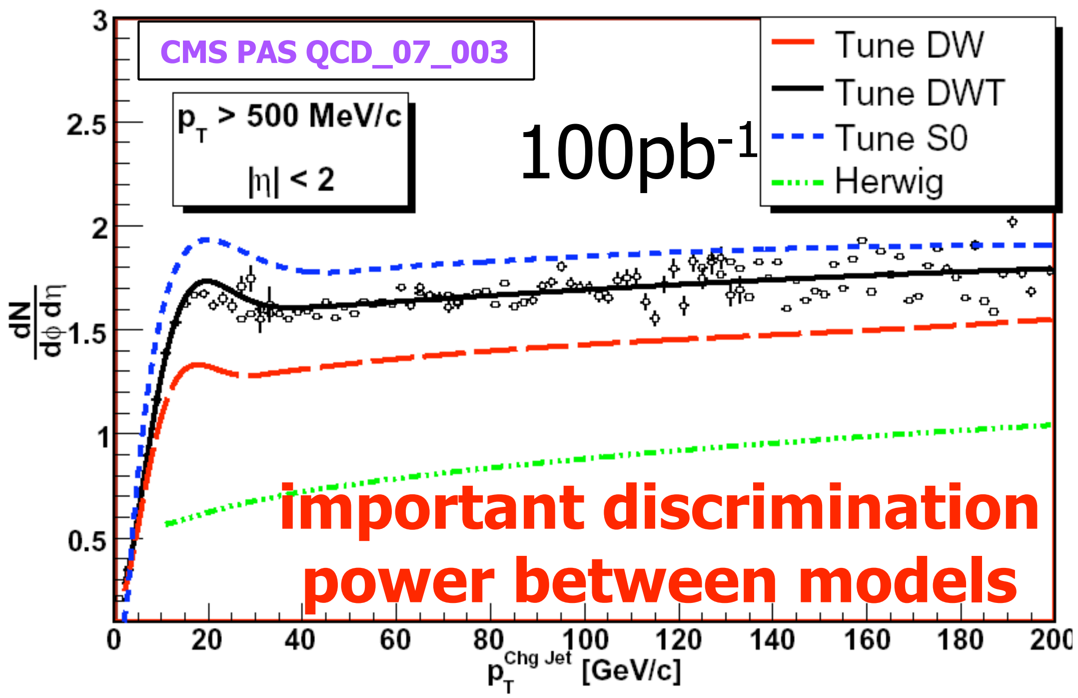
Pre-tabulated probability distribution (as a function of s) in Jimmy. Then in a given event n is chosen according to $P_{n|1}$

Exercise: show that $\langle n \rangle = \frac{\sigma_{\mathrm{inc}}}{\sigma_{\mathrm{tot,a}}}$

N.B. $\sigma_{\mathrm{tot,a}}$ must be less than the total cross-section, but σ_{inc} must not be

MPI: TeV vs LHC





Very large effects at the LHC

UE models tuned at the Tevatron give different extrapolations at the LHC

Models based on physical ideas, but a lot of assumptions and extrapolations behind, only data can help constraining from and parameters

Hadronization

Partons produced in a hard scattering loose energy via perturbative radiation, then they will pick the flavour and color from the vacuum so as to create an observable hadron

Simplest example: consider b-hadro production. The inclusive jet spectrum of b-flavoured hadrons is given by

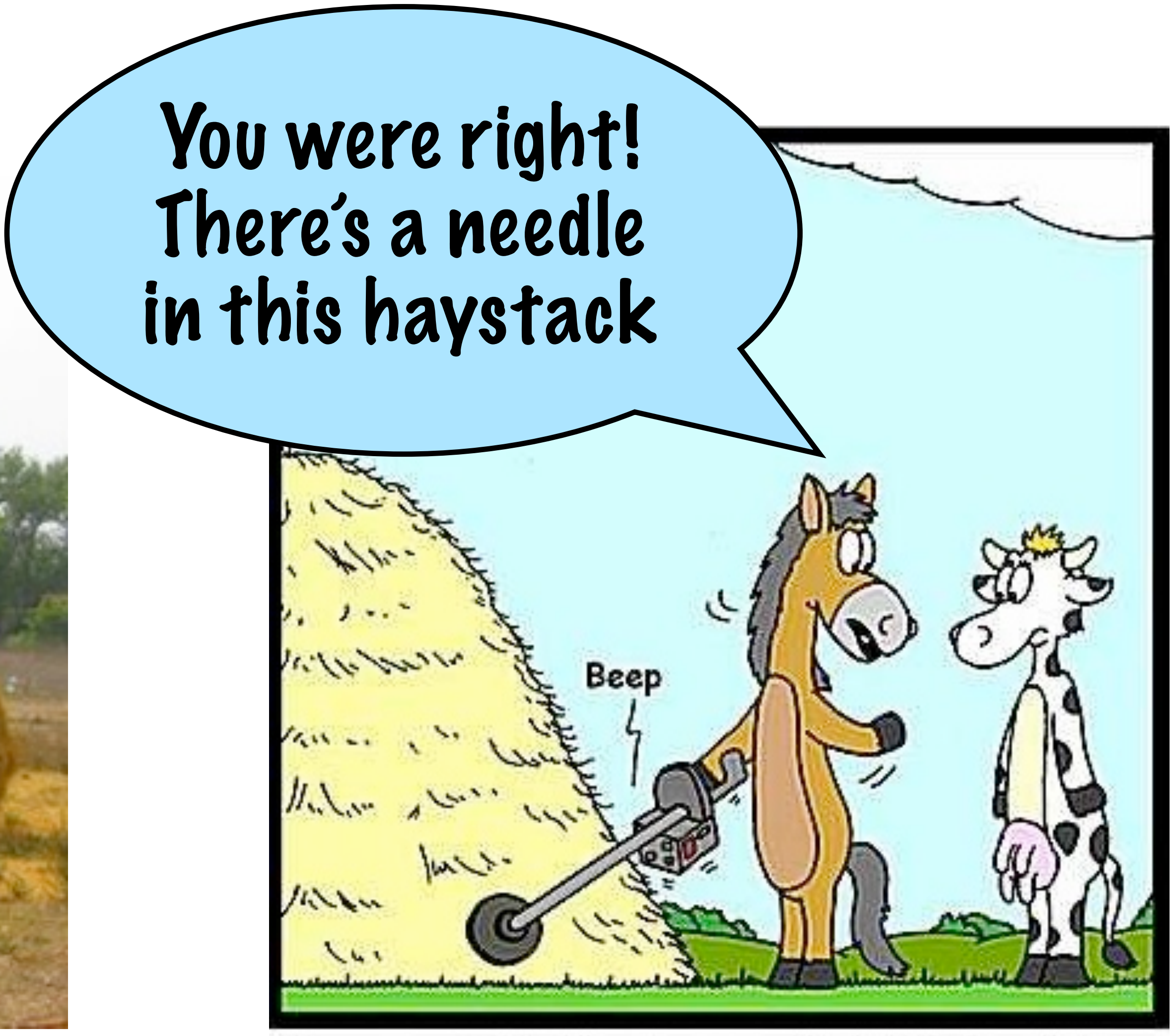
$$\frac{d\sigma_{p_i p_j \to H_b}}{dp_t(H_b)} = \int \frac{dz}{z} D^{b \to H_b}(z) \frac{d\sigma_{p_i p_j \to b}}{dp_t(b)} \qquad p_t(H_b) = z p_t(b)$$

- Fragmentation functions $D^{Q \to H_Q}(z)$ are analogous to PDFs, they can not be computed but are extracted form data (typically in e^+e^-) and are universal
- As for PDFs the functional form is unknown. The parametrization if often a large source of uncertainty which is difficult to estimate

Recap

- higher orders: included only approximately
- parton evolution as branching process from higher to lower x
- parton shower based on Sudakov form factor (Prob. of evolving without branching) with corresponding evolution equation
- branching described by picking randomly 3 numbers (t, x, φ) with the right prob. distributions
- virtuality ordered shower: collinear enhancements
- angular ordering needed for soft enhancements
- parton shower supplemented by hadronization + U.E. (various models $\Rightarrow$ MC tuning) $\Rightarrow$ full event generator
- by construction PS fail to describe multiple hard radiation
- next: improvements

Looking for BSM signals at the LHC is like looking for a needle in a haystack ...



... but, at the end, it is all a matter of having the right tools

UNDERSTANDING QCD CRUCIAL TO DEVELOP THE RIGHT TOOLS!