

Some topics in heavy-ion physics

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i Stavanger



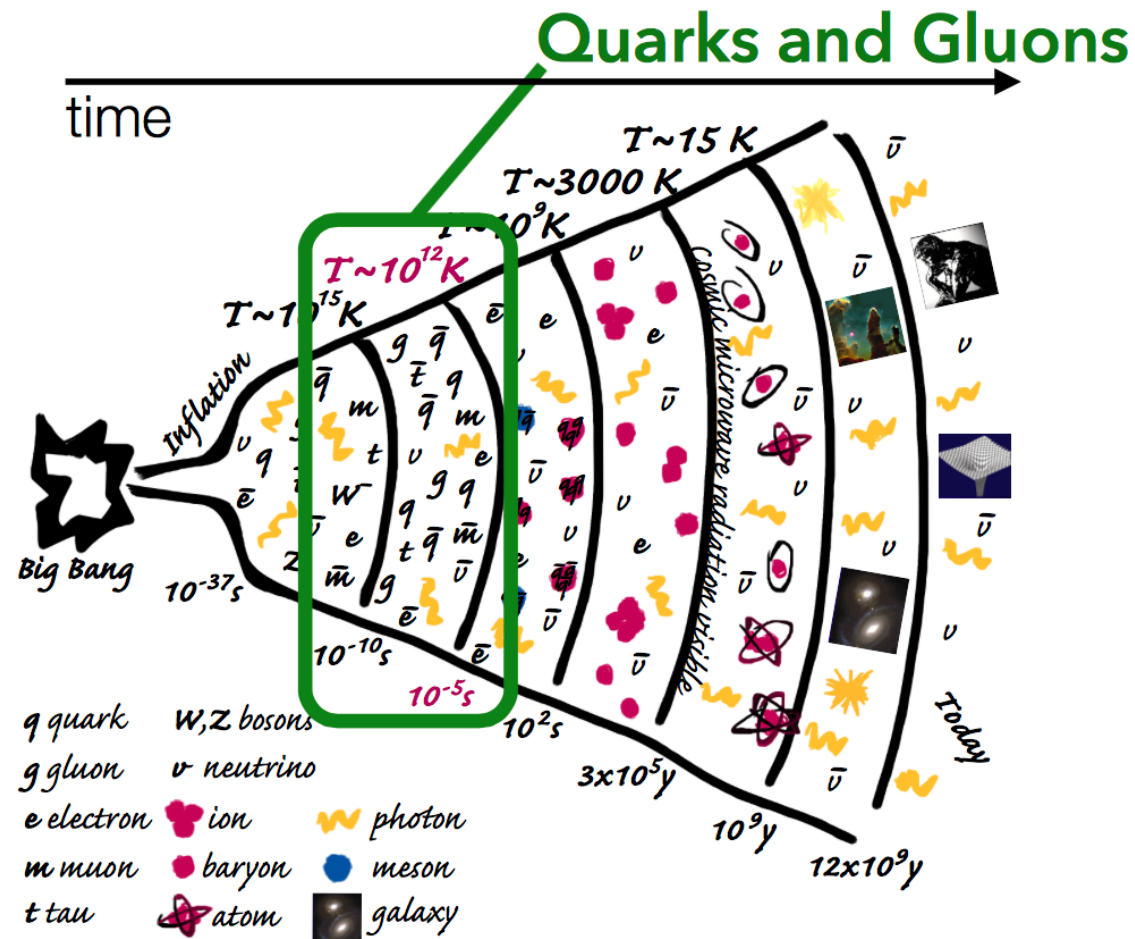
Preface

- QCD is a mature theory of strong interactions with a precision frontier
- Fundamental degrees of freedom quarks and gluons, matter normally in hadronic form
- What happens to QCD if the hadronic structure is broken at high temperature?
- Heavy ion collisions offer an experimental venue to create QCD matter at densities only comparable to cores of neutron stars and early universe
- Ongoing large experimental effort at LHC (CERN) with ALICE, ATLAS, CMS, LHCb and at RHIC (BNL) PHENIX, STAR

Preface

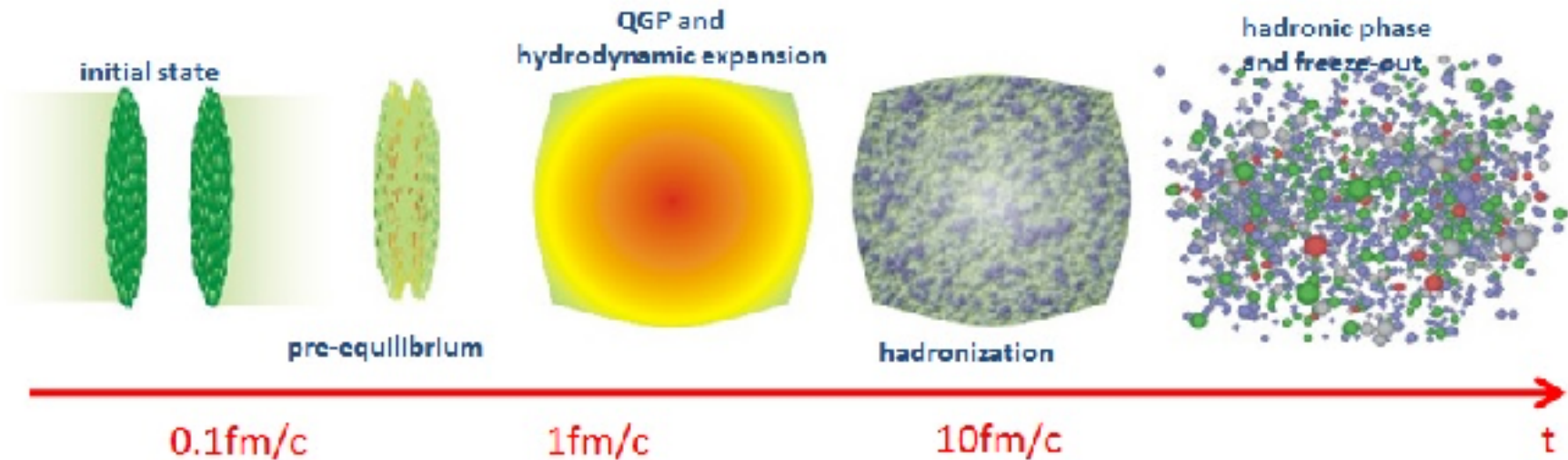
- We want to know: what happens to quantum field theory in extreme conditions. In the presence of a large number of particles, not just perturbations around vacuum.
 - How do collective macroscopic properties arise from microscopic degrees of freedom
 - What are material properties of matter made of quantum fields?
 - Close connection to cosmology: phase transitions, pre/reheating
thermal particle production, etc...
 - Close connection to condensed matter physics: cold atoms,

Quark gluon plasma and cosmology



based on 'History of the Universe' by the Particle Data Group, LBNL ©2000

Evolution of heavy ion collision



- Initial condition: Wave function of highly boosted nucleus.
- Initial scattering and particle production ($0 - 0.1 \text{ fm}/c$)
- Non-equilibrium evolution and thermalization ($0.1 - 1 \text{ fm}/c$)
- Hydrodynamical explosion, expansion, dilution, cool-down ($1 - 10 \text{ fm}/c$)
- Hadronization
- Chemical and kinetic freeze out
- Particle detection ($10^{15} \text{ fm}/c$)

References

- Talks:
 - Jan Fiete Grosse-Oetringhaus, CERN-fermilab school 2015
<https://indico.cern.ch/event/353089/contributions/1762268/attachments/>
 - Stefan Floerchinger, European School of High Energy Physics 2015
<https://indico.cern.ch/event/381289/contributions/1808002/attachments/>
 - Quark Matter student days:
<https://indico.cern.ch/event/355454/timetable/#all.detailed>
 - Jean-Yves Ollitrault, Flow
 - Bjoern Schenke, theory overview

Outline

- Basic theoretical concepts
- Particle production in heavy-ion collisions
- Flow
- Fluid dynamics
- QCD kinetic theory and pre-equilibrium dynamics
- Hard probes

Basic theoretical concepts

QCD

- Lagrangian:

$$\mathcal{L} = -\frac{1}{2}\text{tr } \mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu} - \sum_f \bar{\psi}_f (i\gamma^\mu \mathbf{D}_\mu - m_f) \psi_f$$

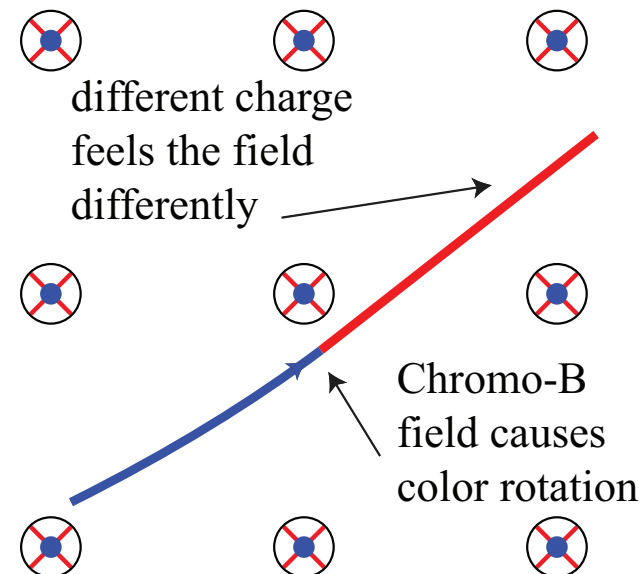
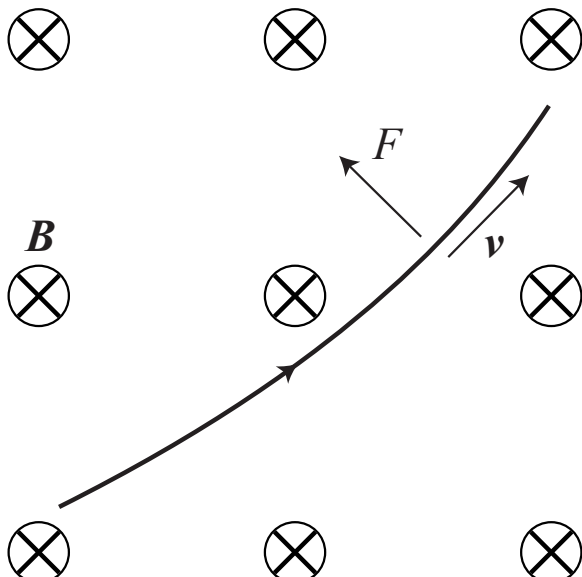
$$\mathbf{F}_{\mu\nu} = \partial_\mu \mathbf{A}_\nu - \partial_\nu \mathbf{A}_\mu - ig[\mathbf{A}_\mu, \mathbf{A}_\nu], \quad \mathbf{D}_\mu = \partial_\mu - ig\mathbf{A}_\mu$$

- Quark masses:

Up	2.3 MeV	Charm	1275 MeV	Top	173 GeV
Down	4.8 MeV	Strange	95 MeV	Bottom	4180 MeV

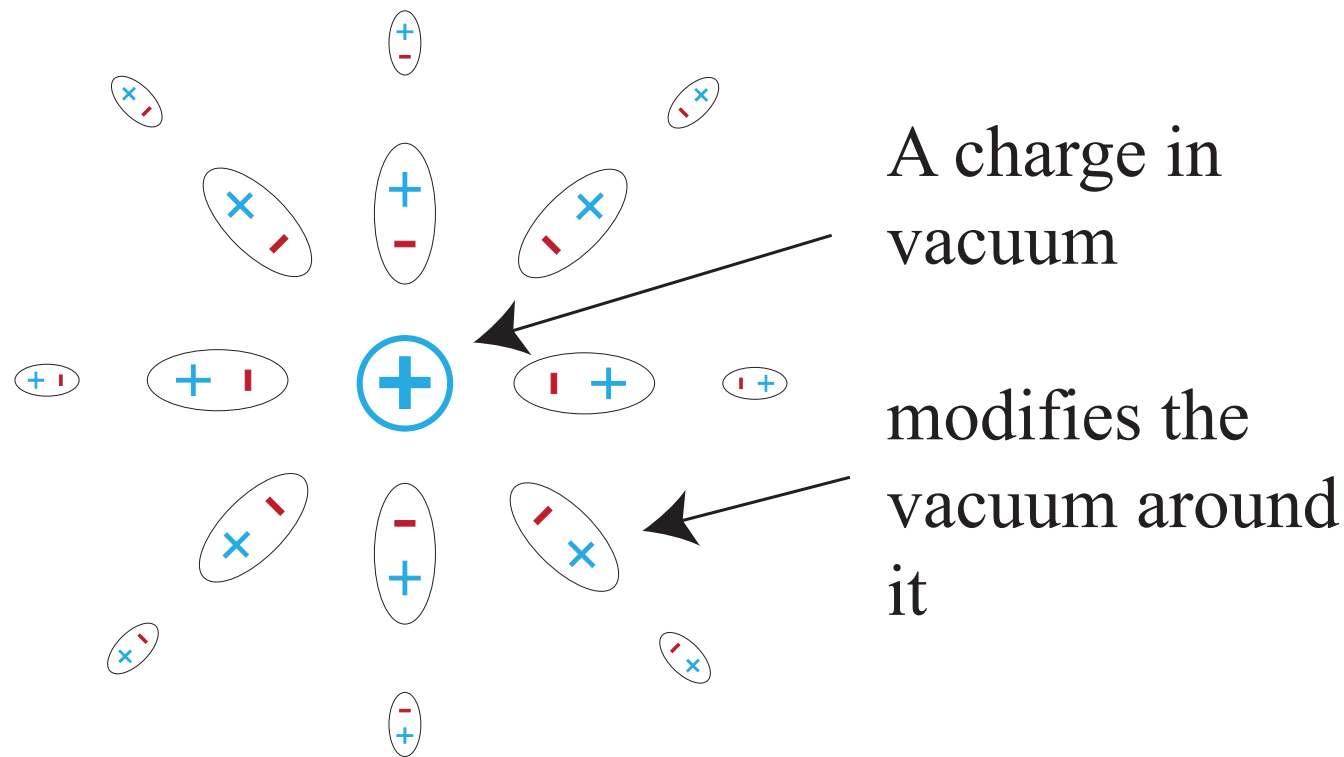
QCD

- Much like QED
 - Quarks like electrons, but come in three colors [rgb]
 - “Chromo-E” and B fields (gluons) like E and B fields (photons) but:
 - E- and B-fields change momentum of particles
 - Chromo-E and -B fields also rotate color (8 different gluon fields: rg, rb,...)
 - Gluons also carry color (charge), interact together



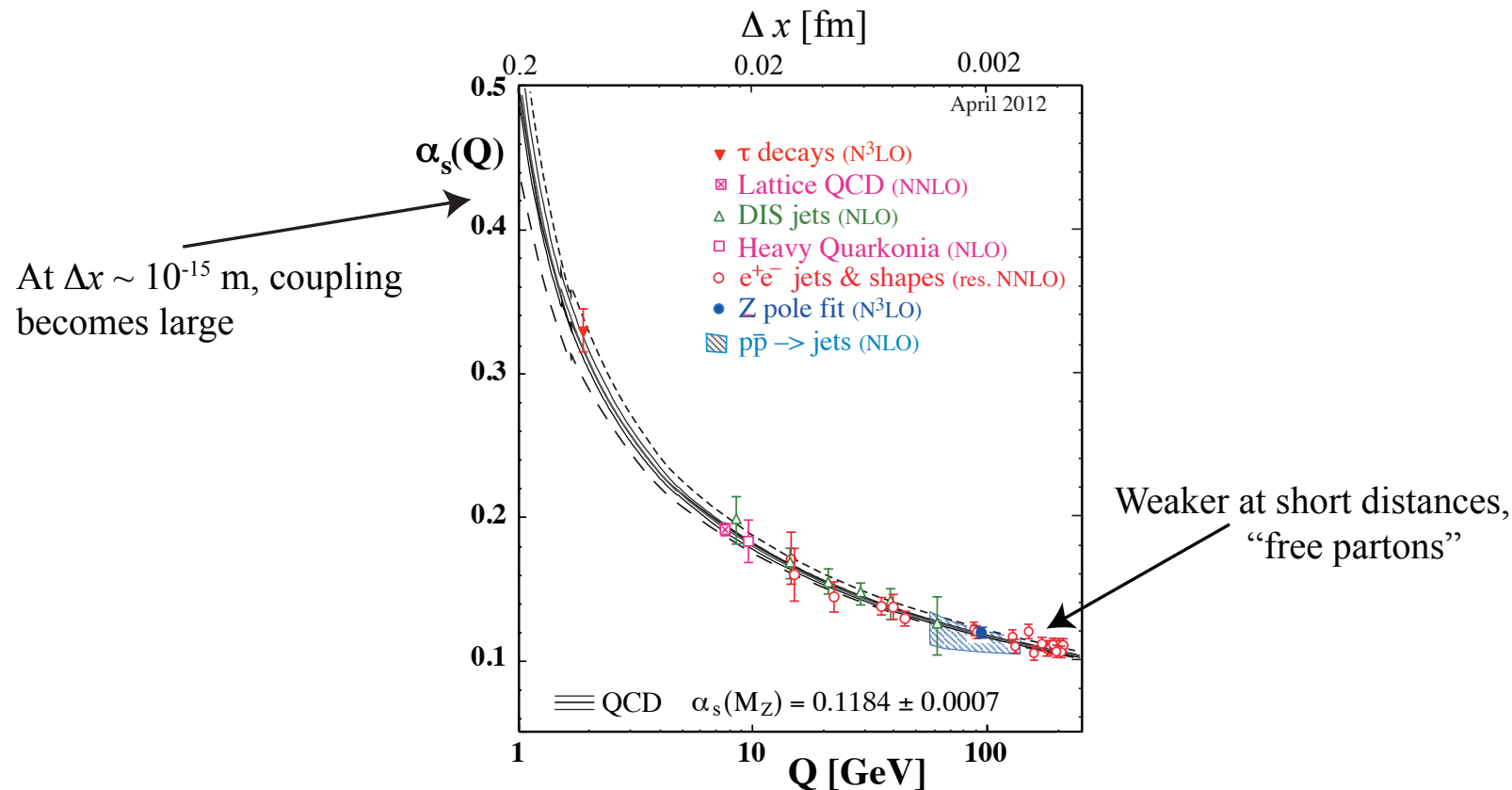
In QFT vacuum is a medium

- QED: vacuum screens charge, strong at short scales



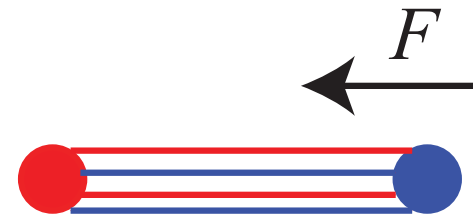
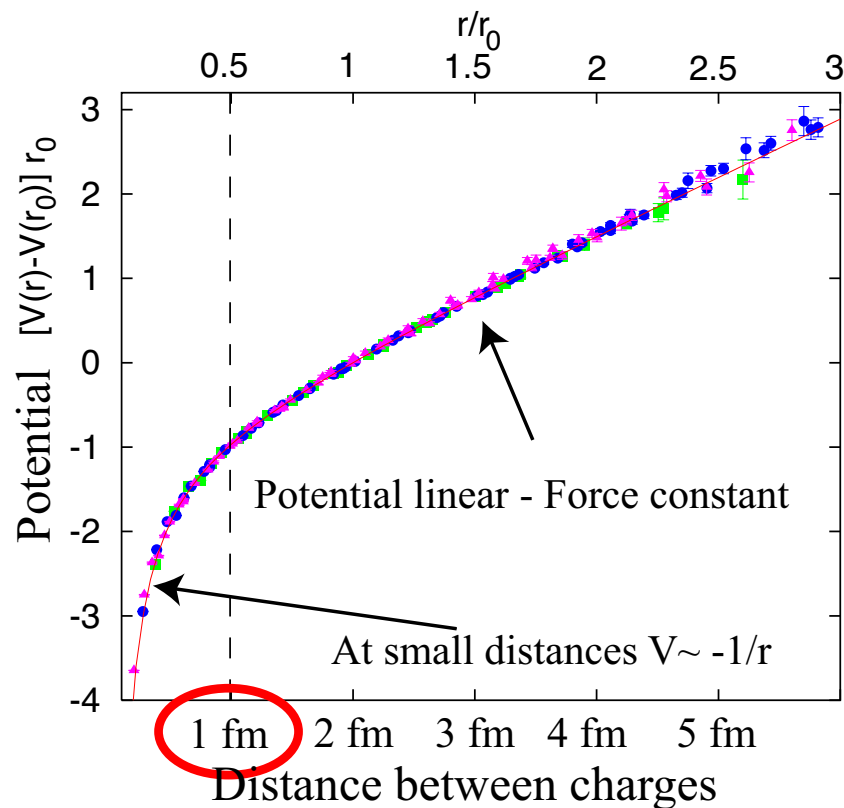
In QFT vacuum is a medium

- QED: vacuum screens charge, strong at short scales
- QCD: vacuum “anti-screens”, weaker at short scales
 - Asymptotic freedom: at short scales “free” q and g



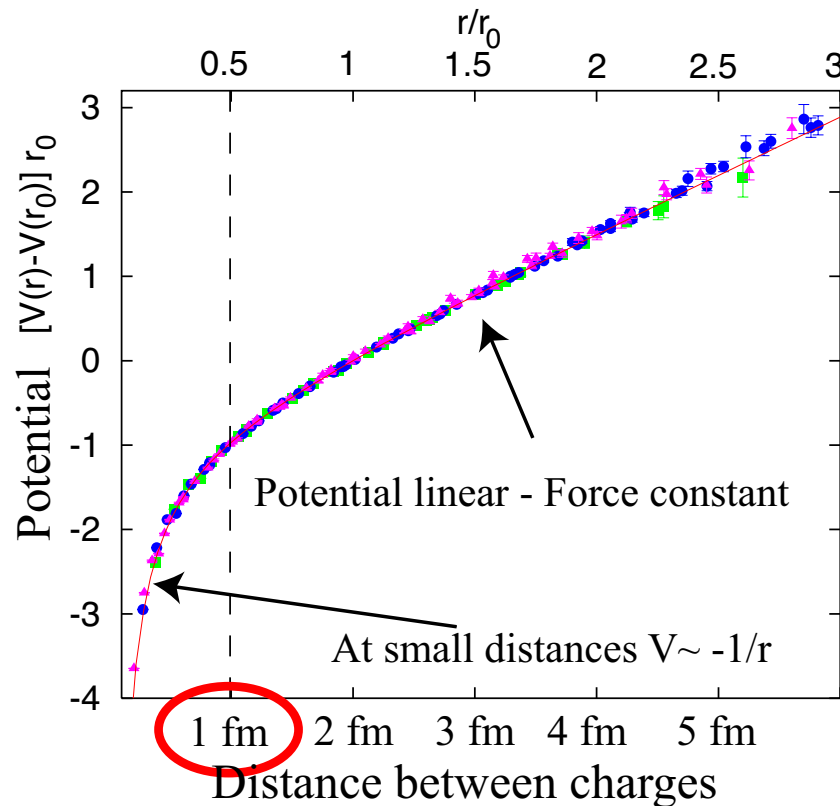
Confinement

- Large coupling reflects linear confinement
- At distances $\Delta x \gtrsim 1 \text{ fm}$, force between color charges independent of distance. “QCD string”

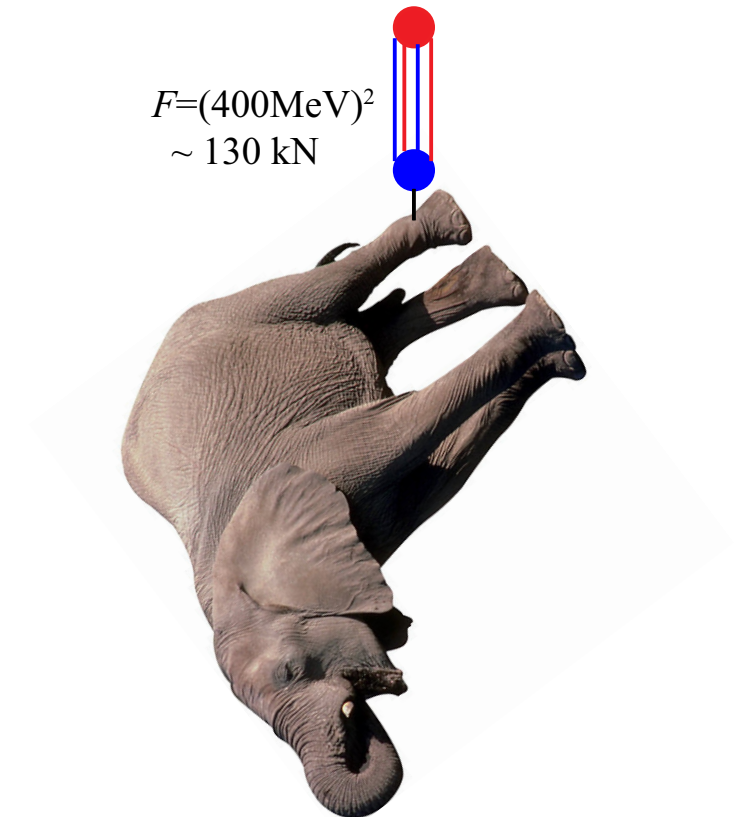


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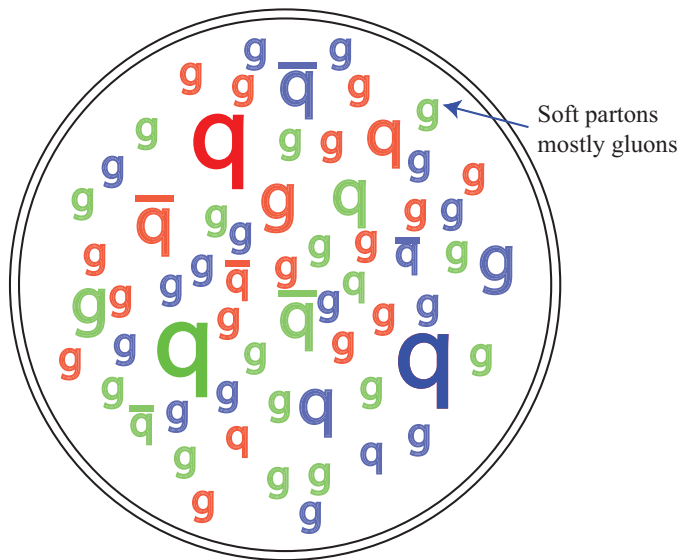


[hep-lat/0001312]

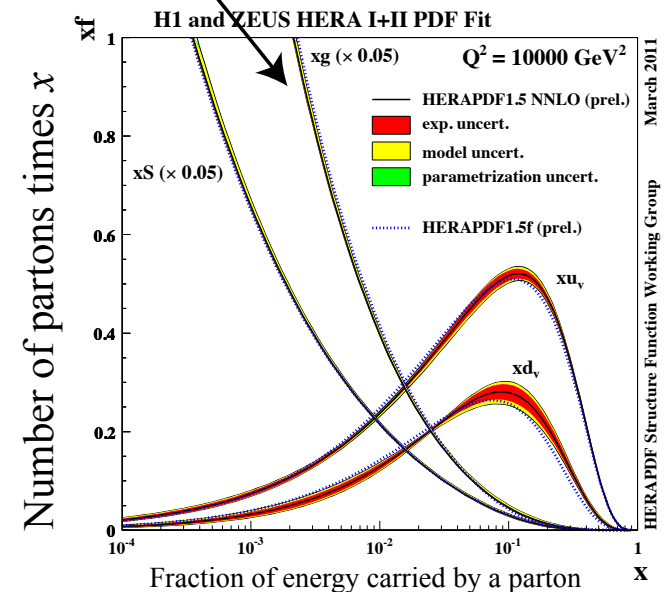


Confinement

- Matter organized in color-neutral lumps: Protons, neutrons, pions, etc. = hadrons
- Quasiparticles of QCD vacuum, not few q and g
- Quark and gluon content can be probed in DIS, pdf's

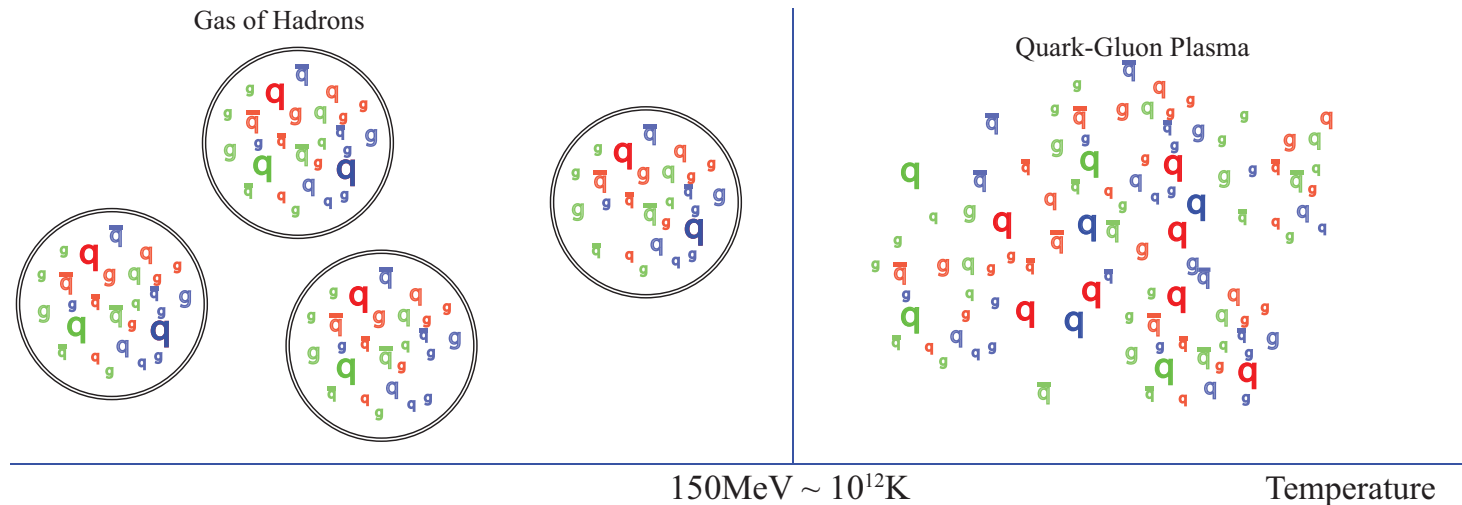


Large number of soft gluons

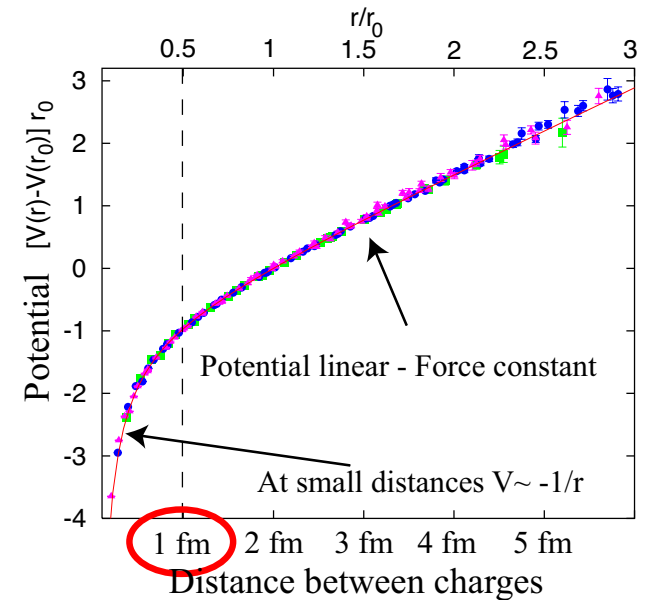


Proton Parton Distribution Functions

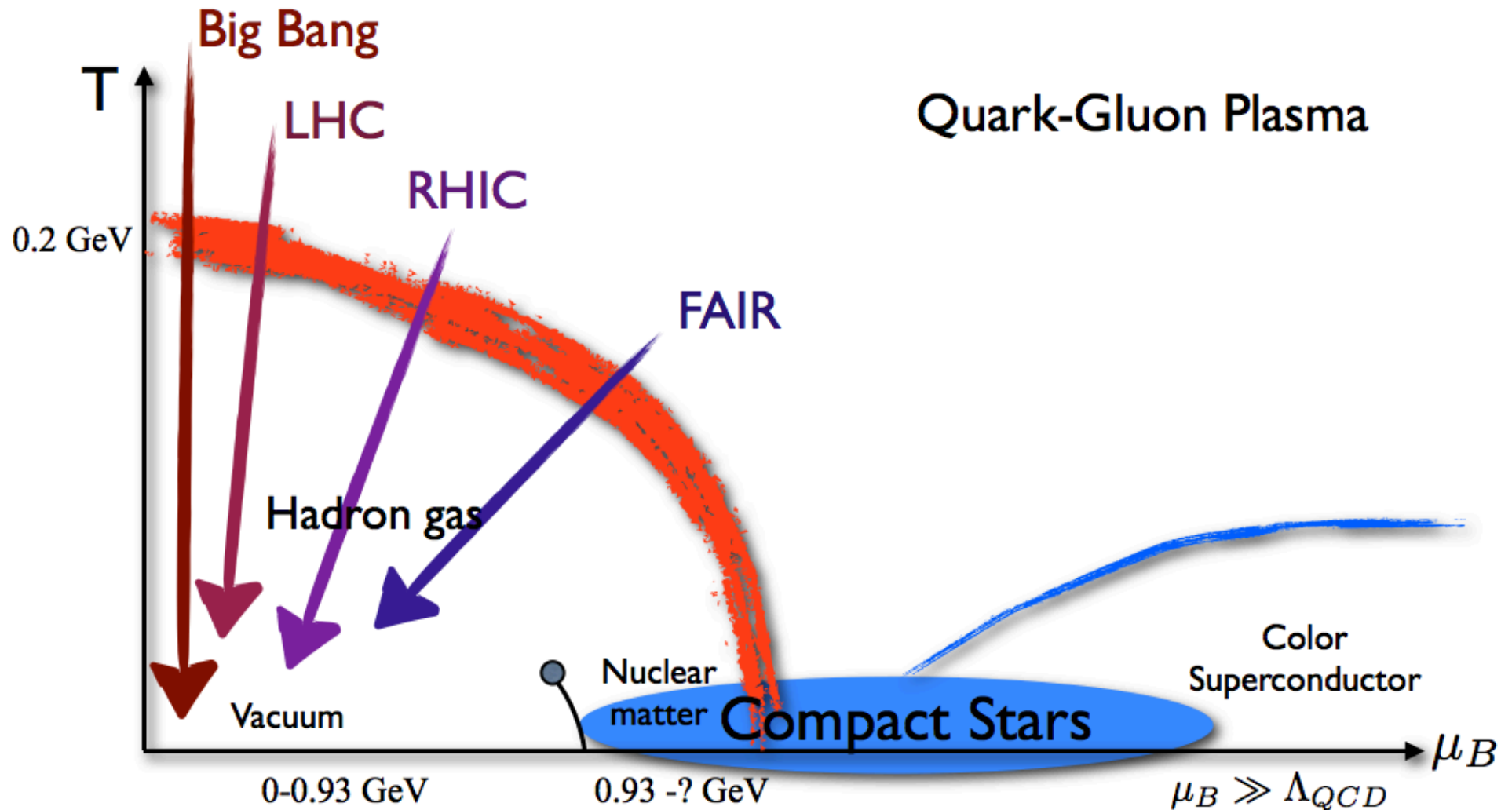
Deconfinement



- At small densities/temperatures: gas of hadrons
- At high densities/temperatures: gas of q and g
- At asymptotically high temperatures, free gas q and g



Phase diagram?



Baryon chemical potential

QCD thermodynamics

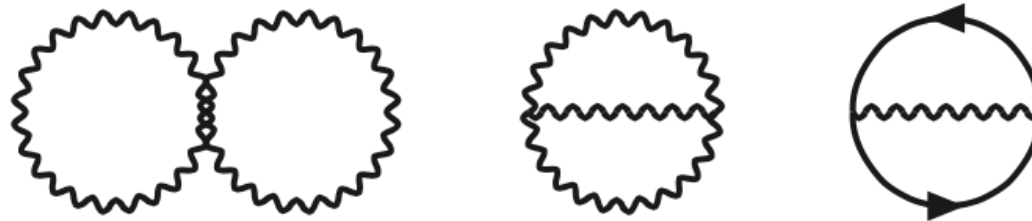
- At asymptotically high temperature: EoS of non-interacting quarks and gluons

$$p(T) = \frac{\pi^2}{90} \left(N_B + \frac{7}{8} N_F \right) T^4$$

$$N_B = 2 \times 8, \quad N_F = 2 \times 2 \times 3 \times 3$$

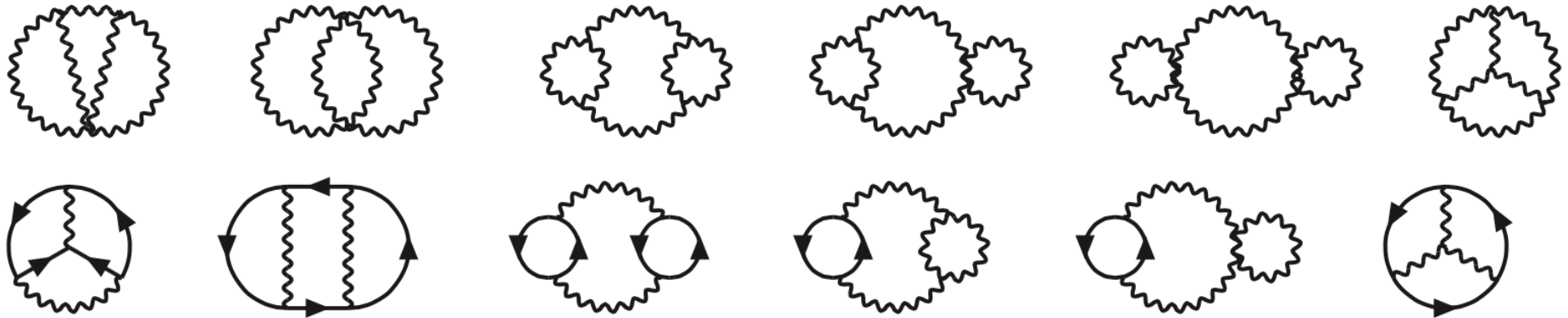
spin color spin particle/antiparticle color flavor: u/d/s

- Corrections through loop diagrams:



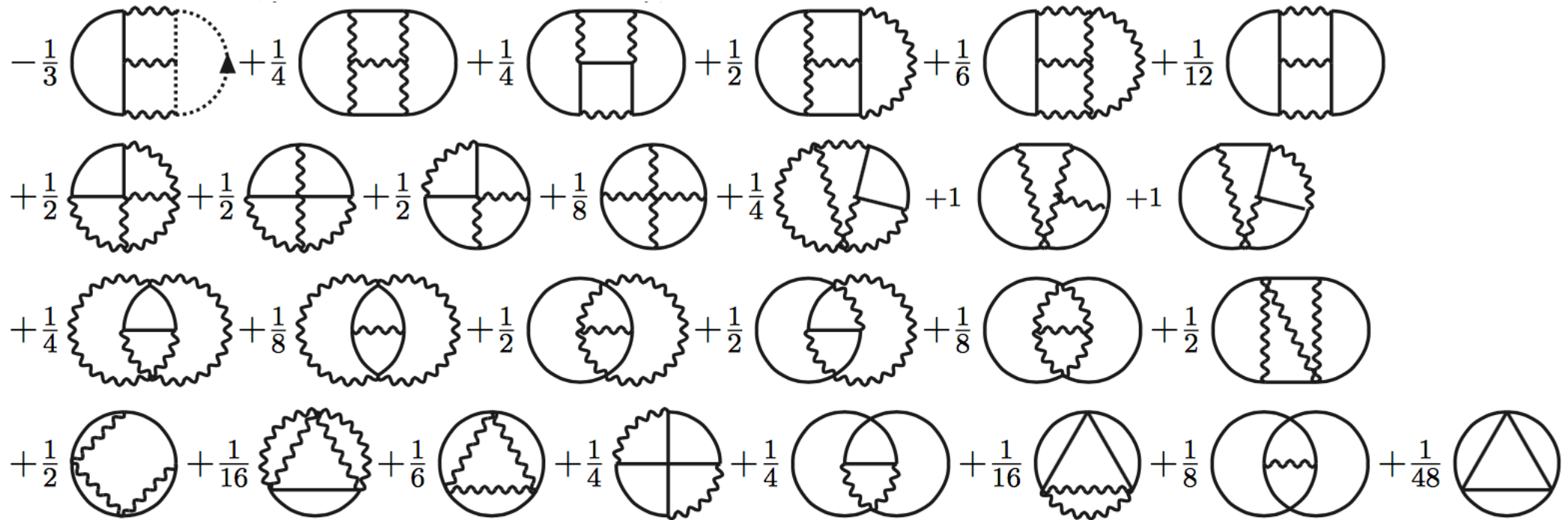
QCD thermodynamics

- ...and



QCD thermodynamics

- ...and



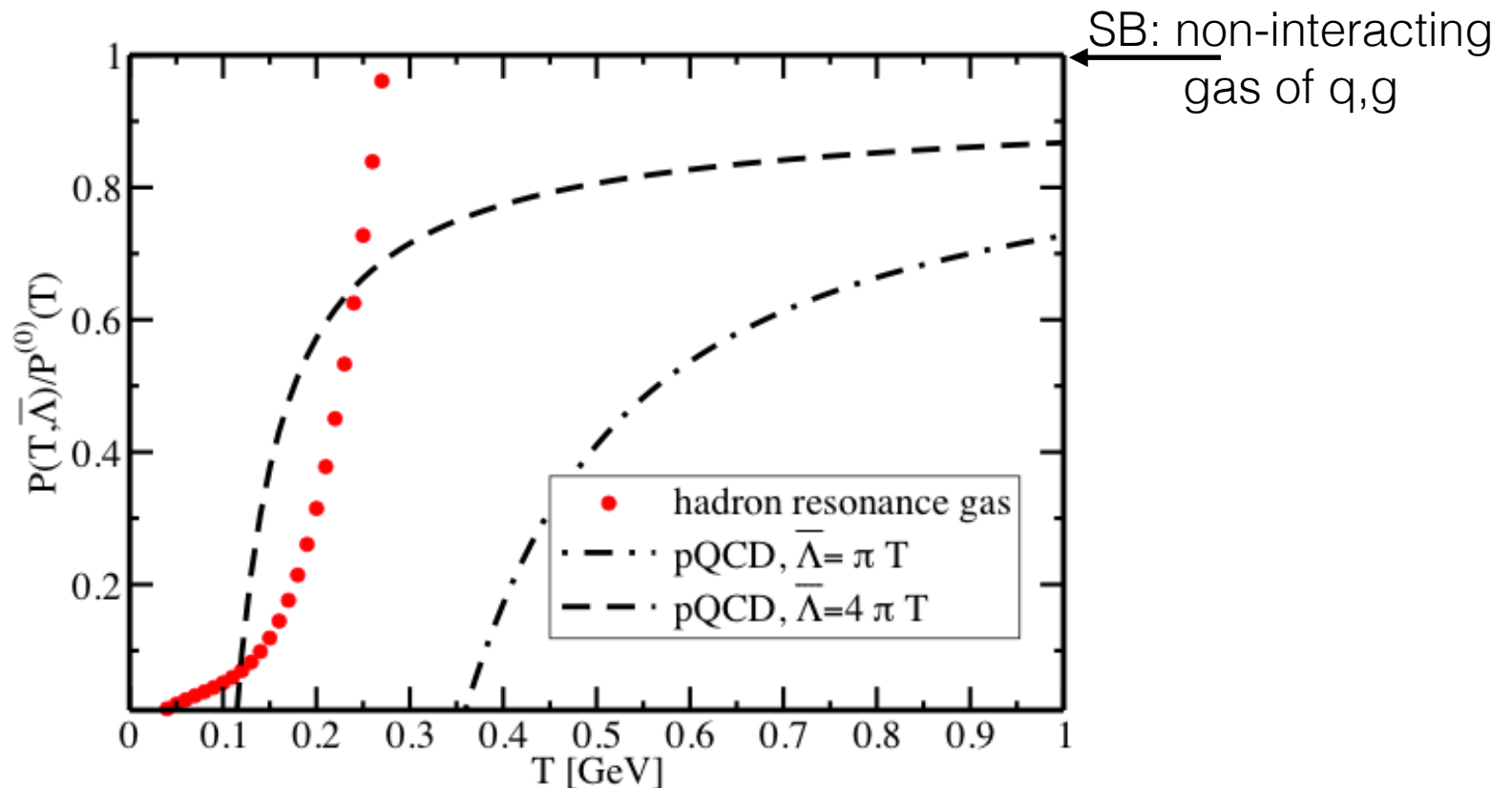
- and so on. Computed to order

$$p \sim 1 + \alpha_s + \alpha_s^{3/2} + \alpha_s^2 + \alpha_s^{5/2} + \dots$$

NNNNNLO?

QCD thermodynamics

- At small temperatures: non-interacting gas of massive hadrons



- In the middle something else!

Lattice QCD

- (Some) equilibrium thermodynamical information numerically calculable nonperturbatively, directly from Lagrangian.
- Discretization of g and q fields on a space time lattice
- Based on a formal analogy of 3+1D quantum theory (in Minkowski) and 4D statistical field theory (Euclidean).

$$Z = \sum_{\psi} \langle \psi | \hat{\rho} | \psi \rangle \quad \hat{\rho} = e^{-\hat{H}/T}$$

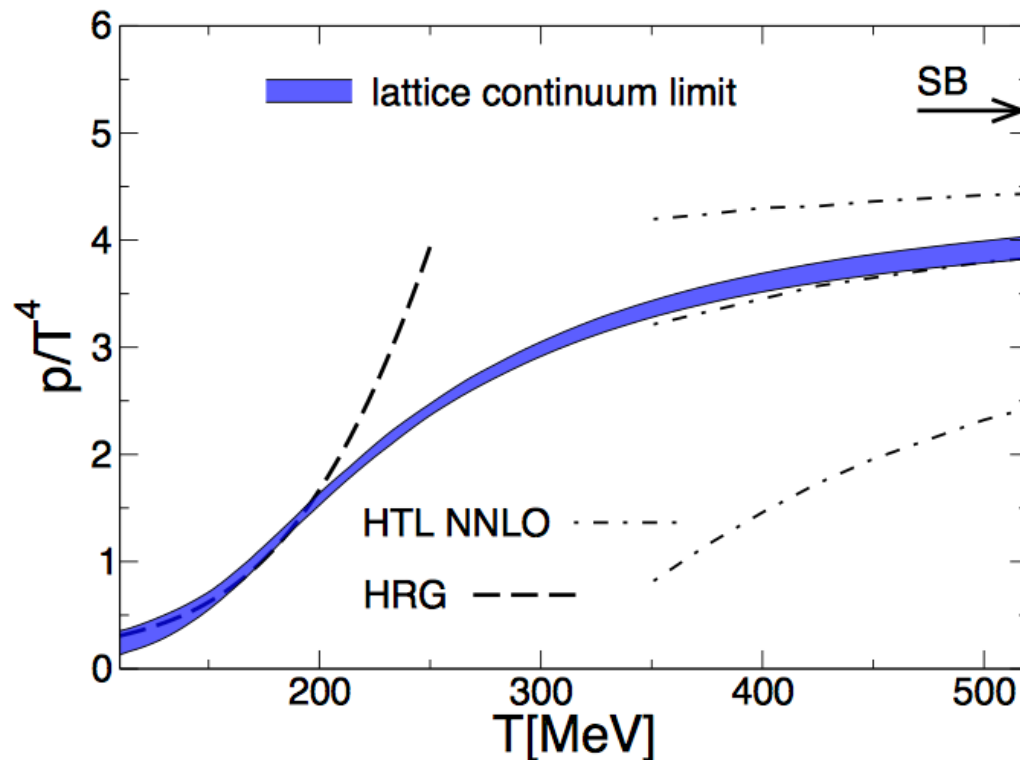
Looks like time evolution operator with imaginary time: $U(0, t) = e^{-i\hat{H}t}$



Lattice QCD

- Applicability limited to static quantities (spacelike correlators) in thermal equilibrium. Not sufficient to model the dynamical evolution in heavy-ion collisions
- Can do: Equation of State, speed of sound, screening masses,
- Can't do: Transport coefficients, non-equilibrium evolution, particle production etc..

Equation of State



- Small temperatures, gas of hadrons
- Large temperatures, ideal gas of q and g
- Pseudocritical temperature of cross-over $T_c \sim 200$ MeV
- At few T_c , still large corrections from ideal gas
- Deviation from ideal gas qualitatively described by thermal perturbation theory with large error bars

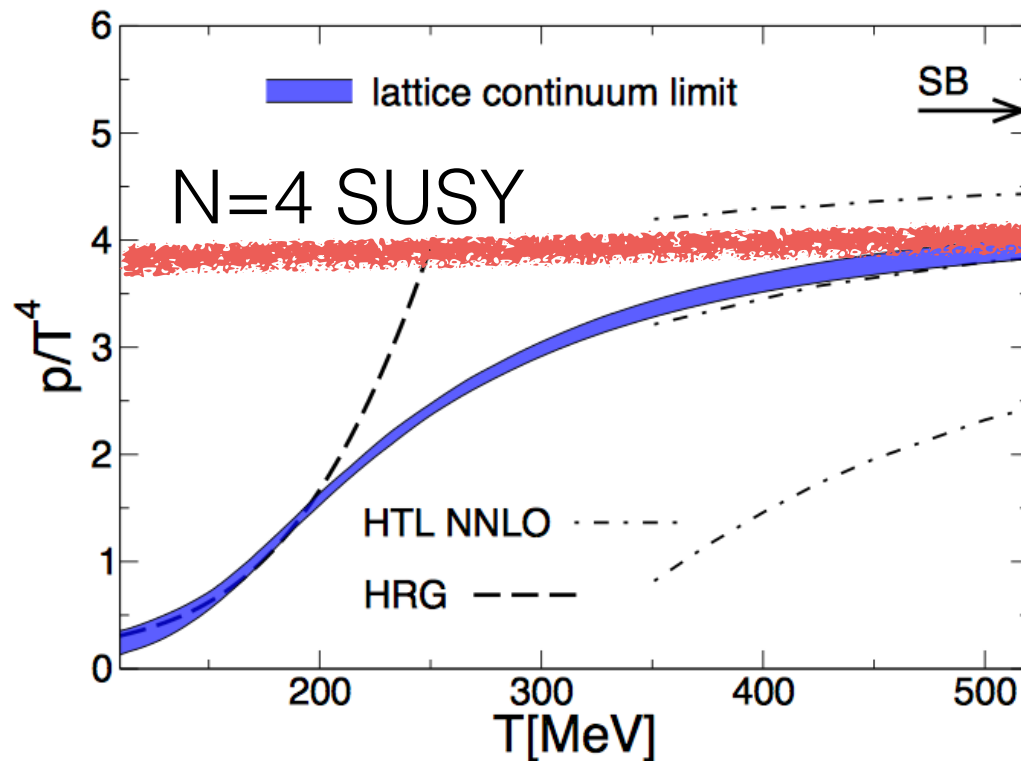
$\mathcal{N} = 4$ Super Yang-Mills theory

- Not QCD
 - Different particle content! Extra scalar, adjoint fermions,
 - Conformal symmetry
 - No confinement, no asymptotic freedom.
- ...But can be solved in the 't Hooft limit using holography (AdS/CFT)

$$N_c \rightarrow \infty, \quad \lambda \equiv 4\pi\alpha_s N_c \rightarrow \infty$$

- Offers a solvable theoretical toy model that is as strongly coupled as it gets.
“The opposite of ideal gas”
- Not to be taken quantitatively predictive but qualitative insight
 - Most results contain a some kind of rescaling (by # of d.o.f etc)

$\mathcal{N} = 4$ Super Yang-Mills theory



Particle production in Heavy-Ion Collisions

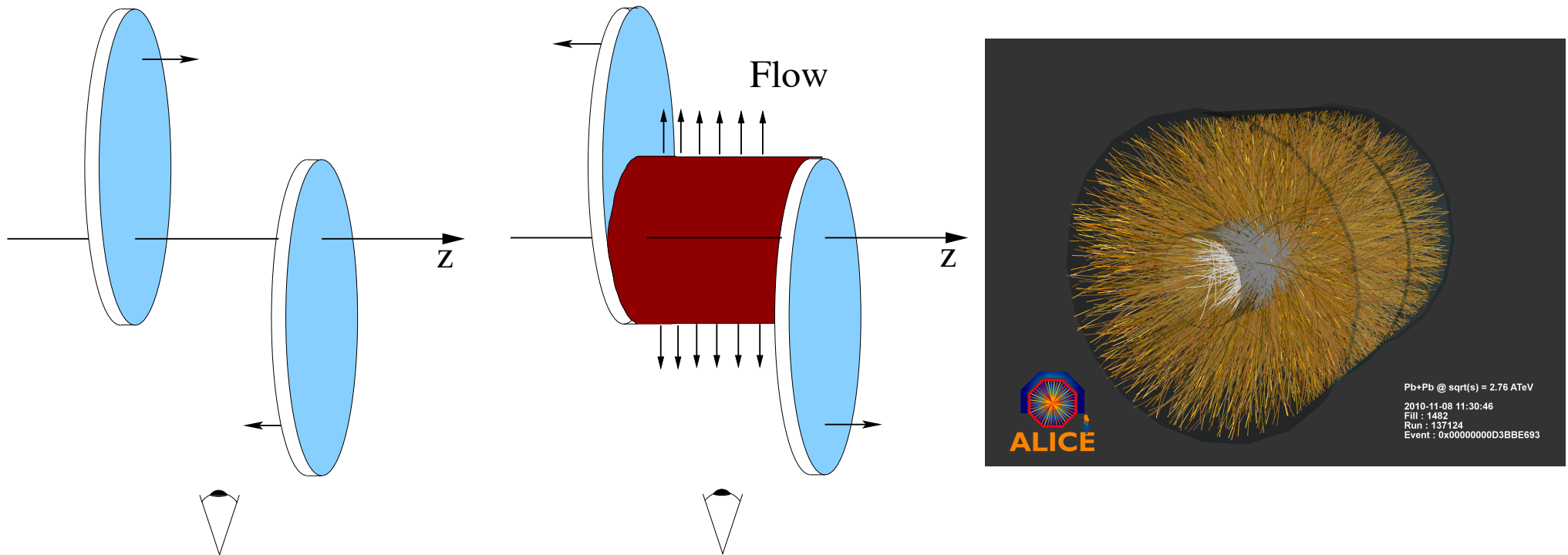
Accelerators

	SPS	RHIC	LHC
\sqrt{s} per nucl. pair (GeV)	17	200	5500
Volume at freeze-out (fm	1200	2300	5000
Energy density (GeV/fm	3-4	4-7	10
Lifetime (fm/c)	4	7	10

- For HIC, centre-of-mass energy per nucleon-nucleon pair is usually indicated $\sqrt{s_{NN}}$
- Total collision energy

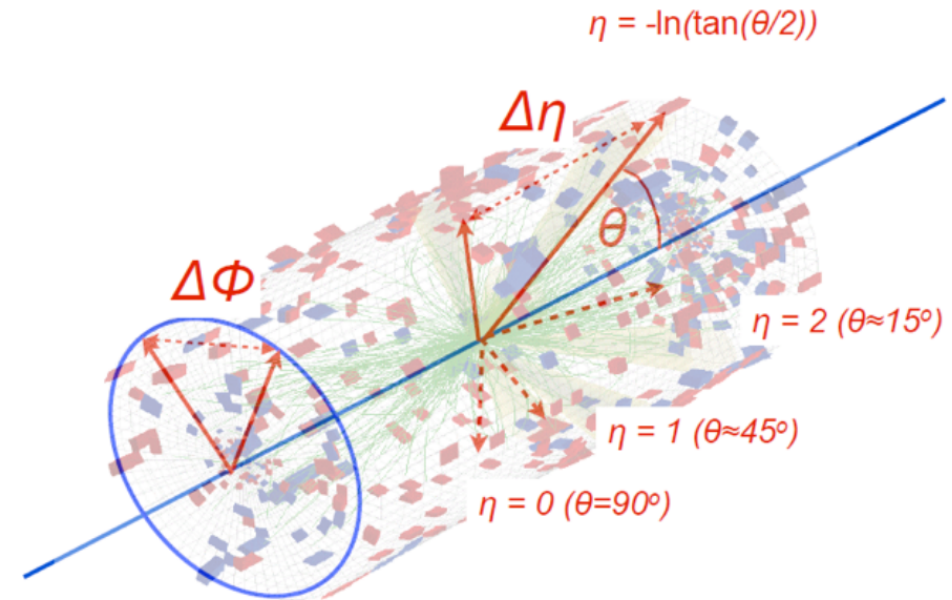
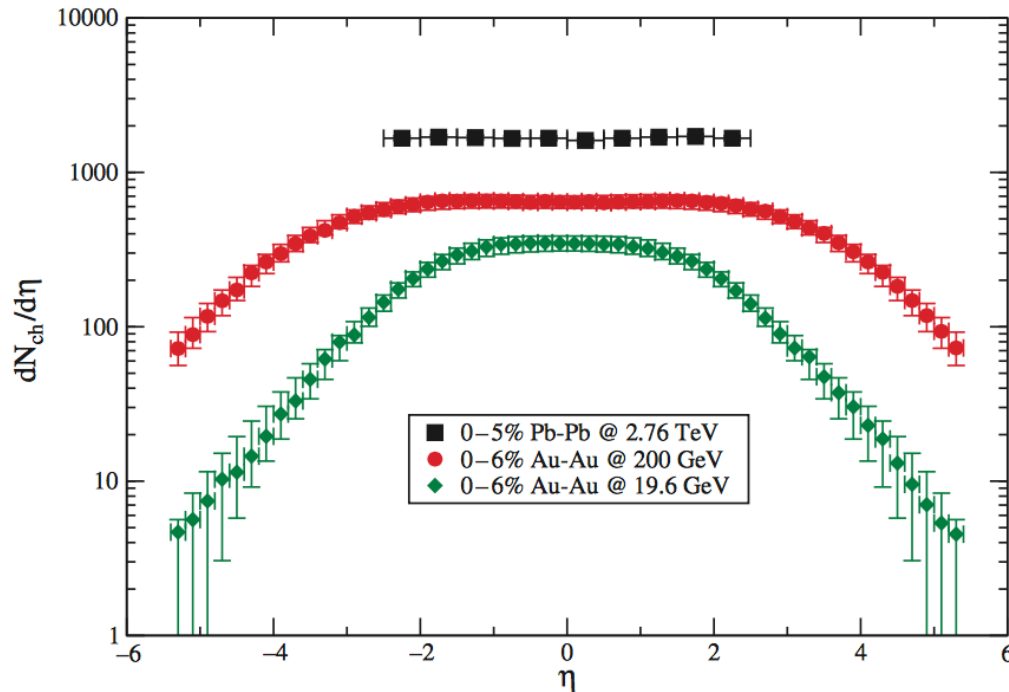
$$\sqrt{s} \approx (82 + 126) \times 5.5 \text{TeV} \approx 1144 \text{TeV}$$

Basic picture



- Most particles and energy continue along beam pipe
- Those that undergo a large angle scattering form a medium and eventually reach detector
- Large Lorentz contraction of the nuclei indicate lack on longitudinal structure: Boost invariance in mid rapidity region

Multiplicities at midrapidity



- Number of charged particles found in detector as a function of pseudorapidity η . Approximately independent of η
- Not all particles charged: $N_{tot} \approx 1.6 \times N_{ch}$
- Total charged:
 - RHIC: ~ 5000 , LHC: ~ 25000

Bjoerken estimate

- The multiplicities give an handle on the energy density and temperature of the fireball
- Consider transverse slab at midrapidity
- Energy density per unit (pseudo)rapidity

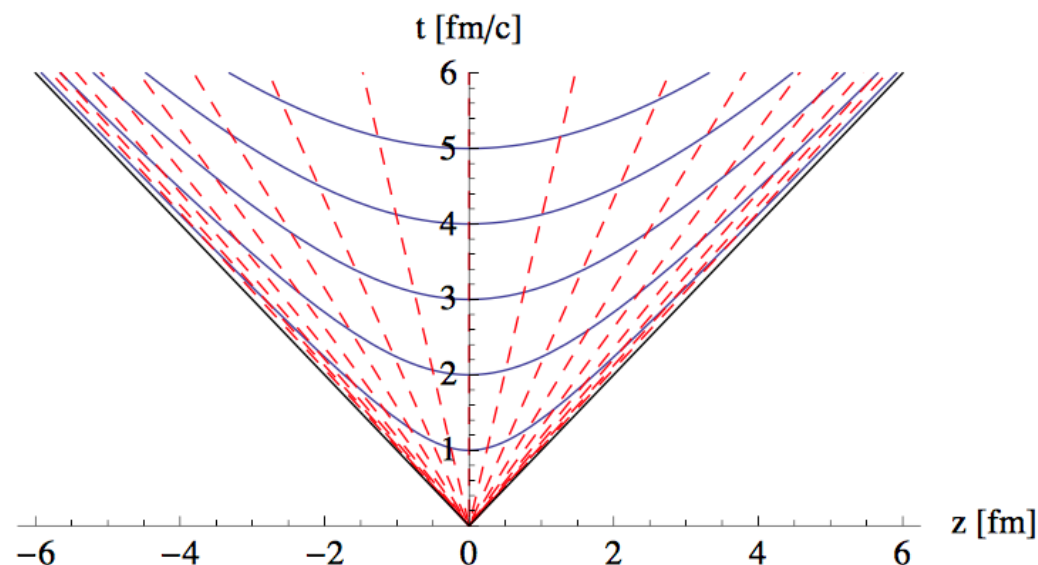
$$\frac{dE_T}{dy} = \frac{dN}{dy} \langle E_T \rangle$$

- At a proper time τ_0 this energy is concentrated in a volume

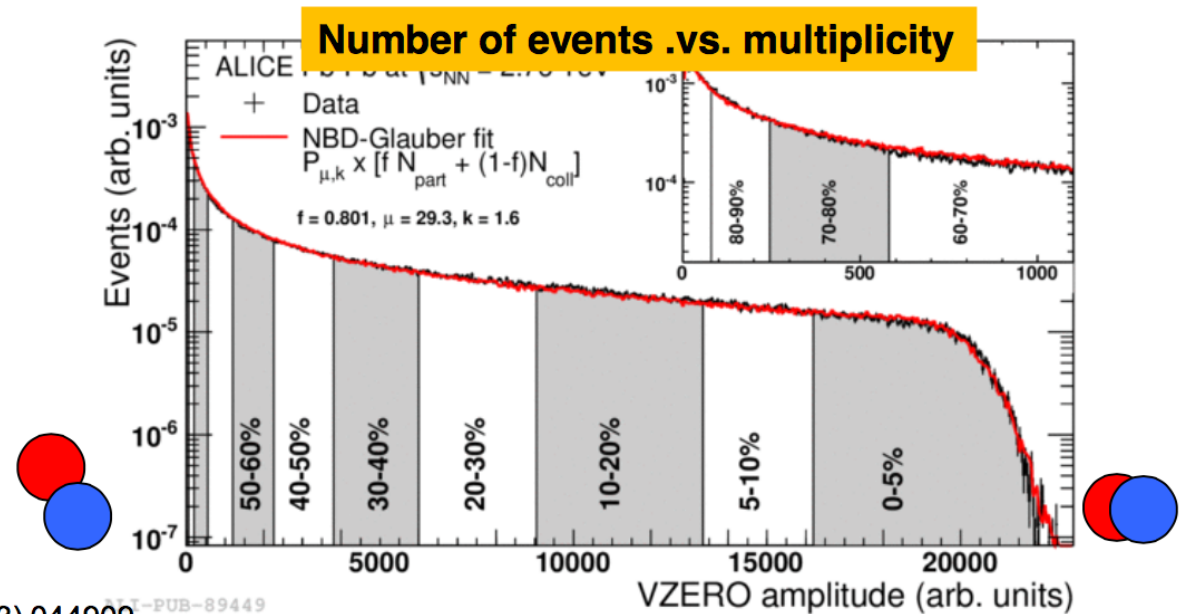
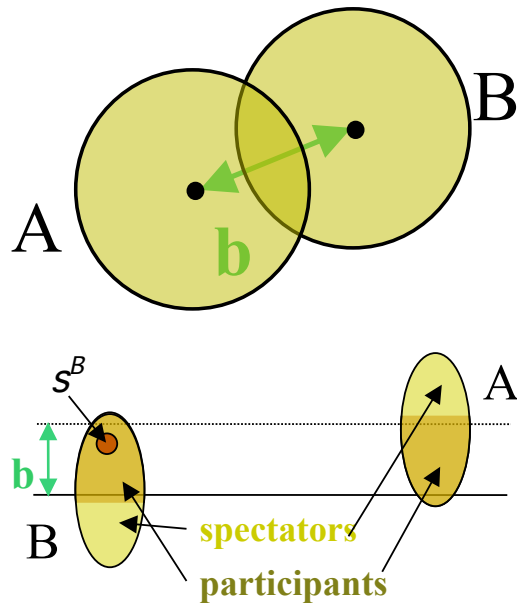
$$\pi R^2 dz = \pi R^2 d\eta \tau_0$$

- The energy density is then

$$\epsilon^{SPS}(\tau_0 = 1 fm/c) \approx 3 - 4 GeV/fm^3, \quad T \sim 3T_c$$



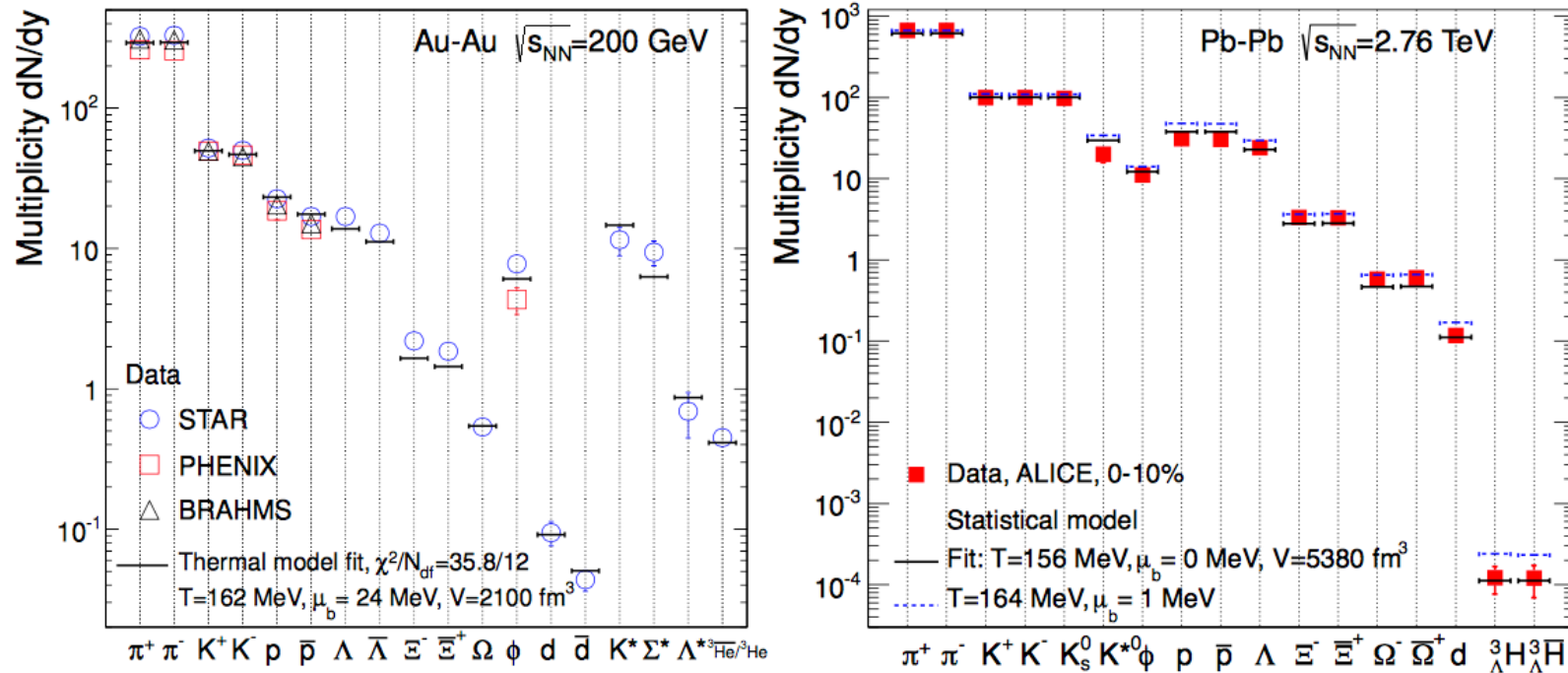
Collision geometry and impact parameter



PRC88 (2013) 044909

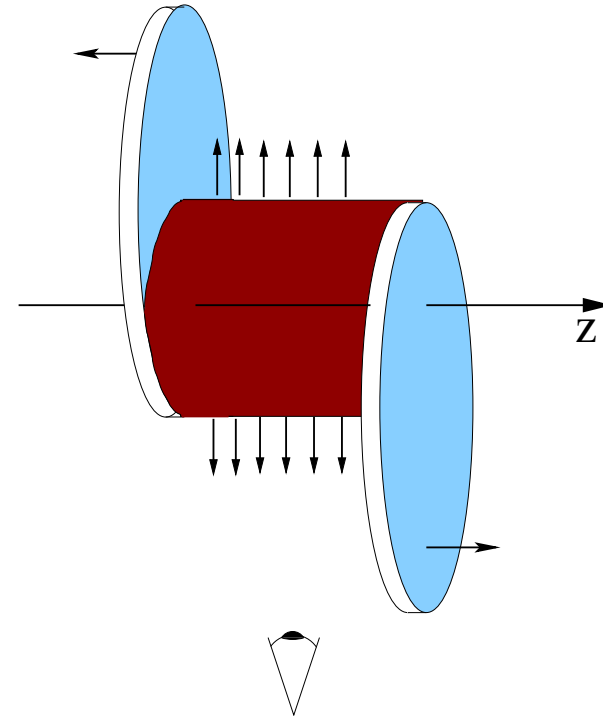
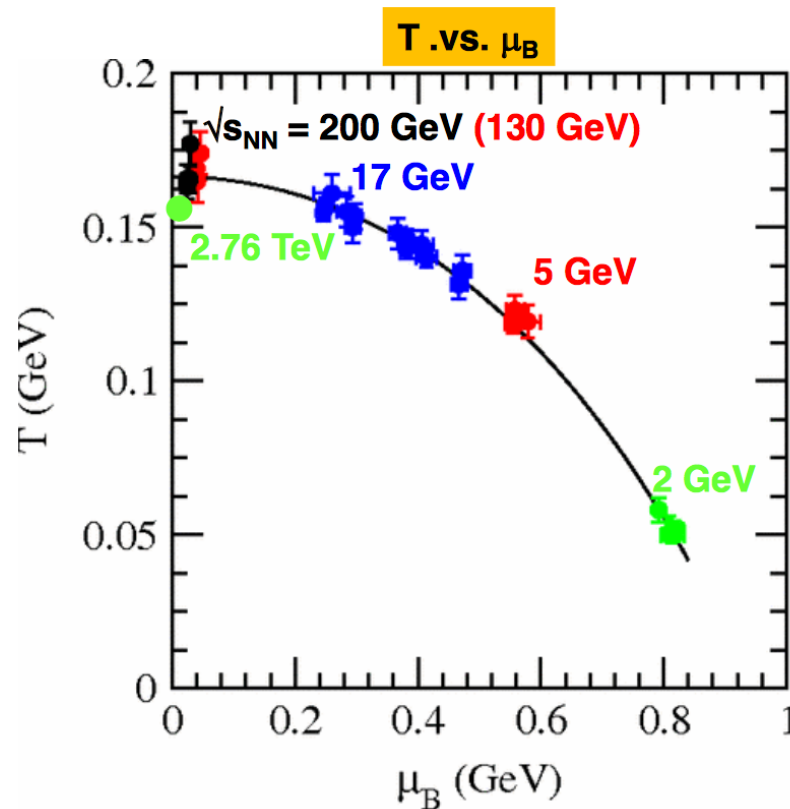
- Centrality class: percentage of the minimum bias cross section
- Multiplicity distribution explained by collision geometry
 -> Impact parameter $b \sim$ multiplicity
- For precise connection modelling necessary

Freeze-out and identified particle multiplicities



- Multiplicities of identified particles well described by statistical model:
 - Hadron gas in thermal and chemical equilibrium
 - Includes all hadronic resonances known to particle data group
 - Interpretation: 1) Matter close to local thermal eq.,
2) cools and interactions fail to keep in thermal
3) Multiplicities frozen to the moment of freeze-out (cf. CMB)
- Evidence of formation of thermalized plasma

Freeze-out and identified particle multiplicities



- Different beam energies correspond to different chemical potentials
- Increasing energy leads to reduced baryochemical potential
 - Transport of baryon number from nuclei to mid rapidity more and more difficult

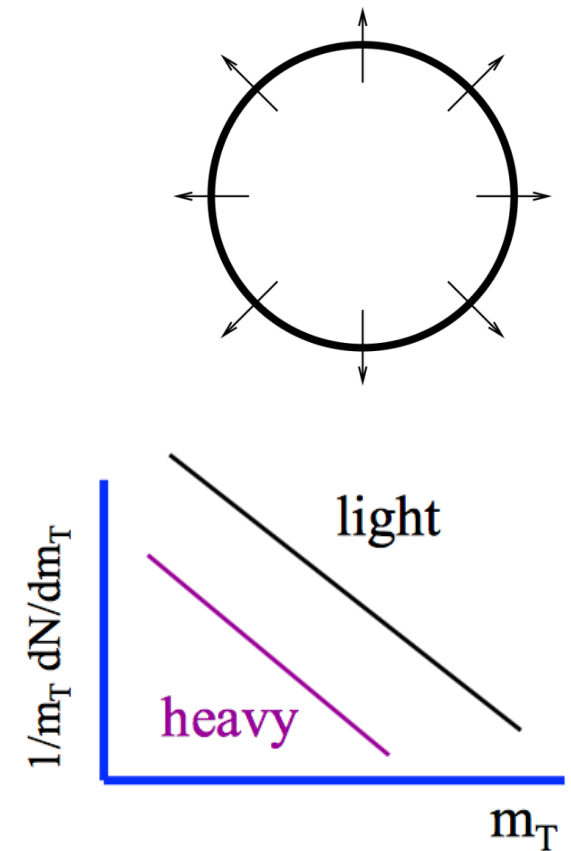
Flow

Radial flow:

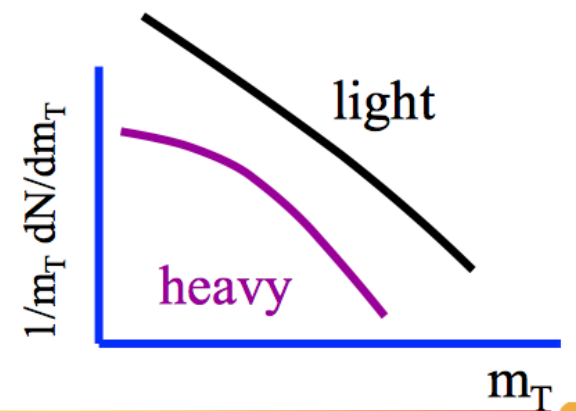
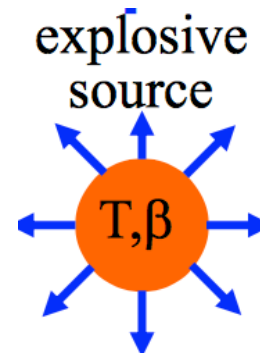
- The explosion of the fire ball leads to *radial flow*
- Emission from thermal source, $\sim e^{-E/T}$

$$E \frac{d^3 N}{d^3 p} = \frac{1}{2\pi m_T} \frac{d^2 N}{dm_T dy} = C m_T K_1 \left(\frac{m_T}{T} \right) \quad \text{purely thermal source}$$

$$\approx C' \sqrt{m_T} e^{-m_T/T} \quad \text{für} \quad m_T \gg T$$

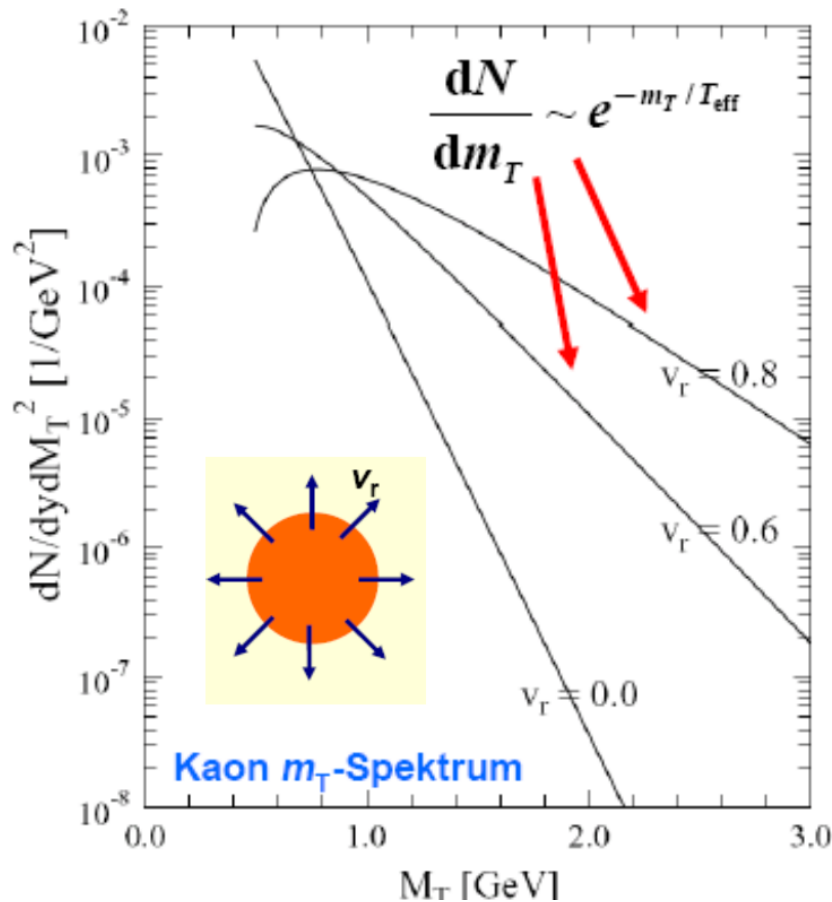


- Shape of the spectrum independent of masses
- Explosive source will blueshift the spectrum
 - Spectrum depends on mass
 - and on the expansion velocity



Radial flow:

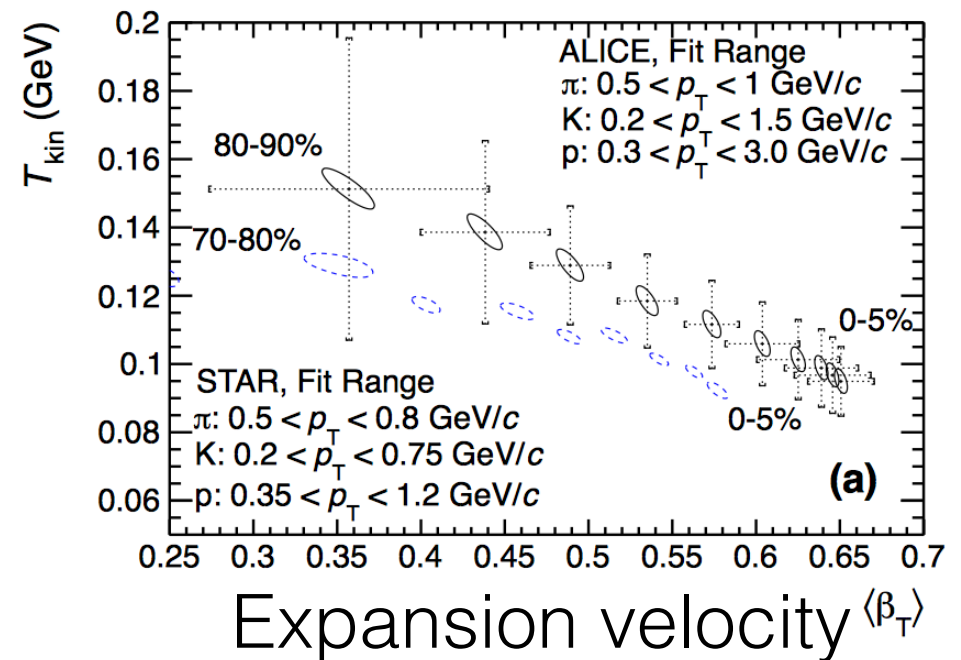
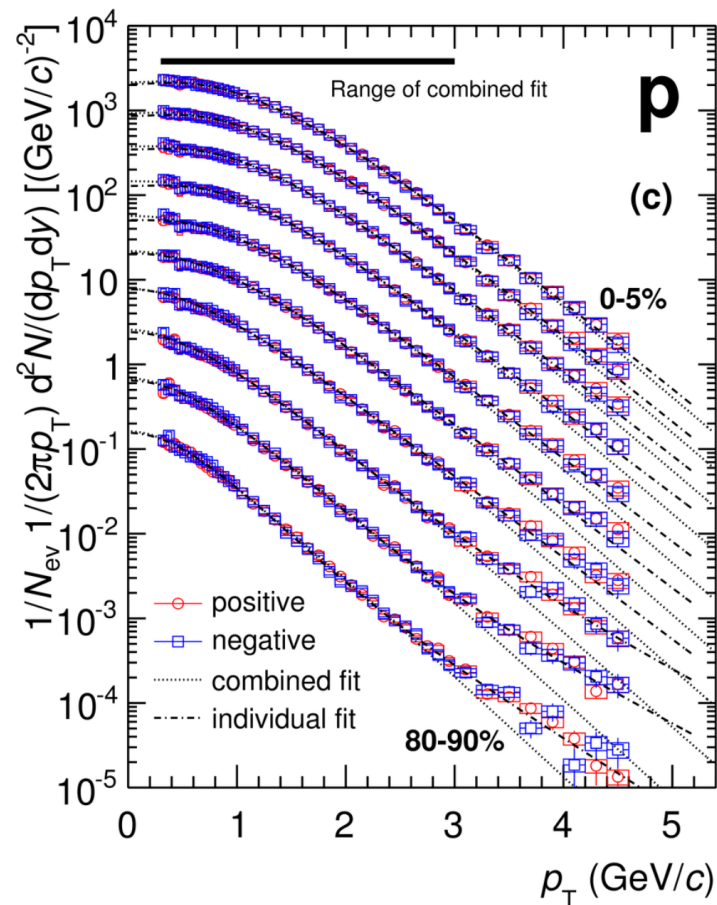
- “Blast wave model”: Hydrodynamics inspired model with a symmetric average geometry
- The inverse slope determines the velocity



$$T_{\text{eff}} = T \sqrt{\frac{1 + v_r}{1 - v_r}}$$

Radial flow:

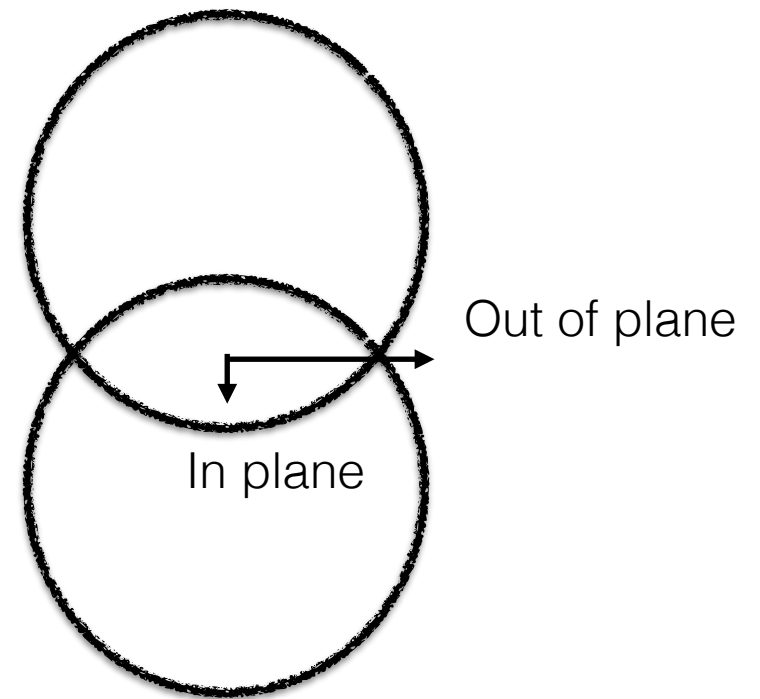
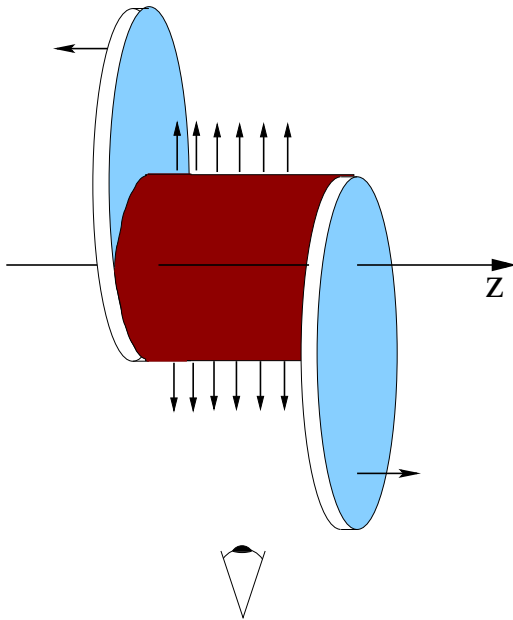
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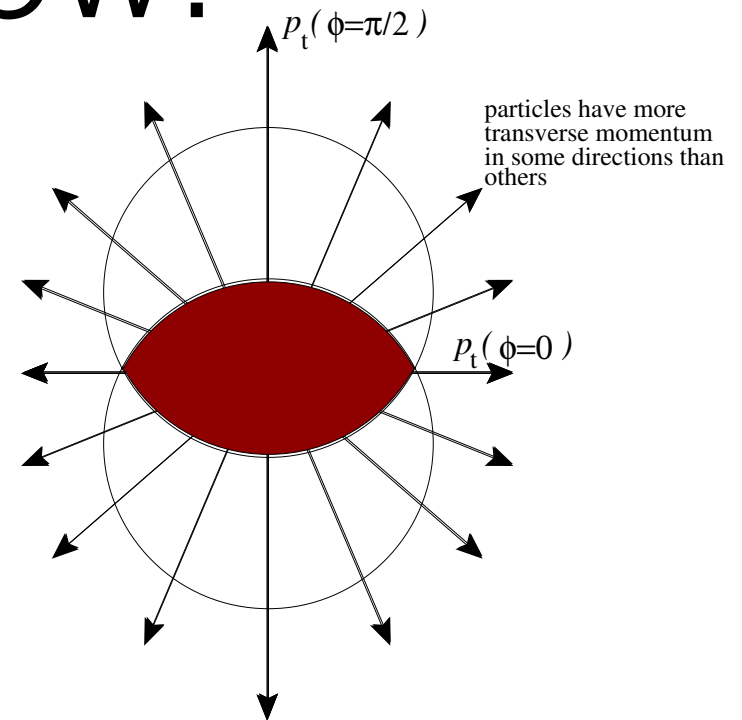
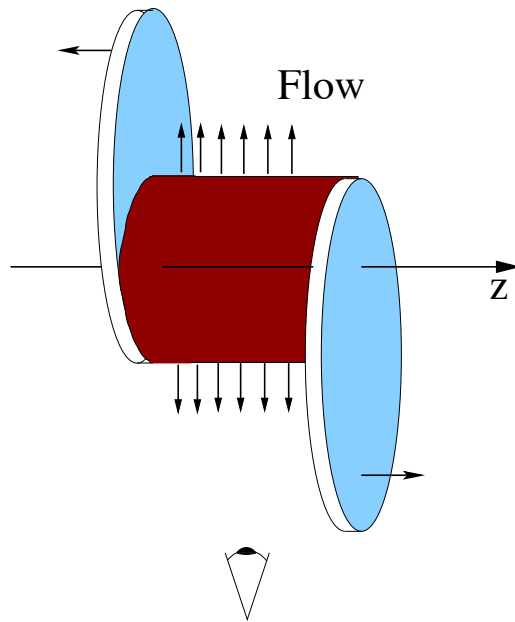
- The inverse slope determines the velocity

Elliptic flow:

- Nuclear overlap area anisotropic in non-central collisions:
Symmetry direction defines *reaction plane*

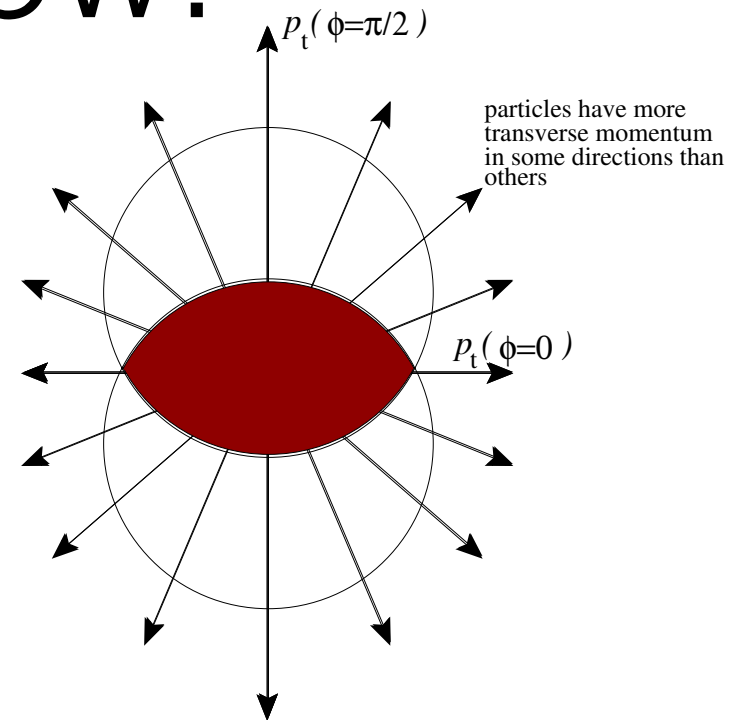
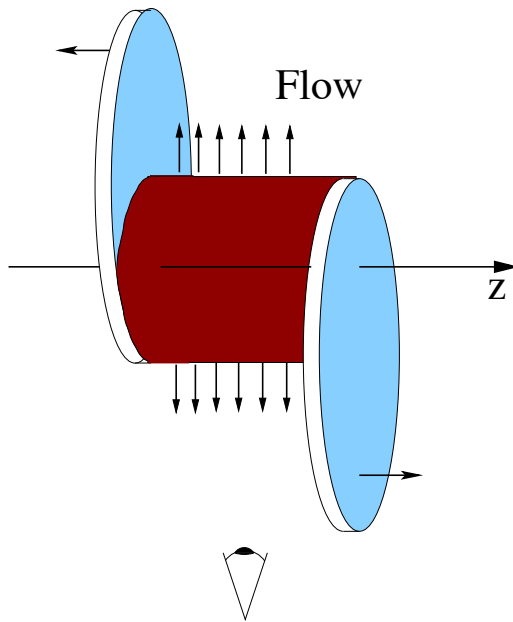


Elliptic flow:




- Hydrodynamical flow converts spatial anisotropy to momentum anisotropy:
 - Pressure gradients larger in the reaction plane
 - Leads larger fluid velocity in this direction, more particles

Elliptic flow:

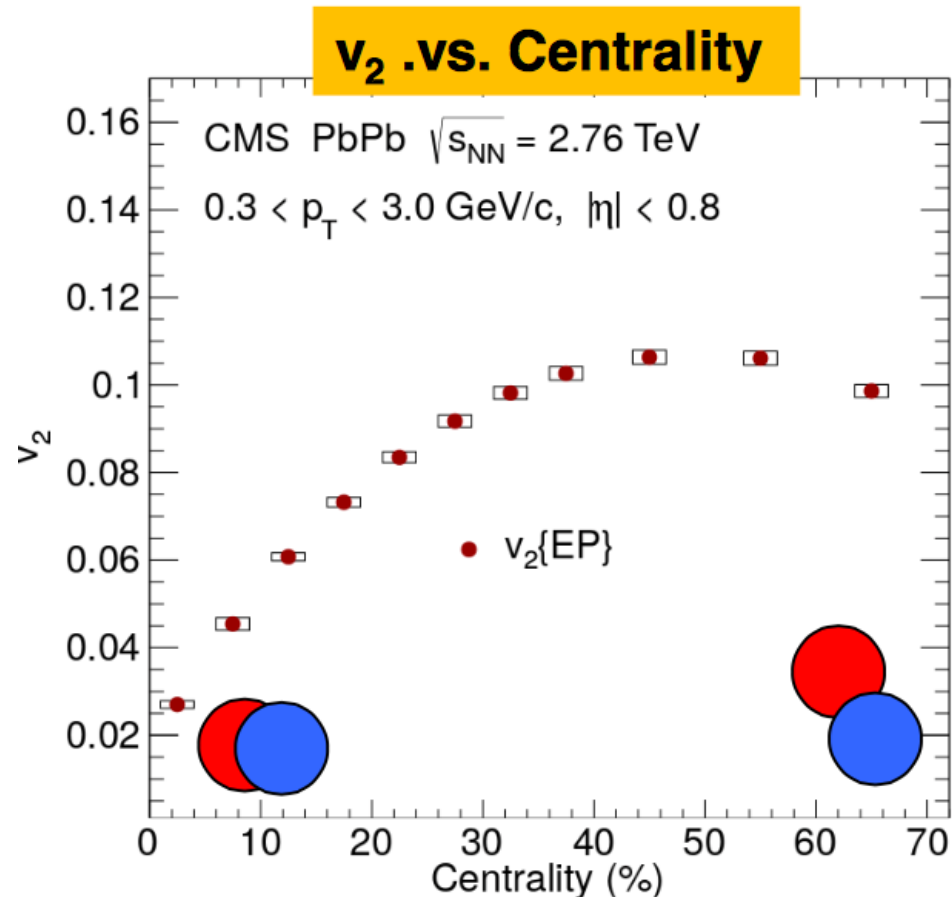


- Quantify anisotropy using Fourier expansion of the azimuthal coordinate:

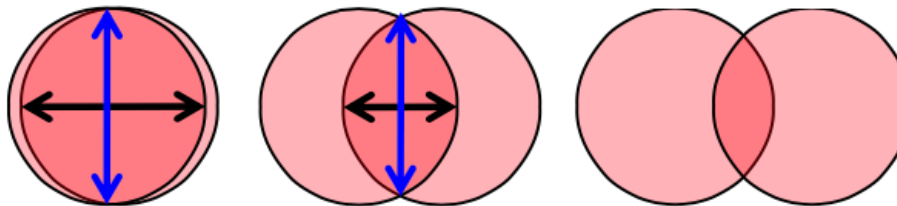
$$\frac{dN}{d\phi} = \frac{N}{2\pi} \left[1 + 2 \sum_m v_m \cos(m(\phi - \psi_R)) \right]$$


 Reaction plane

Elliptic flow



- Strong centrality dependence, largest for 40-50%
- Very small spatial anisotropy in central collisions
- Large anisotropy in midcentral collisions
- Small overlap region in peripheral collisions



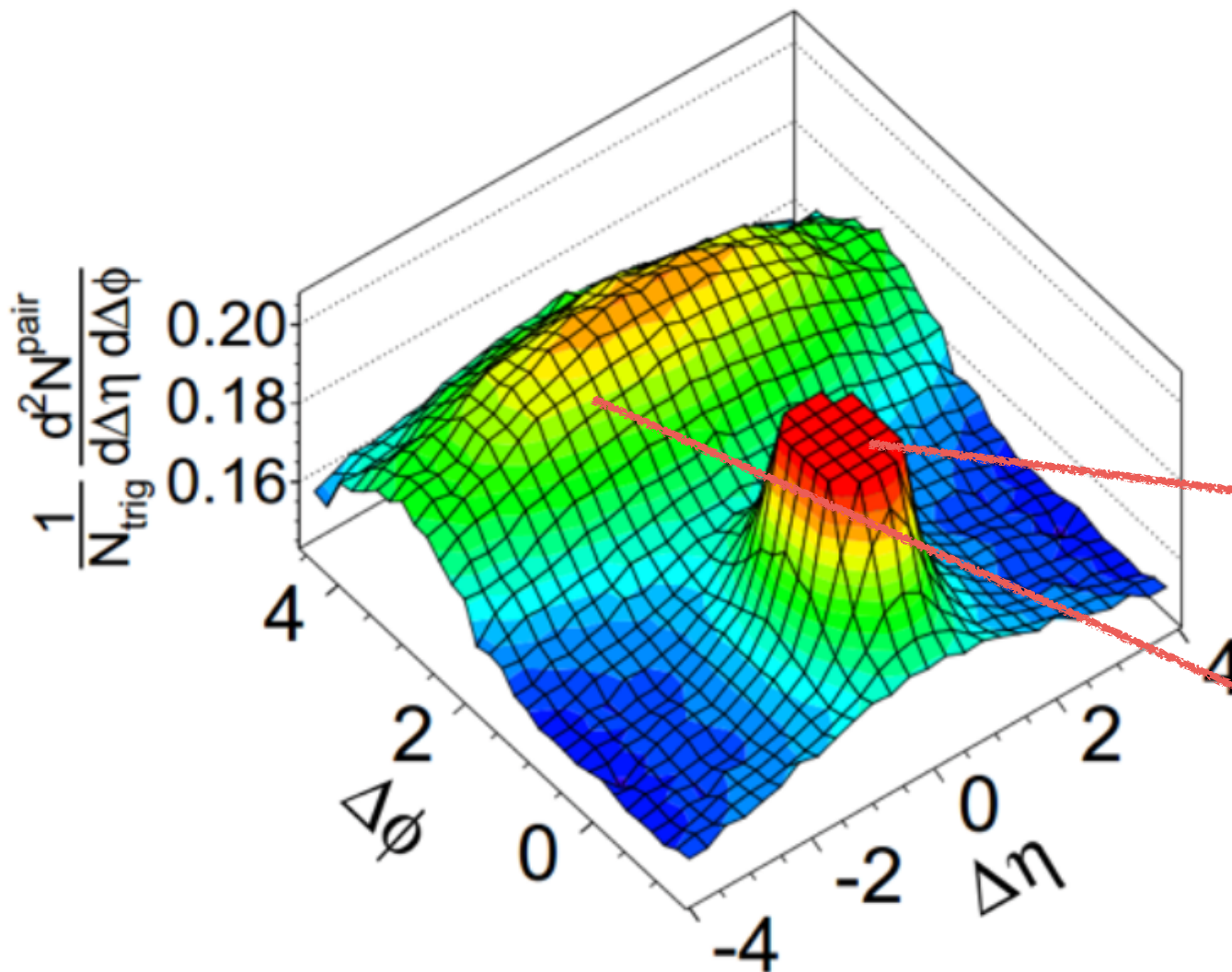
Elliptic flow

- Estimation of the reaction plane even-by-event difficult if multiplicities are small
- Another way to look at same data, pair correlations

$$C(\phi_1, \phi_2) = \frac{\langle \frac{dN}{d\phi_1} \frac{dN}{d\phi_2} \rangle_{\text{events}}}{\langle \frac{dN}{d\phi_1} \rangle_{\text{events}} \langle \frac{dN}{d\phi_2} \rangle_{\text{events}}} = 1 + 2 \sum_m v_m^2 \cos(m(\phi_1 - \phi_2))$$

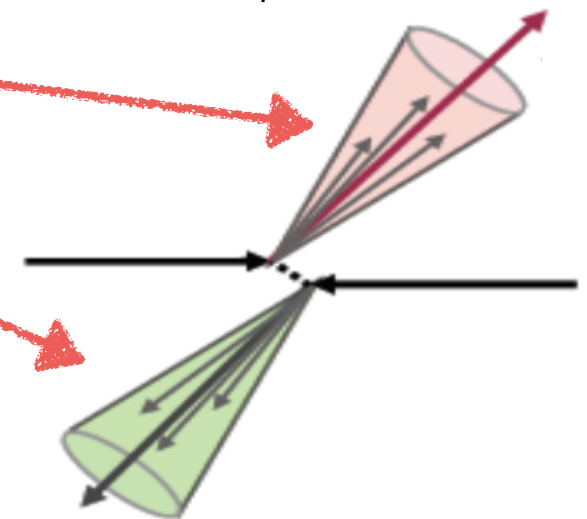
Elliptic flow

- Pair correlations in p-p:



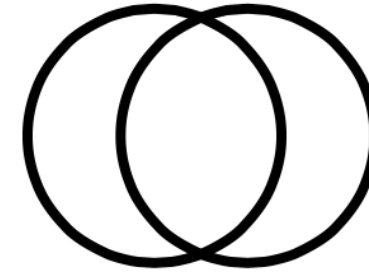
- Nontrivial correlation pattern from elementary processes:

- “Jet peak” from jet fragmentation
- Far side ridge from the back-to-back component



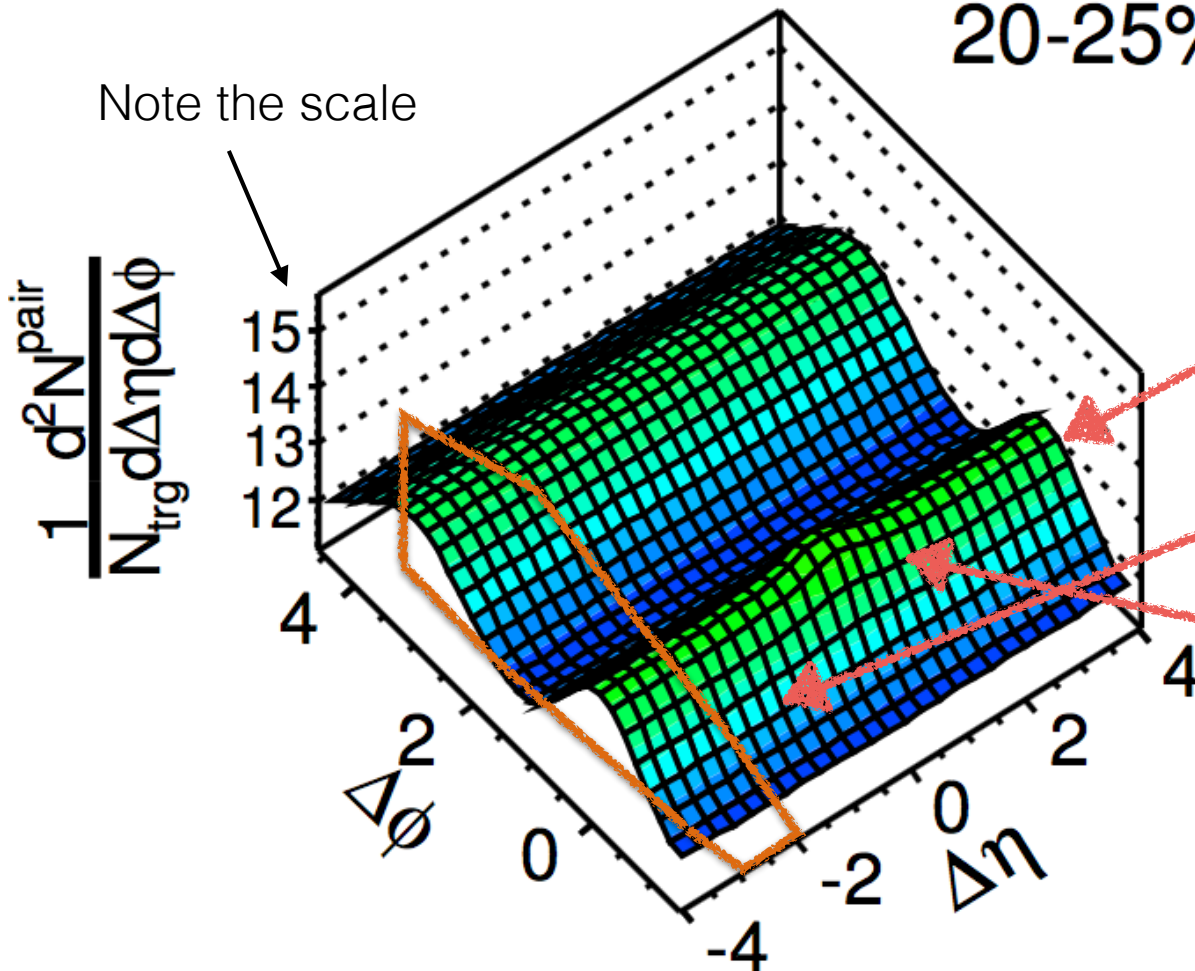
Elliptic flow v_2

- Pair correlations in Pb-Pb:



20-25%

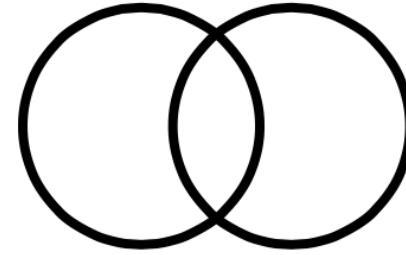
Note the scale



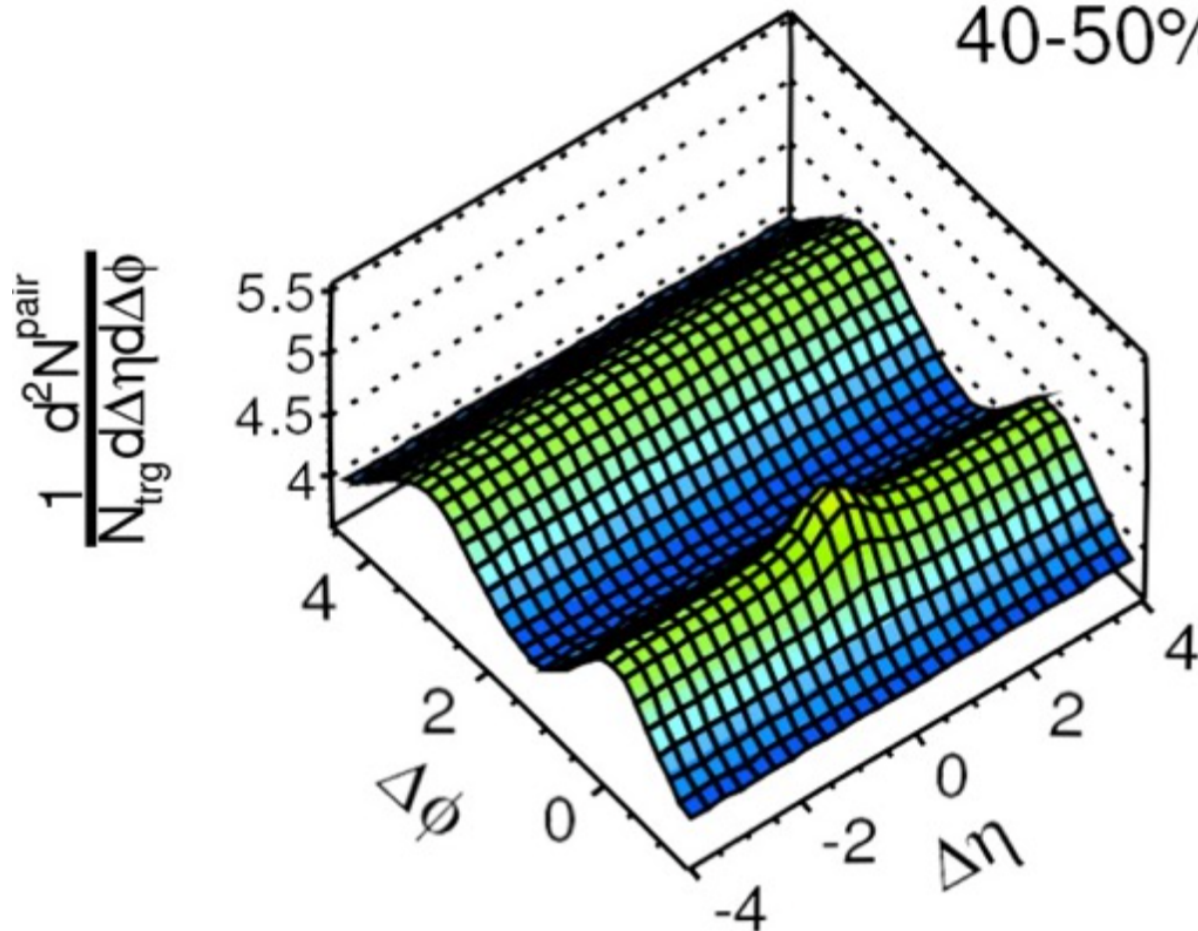
- Ridge: large rapidity correlation from boost invariance
- Azimuthal structure from flow, dominated by v_2
- Tiny remainder of a jet peak

Elliptic flow v_2

- Pair correlations in Pb-Pb:



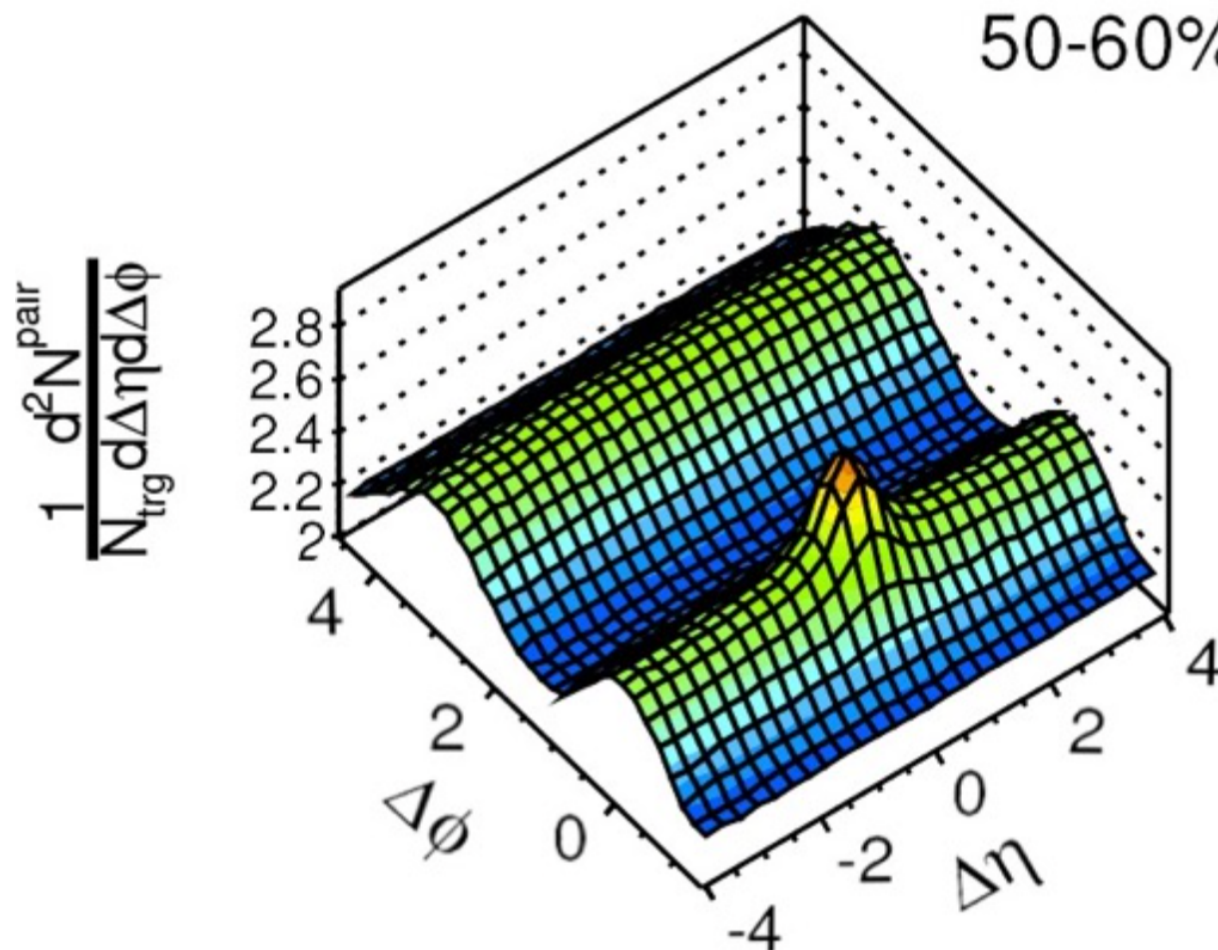
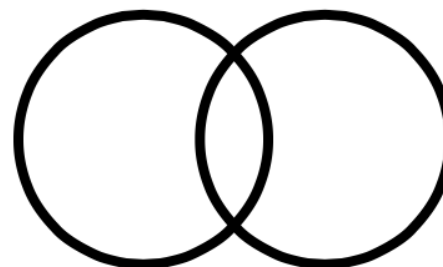
40-50%



Elliptic flow v_2

- Pair correlations in Pb-Pb:

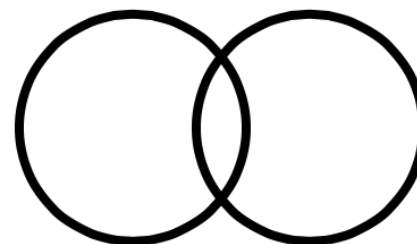
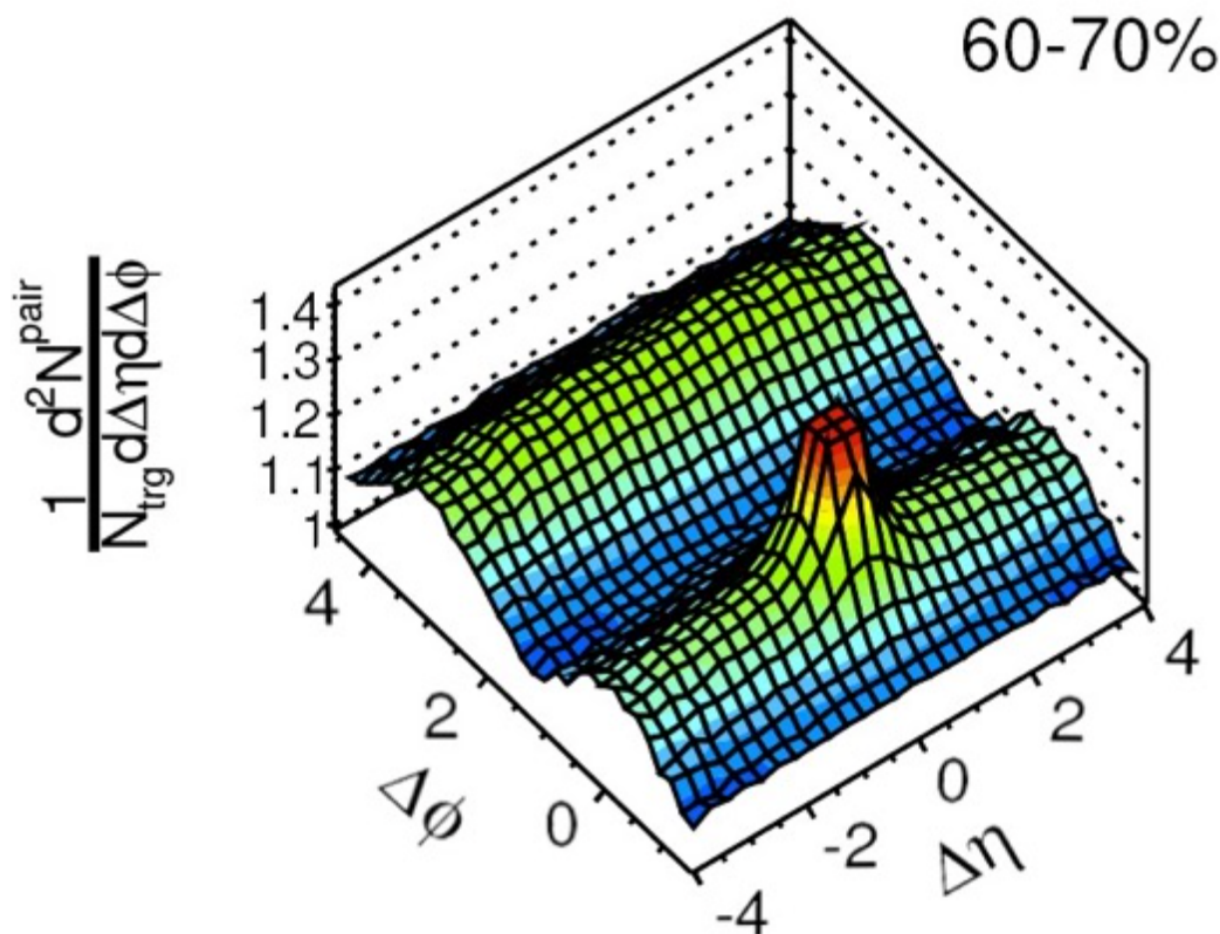
50-60%



- Only the $\Delta\phi$ structure evolves as function of centrality

Elliptic flow v_2

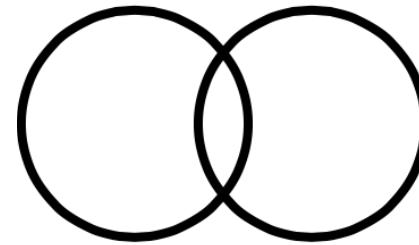
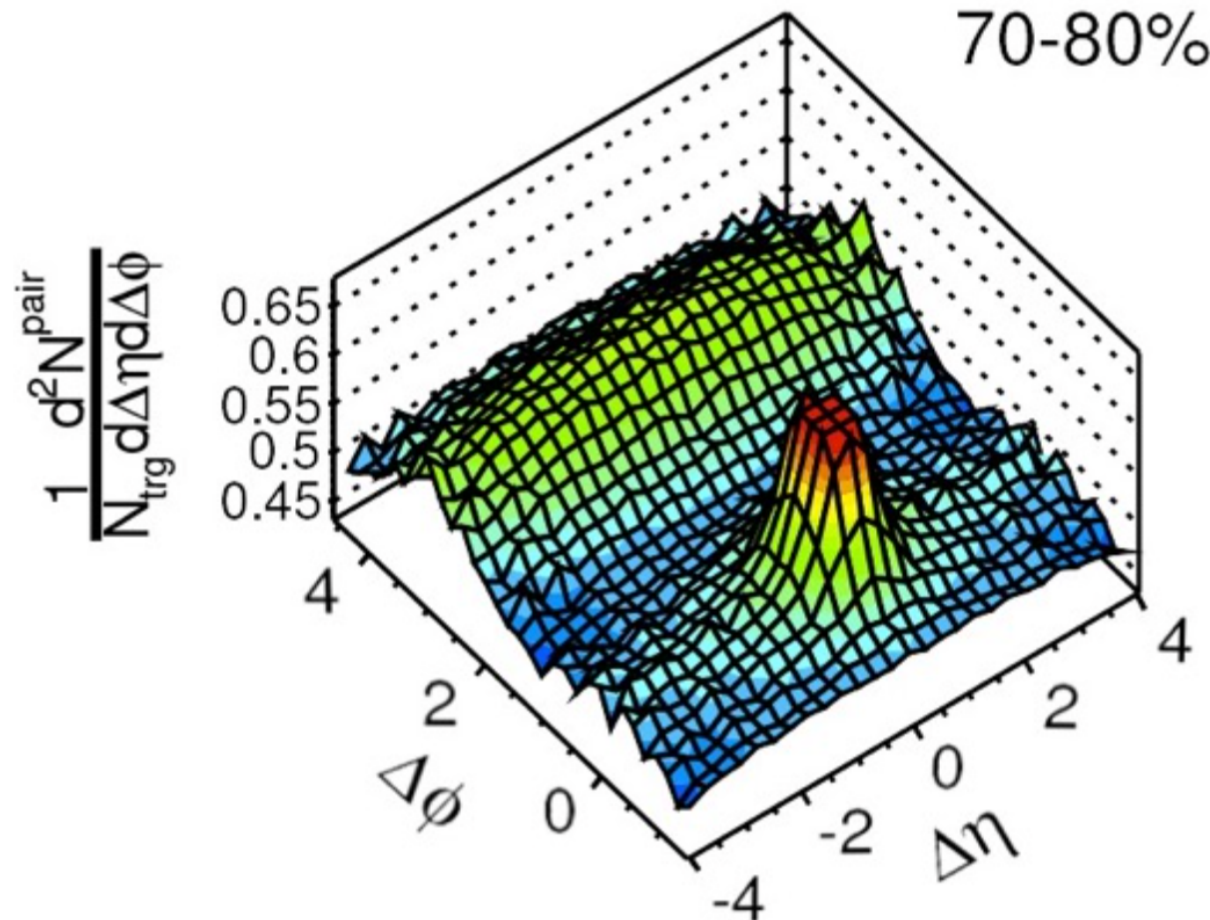
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- Only the $\Delta\phi$ structure evolves as function of centrality

Elliptic flow v_2

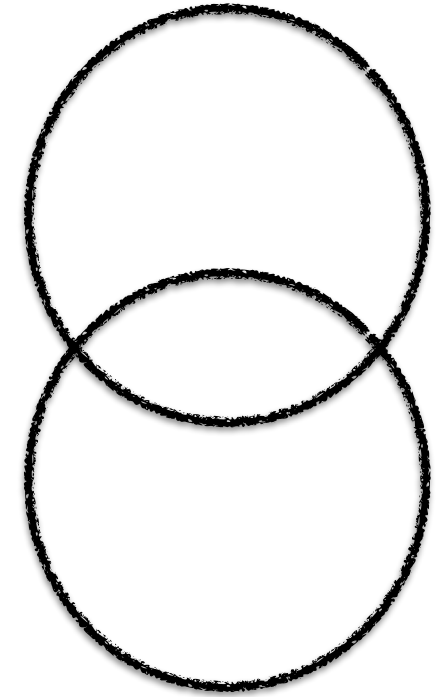
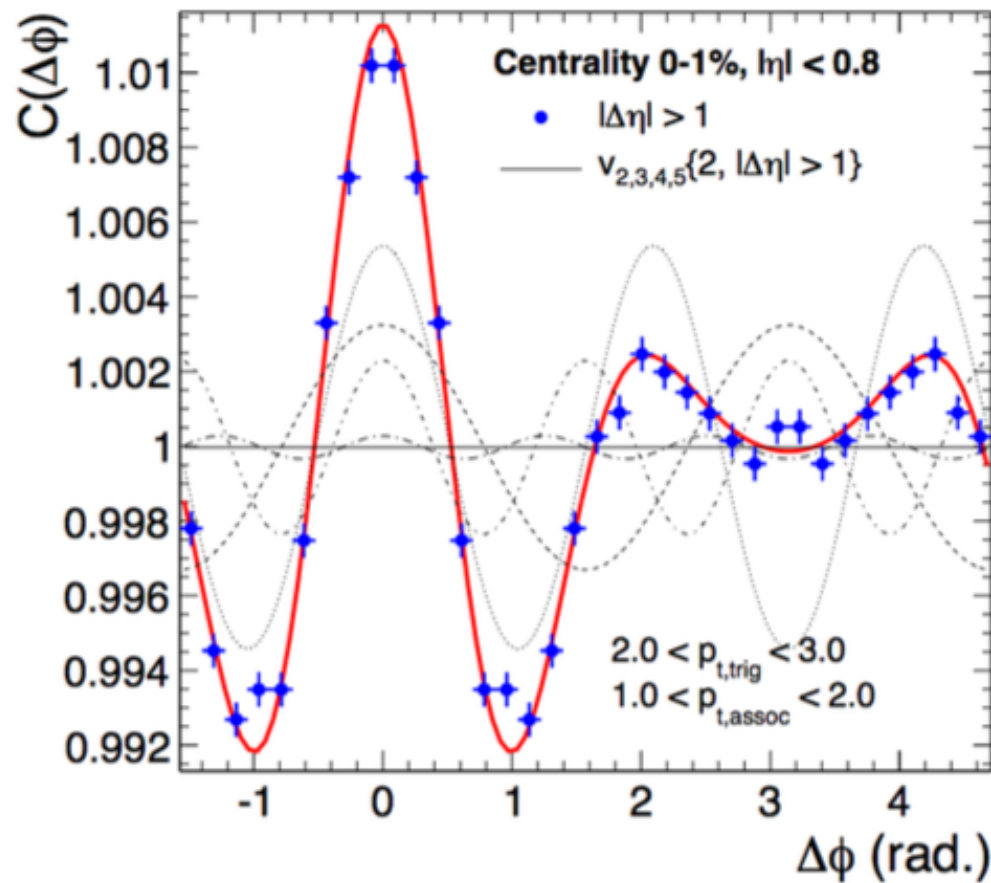
- Pair correlations in Pb-Pb:



- Most peripheral collisions look like p-p collisions again with “near-side jet”

Triangular flow v_3

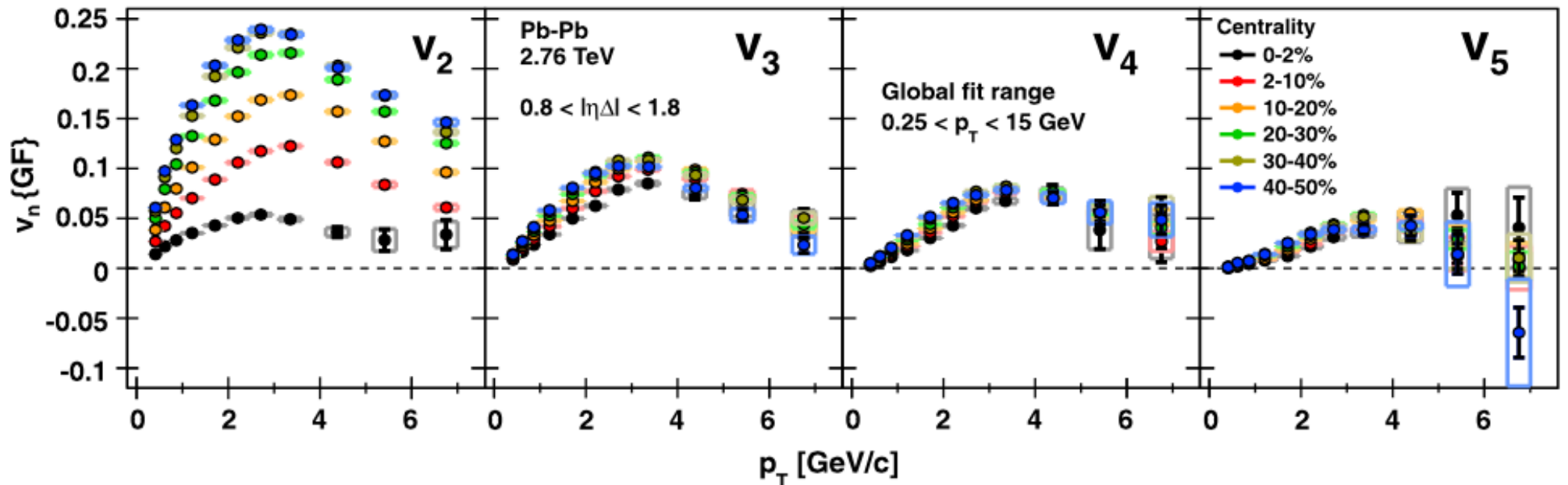
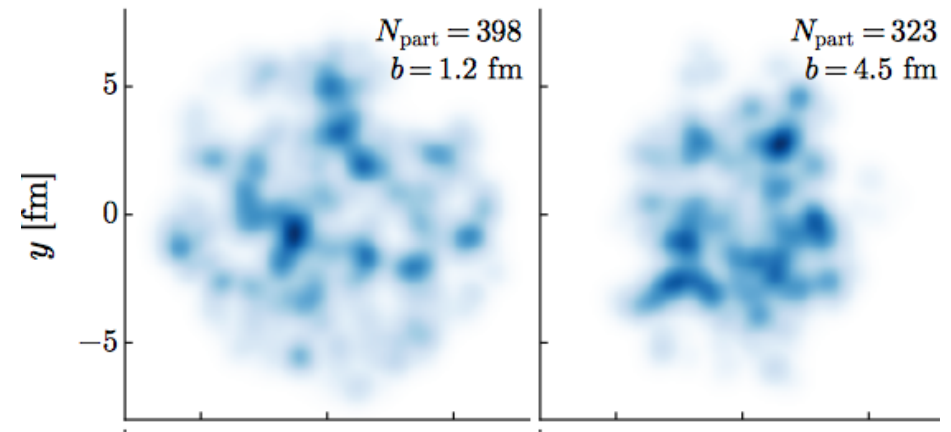
- Naively the geometry has a $\phi \rightarrow \phi + \pi$ symmetry
- Expect v_3, v_5, \dots to be zero



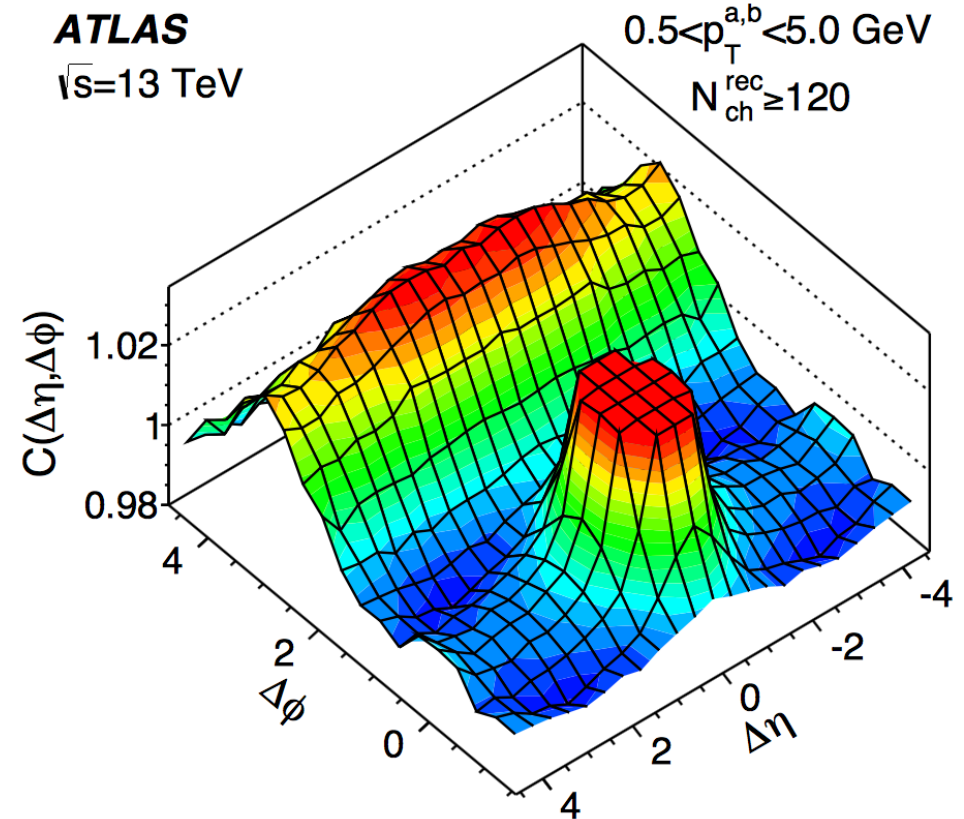
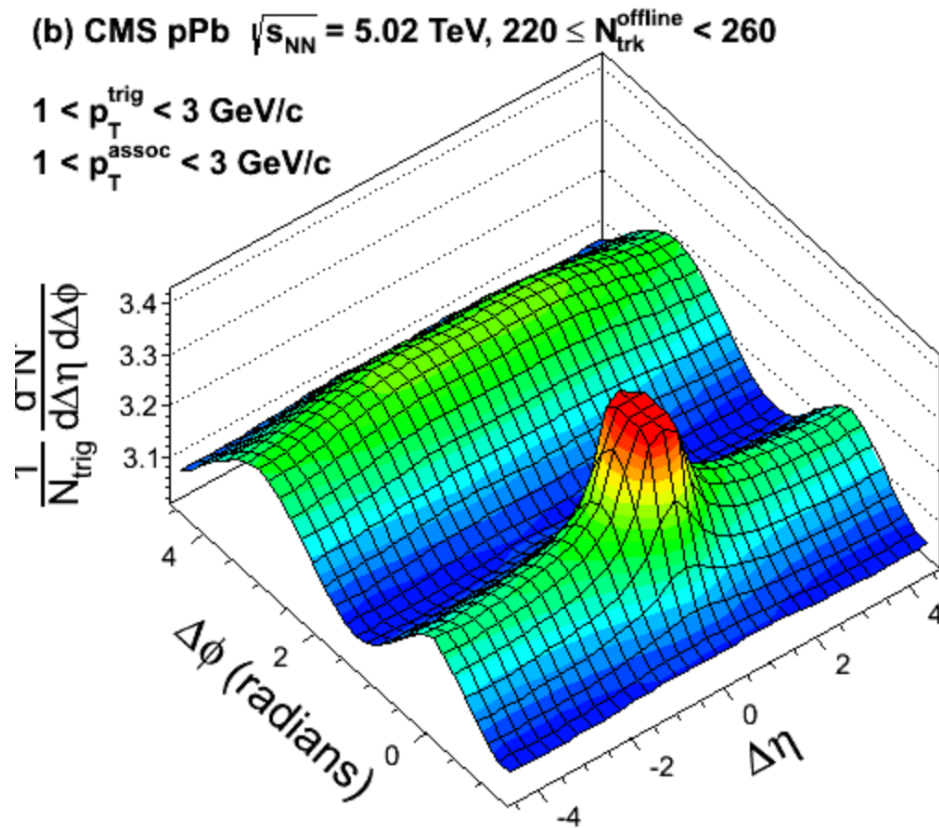
- All coefficients seem to be non-zero!

Triangular flow

- Nuclear geometry not smooth, event-by-event fluctuations of locations of nuclei
- Triangular flow driven by fluctuations
- Picture corroborated by insensitivity to centrality:



Flow in small systems

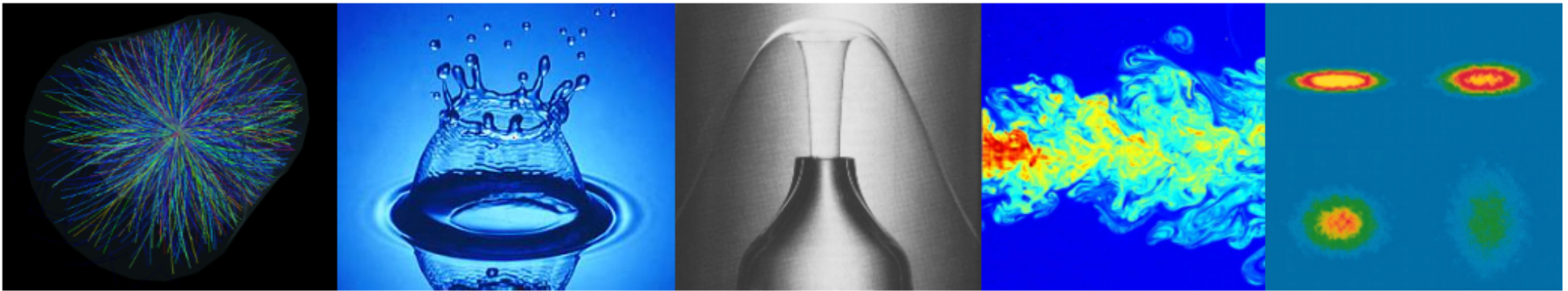
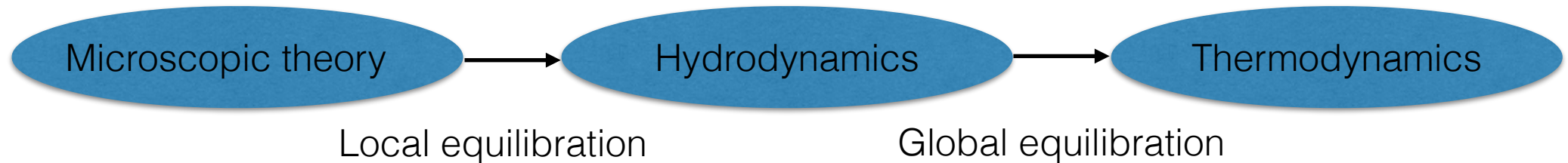


- Ridge-like pattern clearly visible in p-Pb collisions! A formation of a very small liquid?
- Ridge even present at very high multiplicity p-p collisions! What to make out of this? Flow in p-p? At most a very small correction...

Fluid dynamics

Fluid dynamics

- Hydrodynamics is a low energy effective theory describing long distance, late time behaviour of averaged macroscopic features of the system
 - Applicable to a very generic set of theories
 - Assumes that matter is close to local thermal equilibrium
 - Microscopic details of the theory are encapsulated by the inputs of hydrodynamics:
 - EoS, shear viscosity η , bulk viscosity ζ , ...



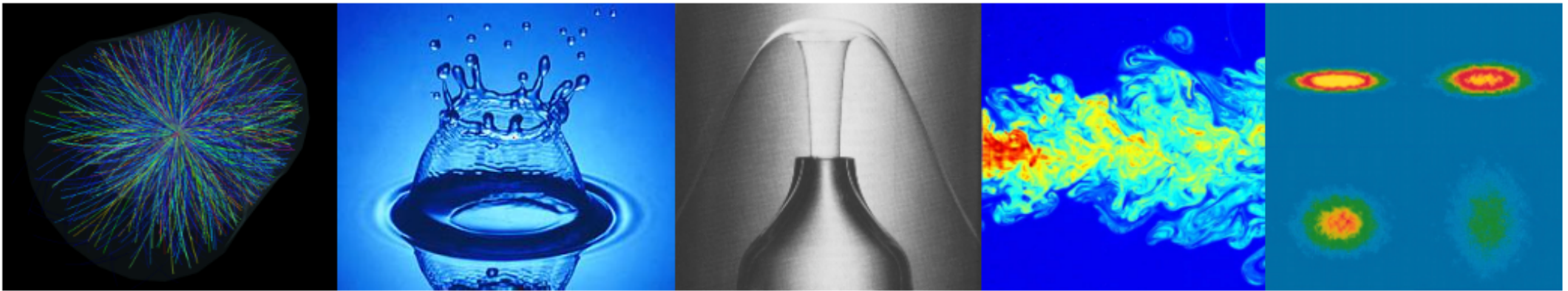
Fluid dynamics

- Requirement: no explicit or spontaneous breaking of Lorentz symmetry
 - Distances larger than mean free path, times larger than scattering rate:

$$\Delta x \gg l_{mfp}, \Delta t \gg \tau_{sc}$$

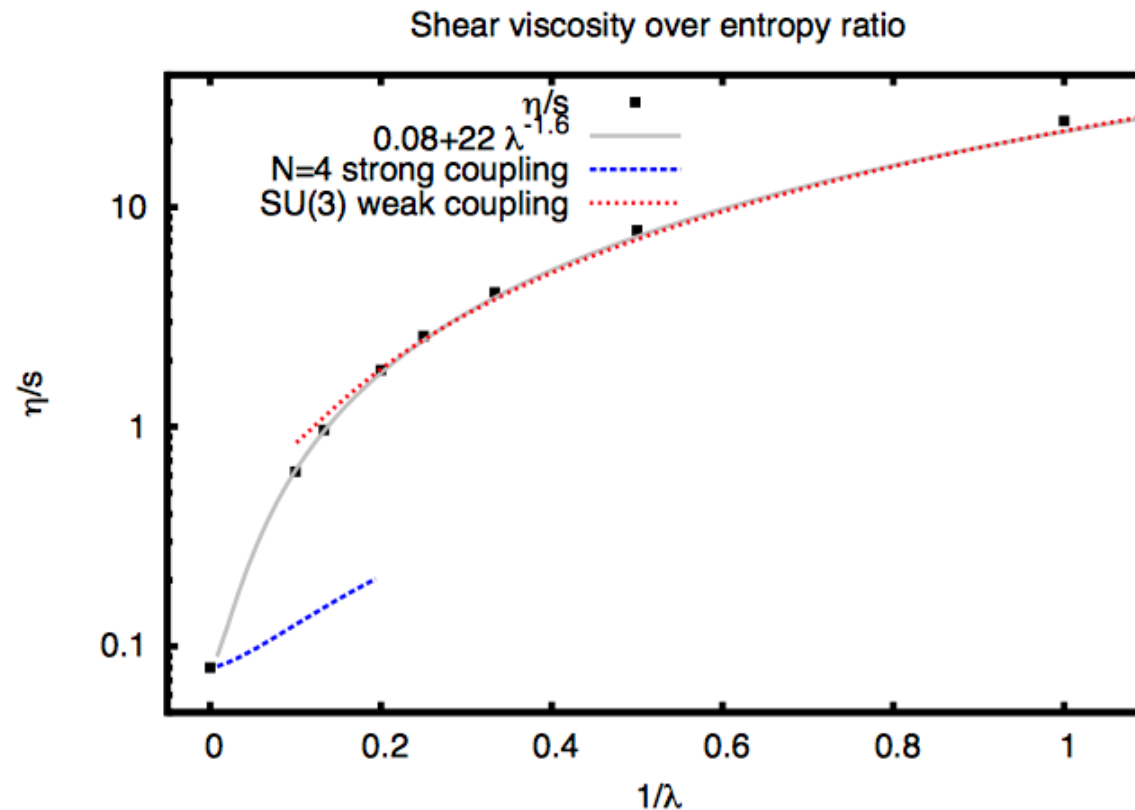
- Systems with sufficiently smooth variation

$$\partial_i \epsilon \ll l_{mfp}^{-1} \epsilon$$



Fluid dynamics

- Strategy in heavy ion collisions:
 - Determine the material properties of QGP by varying the material properties and matching to data
 - Compare the deduced material properties with approximate analytical calculations



Fluid dynamics

- Consider matter in global thermal equilibrium, described by an energy momentum tensor

$$T^{\mu\nu}$$

- What can the energy momentum tensor look like?
- Energy momentum tensor rank-2 tensor. Must be constructed from available rank-2 tensors and vectors
- Available structures are

$$g^{\mu\nu} \quad \text{metric}$$

$$u^{\mu} \quad \text{flow velocity, defining rest frame of the fluid}$$

- The coefficients of the operators from comparing with thermodynamics

$$T^{\mu\nu} = \epsilon u^{\mu} u^{\nu} + p(g^{\mu\nu} + u^{\mu} u^{\nu})$$

- In thermal equilibrium energy density and pressure related through the equation of state (available from lattice!)

Fluid dynamics

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + p(g^{\mu\nu} + u^\mu u^\nu)$$

\uparrow
fluid flow velocity

- Now, introduce a small deviation from equilibrium, such that the energy density and the flow velocities are smooth functions of the coordinate

$$\epsilon = \epsilon(x) \quad u^\mu = u^\mu(x)$$

- If the gradients are smooth enough, the system still stays in *local thermal equilibrium*.
- Then, independent of the microphysics, the evolution of energy momentum tensor is dictated by energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0 \quad \longrightarrow \quad \begin{aligned} u^\mu \partial_\mu \epsilon + (\epsilon + p) \nabla_\mu u^\mu &= 0, \\ (\epsilon + p) u^\mu \nabla_\mu u^\nu + (g^{\nu\mu} + u^\nu u^\mu) \partial_\mu p &= 0. \end{aligned}$$

Ideal hydrodynamics!

Fluid dynamics

$$\begin{aligned}u^\mu \partial_\mu \epsilon + (\epsilon + p) \nabla_\mu u^\mu &= 0, \\(\epsilon + p) u^\mu \nabla_\mu u^\nu + (g^{\nu\mu} + u^\nu u^\mu) \partial_\mu p &= 0.\end{aligned}$$

- In the nonrelativistic limit $u^\mu \approx (1, 0, 0, v)$, $\epsilon \approx \rho$ mass density

$$u^\mu \partial_\mu \approx \partial_t + \vec{v} \cdot \vec{\partial} + \mathcal{O}(|\vec{v}|^2) \quad (g^{i\nu} + u^i u^\nu) \partial_\nu \approx \partial^i + \mathcal{O}(|\vec{v}|)$$

Euler equations:

$$\begin{aligned}\partial_t \vec{v} + (\vec{v} \cdot \vec{\partial}) \vec{v} &= -\frac{1}{\rho} \vec{\partial} p, \\ \partial_t \rho + \rho \vec{\partial} \cdot \vec{v} + \vec{v} \cdot \vec{\partial} \rho &= 0.\end{aligned}$$

Fluid dynamics

- Perturb the system now with larger gradients. Energy momentum tensor receives corrections to the thermal form $\Delta^{\mu\nu} = (g^{\mu\nu} + u^\mu u^\nu)$

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + p \Delta^{\mu\nu} + \Pi^{\mu\nu}$$

↑
“viscous” stress tensor

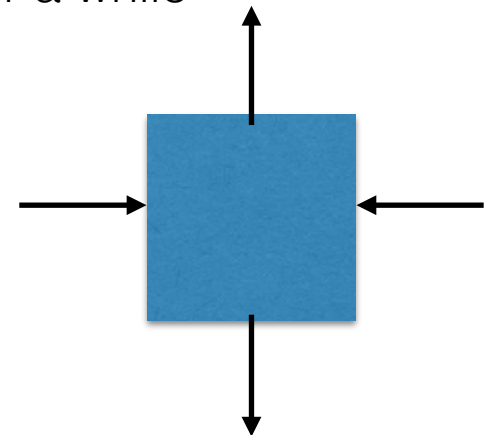
- Customary to decompose to traceless and remainder

$$\Pi^{\mu\nu} = \pi^{\mu\nu} + \Delta^{\mu\nu} \Pi$$

↗
“Shear stress”

↑
“Bulk viscous pressure”

- Bulk viscous pressure: Perform rapid compression to fluid, for a while pressure below thermodynamical pressure
- Shear stress: Anisotropic pressure caused by flow



Fluid dynamics

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + p \Delta^{\mu\nu} + \Pi^{\mu\nu}$$

$$\Pi^{\mu\nu} = \pi^{\mu\nu} + \Delta^{\mu\nu} \Pi$$

- Viscous stress tensor must be also constructed from $g^{\mu\nu}$, u^μ and from additional gradient vector ∂_μ
- Hydrodynamical gradient expansion: grade terms in powers of ∂_μ
Constitutive equations:

- No derivatives: Ideal hydro, $\Pi^{\mu\nu} = 0$
- One derivative: “viscous hydro”

$$\begin{aligned} \Pi &= -\zeta \nabla_\mu u^\mu + \dots, \\ \pi^{\mu\nu} &= -2\eta \left(\frac{1}{2} \Delta^{\mu\alpha} \Delta^{\nu\beta} + \frac{1}{2} \Delta^{\mu\beta} \Delta^{\nu\alpha} - \frac{1}{3} \Delta^{\mu\nu} \Delta^{\alpha\beta} \right) \nabla_\alpha u_\beta + \dots \end{aligned}$$

Bulk viscosity Shear viscosity = 1st order transport coefficients

- Two derivatives: “2nd order hydro”, $\tau_\pi, \lambda_1, \lambda_2, \lambda_3, \dots$

Fluid dynamics

- Transport coefficients properties of equilibrium system and can in principle be obtained from microscopic theory.

$$G_{xy,xy}^R(\omega,0) \equiv \int dt dx e^{i\omega t} \Theta(t) \left\langle [T_{xy}(t,x), T_{xy}(0,0)] \right\rangle_{eq}$$

$$\eta \equiv -\lim_{\omega \rightarrow 0} \frac{1}{\omega} \text{Im} G_{xy,xy}^R(\omega,0)$$

- Correlation function required at non-zero frequency (corresponding to time-like separation of the operators). No reliable lattice calculation available.
- Has been calculated in
 - perturbative QCD using effective kinetic theory methods (more in this later)

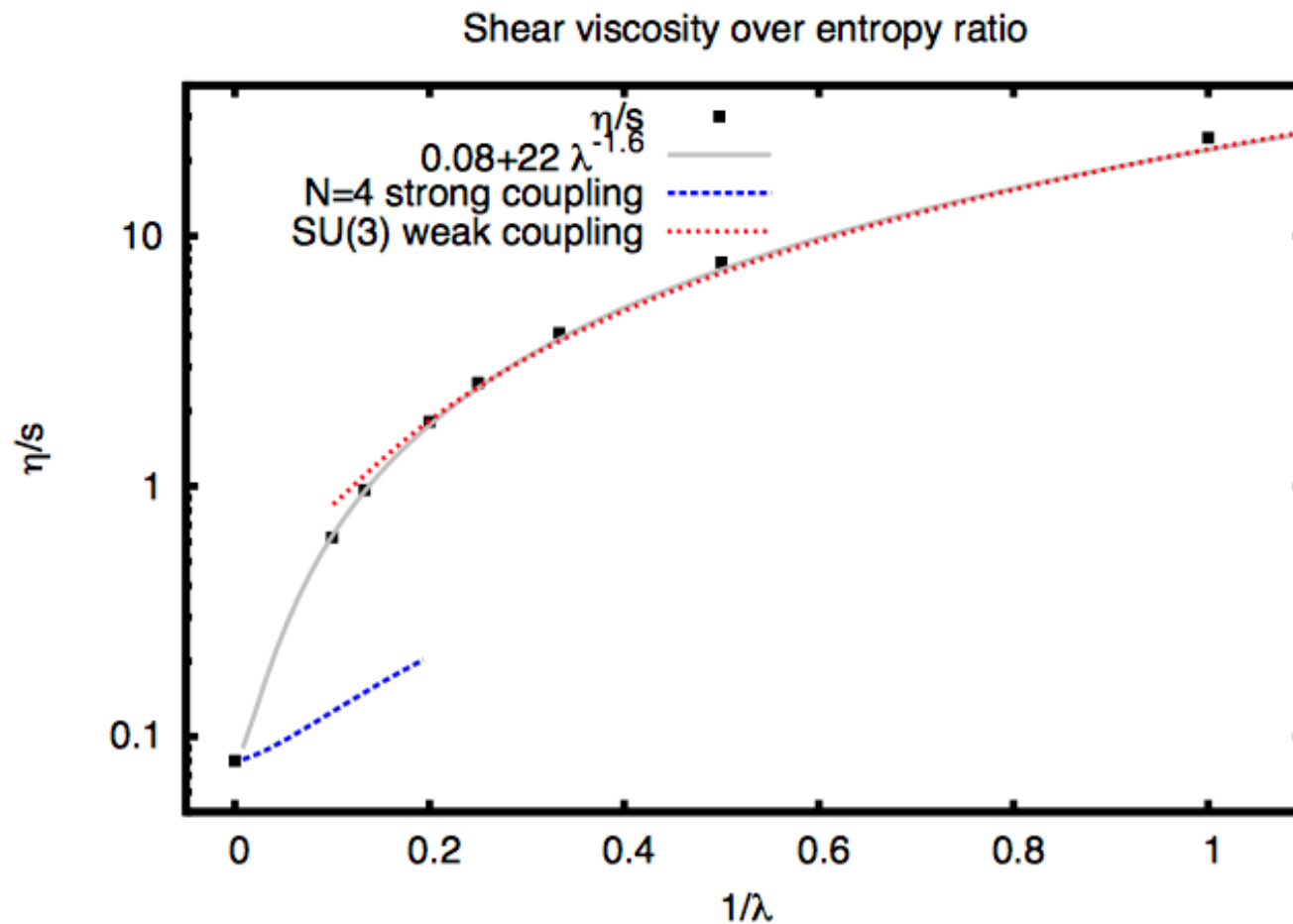
$$\frac{\eta}{s} = \frac{34.784}{\lambda^2 \log(4.789/\sqrt{\lambda})} \quad \lambda = g^2 N_c$$

- In N=4 Super-Yang Mills theory at the limit of large N_c and large 't Hooft coupling $\lambda = g^2 N_c$

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

- Bulk viscosity vanishes for both theories

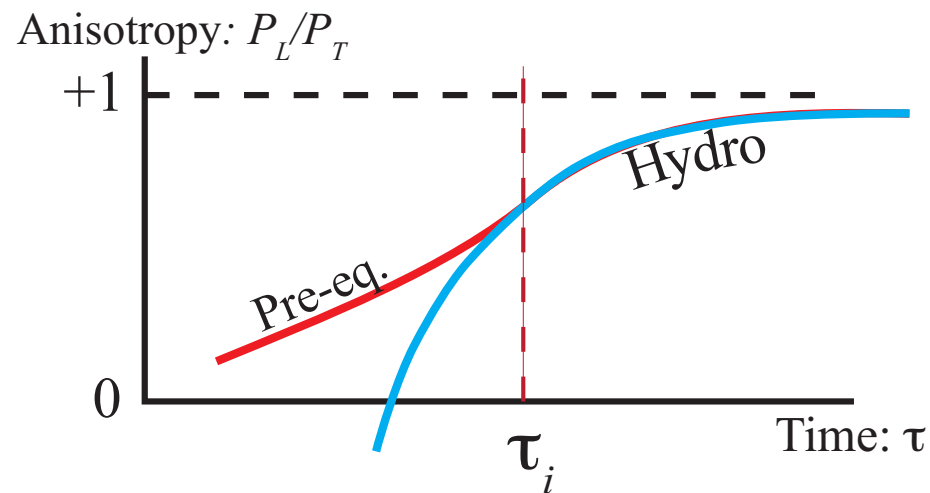
Fluid dynamics



Viscosity as a measure of the interaction strength of the plasma

Fluid dynamics

- Hydrodynamical equations can be solved at some given initial conditions
 - Viscous hydro in fact acausal, need a “resummation” Israel-Stuart theory
- The collision geometry is such that the gradients diverge at early times, and corrections to ideal hydro become large
- The hydrodynamical simulation then has to be initialized at a time τ_i , when the system is sufficiently close to local thermal equilibrium
- It is not completely understood what the correct initial conditions are for heavy ion collisions, they affect any determination of transport coefficients



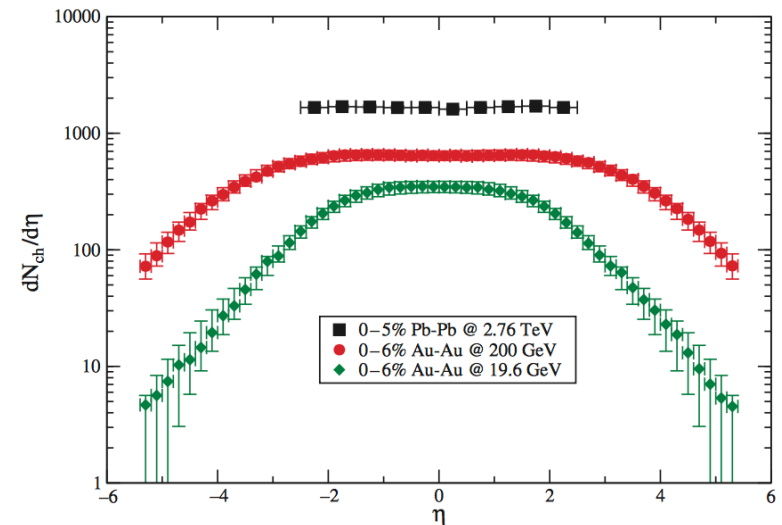
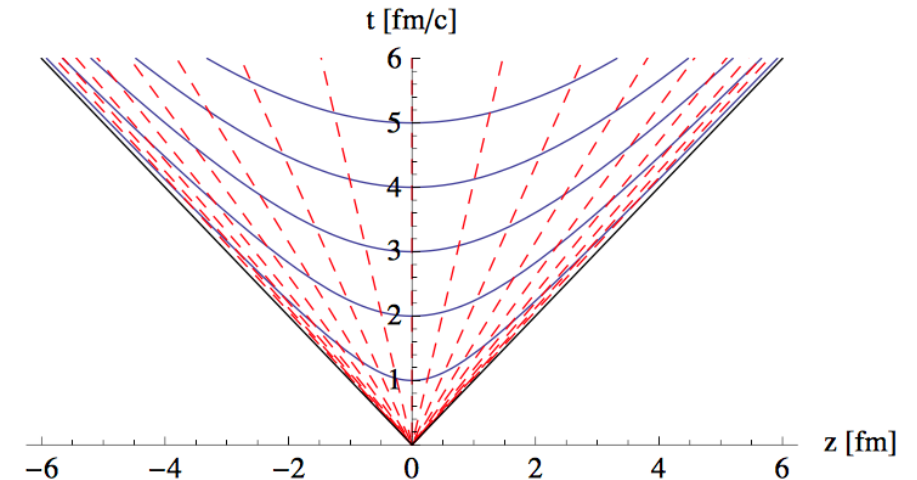
Bjorken boost invariance

- Bjorken's guess:
$$v_z(t, x, y, z) = z/t$$
- Leads to boost invariance in z-direction
- Coordinates:

$$\tau = \sqrt{t^2 - z^2}$$

$$\eta = \operatorname{arctanh}(z/t)$$

- Quantities independent of η
$$\epsilon = \epsilon(\tau, x, y)$$
- Boost symmetry an idealization but it is reasonable accurate close to midrapidity



Bjoerken model

- Further assuming translational invariance, the system can be solved analytically
- A simple model of the inner region of central collision at early times
- Start with initial condition (no transverse flow)

$$\epsilon = \epsilon(\tau_0), \quad u^\mu = (1, 0, 0, 0)$$

- The viscous (first order) hydro equations simplify to

$$\partial_\tau \epsilon + (\epsilon + p) \frac{1}{\tau} - \left(\frac{4}{3} \eta + \zeta \right) \frac{1}{\tau^2} = 0$$

- Solution depends on equation of state and viscosities $p(\epsilon), \eta(\epsilon), \zeta(\epsilon)$
- In terms of effective temperature $\epsilon \sim T^4$

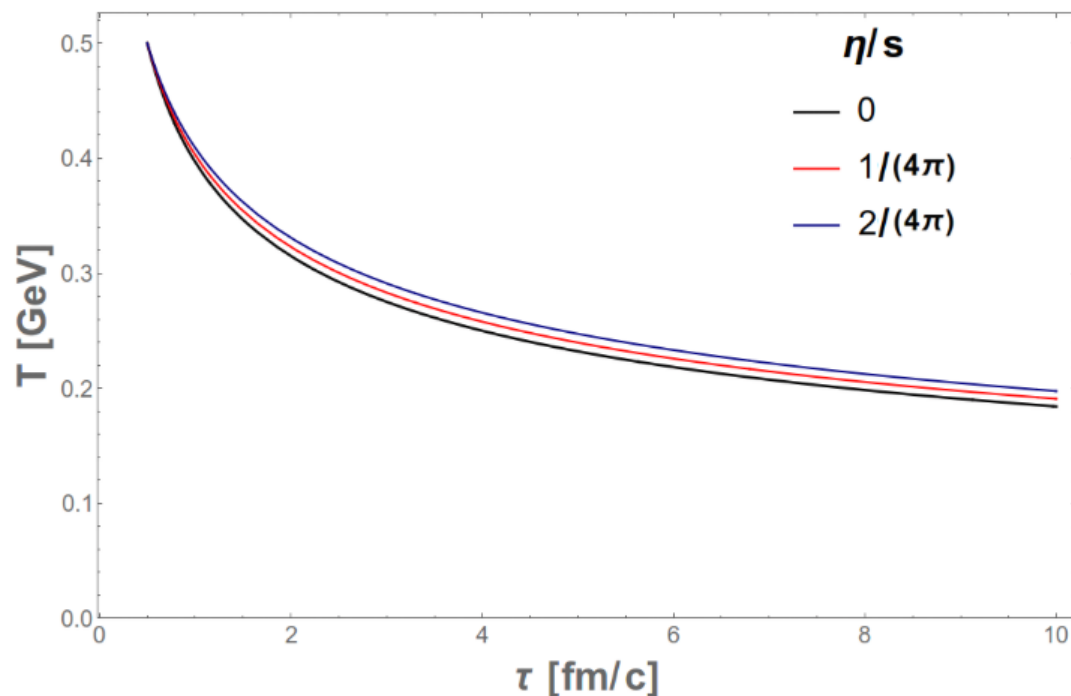
$$\partial_\tau T + \frac{T}{3\tau} \left(1 - \frac{4\eta/3 + \zeta}{sT\tau} \right) = 0$$

Bjoerken model

- Solution for constant η/s , and $\zeta = 0$, and ideal EoS $\epsilon = 3p$

$$T(\tau) = T(\tau_0) \left(\frac{\tau_0}{\tau} \right)^{1/3} \left[1 + \frac{2}{3\tau_0 T(\tau_0)} \frac{\eta}{s} \left(1 - \left(\frac{\tau_0}{\tau} \right)^{2/3} \right) \right]$$

↑
Ideal hydro
↑
Viscous correction

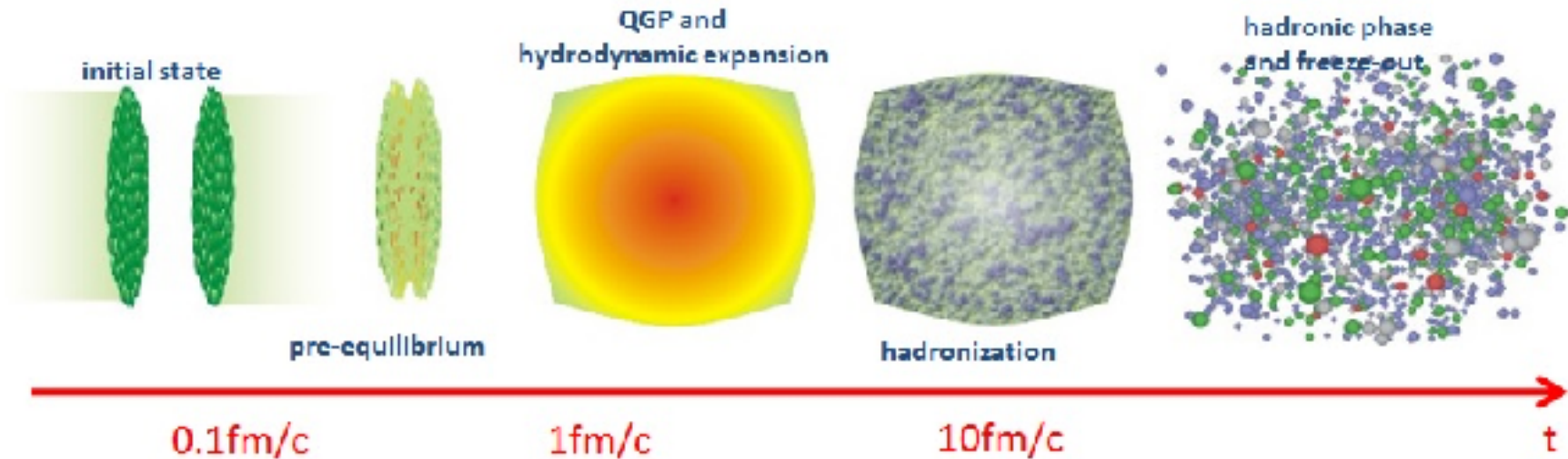


Total entropy

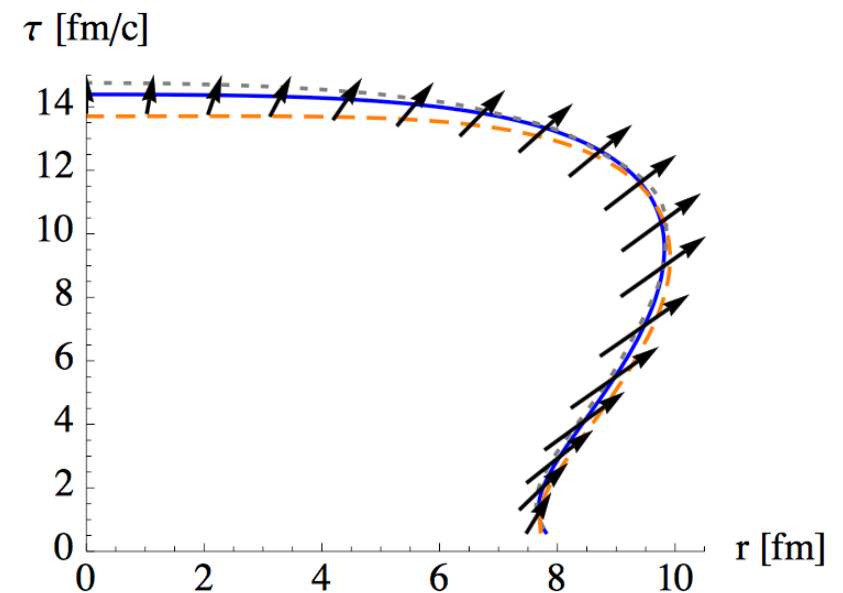
$$\tau S = \tau \frac{\epsilon + P}{T} \sim T^3 \tau$$

- No entropy generation by ideal hydro
- Small amount of entropy generation (heating) from viscous effects

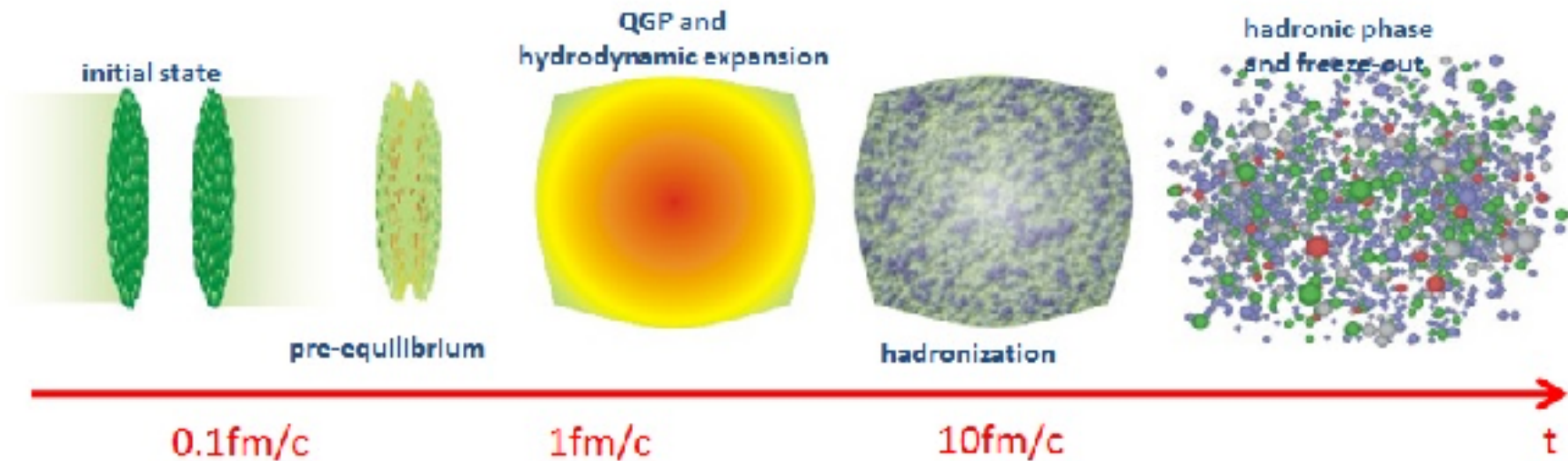
Kinetic freeze-out



- Eventually temperature low and scattering becomes infrequent, the fluid dynamics smoothly goes to free streaming of hadrons \rightarrow kinetic Freeze-out
- Freeze-out takes place at $T \lesssim T_c$ perhaps can get away from hadronization

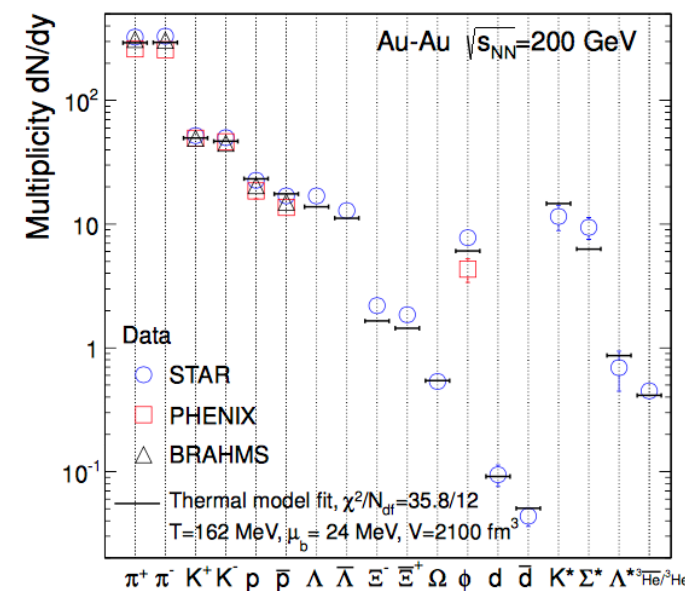


Kinetic freeze-out



- Eventually temperature low and scattering becomes infrequent, the fluid dynamics smoothly goes to free streaming of hadrons -> kinetic Freeze-out
- Freeze-out takes place at $T \lesssim T_c$ perhaps can get away from hadronization
- Guess what the particle distribution is based on hydrodynamical fields

$$f_i = c_i e^{\frac{u_\mu(x) p^\mu}{T(x)}}$$

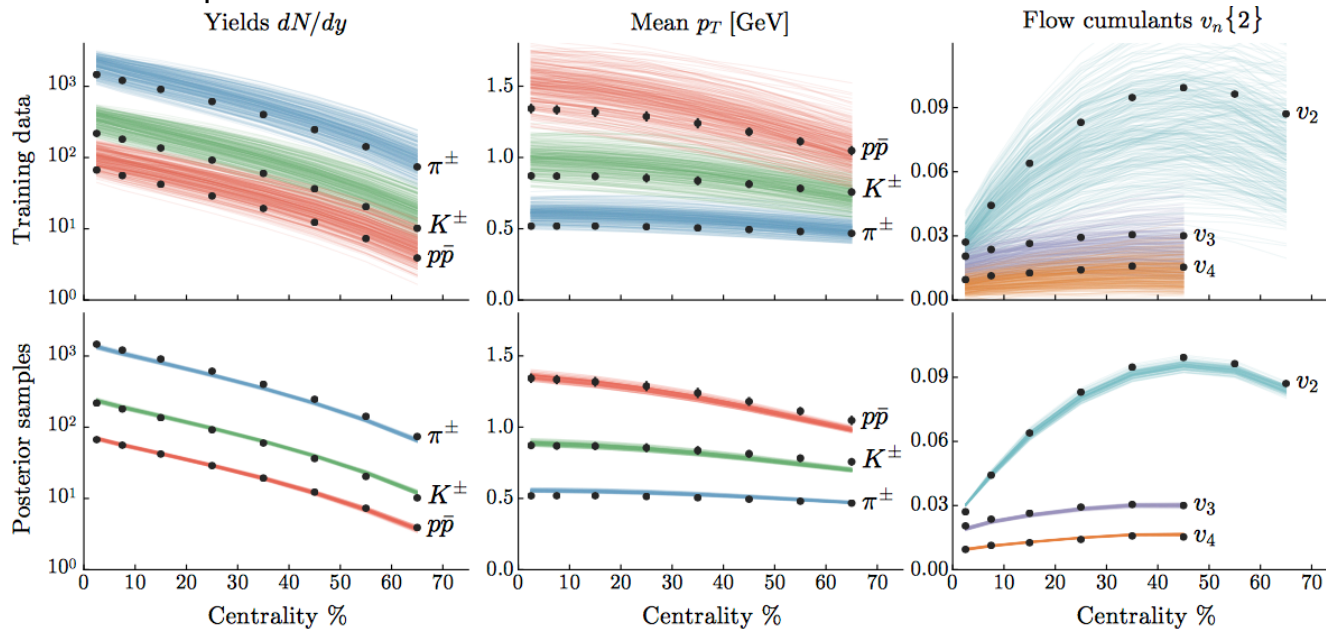
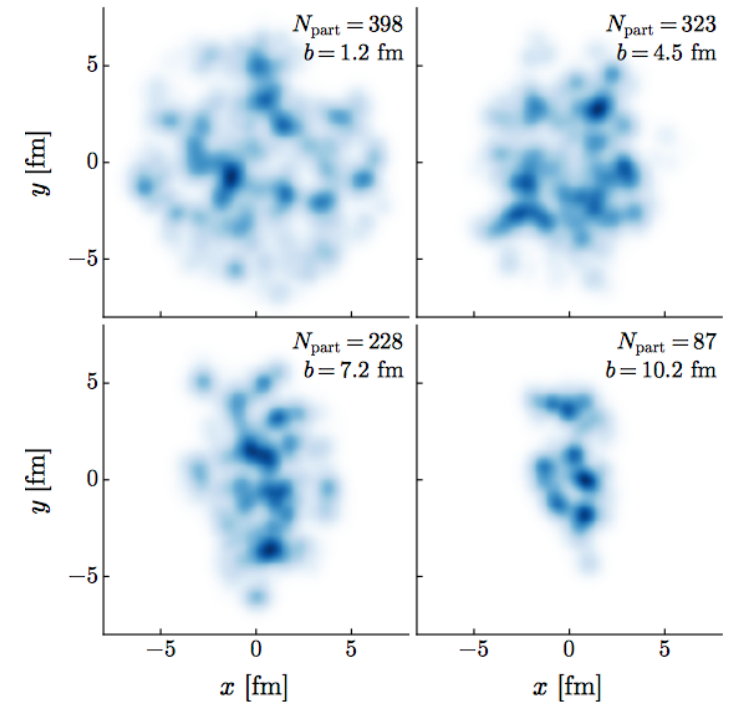


Parameter estimation

- Massively parallel numerical hydro simulations with realistic initial conditions
- Wide range of hydrodynamical parameters

$$(\eta/s)(T) = \begin{cases} (\eta/s)_{\min} + (\eta/s)_{\text{slope}}(T - T_c) & T > T_c \\ (\eta/s)_{\text{hrg}} & T \leq T_c \end{cases}$$

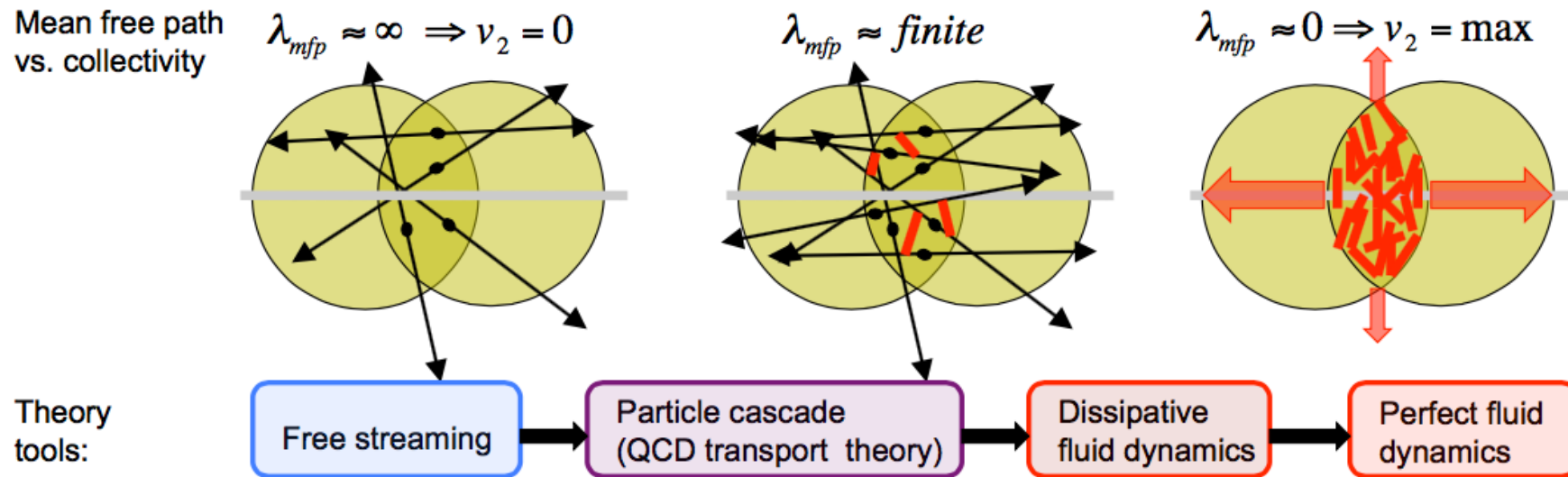
- A large set of observables
- Bayesian analysis to determine the most likely parameter values



Parameter	Calibrated to:	
	Identified	Charged
Normalization	$120.^{+8.}_{-8.}$	$132.^{+11.}_{-11.}$
p	$-0.02^{+0.16}_{-0.18}$	$0.03^{+0.16}_{-0.17}$
k	$1.7^{+0.5}_{-0.5}$	$1.6^{+0.6}_{-0.5}$
w [fm]	$0.48^{+0.10}_{-0.07}$	$0.51^{+0.10}_{-0.09}$
η/s min	$0.07^{+0.05}_{-0.04}$	$0.08^{+0.05}_{-0.05}$
η/s slope [GeV ⁻¹]	$0.93^{+0.65}_{-0.92}$	$0.65^{+0.77}_{-0.65}$
ζ/s norm	$1.2^{+0.2}_{-0.3}$	$1.1^{+0.5}_{-0.5}$
T_{switch} [GeV]	$0.148^{+0.002}_{-0.002}$	—

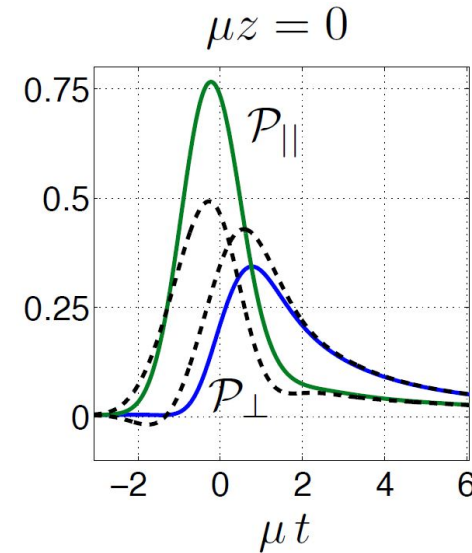
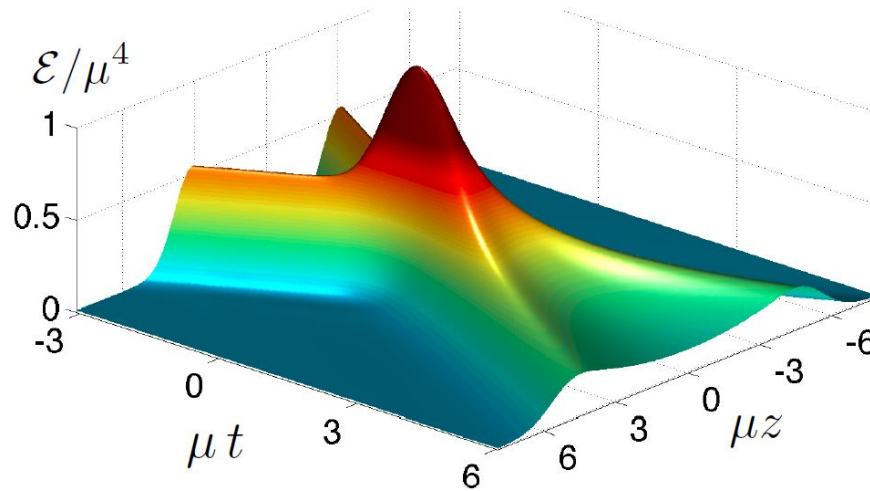
QCD kinetic theory and pre-equilibrium dynamics

Pre-equilibrium dynamics



- At early times, large gradients. Hydrodynamics fails.
- The longer it takes for a pressure to build up, less radial flow, less v_2 etc
- Affects determination of transport coefficients
- Neglected or crudely modelled in most of the current large scale simulations
 - Free streaming, classical YM fields etc...
 - Results sensitive to the initialisation time of hydrodynamical simulation

Pre-equilibrium dynamics

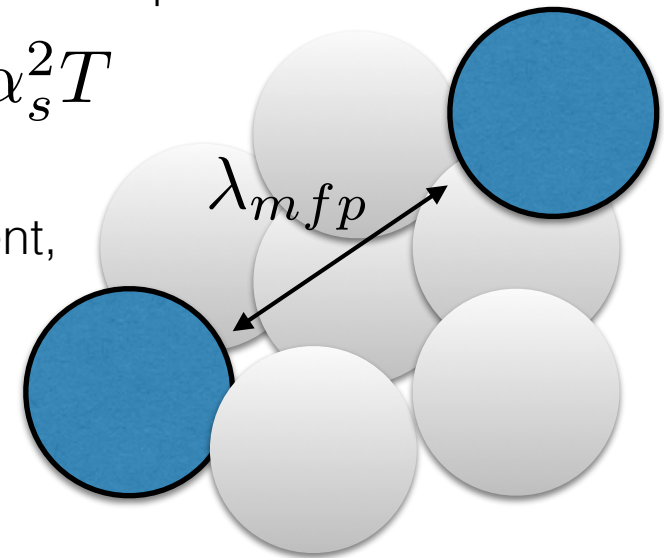
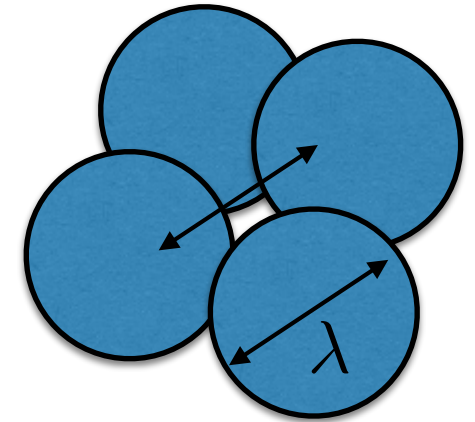


- Pre-equilibrium evolution can be solved for N=4 SUSY at strong coupling,
 - Collision of shock waves in 5 dimensions
 - Onset of hydrodynamics associated with creation of a black hole in the 5th dimension
 - Hydrodynamical flow reached in the mid rapidity region rapidly

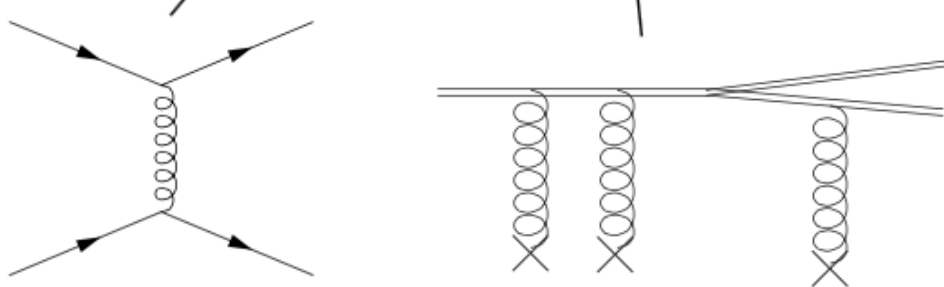
$$\tau_{therm} \sim 1/T$$

QCD kinetic theory

- Consider thermal QCD matter at weak coupling $\alpha_s \rightarrow 0$
- Typical scales:
 - Typical momentum $p \sim T$
 - Interparticle distance $\Delta x \sim 1/T$
 - Thermal wavelength $\lambda \sim 1/p \sim 1/T$
- At weak coupling, scattering only with α_s fraction of particles
 - Mean free path $\lambda_{mfp} \sim 1/\alpha_s^2 T$
- Scale separation implies kinetic theory treatment, can be derived from the Lagrangian in
- Approach to hydrodynamics and transport, jet quenching etc.
- Recent effort to elevate to NLO



QCD kinetic theory

$$\frac{df}{dt} = -C_{2\leftrightarrow 2}[f] - C_{1\leftrightarrow 2}[f] \quad f = \frac{dN_{gluon}}{d^3p d^3x}$$


The diagram shows two Feynman diagrams. The left diagram represents a 2-to-2 scattering process (gluon exchange), with an arrow pointing to the $C_{2\leftrightarrow 2}[f]$ term in the equation. The right diagram represents a 1-to-2 scattering process (gluon emission/absorption), with an arrow pointing to the $C_{1\leftrightarrow 2}[f]$ term in the equation.

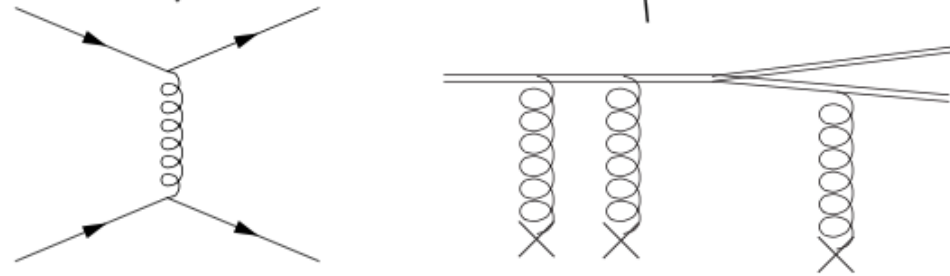
- Evolution equation for (color averaged) quark and gluon distribution
- Contains effective 2to2 and n to (n+1) processes
- Both terms have a non-trivial structure arising from the underlying physical divergences of the underlying processes
- The scattering kernels contain information of a dynamically generated scale

$$m_{screen}^2 \sim \int d^3p \frac{f(p)}{p} \sim \alpha T$$

- Plasma rearranges to screen color charge, $1/m_{screen}$ related to how far a color-electric charge is visible

QCD kinetic theory

$$\frac{df}{dt} = -C_{2\leftrightarrow 2}[f] - C_{1\leftrightarrow 2}[f] \quad f = \frac{dN_{gluon}}{d^3p d^3x}$$



$$C_{2\leftrightarrow 2}[f] = \int_{k,p',k'} |M|^2 [f_p f_k (1 + f_{p'}) (1 + f_{k'}) - f_{p'} f_{k'} (1 + f_p) (1 + f_k)]$$

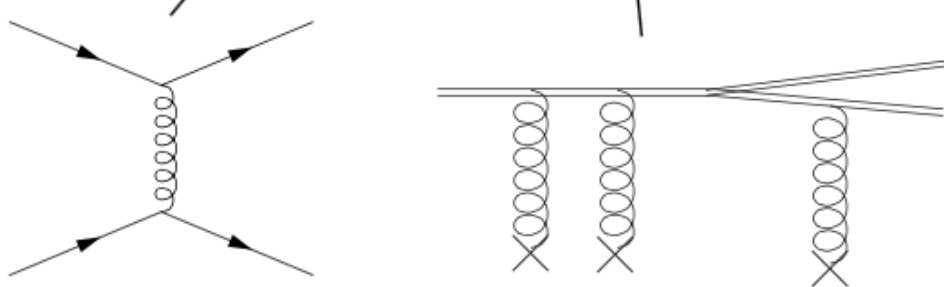
Effective matrix element

Phase space integral

Initial state factors

Quantum mechanical final state factors
+ bose enhancement (- for fermions)

QCD kinetic theory

$$\frac{df}{dt} = -C_{2\leftrightarrow 2}[f] - C_{1\leftrightarrow 2}[f] \quad f = \frac{dN_{gluon}}{d^3p d^3x}$$


$$C_{2\leftrightarrow 2}[f] = \int_{k,p',k'} |M|^2 [f_p f_k (1 + f_{p'}) (1 + f_{k'}) - f_{p'} f_{k'} (1 + f_p) (1 + f_k)]$$

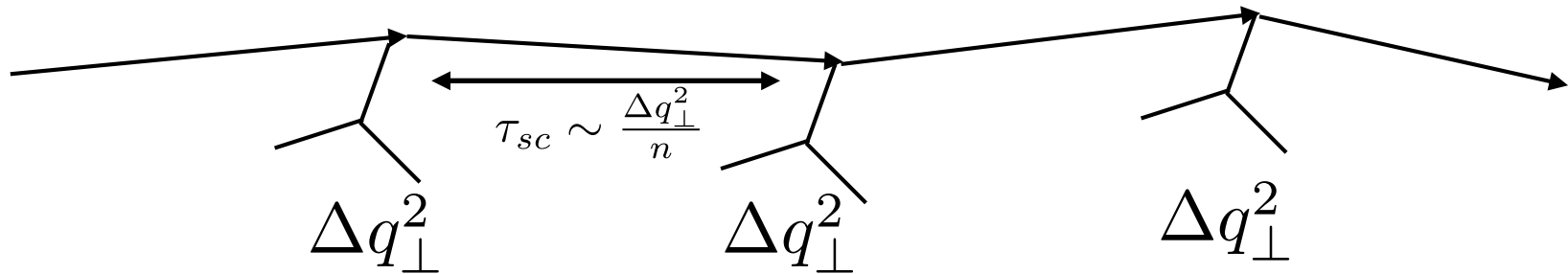
- Naively $|M|^2 = 9 + \frac{(t-u)^2}{s^2} + \frac{(s-u)^2}{t^2} + \frac{(s-t)^2}{u^2}$
- However, in t and u channels Coulombic IR divergence:

- Total scattering rate: $\int |M|^2 \int_p f_p (1 + f_p) \sim n \int d^2q_{\perp} \frac{\alpha_s^2}{(q_{\perp}^2)^2}$

- Regulated by the screening: $\longrightarrow \frac{1}{(q_{\perp}^2 + m_{screen}^2)^2}$

QCD kinetic theory

- What does the infrared sensitivity mean?
- Consider a hard particle travelling in medium and undergoing uncorrelated kicks (“collisional energy loss”)

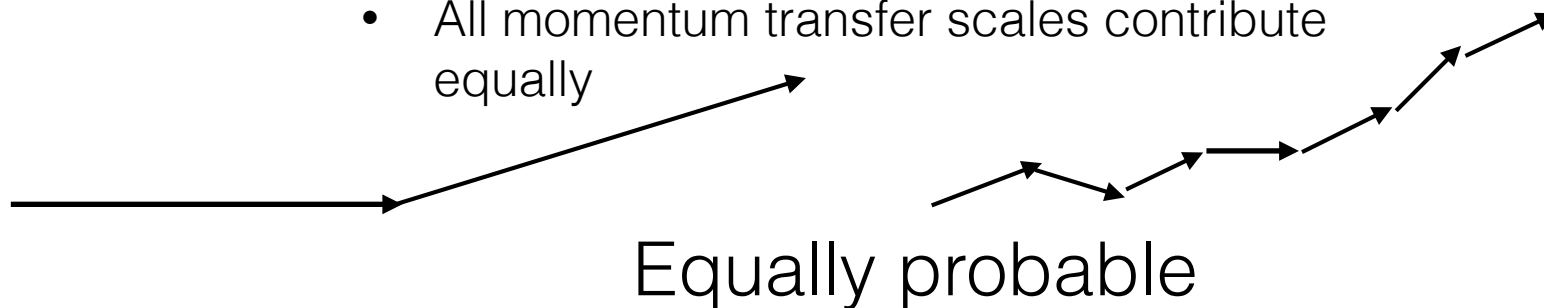


- Total momentum transfer from uncorrelated kicks:

$$Q_{\perp}^2 \sim \sum_i \Delta q_{\perp}^2 \sim \frac{\tau}{\tau_{sc}} \Delta q_{\perp}^2 \equiv \hat{q} \tau$$

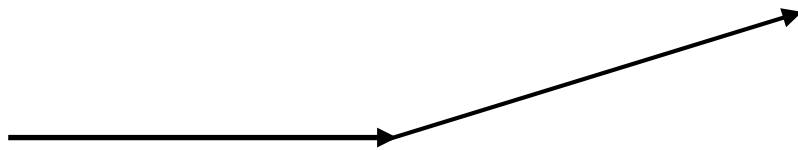
$$\hat{q} \sim \alpha^2 n \int d^2 q_{\perp} \frac{q_{\perp}^2}{(q_{\perp}^2)^2} \sim \alpha^2 n \log(T/m_{screen})$$

- All momentum transfer scales contribute equally

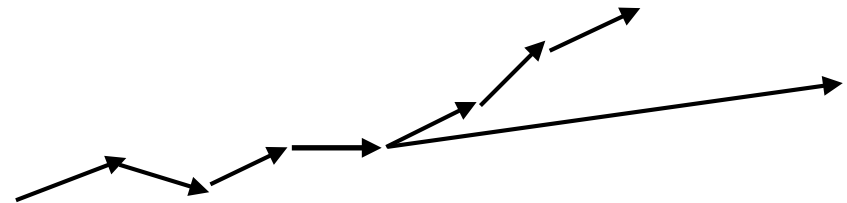


QCD kinetic theory “EKT”

- What does fast soft scattering imply?
- With each scattering, α_s probability of radiation



$$\tau_{sc}(\Delta q_{\perp}^2 \sim T^2) \sim \frac{1}{\alpha^2 T}$$



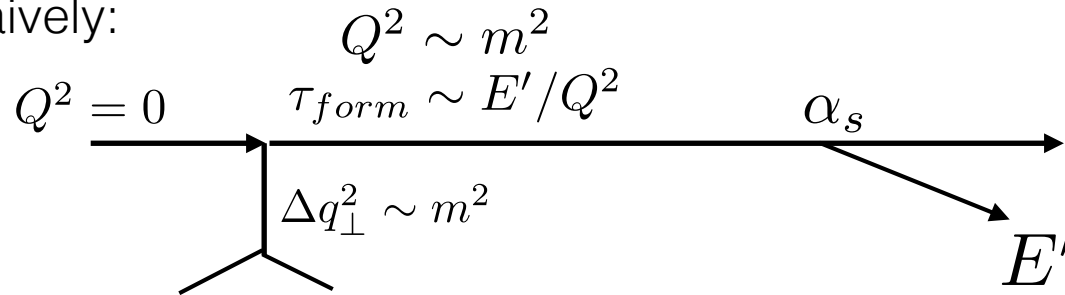
$$\tau_{split}(\Delta q_{\perp}^2 \sim T^2) \sim \frac{1}{\alpha_s} \tau_{sc}(m^2) \sim \frac{1}{\alpha} \frac{1}{\alpha T}$$

- Elastic scattering and collinear splitting equally fast, both need to be included

$$\frac{df}{dt} = -C_{2 \leftrightarrow 2}[f] - C_{1 \leftrightarrow 2}[f]$$

QCD kinetic theory “EKT”

- Collinear splitting and Landau-Pomeranchuk-Migdal suppression
- Naively:



- The soft scattering and the splitting far apart in time, diagram factorizes

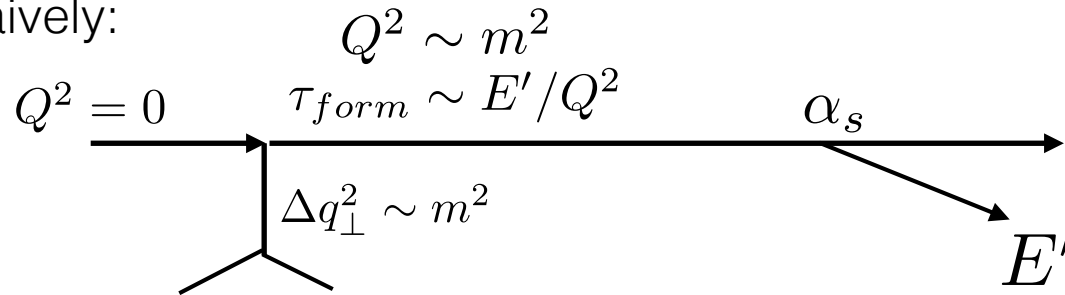
$$C_{1 \leftrightarrow 2} \sim \int dp \, \gamma_{k, p-k}^p [f_p(1 + f_k)(1 + f_{p-k}) - f_k f_{p-k}(1 + f_p)]$$

$$\gamma \sim \frac{1}{\tau_{sc}(m^2)} \times \frac{1}{p}$$

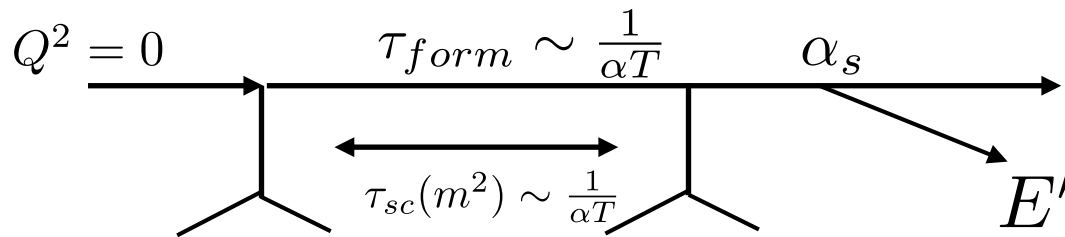
- Bether-Heitler rate

QCD kinetic theory “EKT”

- Collinear splitting and Landau-Pomeranchuk-Migdal suppression
- Naively:



- The soft scattering and the splitting far apart in time, diagram factorizes
- However, if $E' \gtrsim T$, several scatterings during formation

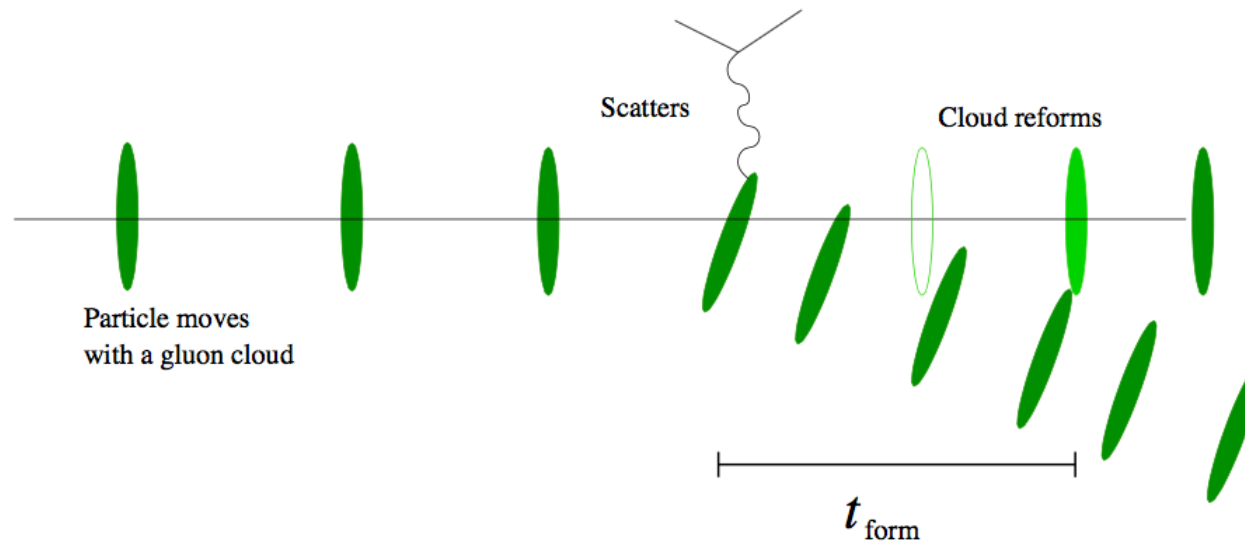


- Leads to interference between diagrams: LPM suppression

Diagram illustrating LPM suppression. On the left, a diagram shows a parton (wavy line) interacting with a medium (represented by a series of vertical lines with loops). On the right, the expression is given as the real part of the product of two diagrams. The first diagram shows a parton interacting with a medium, and the second diagram shows a parton interacting with a medium, with the entire expression enclosed in large parentheses.

QCD kinetic theory “EKT”

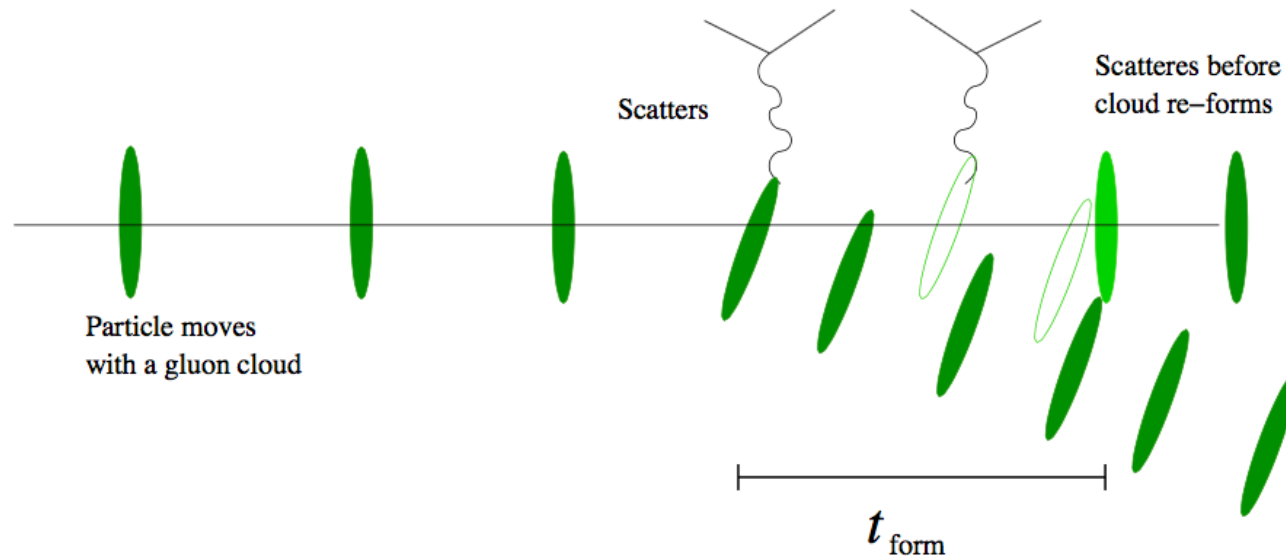
- Landau-Pomeranchuk-Migdal suppression qualitatively



$$t_{\text{form}} \sim \frac{\text{trans.size}}{\text{trans.vel}} \sim \frac{1/q_{\perp}}{q_{\perp}/E'} \sim \frac{E}{q_{\perp}^2}$$

QCD kinetic theory “EKT”

- Landau-Pomeranchuk-Migdal suppression qualitatively



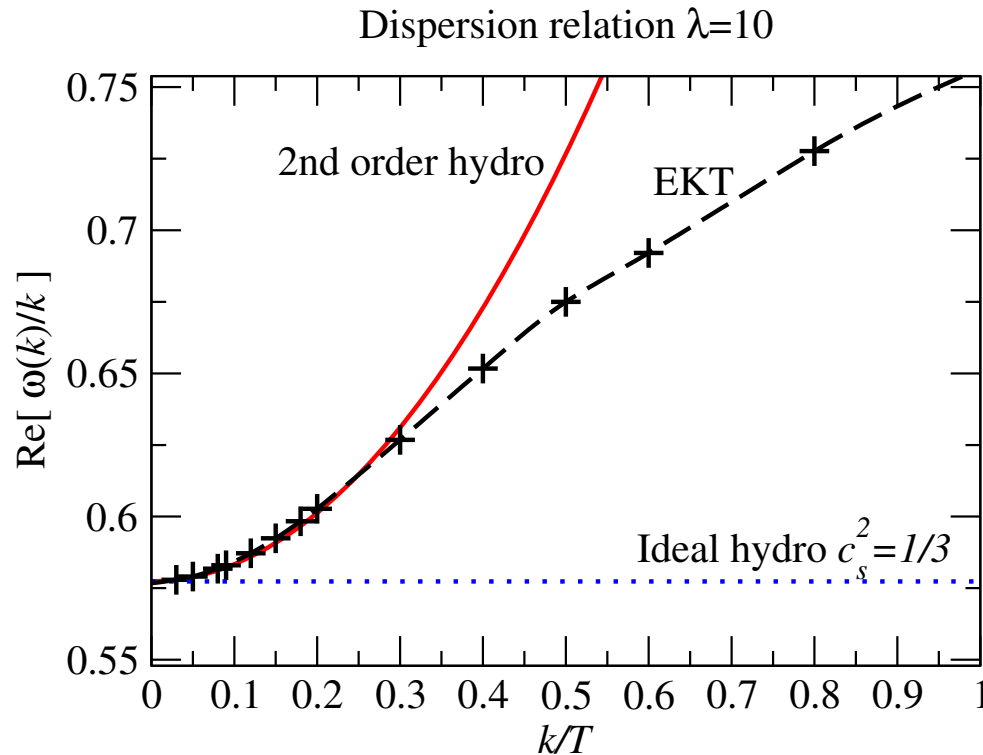
$$\tau_{form} \sim \frac{\text{trans.size}}{\text{trans.vel}} \sim \frac{1/q_{\perp}}{q_{\perp}/E'} \sim \frac{E}{q_{\perp}^2}$$

- If a new scattering occurs while the emission has not formed, it is not resolved. The two scatterings act as a single scattering -> reduced rate
- Assuming that the net effect during τ_{form} from multiple scatterings is

$$q_{\perp}^2 \sim \hat{q} \tau_{form} \Rightarrow \tau_{form} \sim \frac{E}{\hat{q} \tau_{form}} \sim \sqrt{\frac{E}{\hat{q}}}$$

$$\tau_{sc} \sim \frac{1}{\alpha_s} \tau_{form} \sim \frac{1}{\alpha_s} \sqrt{\frac{E}{\hat{q}}}$$

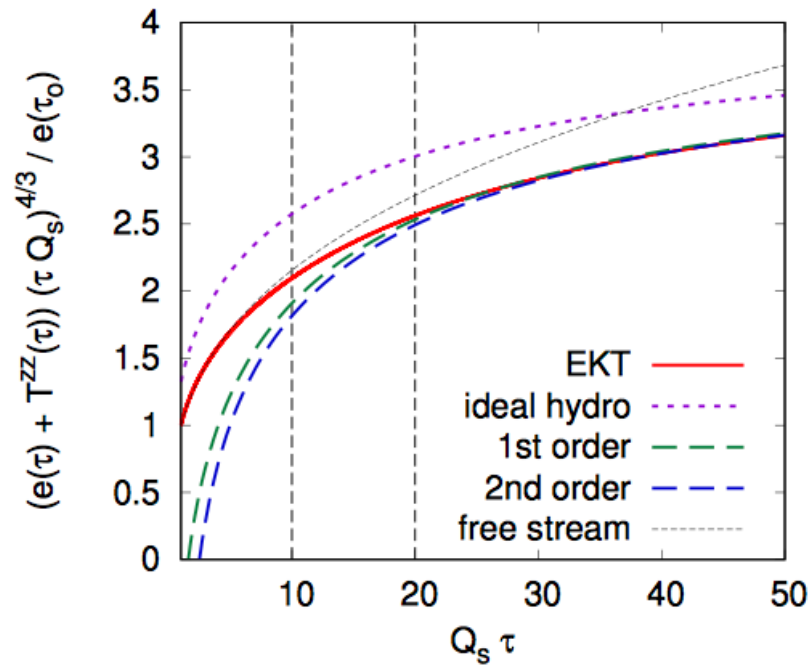
QCD kinetic theory



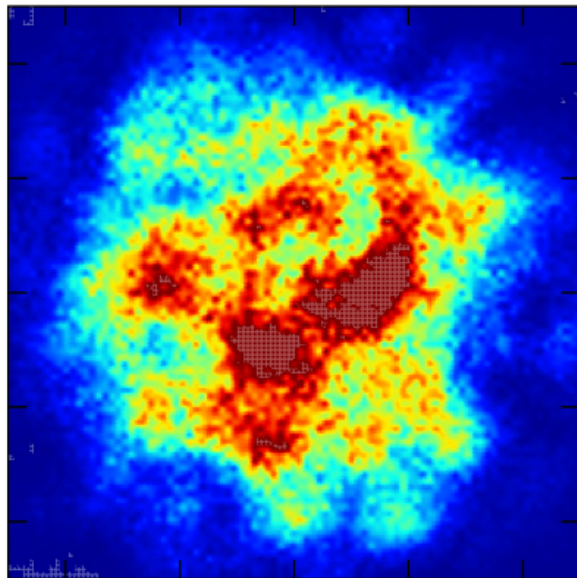
$$\omega = c_s k + \frac{4}{3} \frac{\eta}{e + p} \left(c_s \tau_\pi - \frac{2}{3 c_s} \frac{\eta}{e + p} \right) k^3$$

- Dispersion relation of sound reproduces ideal hydro for long wave lengths (small gradients)
- Deviations from hydrodynamics for small wavelengths

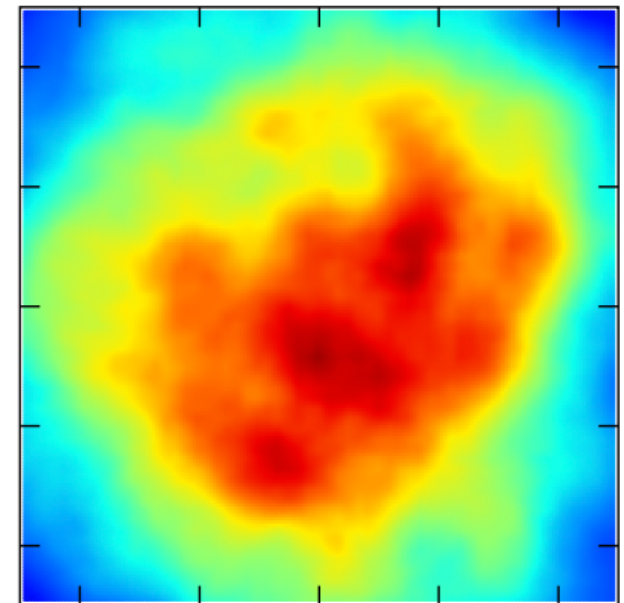
QCD kinetic theory



- The kinetic theory interpolates between free streaming and hydrodynamics
- Can be used to bring the initial condition to time where fluid dynamics is applicable

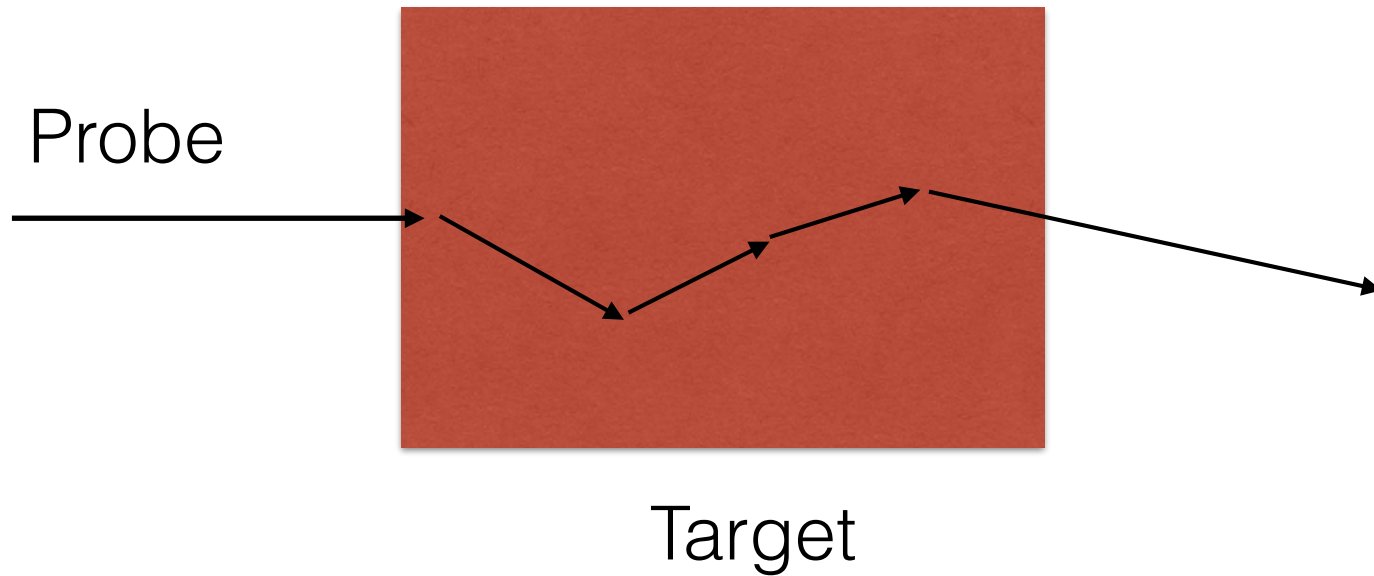


Pre-equilibrium
smearing
and
generation of
preflow



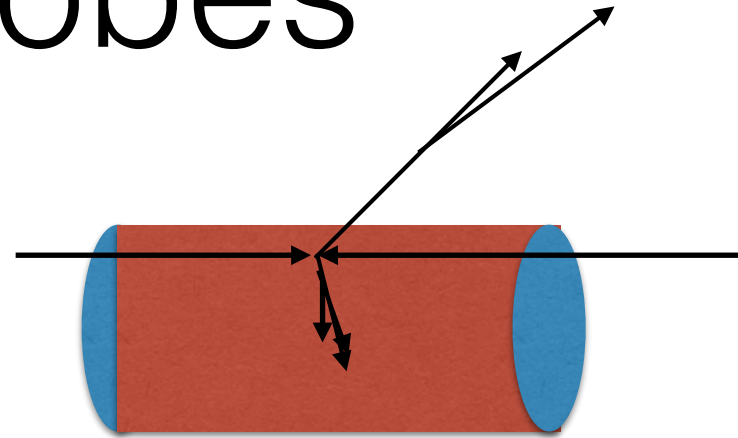
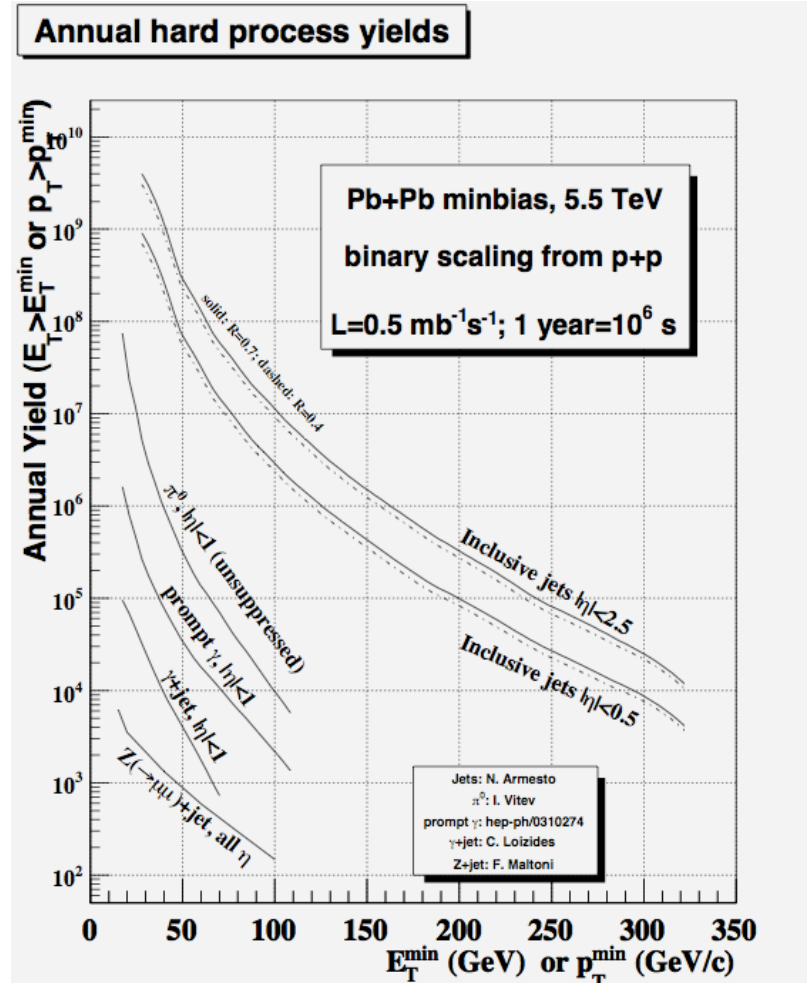
Hard probes

Hard probes



- Classic experiment: shoot a probe through medium to learn about the properties of the medium

Hard probes



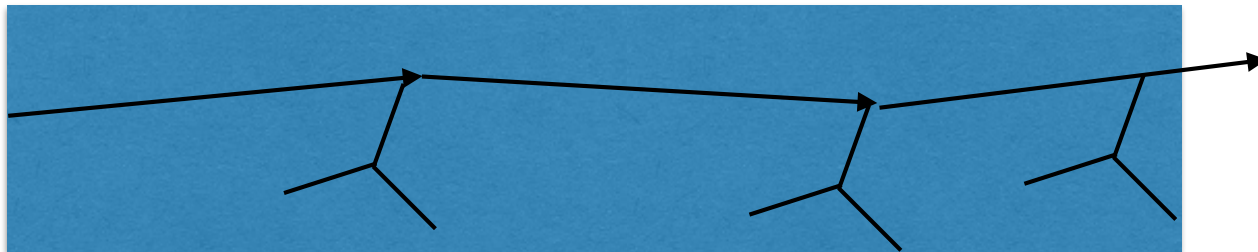
- Hard probes= Rare hard processes embedded in the hot plasma “self generated probes”
- Emission rates of hard particles largely unmodified by the medium and well known. “Calibrated probes”
- Created abundantly at the LHC

Jet quenching

- Thermal background + jet is a non-equilibrium system -> use EKT (AMY)
- Several other formalisms, all do basically the same. Slightly different kinematical approximations. BDMPS, Z, AMY,...
- Key question: How fast an energetic parton loses energy when traversing medium of length L

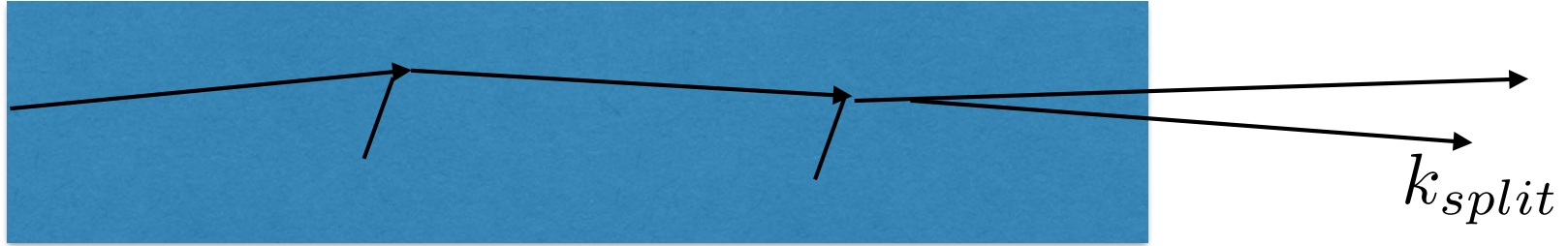
- 2 to 2 scattering: collisional energy loss: $\Delta E \sim \hat{e}L$

- Drag coefficient \hat{e} related to $\hat{q} = 2T\hat{e}$ by Einstein relation



- 1 to 2 splitting: radiational energy loss: $\Delta E \sim \hat{q}L^2$

Radiational energy loss



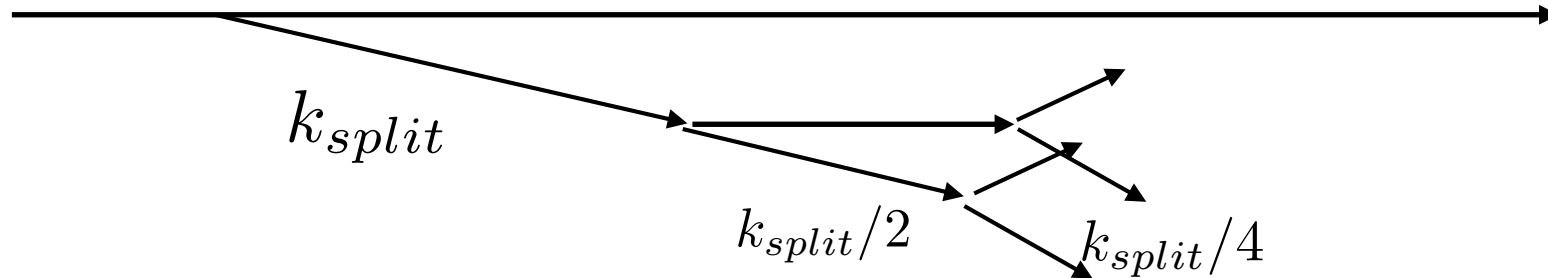
- Relevant rate LPM suppressed

$$\tau_{split}^{-1} \sim \alpha_s \tau_{form}^{-1} \sim \alpha \sqrt{\frac{\hat{q}}{p}}$$

- After a traversing length L , the hardest splitting is given by

$$L \sim \tau_{split}(k_{split}) \quad k_{split} \sim \alpha_s^2 \hat{q} L^2$$

Radiational energy loss



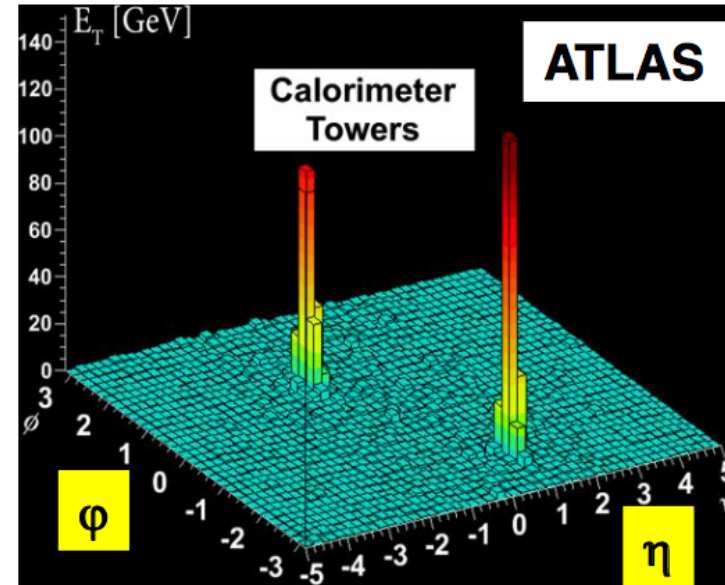
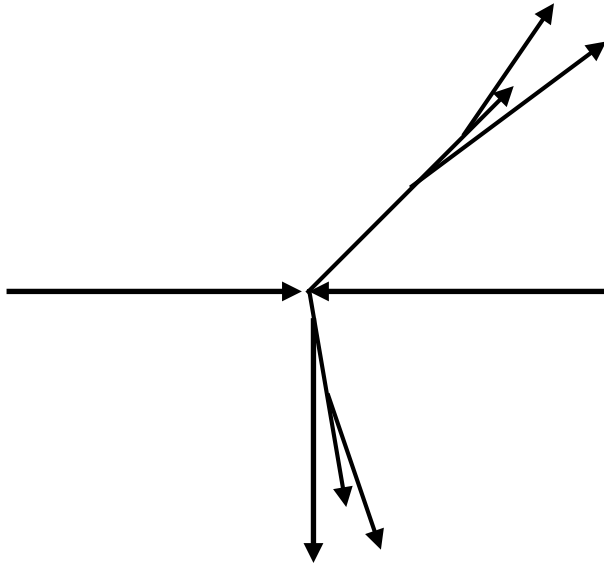
- Those particles that have had time to be emitted have time to re-split again

$$\tau_{split}(k_{split}) > \tau_{split}(k_{split}/2) > \tau_{split}(k_{split}/4)$$

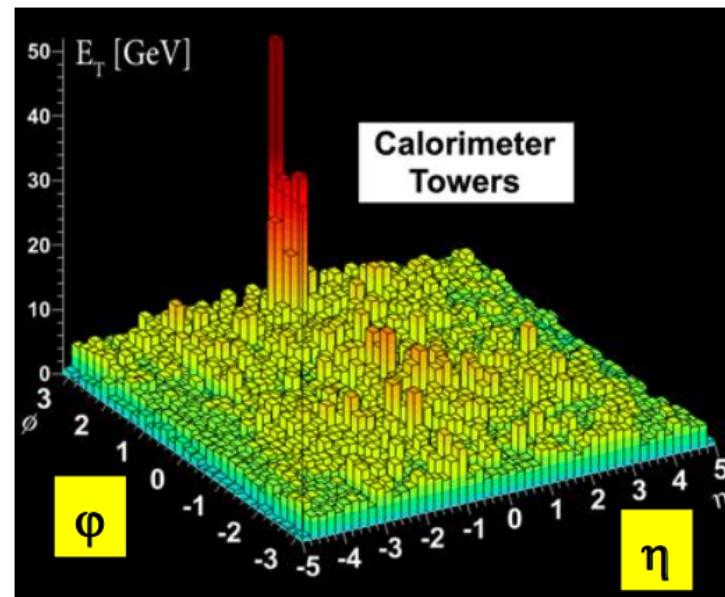
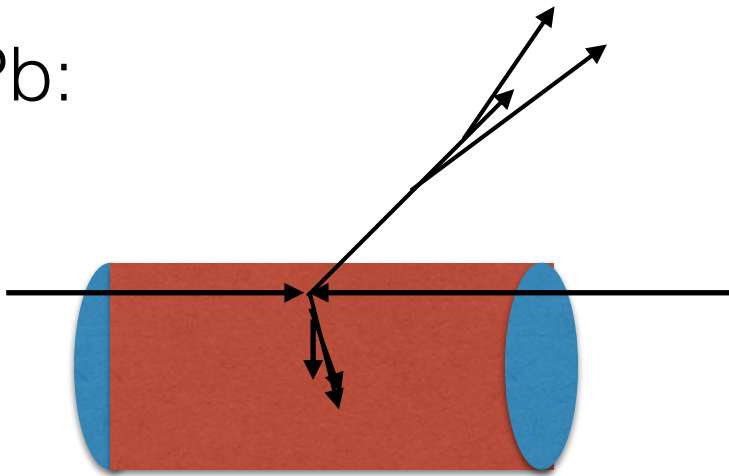
- Emitted particles undergo a radiational cascade until all the energy reaches the thermal scale
- Soft particles undergo large angle scattering more easily and can escape the jet cone
- The jet is fully quenched when $L = \tau_{split}(E)$, $L \sim \sqrt{\frac{E}{\alpha^2 \hat{q}}} \sim \frac{1}{\alpha^2 T} \sqrt{\frac{E}{T}}$

A Back-to-back jet

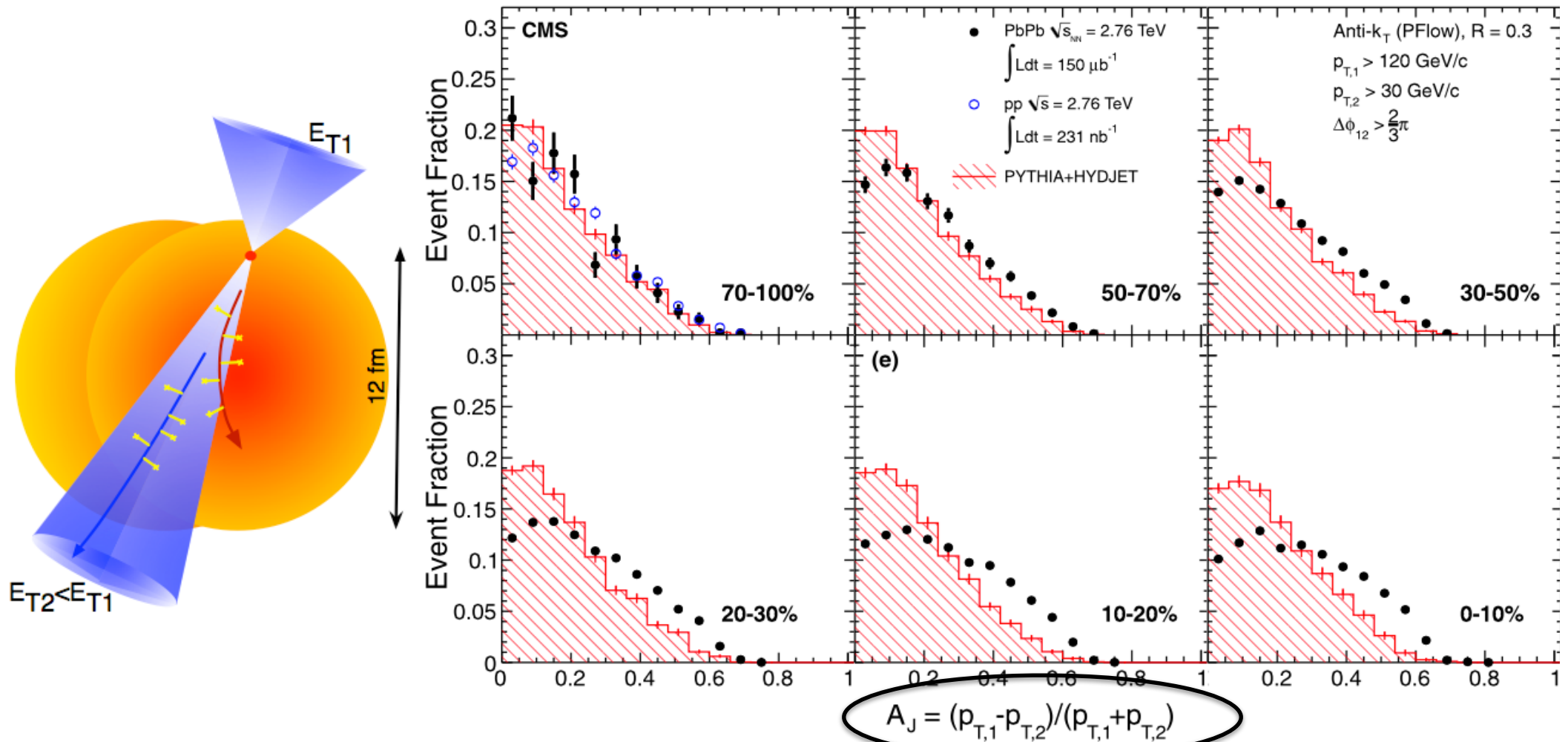
pp:



PbPb:



Dijet asymmetry

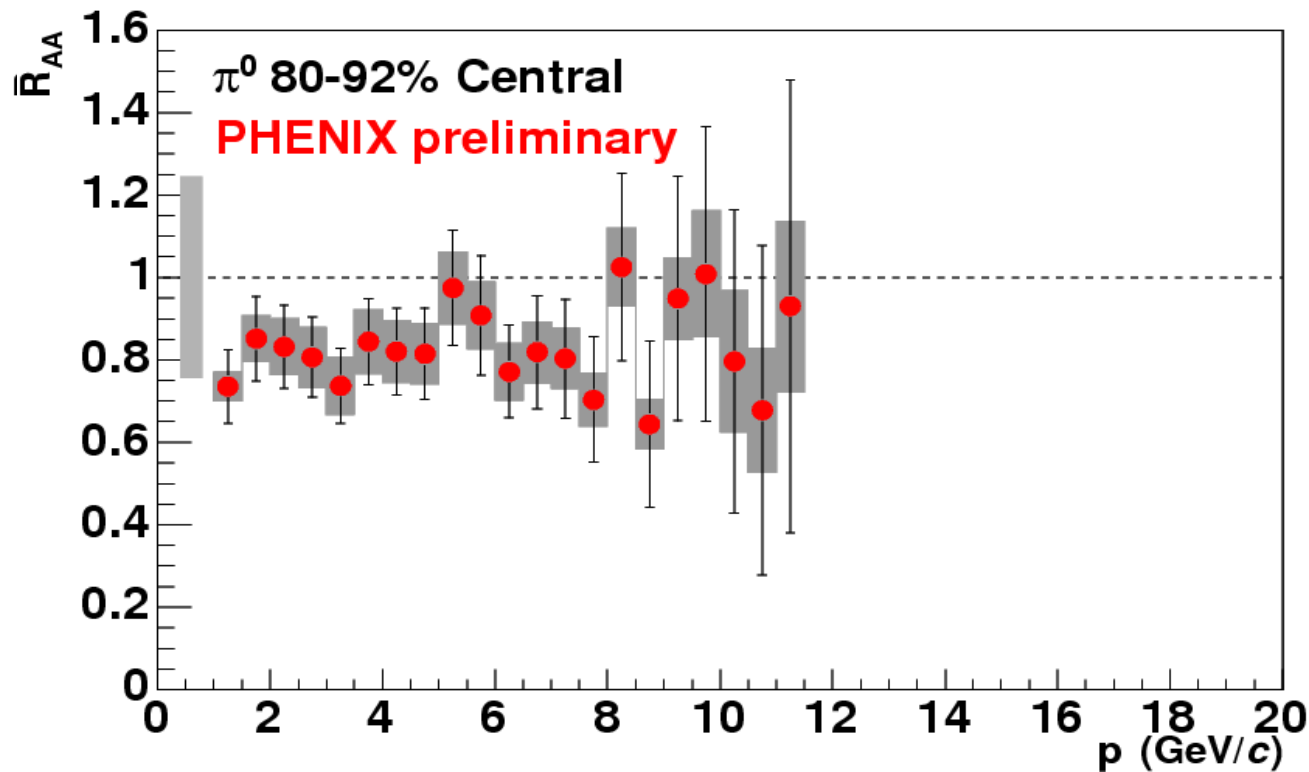
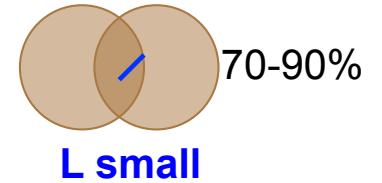
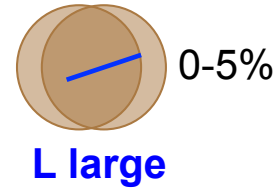


- Peripheral events have an unmodified A_J
- In pp, asymmetry arises from 3-jet events
- Asymmetry much enhanced in central AA

Nuclear modification factor

- Suppression of leading hadron production compared to pp baseline

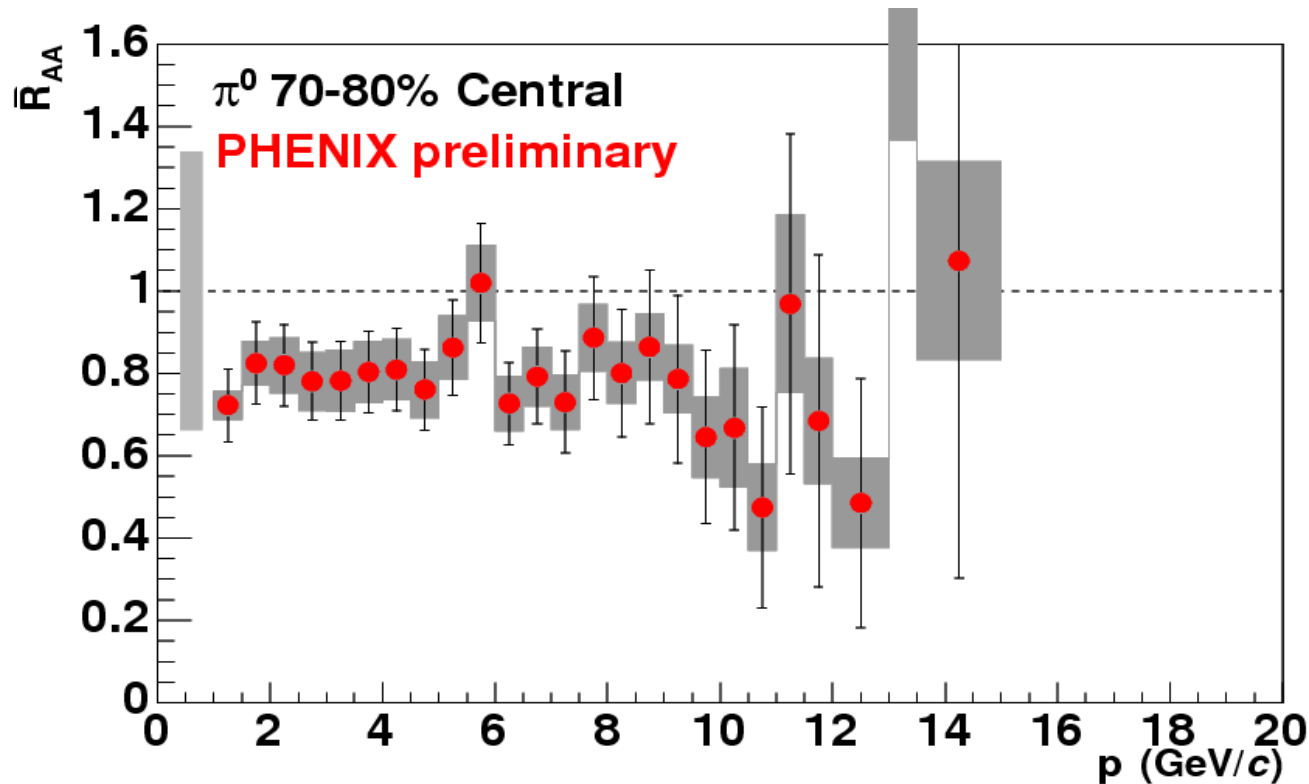
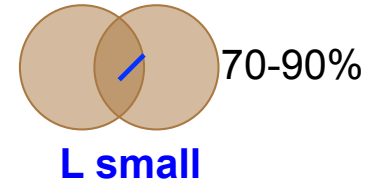
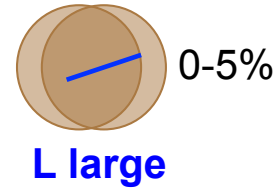
$$R_{AA}(p_T) = \frac{dN^{AA}/dp_T}{n_{coll} dN^{NN}/dp_T}$$



Nuclear modification factor

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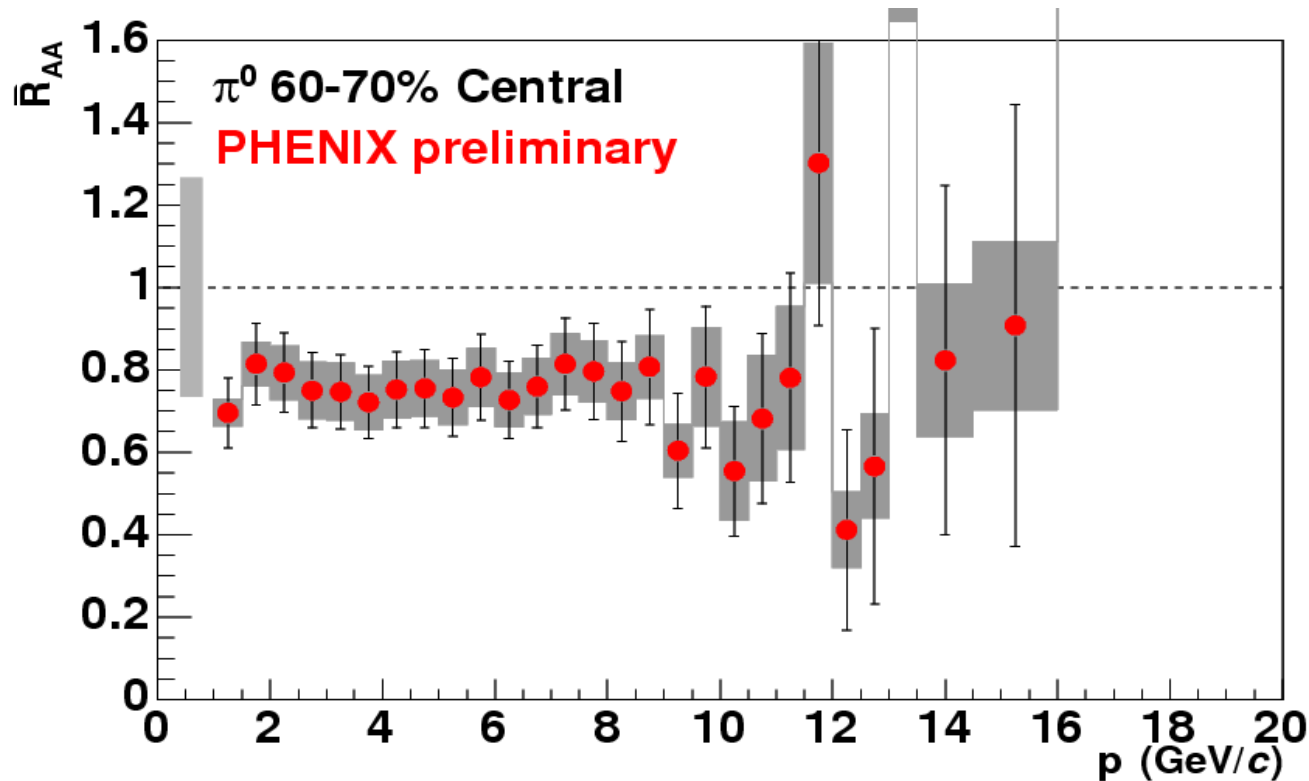
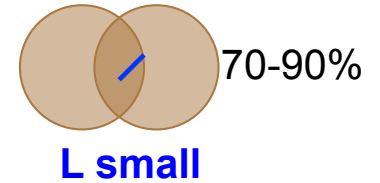
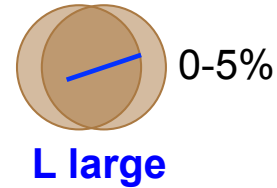
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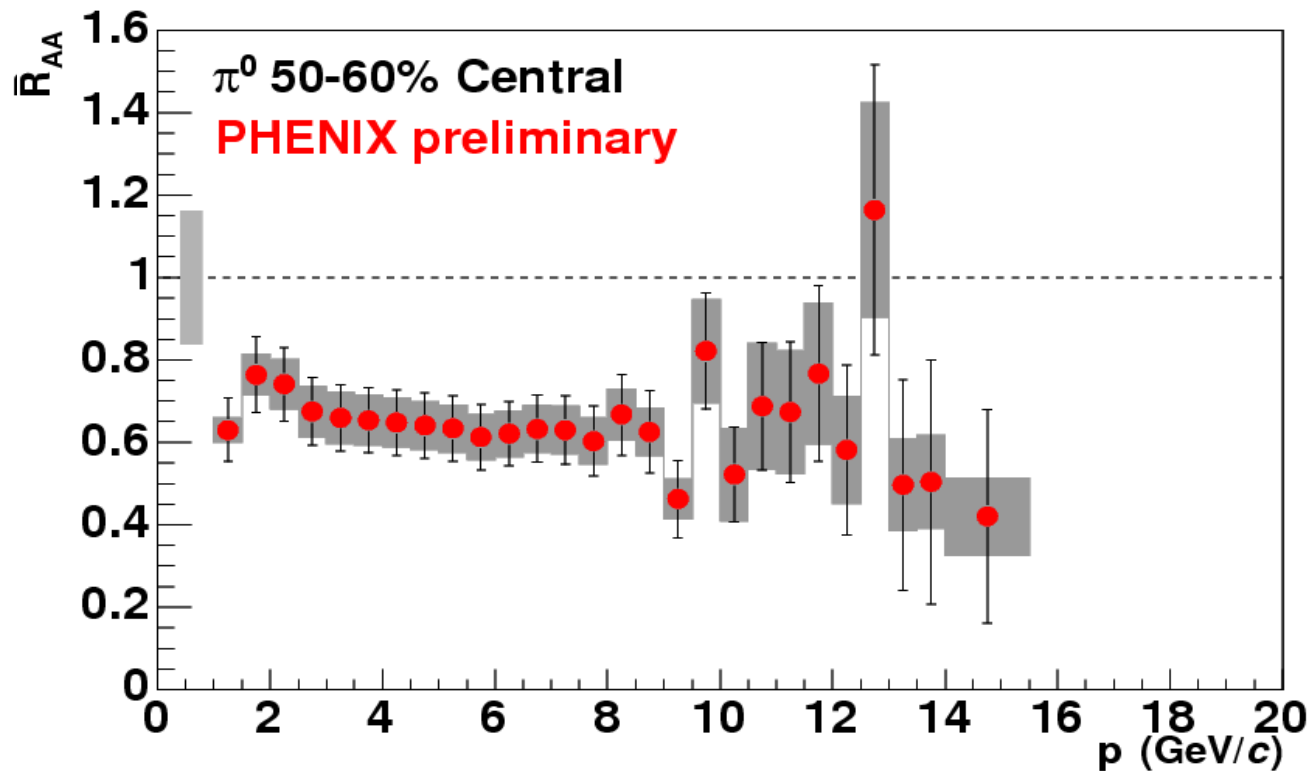
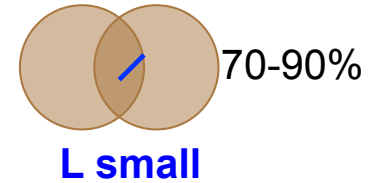
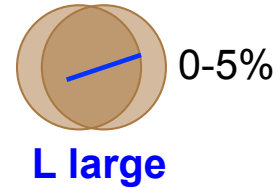
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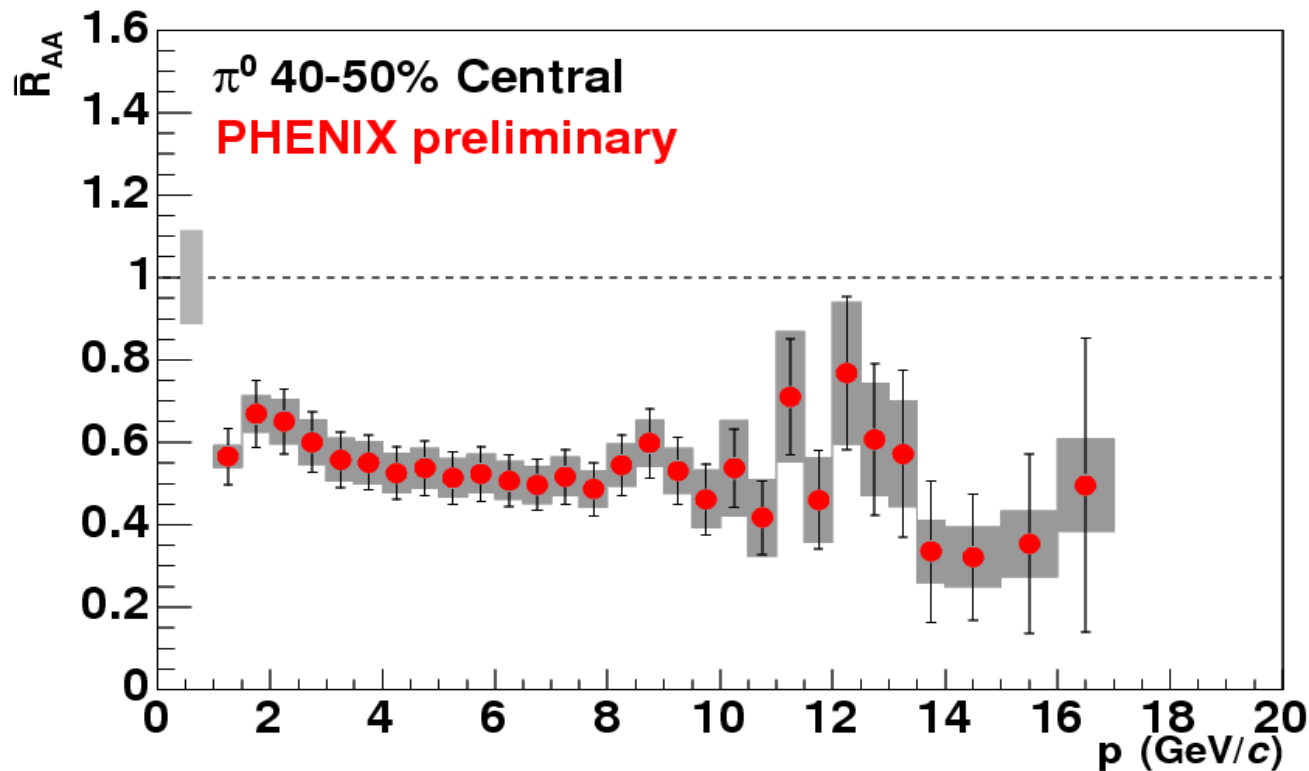
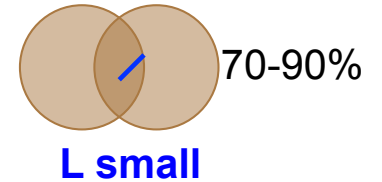
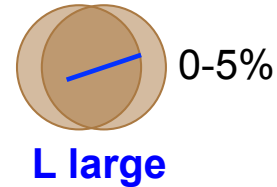
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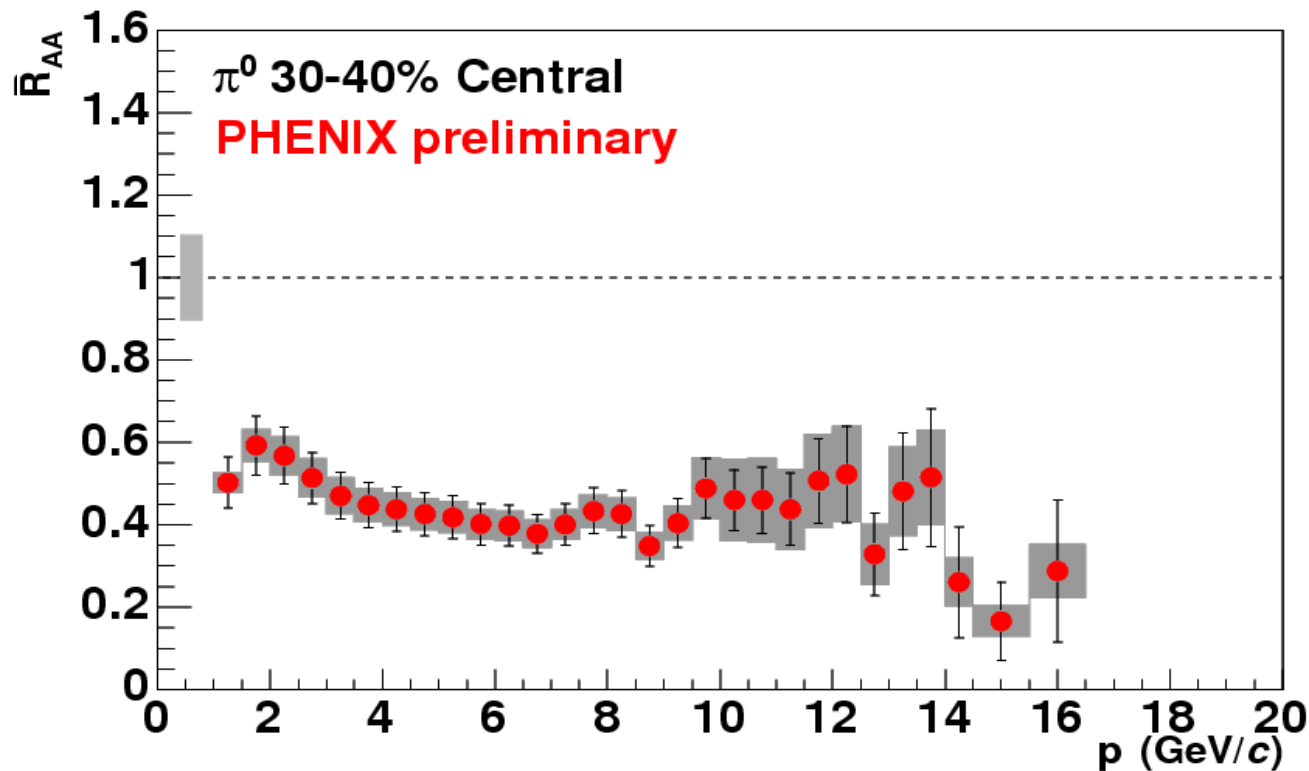
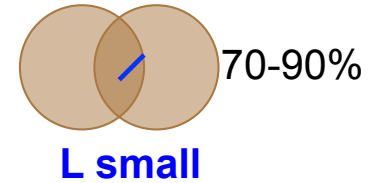
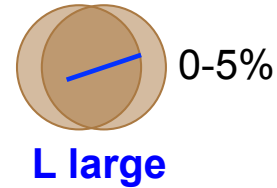
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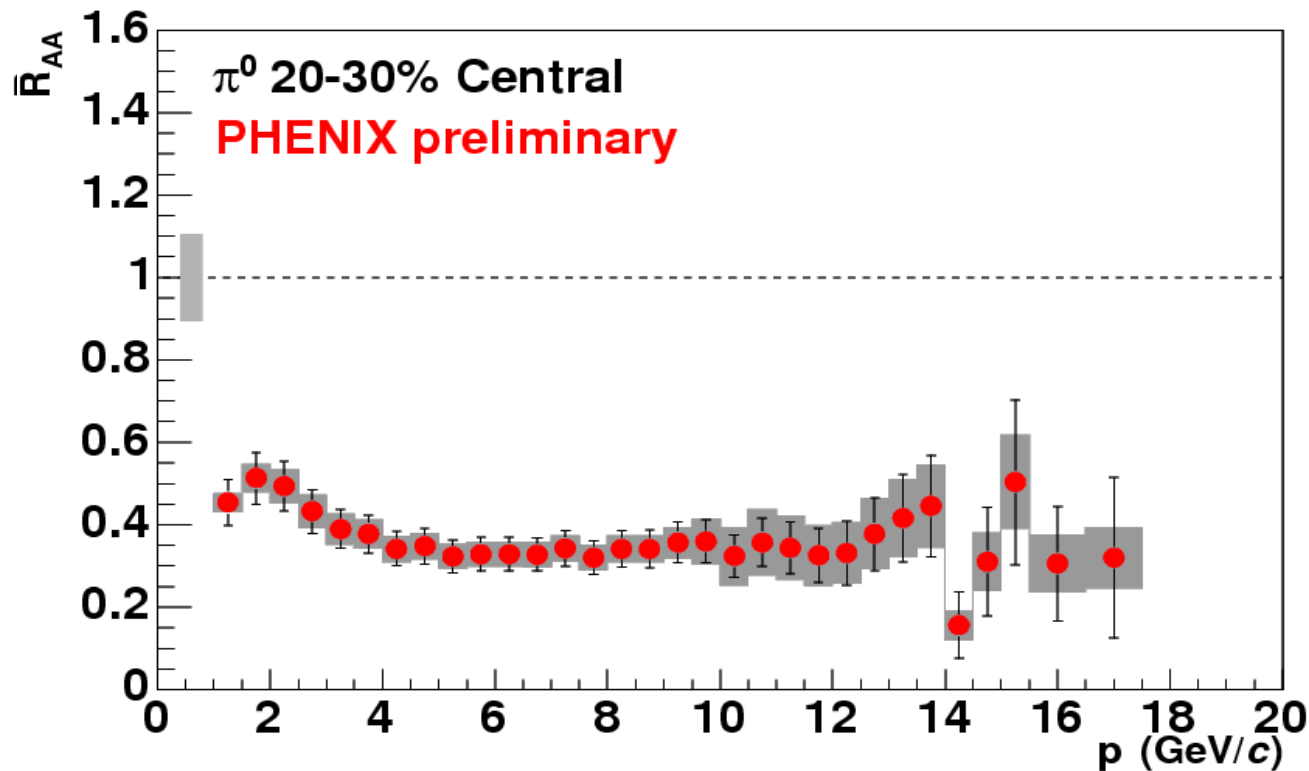
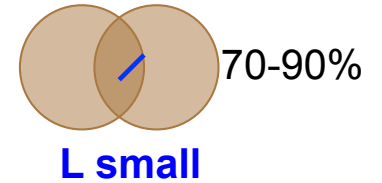
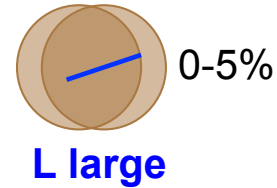
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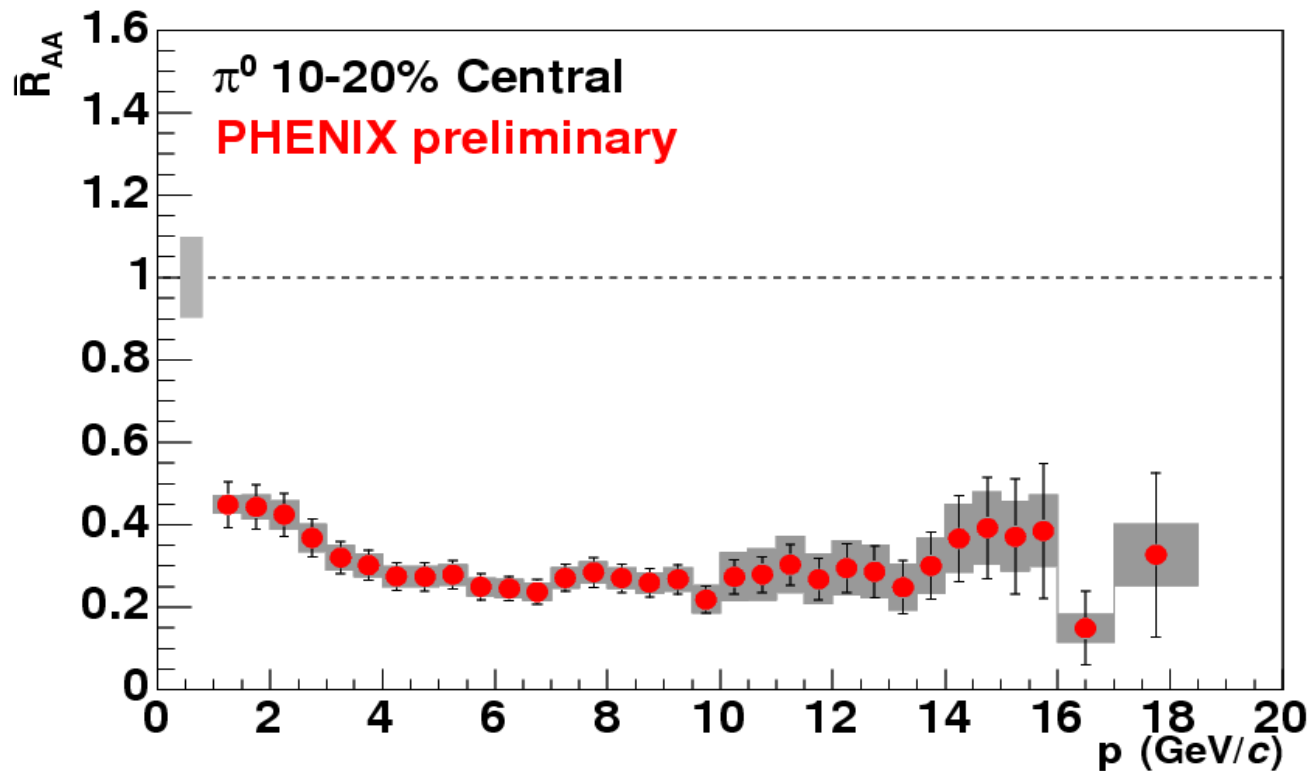
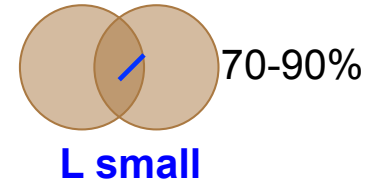
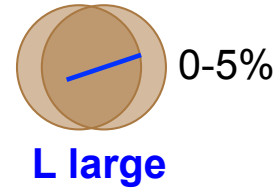
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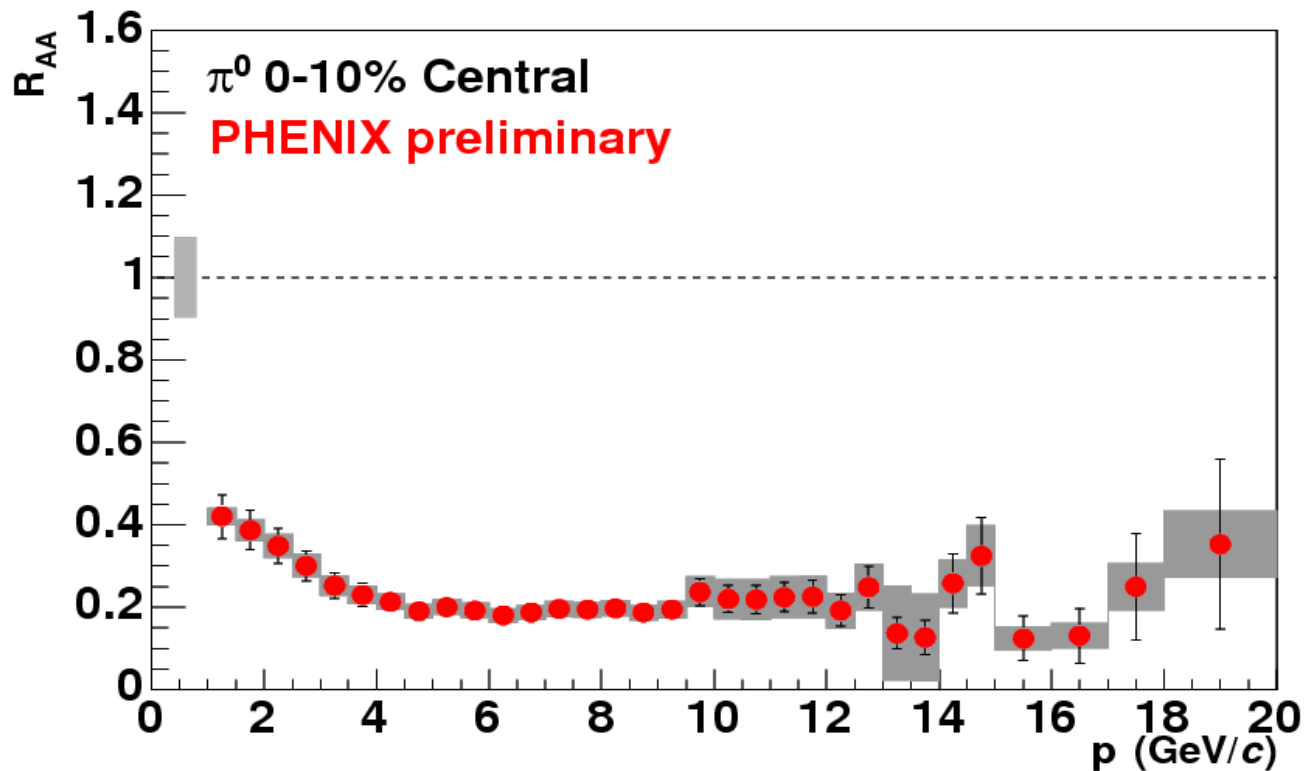
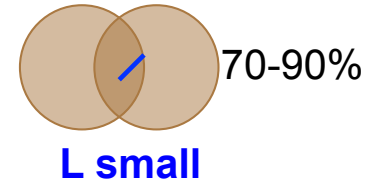
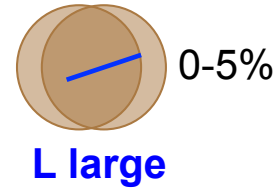
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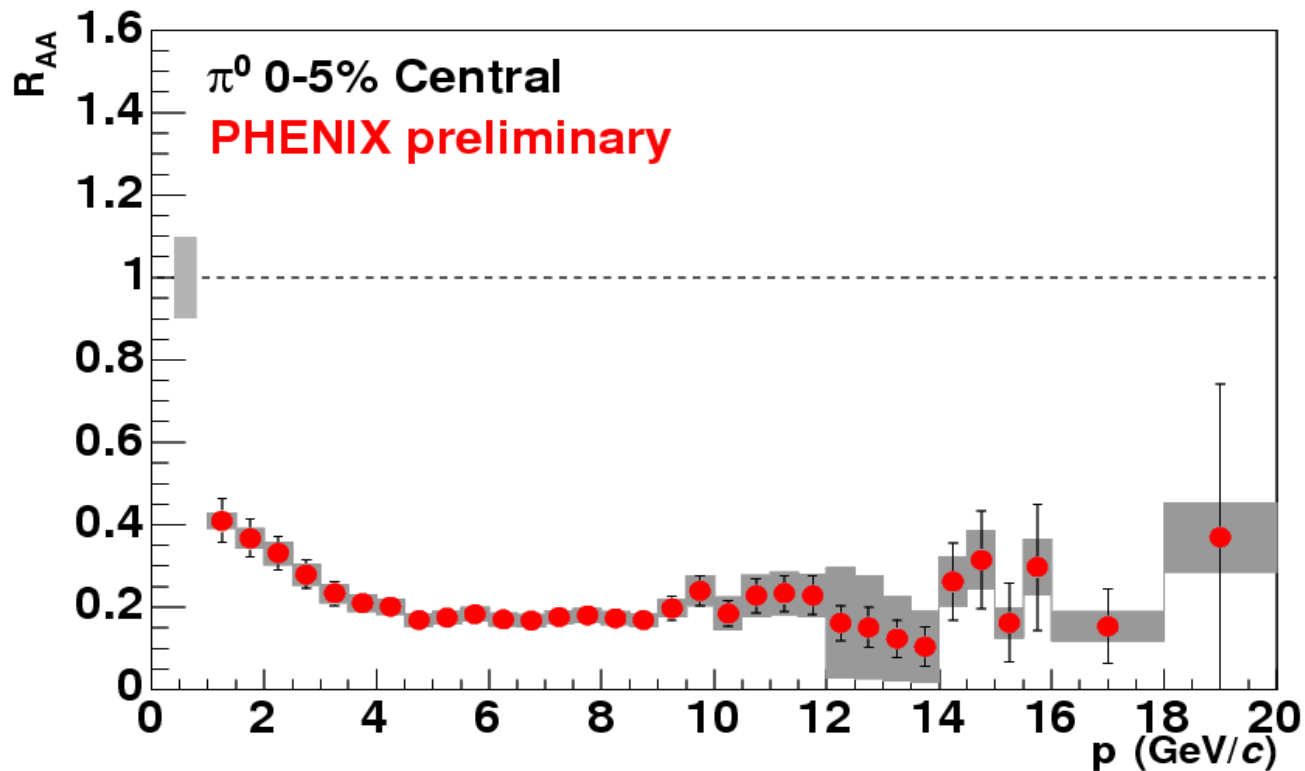
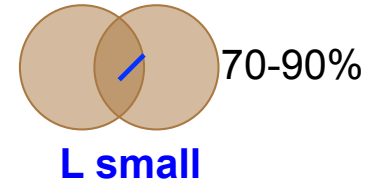
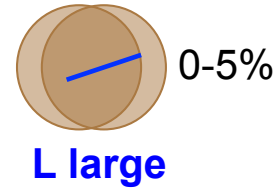
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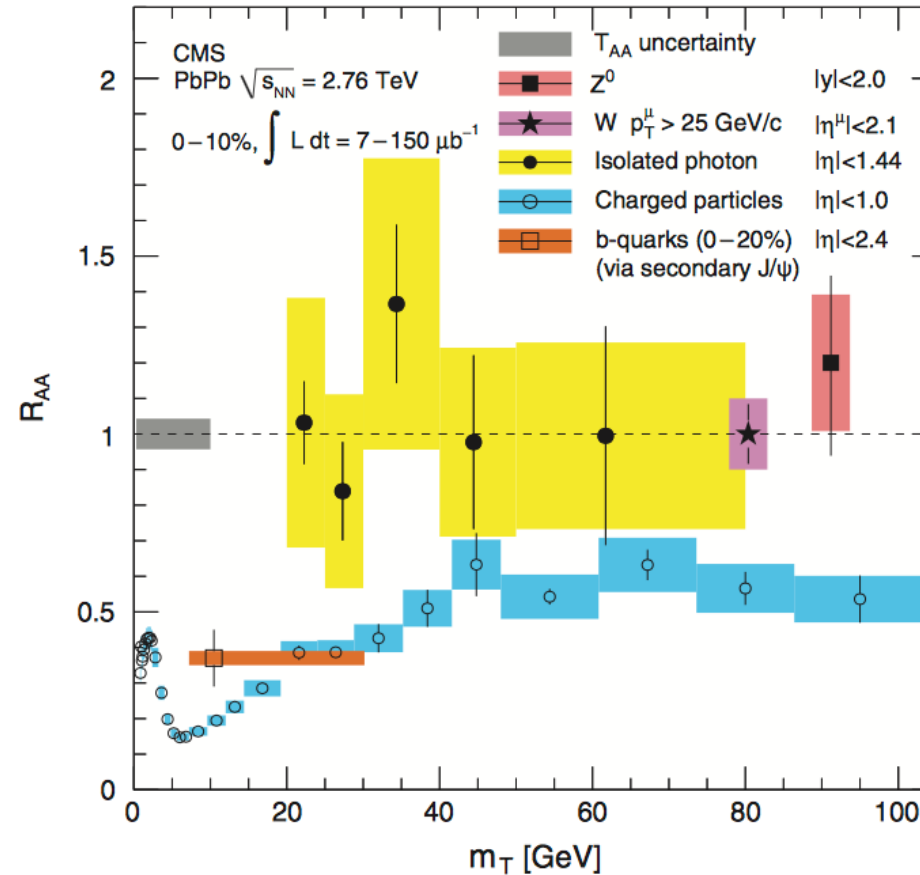
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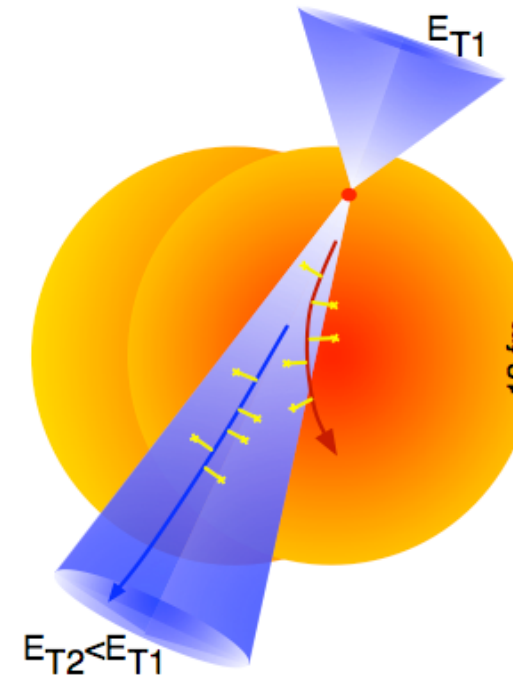
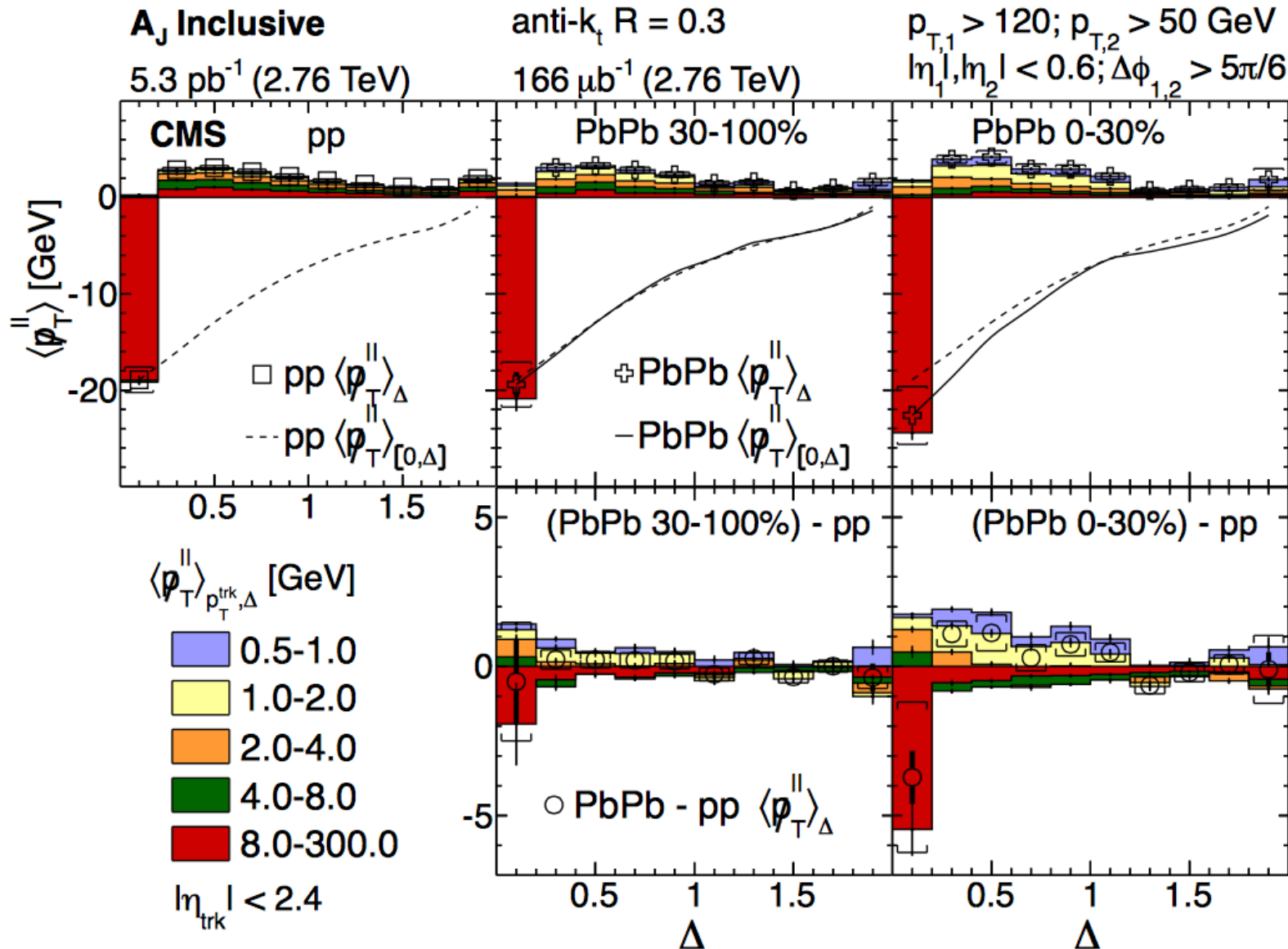
Nuclear modification factor



- Uncolored particles are not quenched: Photons, Z, W

Jet modification

- The energy must go somewhere!



Conclusions

- Heavy-ion collisions allow us to test our understanding of quantum field theory in extreme conditions.
- Many different and independent features of heavy-ion collisions point to the formation of a fire ball that evolves like a liquid
- The dynamical description of the collision is getting increasingly quantitative and precise
- The most striking feature of the liquid is that it seems to be strongly coupled with a very low specific shear viscosity $\eta/s \sim 0.2$