

**Non-Gaussianity of the CMB:
primordial
vs.
non-linearities of the photon-baryon fluid**

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COSMO 09
CERN, 7-11 September 2009

Plan of the talk

Primordial non-Gaussianity: a reminder

CMB anisotropies at second-order in perturbation theory

Contamination to primordial non-Gaussianity

Based on:

N.B., A. Riotto, JCAP 03, 017 (2009)

D. Nitta, E. Komatsu, N.B., S. Matarrese, A Riotto, JCAP 05, 014 (2009)

N.B, S. Matarrese and A. Riotto, JCAP 06, 02 (2006)

JCAP 01, 19 (2007)

Primordial NG

$\Phi(\mathbf{x})$: primordial gravitational potential

A non-vanishing **three point function**, or its Fourier transform, the **bispectrum is an indicator of non-Gaussianity**

$$\langle \Phi(\vec{k}_1)\Phi(\vec{k}_2)\Phi(\vec{k}_3) \rangle = (2\pi)^3 \delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) B(k_1, k_2, k_3) \longrightarrow \left\langle \frac{\Delta T}{T}(n_1) \frac{\Delta T}{T}(n_2) \frac{\Delta T}{T}(n_3) \right\rangle$$

$$B(k_1, k_2, k_3) = f_{NL} F(k_1, k_2, k_3)$$

Amplitude

Shape

f_{NL} and the shape are model-dependent:

e.g. standard single-field slow-roll inflation $f_{NL} \sim \mathcal{O}(\epsilon, \eta) \ll 1$ (Acquaviva et al. 01; Maldacena 01)

Why primordial NG is important?

f_{NL} and shape are model dependent:

e.g. standard single-field slow-roll inflation $f_{\text{NL}} \sim \mathcal{O}(\epsilon, \eta) \ll 1$

Acquaviva et al. 01; Maldacena 01

✓ A crucial example:

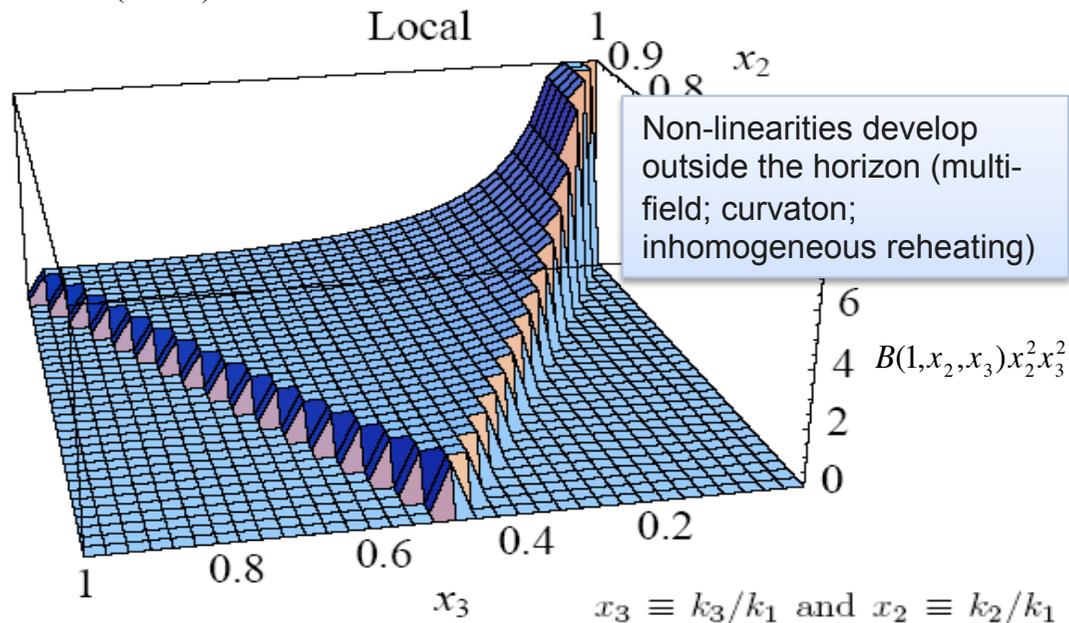
a detection of a primordial $f_{\text{NL}} \sim 10$ would rule out standard single-field models of slow-roll inflation.

non-Gaussianity shape

(see Babich, Creminelli and Zaldarriaga 04; for other shapes see Fergusson and Shellard 08)



Babich et al. (2004)



1. Most signal in squeezed triangles: $k_1 \ll k_2 \sim k_3$

Local Models

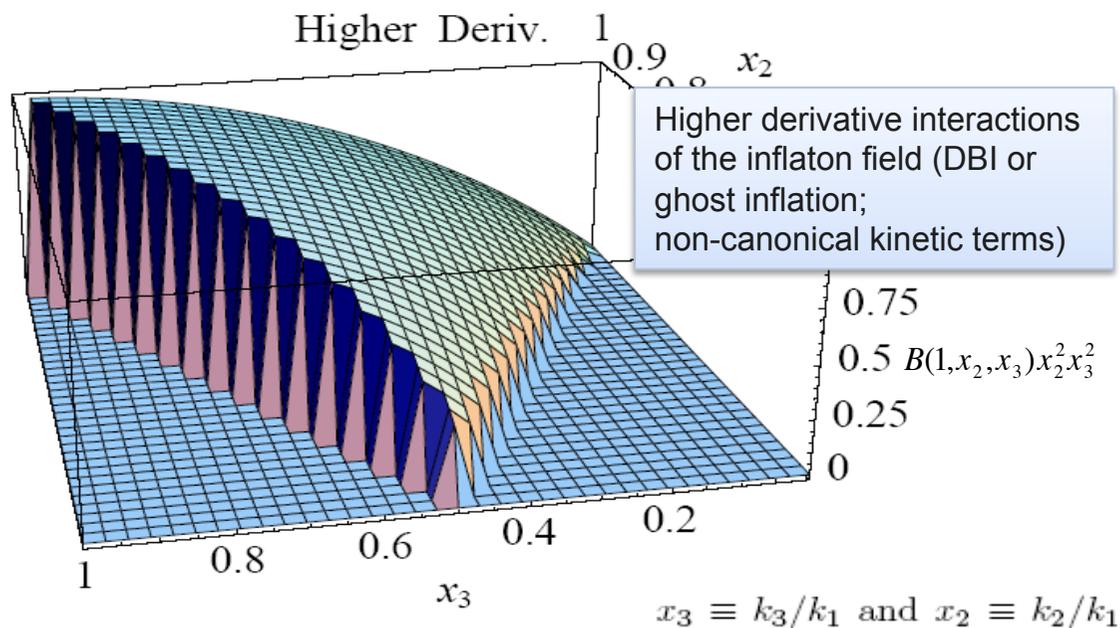
$$\Phi(\mathbf{x}) = \Phi_L(\mathbf{x}) + f_{NL} (\Phi_L^2(\mathbf{x}) - \langle \Phi_L^2(\mathbf{x}) \rangle)$$

$$-4 < f_{NL}^{loc} < 80 \text{ (95\% C.L.)}$$

(Smith et al. 09 from WMAP data)

Planck can reach $f_{NL}^{loc} \sim 5$

Babich et al. (2004)



2. Bispectrum peaks for equilateral triangles: $k_1 = k_2 = k_3$

Equilateral models: operators with gradients

$$-125 < f_{NL}^{equil} < 435 \text{ (95\% C.L.)}$$

(Senatore et al. 09 from WMAP data)

Planck can reach $f_{NL}^{equil} \sim 67$

Non-primordial sources of NG

- ✓ Any non-linearity can contribute to NG and this can contaminate the extraction of the primordial signal

NG = NG primordial + non-linear effects → bias or contamination to primordial

- ✓ Need to specify of which primordial non-Gaussianity we study the contamination from the non-primordial sources (local, equilateral..)

- ✓ We will focus on the non-linearities from Boltzmann equation at second-order for photons, baryons and CDM

In particular: NG from gravity and non-linear dynamics of the photon-baryon fluid at recombination

Less known than other specific secondary effects: e.g.

- reionization: Cooray and Hu 2000
- **ISW-lensing bispectrum**, Goldberg & Spergel '99, Smith & Zaldarriaga 06; Serra and Cooray 08; Hanson et al. 09
- **lensing-Rees Sciama effect**, Mangilli and Verde 09: **a bias $f_{NL} \sim 10$ to local NG**
- Point sources: Komatsu & Spergel 01; Babich & Pierpaoli 08
- ISW-Sunyaev-Zel'dovich bispectrum, Goldberg & Spergel '99, Komatsu & Spergel 01, Smith & Zaldarriaga 06

Second-order CMB Anisotropies

$$\frac{df}{d\eta} = a C[f]$$

Collision term

$$\frac{df}{d\eta} = \frac{\partial f}{\partial \eta} + \underbrace{\frac{\partial f}{\partial x^i} \frac{dx^i}{d\eta} + \frac{\partial f}{\partial p} \frac{dp}{d\eta} + \frac{\partial f}{\partial n^i} \frac{dn^i}{d\eta}}_{\text{Gravity effects}}$$

Gravity effects

Metric perturbations: Poisson gauge

$$ds^2 = a^2(\eta) \left[-e^{2\Phi} d\eta^2 + 2\omega_i dx^i d\eta + (e^{-2\Psi} \delta_{ij} + \chi_{ij}) dx^i dx^j \right]$$

$$\Phi = \Phi^{(1)} + \frac{1}{2} \Phi^{(2)}, \quad \psi = \psi^{(1)} + \frac{1}{2} \psi^{(2)}$$

Example: using the geodesic equation for the photons

$$\frac{1}{p} \frac{dp}{d\eta} = -\mathcal{H} + \Psi' - \Phi_{,i} n^i e^{\Phi+\Psi} - \omega'_i n^i - \frac{1}{2} \chi'_{ij} n^i n^j$$

→ Redshift of the photon
(*Sachs-Wolfe and ISW effects*)

PS: Here the photon momentum is $\mathbf{p} = pn^i$ with $p^2 = g_{ij} P^i P^j$
($P^\mu = dx^\mu(\lambda)/d\lambda$ quadri-momentum vector)

The 2nd-order photon Boltzmann equation

$$\Delta^{(2)'} + n^i \frac{\partial \Delta^{(2)}}{\partial x^i} - \tau' \Delta^{(2)} = S$$

$$\Delta^{(2)}(x^i, n^i, \eta) = \frac{\int dp p^3 f^{(2)}}{\int dp p^3 f^{(0)}}$$

N.B: for a derivation of the Boltzmann equations see also
C. Pitrou CQG 09 (includes polarization);
Senatore, Tassev, Zaldarriaga, arXiv:0812.36523

with $\tau' = -n_e \sigma_T$ a optical depth

Source term $S = S^{(2)} + S^{(I \times I)}$

Second-order baryon velocity

$$S^{(2)} = \underbrace{-\tau'(\Delta_{00}^{(2)} + 4\Phi^{(2)})}_{\text{Sachs-Wolfe effect}} + 4(\Phi^{(2)} + \Psi^{(2)})' - 8\omega'_i n^i - 4\chi'_{ij} n^i n^j - \tau' \left[4\mathbf{v}^{(2)} \cdot \mathbf{n} - \frac{1}{2} \sum_{m=-2}^2 \frac{\sqrt{4\pi}}{5^{3/2}} \Delta_{2m}^{(2)} Y_{2m}(\mathbf{n}) \right]$$

$$S^{(I \times I)} = -8\Delta^{(1)} \left(\Psi^{(1)'} - \Phi_{,i}^{(1)} n^i \right) + 2n^i (\Phi^{(1)} + \Psi^{(1)}) \partial_i (\Delta^{(1)} + 4\Phi^{(1)})$$

$$+ \left[(\Phi_{,j}^{(1)} + \Psi_{,j}^{(1)}) n^i n^j + (\Phi^{,i} - \Psi^{,i}) \right] \frac{\partial \Delta^{(1)}}{\partial n^i} \longrightarrow \text{Gravitational lensing}$$

$$- \tau' \left[2\delta_e^{(1)} \left(4\mathbf{v} \cdot \mathbf{n} + \Delta_0^{(1)} - \Delta^{(1)} + \frac{1}{2} \Delta_2^{(1)} P_2(\hat{\mathbf{v}} \cdot \mathbf{n}) \right) \right] \text{Quadratic-Doppler effect}$$

$$+ 2(\mathbf{v} \cdot \mathbf{n}) \left[\Delta_0^{(1)} + 3\Delta_0^{(1)} - \Delta_2^{(1)} \left(1 - \frac{5}{2} P_2(\hat{\mathbf{v}} \cdot \mathbf{n}) \right) \right] - v \Delta_1^{(1)} (5 + 4P_2(\hat{\mathbf{v}} \cdot \mathbf{n})) + 14(\mathbf{v} \cdot \mathbf{n})^2 - 2v^2$$

Coupling velocity and linear photon anisotropies

CMB angular bispectrum

$$B_{l_1 l_2 l_3}^{m_1 m_2 m_3} \equiv \langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle,$$

$$B_{l_1 l_2 l_3} = \sum_{\text{all } m} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} B_{l_1 l_2 l_3}^{m_1 m_2 m_3},$$

$$\Delta^{(1)} + ik\mu\Delta^{(1)} - \tau'\Delta^{(1)} = S^{(1)}(\mathbf{k}, \hat{\mathbf{n}}, \eta)$$

$$\Delta^{(2)} + ik\mu\Delta^{(2)} - \tau'\Delta^{(2)} = S^{(2)}(\mathbf{k}, \hat{\mathbf{n}}, \eta)$$

$$S_{lm}^{(2)}(\mathbf{k}) = \int \frac{d^3 k'}{(2\pi)^3} \int d^3 k'' \delta^3(\mathbf{k}' + \mathbf{k}'' - \mathbf{k}) \\ \times \mathcal{S}_{lm}^{(2)}(\mathbf{k}', \mathbf{k}'', \mathbf{k}) \zeta(\mathbf{k}') \zeta(\mathbf{k}'')$$

Harmonic components
of the CMB source
function

CMB angular bispectrum (II)

$$a_{lm}^{(1)} = 4\pi(-i)^l \int \frac{d^3k}{(2\pi)^3} g_l(k) Y_{lm}^* \zeta(\mathbf{k})$$

$$\Phi(\mathbf{x}) = \Phi_L(\mathbf{x}) + f_{NL} (\Phi_L^2(\mathbf{x}) - \langle \Phi_L^2(\mathbf{x}) \rangle)$$

$$a_{lm}^{(2)} = \frac{4\pi}{8} (-i)^l \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3k'}{(2\pi)^3} \int d^3k'' \delta^3(\mathbf{k}' + \mathbf{k}'' - \mathbf{k}) \\ \times \sum_{l'm'} F_{lm}^{l'm'}(\mathbf{k}', \mathbf{k}'', \mathbf{k}) Y_{l'm'}^*(\hat{\mathbf{k}}) \zeta(\mathbf{k}') \zeta(\mathbf{k}'')$$

Second-order radiation transfer function

Primordial curvature perturbation

$$F_{lm}^{l'm'}(\mathbf{k}', \mathbf{k}'', \mathbf{k}) = i^l \sum_{\lambda\mu} (-1)^m (-i)^{\lambda-l'} \mathcal{G}_{ll'\lambda}^{-mm'\mu}$$

Nitta, Komatsu, N.B, Matarrese, Riotto 09

$$\times \sqrt{\frac{4\pi}{2\lambda+1}} \int_0^{\eta_0} d\eta e^{-\tau} \mathcal{S}_{\lambda\mu}^{(2)}(\mathbf{k}', \mathbf{k}'', \mathbf{k}) j_{l'}[k(\eta - \eta_0)].$$

CMB source function

A closer look at the CMB source function

Two types of contributions:

- 1) Intrinsically second-order term $\Theta^{(2)} = (\Delta_{00}^{(2)} / 4 + \Phi^{(2)})$
on large scales it gives rise to the Sachs-Wolfe effect;
on small scales it grows as η^2
(pointed out by Pitrou, Uzan, Bernardeau 08)

→ Contamination to equilateral NG

- 2) (first-order)²-terms (oscillating/constant in time)

→ Contamination to local NG

Non-linear dynamics at recombination

On small scales, i.e. modes $k \gg k_{\text{eq}}$, the second-order anisotropies at recombination are dominated by the 2nd-order gravitational potential sourced by dark matter perturbations

$$\Phi^{(2)} \simeq \Psi^{(2)} = \Psi^{(2)}(0) - \frac{1}{14} \left(\partial_k \Phi^{(1)} \partial^k \Phi^{(1)} - \frac{10}{3} \frac{\partial_i \partial^j}{\nabla^2} \left(\partial_i \Phi^{(1)} \partial_j \Phi^{(1)} \right) \right) \eta^2$$

Initial conditions that contain the primordial NG

in Fourier space gives the convolution kernel

$$G(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}) \eta^2 = \left[\mathbf{k}_1 \cdot \mathbf{k}_2 - \frac{10 (\mathbf{k} \cdot \mathbf{k}_2)(\mathbf{k} \cdot \mathbf{k}_1)}{3 k^2} \right] \eta^2$$

As a generalization to the well known expression at linear-order

$$\Theta^{(2)} = \frac{1}{4} \Delta_{00}^{(2)} + \Phi^{(2)} \sim A \cos[kc_s \eta] e^{-(k/k_D)^2} - R\Phi^{(2)} + S \quad (\text{For details see Pitrou et al. 08; see also N.B., Matarrese, Riotto 07})$$

On small scales the combination of the damping effects AND the growth of the potential as η^2 make dominant the term

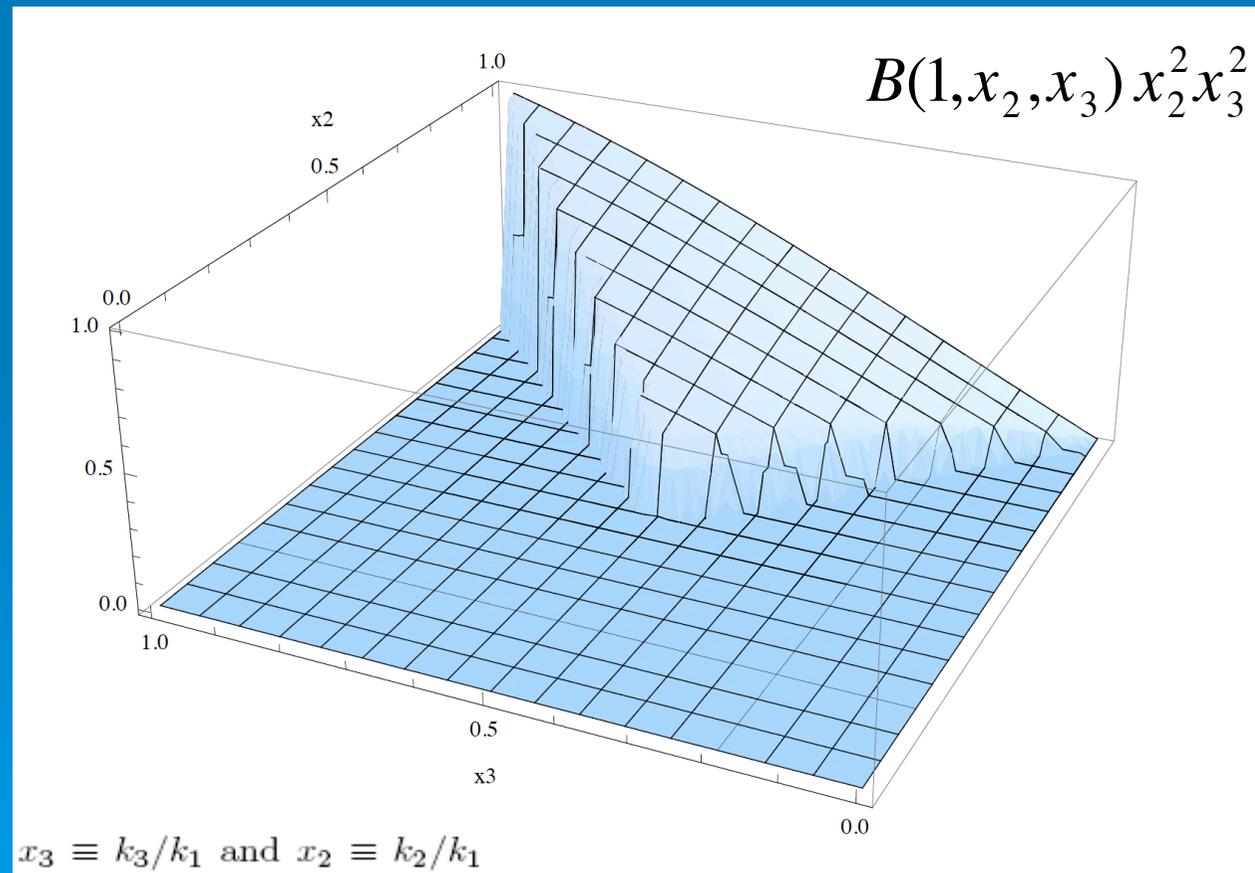
$$-R\Phi^{(2)} = -\frac{R}{14} G(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}) \eta^2 \Phi^{(1)}(\mathbf{k}_1) \Phi^{(1)}(\mathbf{k}_2)$$

Non-Gaussianity from 2nd-order gravitational potential

$$\Theta^{(2)} \cong -R\Phi^{(2)} \cong -\frac{R}{14} G(k_1, k_2, k) \eta^2 \Phi^{(1)}(k_1) \Phi^{(1)}(k_2)$$

This effect is a causal one, i.e. developing on small scales; its origin is gravitational (due to the non-linear growth sourced by dark matter perturbations)

We expect the corresponding CMB bispectrum will be of the equilateral type.



Important comment:

even if at recombination this effect is the dominant one, it will have a minimal contamination to the local primordial NG

Signal to Noise ratio and correlation

See Spergel and Goldberg '99; Cooray and Hu 200; Komatsu and Spergel 2001

$$\chi^2 \equiv \sum_{2 \leq l_1 \leq l_2 \leq l_3} \frac{\left(B_{l_1 l_2 l_3}^{obs} - \sum_i A_i B_{l_1 l_2 l_3}^{(i)} \right)^2}{\sigma_{l_1 l_2 l_3}^2}$$

$$\sigma_{l_1 l_2 l_3}^2 \equiv \langle B_{l_1 l_2 l_3}^2 \rangle - \langle B_{l_1 l_2 l_3} \rangle^2 \approx C_{l_1} C_{l_2} C_{l_3} \Delta_{l_1 l_2 l_3}$$

$B_{l_1 l_2 l_3}^{(i)}$: primordial or secondary bispectra

Fisher matrix

$$F_{ij} \equiv \sum_{2 \leq l_1 \leq l_2 \leq l_3} \frac{B_{l_1 l_2 l_3}^{(i)} B_{l_1 l_2 l_3}^{(j)}}{\sigma_{l_1 l_2 l_3}^2}$$

$$\left(\frac{S}{N} \right)_i \equiv \frac{1}{\sqrt{F_{ii}^{-1}}}$$

$$r_{ij} = \frac{F_{ij}^{-1}}{\sqrt{F_{ii}^{-1} F_{jj}^{-1}}}$$

How similar are two bispectra?

Contamination to primordial f_{NL} of the local type :

We fit the primordial bispectrum template to the 2nd-order bispectrum:

define the best-fitting 'contamination' f_{NL}^{cont} by minimizing

$$\chi^2 = \sum_{2 \leq l_1 \leq l_2 \leq l_3} \frac{(f_{NL}^{\text{cont}} B_{l_1 l_2 l_3}^{\text{prim}} - B_{l_1 l_2 l_3}^{2\text{nd}})^2}{\sigma_{l_1 l_2 l_3}^2}$$



$$f_{NL}^{\text{cont}} = \frac{F_{2\text{nd}, \text{prim}}}{F_{\text{prim}, \text{prim}}} \Big|_{f_{NL}=1} = \frac{1}{N} \sum_{2 \leq l_1 \leq l_2 \leq l_3} \frac{B_{l_1 l_2 l_3}^{2\text{nd}} B_{l_1 l_2 l_3}^{\text{prim}}}{\sigma_{l_1 l_2 l_3}^2}$$

$$N = \sum_{2 \leq l_1 \leq l_2 \leq l_3} \frac{B_{l_1 l_2 l_3}^{\text{prim}^2}}{\sigma_{l_1 l_2 l_3}^2}$$

Correlation to primordial f_{NL} of the intrinsically second-order term $\theta^{(2)} = -R\Phi^{(2)}$

Fisher matrix

$$F_{ij} = \frac{f_{\text{sky}}}{\pi} \frac{1}{(2\pi)^2} \int d^2\ell_1 d^2\ell_2 d^2\ell_3 \delta^{(2)}(\vec{\ell}_{123}) \frac{B^i(\ell_1, \ell_2, \ell_3) B^j(\ell_1, \ell_2, \ell_3)}{6 C(\ell_1) C(\ell_2) C(\ell_3)}$$

For EQUILATERAL primordial $f_{\text{NL}}^{\text{eq}}$

$$\left(\frac{S}{N}\right)_{\text{equil}} = \frac{1}{\sqrt{F_{\text{equil, equil}}^{-1}}} \simeq 12.6 \times 10^{-3} f_{\text{NL}}^{\text{equil}}$$

$$\left(\frac{S}{N}\right)_{\text{rec}} = \frac{1}{\sqrt{F_{\text{rec, rec}}^{-1}}} \simeq 0.1$$

$$r_{\text{rec, equil}} = \frac{F_{\text{rec, equil}}^{-1}}{\sqrt{F_{\text{equil, equil}}^{-1} F_{\text{rec, rec}}^{-1}}} \simeq -0.53$$

$$d_{\text{rec}} = F_{\text{rec, rec}} F_{\text{rec, rec}}^{-1} \simeq 1.4$$

$$d_{\text{equil}} = F_{\text{equil, equil}} F_{\text{equil, equil}}^{-1} \simeq 1.4$$

As a confirmation of our expectations the NG from recombination (governed by the non-linear evolution of the 2nd-order gravitational potential) shows a quite high correlation with an equilateral primordial bispectrum

✓ Degradation of the $1\text{-}\sigma$ uncertainty of $f_{\text{NL}}^{\text{eq}}$

$r_{\text{rec, equil}} = -0.53$ translates into an increase of the minimum detectable value for $f_{\text{NL}}^{\text{eq}}$. We find a minimum value of

$$f_{\text{NL}}^{\text{equil}} \simeq 79$$



$$\Delta f_{\text{NL}}^{\text{equil}} = \mathcal{O}(10)$$

It corresponds to an increase of $\mathcal{O}(10)$: recall: $f_{\text{NL}}^{\text{eq}} = 67$, not accounting for the cross-correlation (i.e. not marginalized over the signal from recombination).

✓ Contamination to primordial f_{NL}

$$\text{Contamination to equilateral} \quad f_{\text{NL}}^{\text{cont}} = \mathcal{O}(10)$$

$$\text{Contamination to local} \quad f_{\text{NL}}^{\text{cont}} \approx 0.3$$

Given the $1\text{-}\sigma$ uncertainty $f_{\text{NL}}^{\text{loc}} \sim 5$ for local, and $f_{\text{NL}}^{\text{eq}} \sim 67$ for equilateral, these numbers are not relevant.

So the contamination from the intrinsically second-order term $\theta^{(2)} = -\mathbf{R}\Phi^{(2)}$ to a primordial local NG is minimal; what is relevant is the degradation in the minimum detectable value of $f_{\text{NL}}^{\text{eq}}$ for primordial equilateral NG

A check for our model: S/N for the primordial equilateral bispectrum

Use the flat sky approximation

$$a(\vec{\ell}) = \int \frac{dk^z}{2\pi} e^{ik^z(\eta_0 - \eta_r)} \Phi(\mathbf{k}') \tilde{\Delta}^T(\ell, k^z)$$

transfer function on small scales
($l \gg l_* \sim 750$; $a \sim 3$)

$$\tilde{\Delta}^T(\ell, k^z) = a(\eta_0 - \eta_r)^{-2} e^{-1/2(\ell/l_*)^{1.2}} e^{-1/2(|k_z|/k_*)^{1.2}}$$

power spectrum
(see also Babich & Zaldarriaga 04)

$$C(\ell) \simeq a^2 \frac{A}{\pi} \frac{\ell_*}{\ell^3} e^{-(\ell/l_*)^{1.2}}$$

Bispectrum

$$B_{\text{equil}}(\ell_1, \ell_2, \ell_3) = \frac{(\eta_0 - \eta_r)^2}{(2\pi)^2} \int dk_1^z dk_2^z dk_3^z \delta^{(1)}(k_{123}^z) B_{\text{equil}}(k'_1, k'_2, k'_3) \tilde{\Delta}^T(\ell_1, k_1^z) \tilde{\Delta}^T(\ell_2, k_2^z) \tilde{\Delta}^T(\ell_3, k_3^z)$$

$$B_{\text{equil}}(k_1, k_2, k_3) = f_{\text{NL}}^{\text{equil}} \cdot 6A^2 \cdot \left(-\frac{1}{k_1^3 k_2^3} - \frac{1}{k_1^3 k_3^3} - \frac{1}{k_2^3 k_3^3} - \frac{2}{k_1^2 k_2^2 k_3^2} + \frac{1}{k_1 k_2^2 k_3^3} + (5 \text{ perm.}) \right)$$

$$B_{\text{equil}}(\ell_1, \ell_2, \ell_3) = \frac{24f_1}{(2\pi)^2} f_{\text{NL}}^{\text{equil}} a^3 A^2 e^{-\ell_T^{1.2}/2\ell_*^{1.2}} \ell_*^2 \left(-\frac{1}{\ell_1^3 \ell_2^3} - \frac{1}{\ell_1^3 \ell_3^3} - \frac{1}{\ell_2^3 \ell_3^3} - \frac{2}{\ell_1^2 \ell_2^2 \ell_3^2} + \frac{1}{\ell_1 \ell_2^2 \ell_3^3} + (5 \text{ perm.}) \right)$$

S/N for the primordial equilateral bispectrum

Signal-to-noise ratio

$$\left(\frac{S}{N}\right)_{\text{equil}}^2 \simeq 8 f_{\text{sky}} A (f_{\text{NL}}^{\text{equil}})^2 \ell_{\text{max}}$$

N.B. & Riotto JCAP 09

For $f_{\text{sky}}=0.8$ and $\ell_{\text{max}}=2000$, for an experiment like Planck gives a minimum detectable

$$f_{\text{NL}}^{\text{equil}} \simeq 66$$

very good agreement with the numerical results of Smith and Zaldarriaga 06, and Liguori 09

N.B. The S/N scale as $(\ell_{\text{max}})^{1/2}$, unlike ℓ_{max} as in the local case;

$(S/N)^2$ receive contribution from the configuration which is peaked at $l_1 \sim l_2 \sim l_3$,

$$(S/N)^2 \propto \int d^2 l_1 d^2 l_2 \delta(l_1 - l_2) / l_1^2 \propto \ell_{\text{max}}$$

Non-Gaussianity from 2nd-order gravitational potential

We have estimated in a semi-analytical way the contribution to the CMB anisotropies coming from

$$\Theta_{SW}^{(2)} \cong -R\Phi^{(2)} \cong -\frac{R}{14} G(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}) \eta^2 \Phi^{(1)}(\mathbf{k}_1) \Phi^{(1)}(\mathbf{k}_2)$$

$$G(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}) = \left[\mathbf{k}_1 \cdot \mathbf{k}_2 - \frac{10}{3} \frac{(\mathbf{k} \cdot \mathbf{k}_2)(\mathbf{k} \cdot \mathbf{k}_1)}{k^2} \right]$$

Bispectrum: in the flat-sky approx.

$$B_{\text{rec}}(\ell_1, \ell_2, \ell_3) = -\frac{4f_2}{(2\pi)^2} \frac{R}{14} A^2 a^2 T_0^2 (k_{\text{eq}} \eta_r)^2 \ell_{\text{eq}}^2 \ell_*^2 e^{-1/2(\ell_1/\ell_*)^{1.2}} e^{-1/2(\ell_2/\ell_*)^{1.2}} \\ \times \frac{1}{\ell_1^5 \ell_2^5} \left[\vec{\ell}_1 \cdot \vec{\ell}_2 - \frac{10}{3} \frac{(\vec{\ell}_3 \cdot \vec{\ell}_1)(\vec{\ell}_3 \cdot \vec{\ell}_2)}{\ell_3^2} \right] + \text{cyclic}.$$

- A: amplitude of the power spectrum of Φ
- $f_2 \approx f_1 = 0.7$
- $R = 3Q_b / 3Q_\gamma \approx 0.3$ at recombination;
- $l_{\text{eq}} \approx 160$

- Matter transfer function

$$\Phi^{(1)}(\mathbf{k}) = T(k) \Phi_0(\mathbf{k})$$

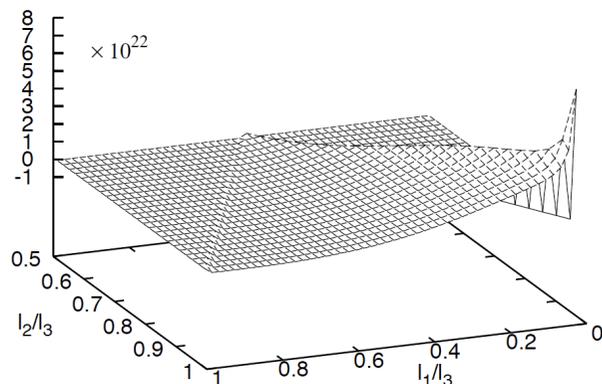
$$T(k) \approx 12(k_{\text{eq}}/k)^2 \ln[k/8k_{\text{eq}}] \equiv T_0(k) \ln[k/8k_{\text{eq}}]$$

$T_0 \approx 11$ for the scales of interest
($750 < l_{\text{max}} \sim 2000$)

Shape of the second-order bispectrum from products of 1st-order terms

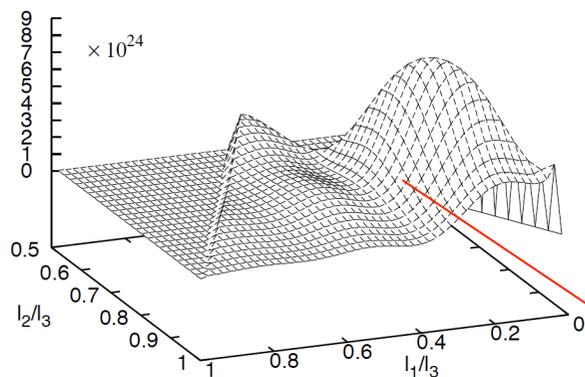
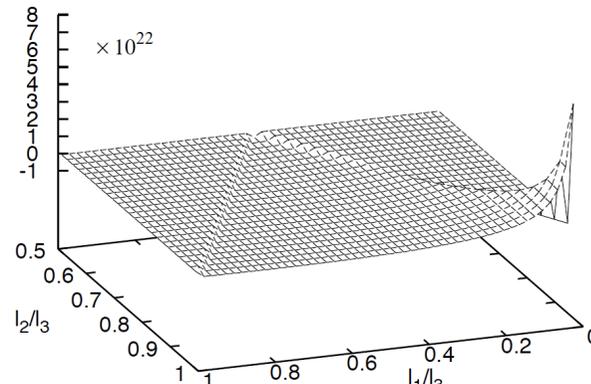
Maximum signal in the squeezed triangles, $l_1 \ll l_2 \sim l_3$, similar to the local primordial bispectrum

$$l_1 l_2 \langle a_{l_1 m_1}^{(1)} a_{l_2 m_2}^{(1)} a_{l_3 m_3}^{(2)} \rangle (\mathcal{G}_{l_1 l_2 l_3}^{m_1 m_2 m_3})^{-1} / (2\pi)^2 \times 10^{22}$$



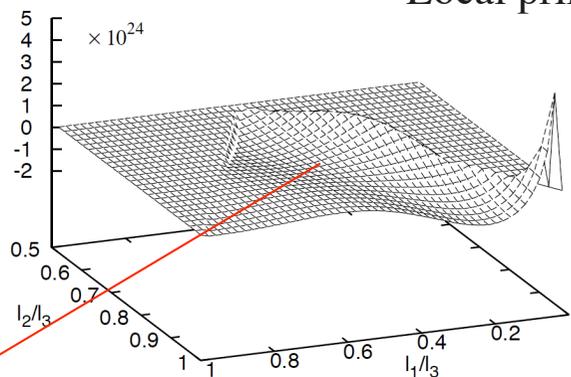
$l_3=200$

Local primordial



$l_3=1000$

Local primordial

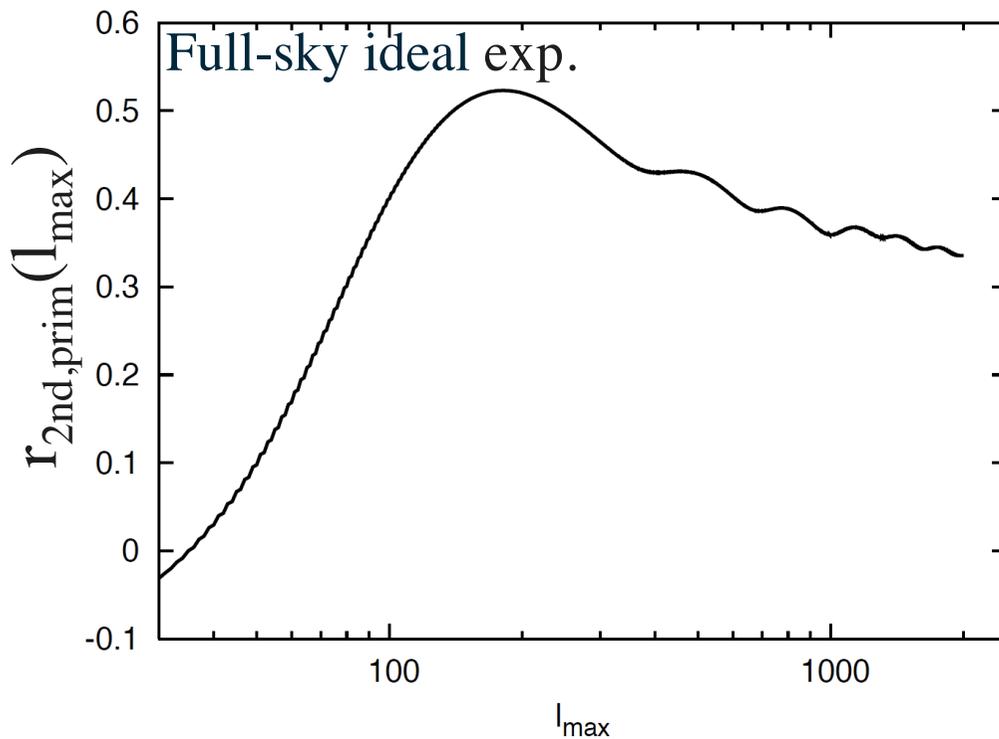


Acoustic oscillations

Cross-correlation

$$r_{ij} = \frac{F_{ij}^{-1}}{\sqrt{F_{ii}F_{jj}}}$$

How similar are 2 bispectra?

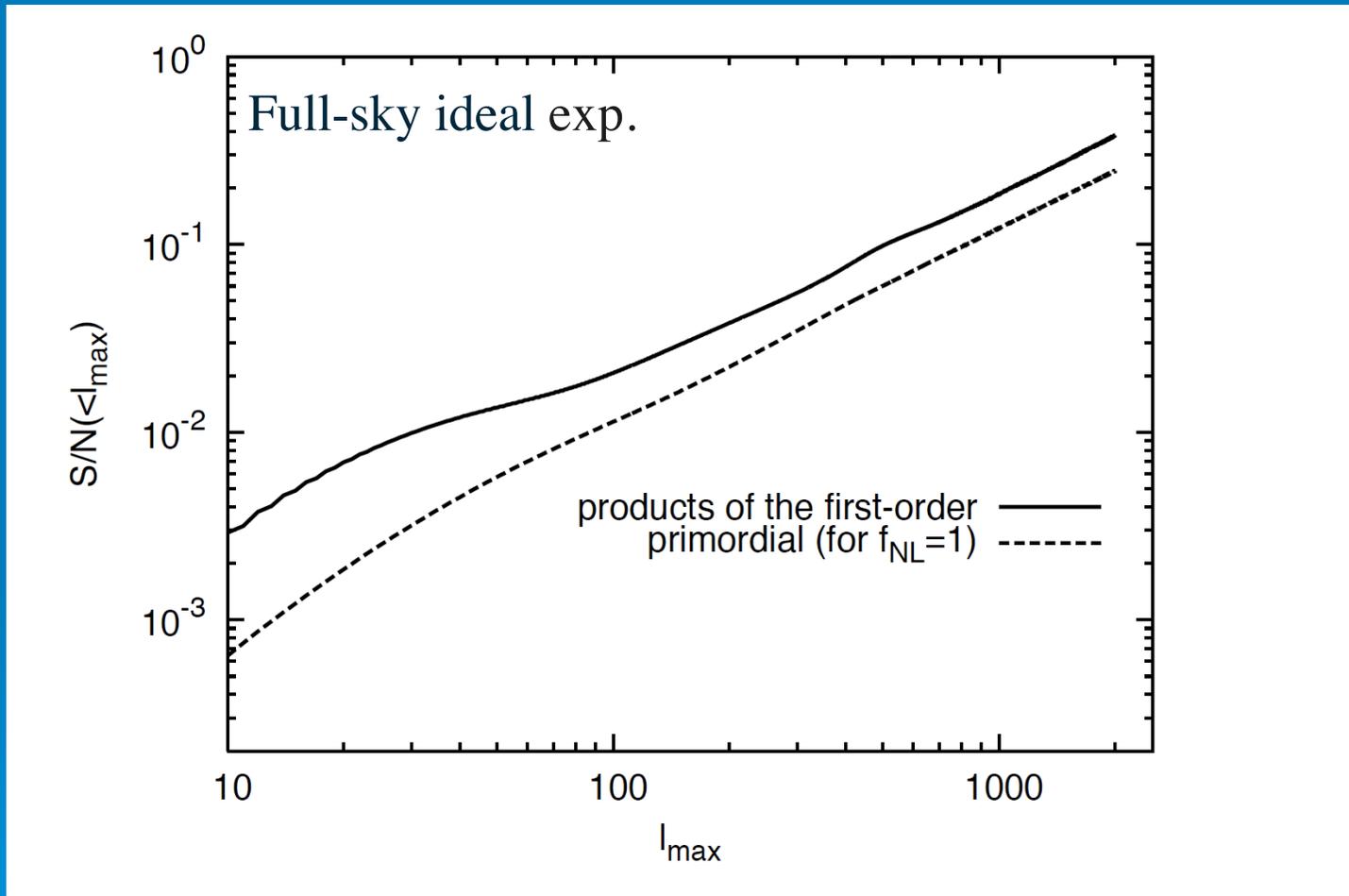


2nd-order bispectrum
and local primordial
are fairly similar

$$r_{2\text{nd,prim}} \sim 0.5 \text{ at } l_{\max} \sim 200$$

$$r_{2\text{nd,prim}} \sim 0.3 \text{ at } l_{\max} \sim 2000$$

Signal to Noise ratio: numerical results for (first-order)²-terms

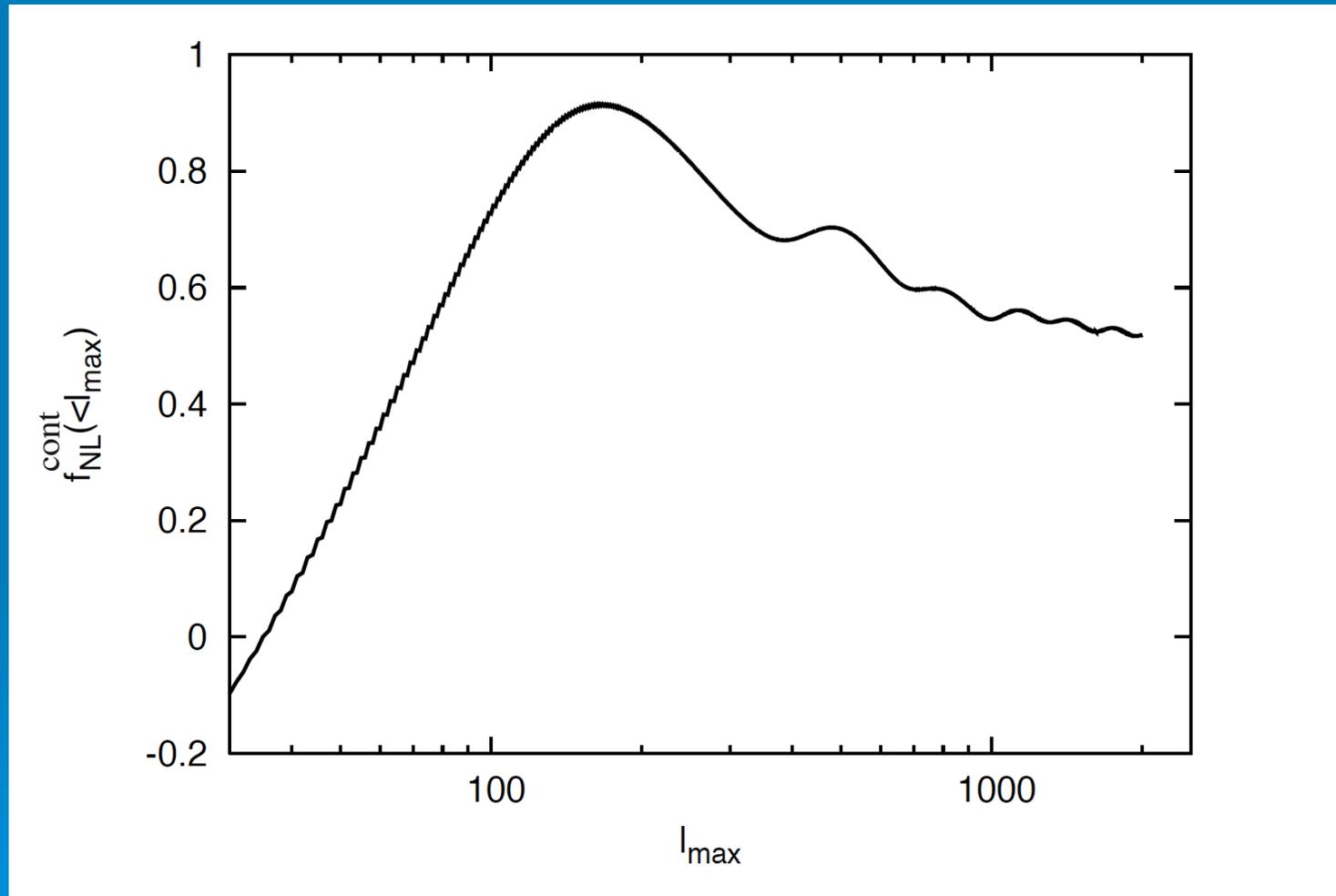


(Nitta, Komatsu,
N.B., Matarrese,
Riotto, JCAP 09)

(S/N) from the (first-order \times first-order) terms is about 0.4
at $l_{max} \approx 2000$ for an ideal full-sky experiment

Contamination to primordial f_{NL} of the local type from (first-order)²-terms

(Nitta, Komatsu, N.B., Matarrese, Riotto, JCAP 09)



Contamination is 0.9 at $l_{\max} \approx 200$ and 0.5 at $l_{\max} \approx 2000$ vs 5 which is the minimum detectable value for Planck: good news

Conclusions

- ✓ Both a positive measurement of primordial non-Gaussianity or an upper limit on its amplitude will represent a crucial observational discriminant between competing models for the primordial perturbation generation
- ✓ A detection of $f_{\text{NL}} \sim 10$ would rule out all standard single-field models of inflation
- ✓ To assess a detection of primordial NG: take care of systematics, residual foregrounds and any secondary signal that can mimick a primordial NG signal
- ✓ **We are assessing precisely the level and shape of NG from (previously unknown) 2nd-order perturbation effects and their contamination to primordial NG for an experiment like Planck: work in progress!**
- ✓ Future techniques exploiting the CMB polarization and additional independent statistical estimators might help NG detection down to $f_{\text{NL}} \sim 3$; with LSS one can hope to reach $f_{\text{NL}} \sim 1$ (see Carbone, Verde, Matarrese 08) need to compute exactly the *predicted amplitude and shape of NG from the post-inflationary evolution of perturbations.*

Finally.....session poster

- ✓ Primordial NG: from preheating phase (Huang)
from DM isocurvature fluctuations (Yokoyama)
in axion monodromy Inflation (Haugboelle)
from cosmic strings (Naruko)
Techniques: beyond delta N formalism (Takamizu)

- ✓ Second-order perturbations: poster by D'Amico

On large-scales: Sachs-Wolfe effect

$$\frac{\delta_{\text{np}} T}{T} = e^{\Phi/3} - 1$$

N.B., S. Matarrese, A. Riotto JCAP 2005

$$\frac{\delta^{(2)} T}{T} = \frac{1}{3} \Phi^{(2)} + \frac{1}{9} (\Phi^{(1)})^2 \quad (\text{N.B., Matarrese, Riotto, Phys. Rev. Lett. 04})$$

+ 2nd - order vector/tensor modes + Rees - Sciama + lensing
(Boubekeur, Creminelli, D'Amico, Norena, Vernizzi 09)

It includes primordial NG

Post-inflation non-linear gravity
common to all scenarios

- ✓ NG and LSS: see posters by Yam Lam on Halo mass function, and by Hiramatsu on non-linear evolution of matter power spectrum
- ✓ Rossi on Peak statistic; Chingangbam, simulated non-Gaussian CMB maps with cubic terms