Flavour physics and CP violation

Lecture 1: Flavour physics in the Standard Model

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Outline

1. **Flavour physics in the Standard Model (SM)**
   - CKM matrix
   - flavour changing neutral currents
   - effective Hamiltonian
2. **Phenomenology of K and B meson decays**
   - neutral meson mixing
   - rare K decays
   - $b \to s$ transitions
3. **Flavour physics beyond the SM**
   - constraints on the scale of new physics
   - Minimal Flavour Violation
   - flavour hierarchies from partial compositeness

Further reading

- Gino Isidori – Flavor physics and CP violation
  https://arxiv.org/abs/1302.0661
- Yuval Grossman – Introduction to flavor physics
  https://arxiv.org/abs/1006.3534
- Andrzej J. Buras – Flavour Physics and CP Violation
  http://arxiv.org/abs/hep-ph/0505175
- Andrzej J. Buras – Weak Hamiltonian, CP Violation and Rare Decays
  http://arxiv.org/abs/hep-ph/9806471
- ...and many other excellent lectures on INSPIRE/arXiv!

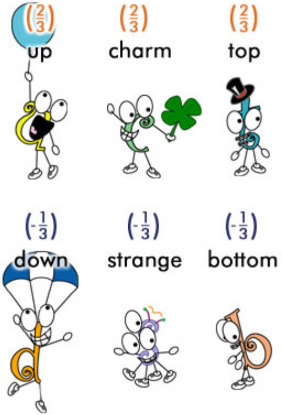
What is flavour physics?

Three generations of quarks (and leptons)

- identical gauge quantum numbers
- different masses

➢ **flavour physics** describes interactions that distinguish between flavours

this lecture: only quark flavour physics, for lepton flavour see lectures by Gabriela Barenboim



Quarks in the SM

Recall: parity violation of electroweak interactions

➢ left-handed quarks are introduced as $SU(2)_L$ doublets

$$Q_j = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

and right-handed quarks as $SU(2)_L$ singlets

$$U_j = u_R, c_R, t_R \qquad D_j = d_R, s_R, b_R$$

Gauge couplings of the quarks

$$\mathcal{L}_{\text{fermion}} = \sum_{j=1}^{3} \bar{Q}_j i \not{D}_Q Q_j + \bar{U}_j i \not{D}_U U_j + \bar{D}_J i \not{D}_D D_j$$

with the covariant derivatives ($Y_Q = 1/6$, $Y_U = 2/3$, $Y_D = -1/3$)

$$\begin{aligned} D_{Q,\mu} &= \partial_\mu + i g_s T^a G^a_\mu + i g \tau^a W^a_\mu + i Y_Q g' B_\mu \\ D_{U,\mu} &= \partial_\mu + i g_s T^a G^a_\mu + i Y_U g' B_\mu \\ D_{D,\mu} &= \partial_\mu + i g_s T^a G^a_\mu + i Y_D g' B_\mu \end{aligned}$$

➢ **flavour universality**: gauge couplings are equal for all three generations

Yukawa couplings

flavour non-universality introduced by Yukawa couplings between the Higgs field and the quarks:

$$\mathcal{L}_{\mathsf{Yuk}} = \sum_{i,j=1}^{3} (-Y_{U,ij}\bar{Q}_{Li}\tilde{H}U_{Rj} - Y_{D,ij}\bar{Q}_{Li}HD_{Rj} + h.c.)$$

where i, j are generation indices and $\tilde{H} = \epsilon H^* = (H^{0*}, -H^-)^T$

➢ replacing H by its vacuum expectation value $\langle H \rangle = (0, v)^T$, we obtain the quark mass terms

$$\sum_{i,j=1}^{3} (-m_{U,ij}\bar{u}_{Li}u_{Rj} - m_{D,ij}\bar{d}_{Li}d_{Rj} + h.c.)$$

with the quark mass matrices given by $m_A = vY_A$ $(A = U, D, E)$.

Diagonalising the fermion mass matrices

- quark mass matrices m_U, m_D, m_L are 3×3 complex matrices in the generation space ("flavour" space) with *a priori* arbitrary entries
- can be diagonalised by bi-unitary field redefinitions

$$u_L = \hat{U}_L u_L^m \qquad u_R = \hat{U}_R u_R^m \qquad d_L = \hat{D}_L d_L^m \qquad d_R = \hat{D}_R d_R^m$$

with ^{m} denoting quarks in the mass eigenstate basis

- in this basis

$$m_U^{\text{diag}} = \hat{U}_L^\dagger m_U \hat{U}_R \qquad m_D^{\text{diag}} = \hat{D}_L^\dagger m_D \hat{D}_R$$

are diagonal

➢ **Is the SM Lagrangian invariant under these field redefinitions?**

The CKM matrix

- transformations of the right-handed quarks are indeed unphysical, i. e. they leave the rest of the Lagrangian invariant
- however, u_{Li} and d_{Li} form the $SU(2)_L$ doublets Q_{Li}
  ➢ kinetic term gives rise to the interaction

$$\frac{g}{\sqrt{2}}\bar{u}_{Li}\gamma_\mu W^{\mu+} d_{Li}$$

- transforming to the mass eigenstate basis, we obtain

$$\frac{g}{\sqrt{2}}\bar{u}_{Li}\hat{U}^\dagger_{L,ij}\hat{D}_{L,jk}\gamma_\mu W^{\mu+} d_{Lk}$$

➢ The combination $\hat{V}_{\text{CKM}} = \hat{U}_L^\dagger \hat{D}_L$ is physical and is called the CKM matrix. It leads to flavour violating charged current interactions.

Parameter counting

How many parameters does the CKM matrix have?

- unitary 3×3 matrix can be parametrised by 3 mixing angles and 6 complex phases.
- however 5 phases are unphysical, as they can be absorbed as unobservable parameters into the up-type and down-type quarks, respectively
  note: overall phase rotation of *all* quarks does not affect the CKM matrix

➢ CKM matrix contains three mixing angles and one physical complex phase

Most common parametrisations

Standard parametrisation $(s_{ij} = \sin\theta_{ij}, c_{ij} = \cos\theta_{ij})$

$$V_{\text{CKM}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

Experimentally: $s_{12} \sim 0.2$, $s_{23} \sim 0.04$, $s_{13} \sim 4 \cdot 10^{-3}$

Wolfenstein parametrisation

$$V_{\text{CKM}} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

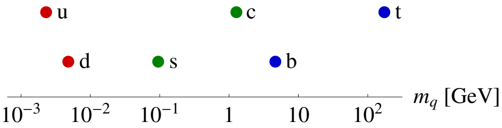
approximate, very useful for estimating the size of flavour violating decays

Present status

CKM matrix is found to be close to the unit matrix

$$V_{\mathrm{CKM}} = \begin{pmatrix} 0.97427 \pm 0.00014 & 0.22536 \pm 0.00061 & 0.00355 \pm 0.00015 \\ 0.22522 \pm 0.00061 & 0.97343 \pm 0.00015 & 0.0414 \pm 0.0012 \\ 0.00886^{+0.00033}_{-0.00032} & 0.0405^{+0.0011}_{-0.0012} & 0.99914 \pm 0.00005 \end{pmatrix}$$

also quark masses exhibit strong hierarchy



where does this hierarchical structure come from?

➢ **flavour hierarchy problem**

The unitarity triangle

- CKM matrix parametrises charged current interactions

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

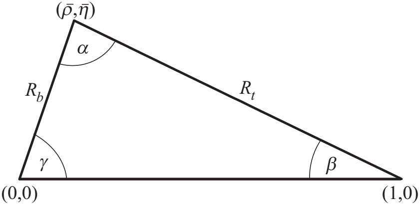
- its unitarity implies various relations among its elements, e. g.

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

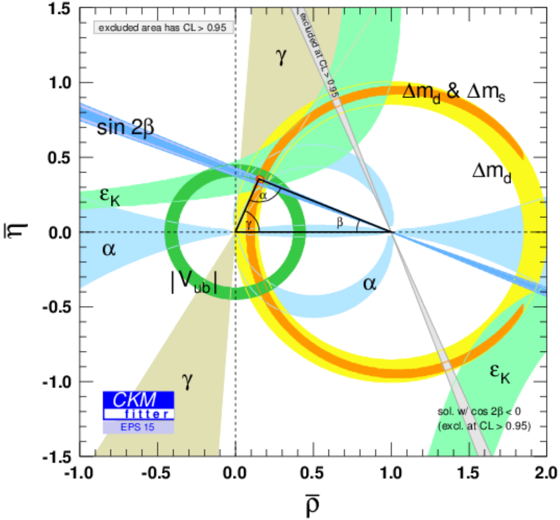
➢ **unitarity triangle** in the complex plane $\bar{\rho} = (1 - \frac{\lambda^2}{2})\rho,\ \bar{\eta} = (1 - \frac{\lambda^2}{2})\eta$

$$R_b = \left|\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right|$$

$$R_t = \left|\frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*}\right|$$



Present status of the unitarity triangle



Flavour symmetry of the SM

- ignoring the Yukawa couplings, the SM quark sector has a global flavour symmetry

$$G_{\text{flavour}} = U(3)_Q \times U(3)_U \times U(3)_D$$

- Yukawa interactions Y_U, Y_D break G_{flavour} to a single $U(1)$ factor, corresponding to the overall phase of the quark fields

  ➢ associated to baryon number conservation

Another way to count parameters:

- Y_U, Y_D have 9 real parameters and 9 phases each: $2 \times (9, 9) = (18, 18)$
- each broken generator of G_{flavour} corresponds to an unphysical parameter

$$3 \times (3, 6) - (0, 1) = (9, 17)$$

- 9 real parameters and one phase are physical:

  6 quark masses, 3 CKM mixing angles, CKM phase δ

CP violation and the complex CKM matrix

CP: combination of parity transformation P and charge conjugation C

- $P : \psi(r) \to \gamma^0\psi(-r)$ transforms left(right)-handed quark into right(left)-handed quark
- $C : \psi \to i(\bar{\psi}\gamma^0\gamma^2)^T$ transforms left(right)-handed quark into left(right)-handed antiquark

weak interactions neither invariant under P nor C ➢ what about CP?

$(g_W = g/\sqrt{2})$

$$\begin{aligned} & g_W \bar{u}_{Li} V_{\text{CKM},ik} \gamma_\mu W^{\mu+} d_{Lk} + h.c. \\ &= g_W \bar{u}_{Li} V_{\text{CKM},ik} \gamma_\mu W^{\mu+} d_{Lk} + g_W \bar{d}_{Lk} V^*_{\text{CKM},ik} \gamma_\mu W^{\mu-} u_{Li} \\ &\xrightarrow{CP} g_W \bar{d}_{Lk} V_{\text{CKM},ik} \gamma_\mu W^{\mu-} u_{Li} + g_W \bar{u}_{Li} V^*_{\text{CKM},ik} \gamma_\mu W^{\mu+} d_{Lk} \\ &= g_W \bar{u}_{Li} V^*_{\text{CKM},ik} \gamma_\mu W^{\mu+} d_{Lk} + h.c. \end{aligned}$$

➢ **CP is violated because the CKM matrix is complex** $(\delta \neq 0, \pi)$

Flavour changing neutral currents

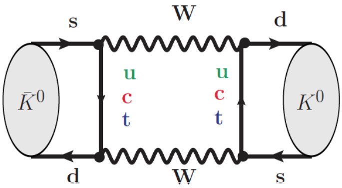
Are there also flavour changing neutral currents (FCNCs)?

- absent at the tree level, because

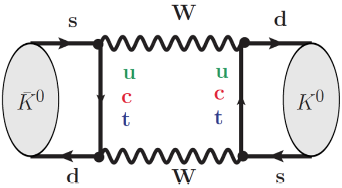
$$g_Z \bar{d}_{Li} \hat{D}^\dagger_{L,ij} \hat{D}_{L,jk} \gamma_\mu Z^\mu d_{Lk} \equiv g_Z \bar{d}_{Li} \gamma_\mu Z^\mu d_{Li}$$

- generated however by loop diagrams with $W^\pm$ boson exchanges

Example: neutral K meson mixing



GIM mechanism



$$\mathcal{M} \propto \sum_{i,j=u,c,t} V_{id}^* V_{is} V_{jd}^* V_{js} F(x_i, x_j)$$

$F(x_i, x_j)$: loop function that depends on mass square ratios $x_i = m_i^2/M_W^2$

CKM unitarity: $\sum_{i=u,c,t} V_{id}^* V_{is} = 0$

➢ **GIM mechanism** Glashow, Iliopoulos, Maiani (1970)
FCNCs are suppressed by quark mass differences $x_i - x_j$

$+$ loop suppression $g^2/(16\pi^2)$

From quarks to mesons

quarks are not free particles, but confined in mesons
(ecxept for the top. . .)

Necessary ingredients for the study of quark flavour violation

- flavour violating transition mediated by weak interactions ($\mu \sim M_W$)
- low energy QCD effects in and between the mesons ($\mu \sim$ GeV)

Challenges

- very different energy scales ➢ renormalisation group running important
- QCD effects non-perturbative at low energies

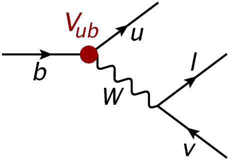
Flavour theory roadmap

Flavour violating meson decays in five steps

1. calculate flavour violating quark decay mediated by weak interactions
2. construct low energy effective Lagrangian by integrating out heavy particles $(W^{\pm}, Z, t)$
3. evaluate renormalisation group running from weak scale $\mu \sim M_W$ to hadronic scale $\mu \sim m_b, 2\,\text{GeV}$
4. gather non-perturbative effects in matrix element involving initial and final state mesons
5. obtain numerical result for matrix element from non-perturbative methods (lattice QCD, QCD sum rules etc.) or extract it from data

Example: Semileptonic charged currents

Consider $b \to u\ell\bar{\nu}$ decay that is relevant for $|V_{ub}|$ measurements



momentum of W boson propagator can be neglected:

$$\frac{g}{\sqrt{2}} V_{ub} (\bar{u}\gamma^\nu P_L b) \frac{g_{\mu\nu}}{p^2 - M_W^2} \frac{g}{\sqrt{2}} (\bar{\ell}\gamma^\mu P_L \nu) \xrightarrow{p^2 \ll M_W^2} -\frac{g^2}{2M_W^2} V_{ub} (\bar{u}b)_{V-A} (\bar{\ell}\nu)_{V-A}$$

➢ effective Hamiltonian with the well-known Fermi constant

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{ub} (\bar{u}b)_{V-A} (\bar{\ell}\nu)_{V-A} \qquad \text{with} \quad \frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

$|V_{ub}|$ from $B \to \pi\ell\nu$

next:

- include known QCD corrections
- evaluate $\langle\pi|(\bar{u}b)_{V-A}|B\rangle$ on the lattice

➢ measuring the $B^0 \to \pi^-\ell^+\nu$ branching ratio determines $|V_{ub}|$

$$|V_{ub}|^{\pi\ell\nu} = (3.72 \pm 0.16) \cdot 10^{-3}$$

Note: tree level transition, therefore highly unlikely affected by new physics

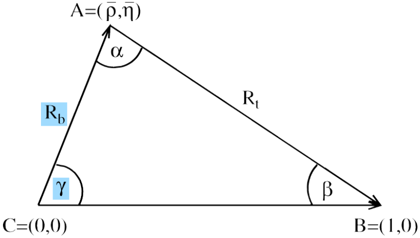
The reference unitariy triangle

- unitary CKM matrix uniquely determined by three mixing angles and one complex phase

$$|V_{us}| \equiv \lambda \qquad |V_{cb}| \qquad |V_{ub}| \qquad \gamma$$

  measured in tree level decays and therefore insensitive to BSM contributions

- compare model-indpendent **reference unitarity triangle** (UT) with model-dependent UT fits to discover BSM physics

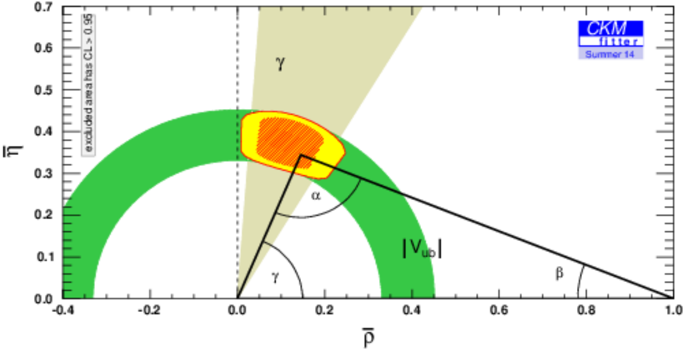


$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

$$R_b = \left(1 - \frac{\lambda^2}{2}\right)\frac{1}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right|$$

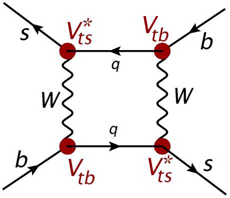
CKM matrix from tree level decays

CKMFITTER (2015)



main uncertainty from $|V_{ub}|$ and γ

Example 2: $B_s - \bar{B}_s$ mixing



box diagram mediating $B_s - \bar{B}_s$ mixing:

$$\frac{G_F^2}{16\pi^2} M_W^2 \sum_{i,j=u,c,t} V_{ib}^* V_{is} V_{jb}^* V_{js} F(x_i, x_j)$$

Simplifications:

- external quark momenta negligible
- GIM mechanism: mass-independent piece of $F(x_i, x_j)$ drops out
- $m_i \ll m_t$ and $|V_{ib}^* V_{is}| \ll |V_{tb}^* V_{ts}|$ $(i = u, c)$ ➢ only top quark contribution relevant
- perturbative QCD corrections can be included by adding a factor η_B (known from tedious calculations)

$B_s - \bar{B}_s$ mixing continued

➢ effective Hamiltonian for $B_s - \bar{B}_s$ mixing:

$$\mathcal{H}_{\text{eff}}^{B_s - \bar{B}_s} = \frac{G_F^2}{16\pi^2} M_W^2 \eta_B (V_{tb}^* V_{ts})^2 S_0(x_t) (\bar{b}s)_{V-A} (\bar{b}s)_{V-A} + h.c.$$

$$\text{with } S_0(x_t) = \frac{4x_t - 11x_t^2 + x_t^3}{4(1-x_t)^2} - \frac{3x_t^3 \ln x_t}{2(1-x_t)^3}$$

now sandwich $\mathcal{H}_{\text{eff}}^{B_s - \bar{B}_s}$ between initial and final state meson to obtain mixing matrix element:

$$M_{12} = \frac{1}{2m_{B_s}} \left\langle \bar{B}_s \left| \mathcal{H}_{\text{eff}}^{B_s - \bar{B}_s} \right| B_s \right\rangle^*$$

with the hadronic matrix element $\left\langle \bar{B}_s \left| (\bar{b}s)_{V-A} (\bar{b}s)_{V-A} \right| B_s \right\rangle$ calculated on the lattice

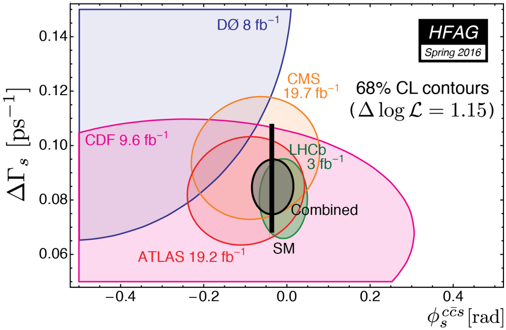
Comparing with the data

mass difference $\Delta M_s = 2|M_{12}|$

$$(\Delta M_s)^{\text{LHCb}}_{\text{exp}} = (17.757 \pm 0.021)\ \text{ps}^{-1}$$

$$(\Delta M_s)^{\text{Fermilab-MILC}}_{\text{SM}} = (19.6 \pm 1.6)\ \text{ps}^{-1}$$

CP-violating phase $\phi_s = \arg M_{12}$



Summary

- in the SM, flavour violation is governed by the Yukawa couplings
- quark masses and CKM matrix with very hierarchical structure
  ➢ flavour hierarchy problem
- strong suppression of FCNC effects by CKM hierarchy, loop factor and GIM mechanism
  ➢ excellent place to look for new physics