Accelerated expansion from structure formation

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A factor of 2

- The early universe is well described by a model which is homogeneous and isotropic, contains only ordinary matter and evolves according to ordinary general relativity.
- However, such a model underpredicts the distances measured in the late universe by a factor of 2.
- This is interpreted as faster expansion.
- There are three possibilities:

  1) There is matter with negative pressure.

  2) General relativity does not hold.

  3) The universe is not homogeneous and isotropic.

Our clumpy universe

- At late times, the universe is only *statistically* homogeneous and isotropic, on scales >100 Mpc.
- The average evolution of an inhomogeneous and/or anisotropic spacetime is not the same as the evolution of the corresponding smooth spacetime, a feature known as **backreaction**.
- Describing the average behaviour of a clumpy universe was termed **the fitting problem** by George Ellis in 1983.
- Clumpiness affects the expansion rate, light propagation and their relationship.

1. The effect of structures on the expansion rate
2. The meaning of average acceleration
3. Estimating the magnitude of the effect
4. Light propagation

Backreaction, exactly

- Consider a dust universe. The Einstein equation is

$$G_{\alpha\beta} = 8\pi G_N \rho u_\alpha u_\beta .$$

- The dynamics can be written in terms of the gradient

$$\nabla_\beta u_\alpha = \frac{1}{3} h_{\alpha\beta}\theta + \sigma_{\alpha\beta} + \omega_{\alpha\beta} .$$

- The scalar part of the evolution equations is:

$$\begin{cases} \dot{\theta} + \frac{1}{3}\theta^2 = -4\pi G\rho - 2\sigma^2 + 2\omega^2 \\ \frac{1}{3}\theta^2 = 8\pi G\rho - \frac{1}{2}{}^{(3)}R + \sigma^2 - \omega^2 \end{cases}$$

$$\dot{\rho} + \theta\rho = 0.$$

- Here θ is the expansion rate, ρ is the energy density, $\sigma^2 \geq 0$ is the shear, $\omega^2 \geq 0$ is the vorticity and ${}^{(3)}R$ is the spatial curvature. We take $\omega^2 = 0$.

- The Buchert equations (gr-qc/9906015):
- The FRW equations:

$$\begin{cases} 3\dfrac{\ddot{a}}{a} = -4\pi G\langle\rho\rangle + Q \\ 3\dfrac{\dot{a}^2}{a^2} = 8\pi G\langle\rho\rangle - \dfrac{1}{2}\langle {}^{(3)}R\rangle - \dfrac{1}{2}Q \\ \partial_t\langle\rho\rangle + 3\dfrac{\dot{a}}{a}\langle\rho\rangle = 0 \end{cases}$$

$$\dot{\theta} + \frac{1}{3}\theta^2 = -4\pi G\rho - 2\sigma^2$$
$$\frac{1}{3}\theta^2 = 8\pi G\rho - \frac{1}{2}{}^{(3)}R + \sigma^2$$
$$\dot{\rho} + \theta\rho = 0$$

- Here $a(t) \propto \left(\int d^3x\sqrt{{}^{(3)}g}\right)^{1/3} \Leftrightarrow \langle\theta\rangle = 3\dfrac{\dot{a}}{a}.$

$$\langle f\rangle \equiv \frac{\int d^3x\sqrt{{}^{(3)}g}\,f}{\int d^3x\sqrt{{}^{(3)}g}}$$

- The backreaction variable is

$$Q \equiv \frac{2}{3}\left(\langle\theta^2\rangle - \langle\theta\rangle^2\right) - 2\langle\sigma^2\rangle.$$

- The average expansion can accelerate, even though the local expansion decelerates.

Understanding acceleration

- The average expansion rate can increase, because the fraction of volume taken by faster regions grows.
- Structure formation involves overdense regions slowing down more and underdense regions decelerating less.
- Acceleration can be demonstrated with a toy model which has one overdense and one underdense region(astro-ph/0607626) or LTB models (astro-ph/0512651, gr-qc/0605120, astro-ph/0605195).

$$H \equiv \frac{\dot{a}}{a} = \frac{a_1^3}{a_1^3 + a_2^3} H_1 + \frac{a_2^3}{a_1^3 + a_2^3} H_2 = v_1 H_1 + v_2 H_2$$

$$\frac{\ddot{a}}{a} = v_1 \frac{\ddot{a}_1}{a_1} + v_2 \frac{\ddot{a}_2}{a_2} + 2 v_1 v_2 (H_1 - H_2)^2$$

Towards reality

- Acceleration due to structures is possible: is it realised in the universe?
- The non-linear evolution is too complex to follow exactly.
- Because the universe is statistically homogeneous and isotropic, a statistical treatment is sufficient.
- One can evaluate the expansion rate with an evolving ensemble of regions.

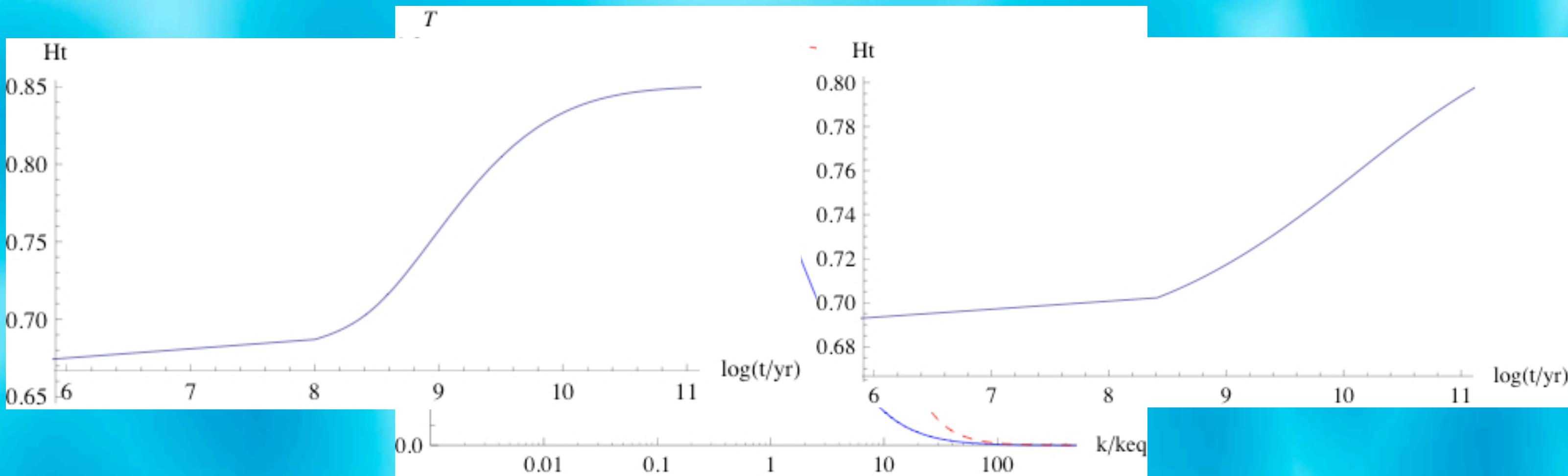
The peak model (arXiv:0801.2692)

- Start from the spatially flat FRW dust background with an initial Gaussian linear density field.
- Identify structures with spherical isolated peaks of the smoothed density field (BBKS 1986), keeping the smoothing threshold fixed at $\sigma(t,R)=1$.
- Each peak evolves like a separate FRW universe.
- The peak number density as a function of time is determined by the primordial power spectrum and the transfer function.
- Take a scale-invariant spectrum with CDM transfer function.

- The expansion rate is $H(t) = \int_{-\infty}^{\infty} d\delta\, v_\delta(t) H_\delta(t).$

- There are no free parameters to adjust.

- We consider two approximate transfer functions.
  Bonvin, Durrer, Gasperini BBKS (with f_b=0.2)



Ht as a function of time (with t_{eq}=50 000 yr)

Light propagation (arXiv:0812.2872)

- The average expansion rate is evaluated on the spatial hypersurface of proper time.
- Most cosmological observations are made along the past lightcone, and measure the redshift and the angular diameter distance.
- In a general spacetime, these quantities are not determined only by expansion.
- However, in a statistically homogeneous and isotropic dust universe, the average expansion rate does give the redshift and the distance.

Light and speed

- Assuming statistical homogeneity and isotropy, the redshift is given by the spatial volume(arXiv:0812.2872):

$$1+z=\exp\left[\int_t^{t_o} dt\left(\frac{1}{3}\theta+\sigma_{\alpha\beta}e^\alpha e^\beta\right)\right]\approx\exp\left(\int_t^{t_o} dt\,\frac{1}{3}\langle\theta\rangle\right)=a(t)^{-1}.$$

- For the angular diameter distance, we have (with $3H\equiv\langle\theta\rangle$)

$$H\partial_{\bar{z}}\left[(1+\bar{z})^2 H\partial_{\bar{z}}\bar{D}_A\right]=-4\pi G_N\langle\rho\rangle\bar{D}_A\,.$$

- Due to conservation of mass, $\langle\rho\rangle\propto a^{-3}\propto(1+\bar{z})^3$.

- The distance in terms of *H* is the same as in FRW ΛCDM.
- If *H* deviates from the FRW ΛCDM model, the relation between *H* and D_A is different than in FRW models.
- Independent observations of *H* and D_A provide a test of backreaction, and of the FRW metric(arXiv:0712.3457).

Summary

- Observations of the late universe are inconsistent with homogeneous and isotropic models with ordinary matter and gravity.
  - FRW models do not include non-linear structures.
- The Buchert equations show that the average expansion of a clumpy dust space can accelerate.
  - The acceleration is understood physically.
  - The right order of magnitude and the timescale $10^5\, t_{eq}$ emerge from a simple model of structures.
  - The relationship of the average expansion rate to distance observations has been determined.
- More work is necessary to quantify the effect of non-linear structures before it could be concluded that new physics is needed.