Anisotropic Spherically Symmetric Collapsing Star From Higher Order Derivative Gravity Theory

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ABSTRACT- We add linear combinations $R^2, R_{\mu\nu}R^{\mu\nu}$ and $R_{\mu\nu\eta\delta}R^{\mu\nu\eta\delta}$ with Einstein-Hilbert action and obtain interior metric of an anisotropic spherically symmetric collapsing stellar cloud. We solved linearized metric equation via perturbation method and obtained 12 different kinds of metric solutions $P_1, P_2, \cdots P_{12}$. Calculated Ricci and Kretschmann scalars of our metric solutions are non-singular at beginning of the collapse for 2 kinds of them only. Event and apparent horizons are formed at finite times for two kinds of singular metric solutions while 3 metric solutions exhibit with event horizon only with no formed apparent horizon. There are obtained 3 other kinds of the metric solutions which exhibit with apparent horizon with no formed event horizon. Furthermore 3 kinds of our metric solutions do not exhibit with horizons. Our solutions satisfy different regimes such as domain walls (6 kinds), cosmic string (2 kinds), dark matter (2 kinds), anti-matter (namely negative energy density) (1 kind) and stiff matter (1 kind). Calculated time dependent radial null geodesics expansion parameter $\Theta^*_i(T); i = 1, 2, \cdots 12$ takes positive (negative) values for 4 (8) kinds of our solutions which means the collapse ended to a naked (covered) singularity at end of the collapse.

Keywords-Higher order derivative gravity, Anisotropic fluid, Spherically symmetric collapse, Non-singular models, Naked singularity, Domain walls, Cosmic string, Dark matter, Anti-matter, Stiff matter, Trapped surface.

Higher order derivatives gravity models are made from extension of the Einstein metric equation via 'R^2', $\Box R$, '$R_{\mu\nu}R^{\mu\nu}$' and '$R_{\mu\nu\gamma\delta}R^{\mu\nu\gamma\delta}$', as $G_{\mu\nu} = 8\pi T_{\mu\nu} = -(\alpha H^{(1)}_{\mu\nu} + \beta H^{(2)}_{\mu\nu})$ where we used units '$G = c = \hbar = 1$' and defined $\alpha = \zeta - \xi, \beta = \eta + 4\xi, H^{(1)}_{\mu\nu} = 2(\nabla_\mu\nabla_\nu R + RR_{\mu\nu}) - g_{\mu\nu}(2\Box R + \frac{1}{2}R^2)$ and $H^{(2)}_{\mu\nu} = \nabla_\mu\nabla_\nu R - \Box R_{\mu\nu} + 2R^{\alpha\beta}R_{\alpha\beta\mu\nu} - \frac{1}{2}g_{\mu\nu}(\Box R + R^{\alpha\beta}R_{\alpha\beta})$. '$R$', '$R_{\mu\nu}$' and '$R_{\mu\nu\gamma\delta}$' given in the above equations are Ricci scalar, Ricci tensor and Kretschmann scalar respectively [1]. The coupling constants 'ζ', 'η' and 'ξ' come from dimensional regularization of interacting quantum matter fields. The basic motivation for studying these *Higher Derivative gravity theories* comes from the fact that they provide one possible approach to an as yet unknown quantum theory of gravity [2]. However, the structure of classical solutions of higher derivative gravity may provide a better approximation to some metric solutions with respect to those provided by general relativity. Some applications are studied at classical Robertson-Walker cosmology [3,4] and its quantum approach also [5]. In the present work we want to study physical effects of these higher order terms on collapse of anisotropic spherically symmetric stellar cloud with general from of internal line element as $ds^2 = -e^{a(t)}dt^2 + e^{b(t)}dr^2 + t^2e^{c(t)}(d\theta^2 + \sin^2\theta d\varphi^2)$ where $a(t), b(t), c(t)$ are determined by solving the metric equation. It should be pointed that 2-sphere spatial part of the above metric is inhomogeneous because of absence of r^2 term. Inserting the above line element the components of the Einstein metric equation become nonlinear and so we solve them via perturbative analytical methods. Setting $e^{a(t)} = e^{a_0}\{1+\epsilon a_1(t)+O(\epsilon^2)\}$,$e^{b(t)} = e^{b_0}\{1+\epsilon b_1(t)+O(\epsilon^2)\}$ and $e^{c(t)} = e^{c_0}\{1+\epsilon c_1(t)+O(\epsilon^2)\}$ where a_0, b_0, c_0 are constants and ϵ is a suitable dimensionless order parameter of the series expansion, zero order approximation of the metric equation become $e^{a_0} = -e^{c_0}$ and first order part of nonzero $tt, rr, \theta\theta$, components of the metric equation leads to linear differential equations which has solutions as: $a_1(T) \simeq AT^\mu$, $b_1(T) \simeq BT^\mu$, and $c_1(T) \simeq ET^\mu$ where $T = t/\sqrt{\alpha}$ and numerical values of the parameters A, B, E and μ and $\omega = \frac{\beta}{\alpha}$ are given at table 1 for 12 kinds of our metric solutions $P_i = (\mu_i, \omega_i); i = 1, 2, \cdots 12$ with fluid characters as: $\frac{\rho(T)}{\rho(\sqrt{\alpha})} \approx T^{\mu-2} = \frac{p_r(T)}{p_r(\sqrt{\alpha})} = \frac{p_t(T)}{p_t(\sqrt{\alpha})} = \frac{p(T)}{p(\sqrt{\alpha})} = \frac{R^\lambda_\lambda(T)}{R^\lambda_\lambda(\sqrt{\alpha})} = \sqrt{\frac{K(T)}{K(\sqrt{\alpha})}}$. Here ρ, p_r, p_t, p are density, and pressures (radial, tangential and itself). R^λ_λ and K are Ricci and Kretschmann scalar respectively. We evaluated also barotropic index $\gamma(T) = p(T)/\rho(T)$, anisotropy index $\Delta(T) = (p_t - p_r)/\rho$, as constants, time formation of horizons (apparent and event) and also possible trapped surfaces for all 12 kinds of our metric solutions. Our numerical results are collected at the following tables and present that geometrical source treats as domain walls (6 kinds), cosmic string (2 kinds), dark matter (2 kinds), stiff matter (1 kind) and anti-matter (negative energy density) with 1 kinds. In summary 7 kinds of our solutions reach to compact object with covered singularity (the black hole) but 5 solutions reach to naked singular metric at end of the collapse and hence cosmic censorship conjecture maintain valid for 7 kinds of our 12 metric solutions only.

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P_i	μ_i	ω_i	$(A/B)_i$	$(E/B)_i$	R_i^*	K_i^*
P_1	+2.433	-0.719	+1.450	+1.019	-25.619	+210.786
P_2	+2.057	+0.968	+0.029	-0.015	+6.392	+13.345
P_3	+1.888	-0.607	-0.174	-0.188	+9.583	+14.424
P_4	+1.616	-1.648	+1.633	+1.568	-25.774	+199.272
P_5	+0.644	-0.086	-0.001	+0.295	-0.531	+6.690
P_6	+0.521	+1.883	+1.478	-4.782	+18.432	+1875.170
P_7	+0.508	-1.713	-0.304	+1.086	-3.257	+92.697
P_8	-0.041	+3.285	-0.028	-0.009	-0.002	+0.039
P_9	-0.201	+0.850	+0.389	-0.672	+0.076	+20.787
P_{10}	-0.270	-1.986	-0.044	-0.047	-0.083	+0.269
P_{11}	-3.284	-2.013	+3.467	-1.339	+37.305	+456.394
P_{12}	-6.206	+0.645	-9.146	-2.433	+68.973	+21412.731

Table 1. Characteristics of metric parameters

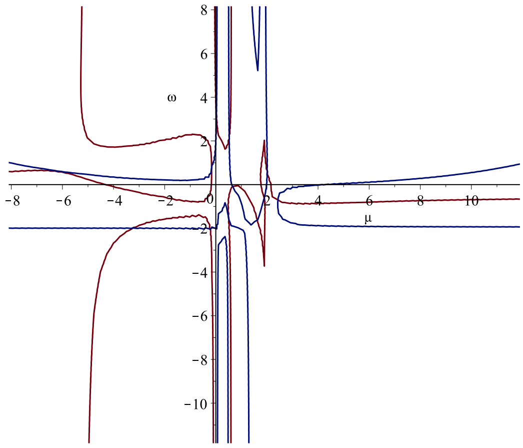

FIG. 1: Numerical values of the parameters $P_i = (\mu_i, \omega_i)$ are crossing points of the diagrams collected at the table 1.

P_i	$\rho_i{}^*$	p_{ir}^*	p_{it}^*	$p_i{}^*$	$\Delta_i(t)$
P_1	+3.049	-7.033	-5.452	-7.298	+1.069
P_2	+0.925	+0.230	-2.038	-1.317	-0.616
P_3	+0.633	+1.061	-1.435	-0.498	-1.198
P_4	+3.468	-6.456	-3.300	-5.253	+1.081
P_5	+1.486	-0.799	-0.364	-0.410	+0.207
P_6	-7.755	+13.316	+2.145	+3.525	+1.117
P_7	+2.942	-2.928	-0.622	-0.876	+0.695
P_8	+1.037	-0.036	-0.0001	+0.0002	-8.917
P_9	+0.074	+0.740	-0.113	-0.157	+0.157
P_{10}	+1.010	-0.007	-0.035	-0.031	-0.527
P_{11}	+0.590	-0.936	-6.065	-2.162	-0.753
P_{12}	+22.814	+113.561	+48.430	+53.458	-7.776

Table 2. Fluid characteristics of stellar cloud.

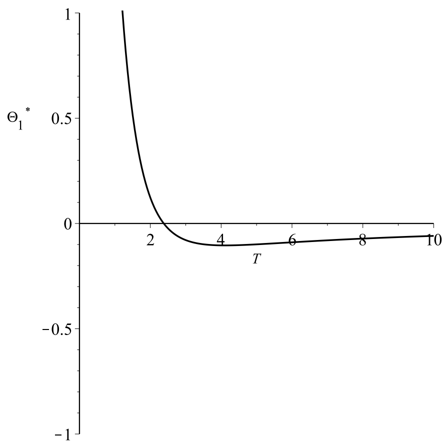

FIG. 2: Diagram of radial null geodesics expansion parameter is plotted against T for metric solution P_1

P_i	$T_{EH}/\sqrt{\alpha}$	$T_{AH}/\sqrt{\alpha}$	$R_{EH}/\sqrt{\alpha}$	$R_{AH}/\sqrt{\alpha}$
P_1	-	-	-	-
P_2	-	+5.453	-	+3.889
P_3	+2.526	+1.707	+2.752	+1.188
P_4	-	-	-	-
P_5	36692.907	-	+38236.634	-
P_6	-	+0.032	-	+0.015
P_7	+10.427	-	+3.054	-
P_8	1.372×10^{-38}	-	1.342×10^{-38}	-
P_9	-	0.082	-	-
P_{10}	9.473×10^{-6}	7.099×10^{-6}	$+9.677 \times 10^{-6}$	-
P_{11}	-	-	-	-
P_{12}	+1.429	-	+2.647	-

Table 3. Time and radius of formed event and apparent horizons

P_i	$\gamma_i(t)$	Trapped surfaces	Phase of fluid	Nakedness
P_1	-0.669	Full	Domain walls	Covered
P_2	-0.323	Quasi	Cosmic sting	Naked
P_3	-0.174	Full	Dark matter	Covered
P_4	-0.796	No	Domain walls	Naked
P_5	-0.563	No	Domain walls	Naked
P_6	-0.822	Quasi	Anti-matter	Covered
P_7	-0.704	No	Domain walls	Naked
P_8	+1.092	No	Stiff matter	Naked
P_9	-0.691	Quasi	Domain walls	Covered
P_{10}	-0.438	Quasi	Quasi-cosmic string	Covered
P_{11}	-0.178	Quasi	Dark matter	Covered
P_{12}	-0.552	Quasi	Quasi-domain walls	Covered

Table 4. Nakedness, trapped surfaces and phase of fluid

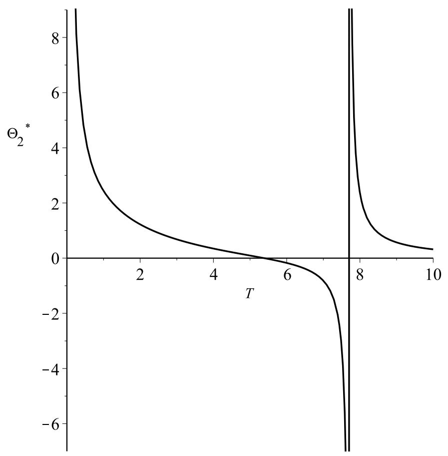

FIG. 3: Diagram of radial null geodesics expansion parameter is plotted against T for metric solution P_2

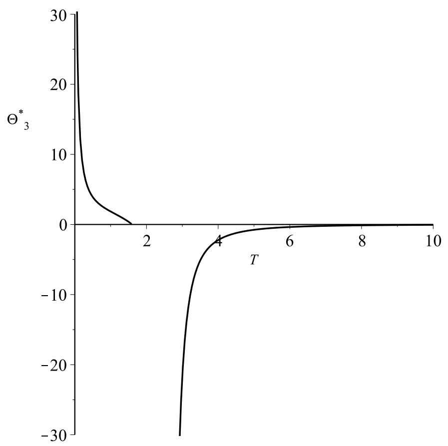

FIG. 4: Diagram of radial null geodesics expansion parameter is plotted against T for metric solution P_3

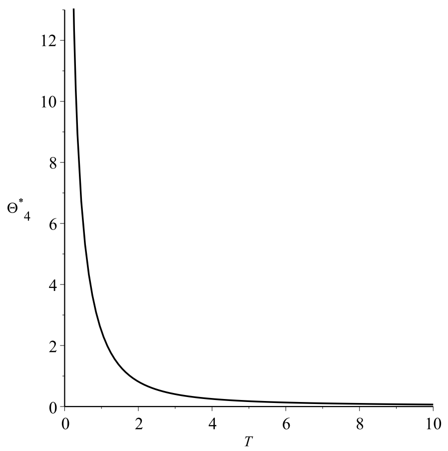

FIG. 5: Diagram of radial null geodesics expansion parameter is plotted against T for metric solution P_4

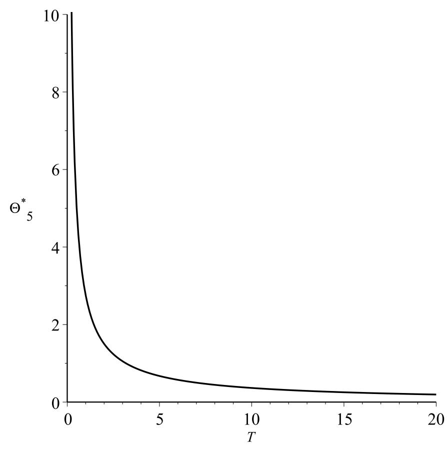

FIG. 6: Diagram of radial null geodesics expansion parameter is plotted against T for metric solution P_5

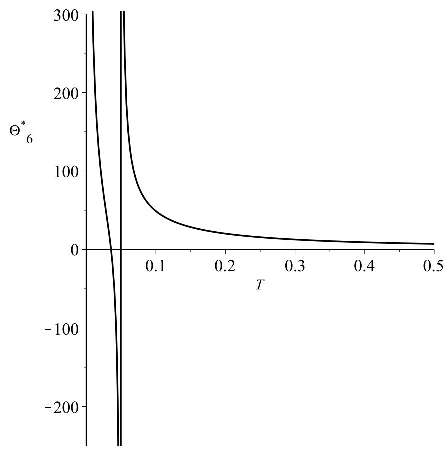

FIG. 7: Diagram of radial null geodesics expansion parameter is plotted against T for metric solution P_6

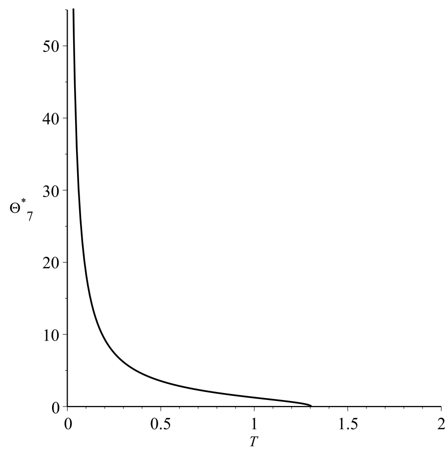

FIG. 8: Diagram of radial null geodesics expansion parameter is plotted against T for metric solution P_7

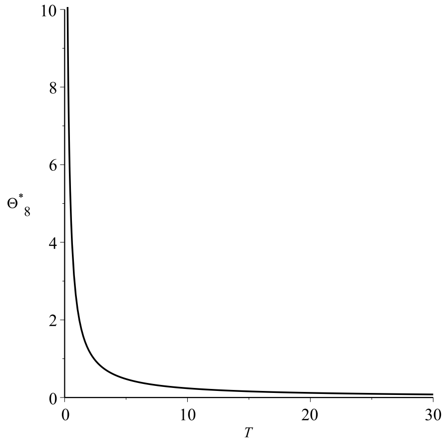

FIG. 9: Diagram of radial null geodesics expansion parameter is plotted against T for metric solution P_8

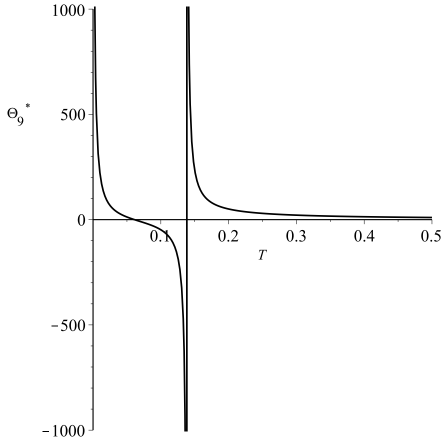

FIG. 10: Diagram of radial null geodesics expansion parameter is plotted against T for metric solution P_9

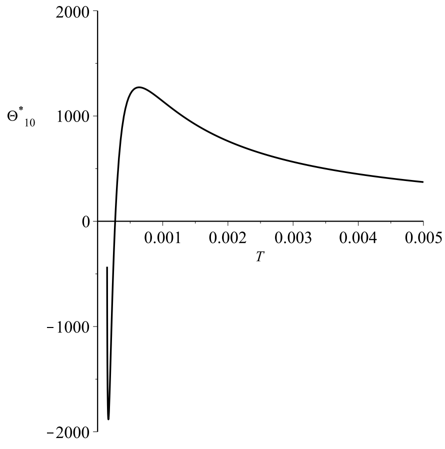

FIG. 11: Diagram of radial null geodesics expansion parameter is plotted against T for metric solution P_{10}

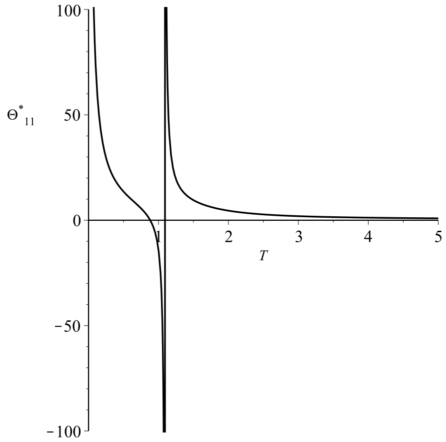

FIG. 12: Diagram of radial null geodesics expansion parameter is plotted against T for metric solution P_{11}

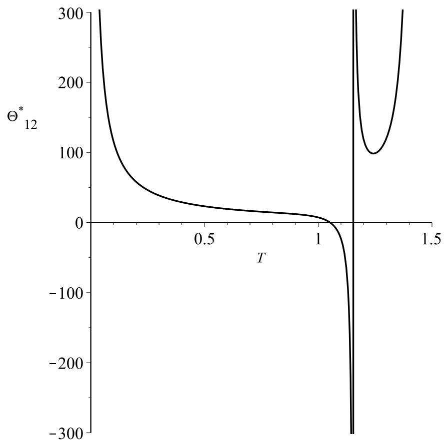

FIG. 13: Diagram of radial null geodesics expansion parameter is plotted against T for metric solution P_{12}

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