COSMO 09, CERN, Geneva

Breaking the scale invariance of primordial spectrum in Horava-Lifshitz cosmology

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Work with
Kazuhiro Yamamoto and Gen Nakamura (Hiroshima University)
Phys. Rev. D, to appear; **arXiv: 0907.1549**

Quantum gravity & Cosmology

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- Horava's gravity Horava, 2009
  - A new theory of quantum gravity proposed recently
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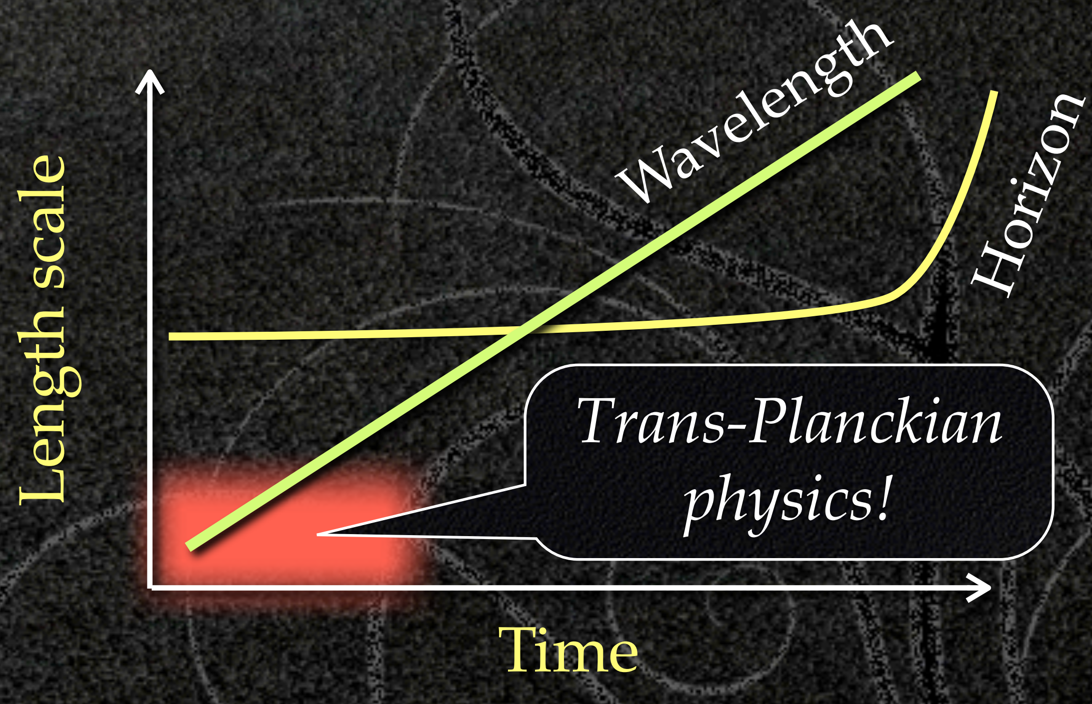
- Cosmological window on quantum gravity phase
  - Fluctuations are stretched by the expansion of the universe
  - **Can probe high energy physics with cosmological perturbations**



Quantum gravity & Cosmology

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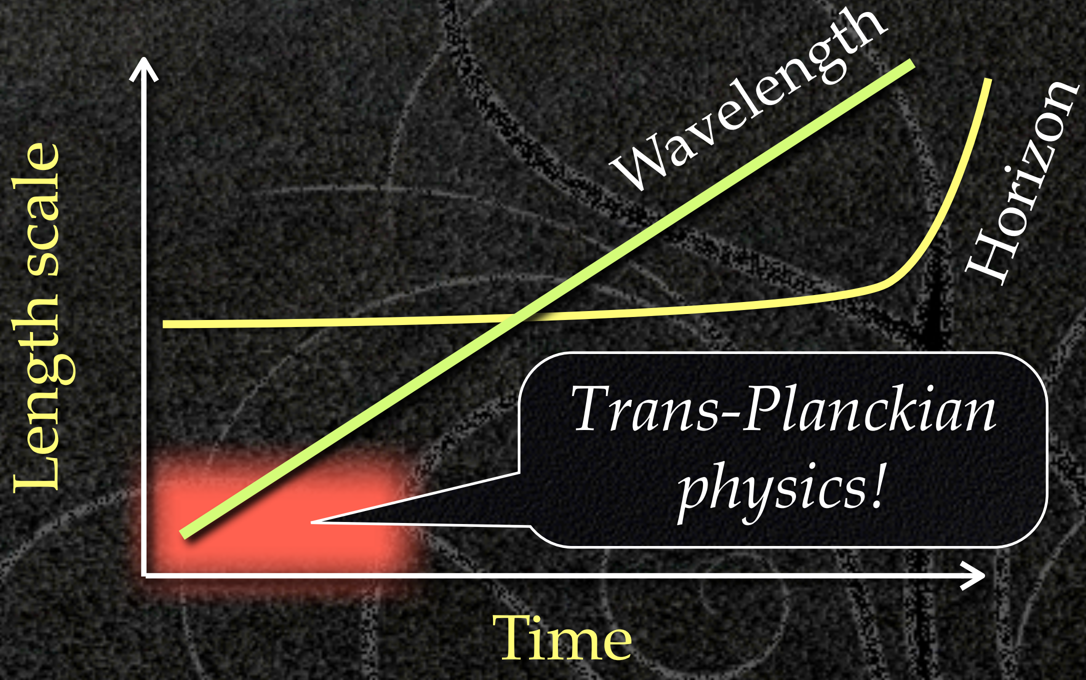
Why don't we study cosmological perturbations in Horava's gravity?

- Fluctuations are stretched by the expansion of the universe
- Can probe high energy physics with cosmological perturbations

Gao, Wang, Brandenberger & Riotto;
Wang & Maartens; Koh;
T.K., Urakawa & Yamaguchi, 2009; ....



Dispersion relation modified in the UV

Essence of Horava's gravity

Horava, 2009

- Action is invariant under the Lifshitz-like anisotropic scaling:

  $t \rightarrow \ell^z t, \quad \vec{x} \rightarrow \ell \vec{x}$ with **z = 3** in the UV

  - flows to $z = 1$ in the IR

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- Consider a mode equation (for GWs and a scalar field fluctuation)

$$\varphi_k'' + \left[k_{\rm eff}^2(\eta) - \frac{a''}{a}\right]\varphi_k = 0$$

- w/ dispersion relation $\omega^2 = k_{\rm eff}^2(\eta)/a^2$

Scale-invariant spectrum from z = 3

Kiritsis, Kofinas, 2009; Mukohyama, 2009

- In the very early universe, set $k^2_{\rm eff}(\eta) = \dfrac{k^6}{M^4 a^4}$ z=3 UV

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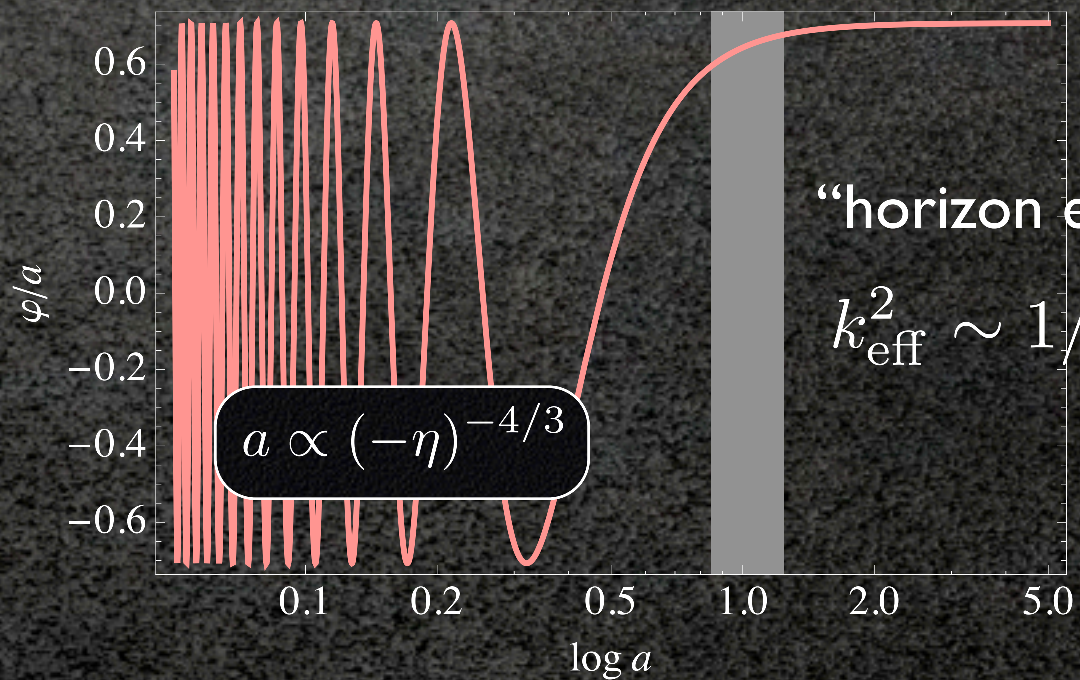
- Normalized mode function: $\dfrac{\varphi_k}{a} = \dfrac{M}{\sqrt{2k^3}} \exp\left[-i\dfrac{k^3}{M^2}\int \dfrac{d\eta}{a^2}\right]$
- Scale-invariant spectrum $\mathcal{P}_\varphi^{1/2} = \dfrac{M}{2\pi}$ (if $\displaystyle\int^{\eta_\infty} \dfrac{d\eta}{a^2}$ converges)



"horizon exit" at

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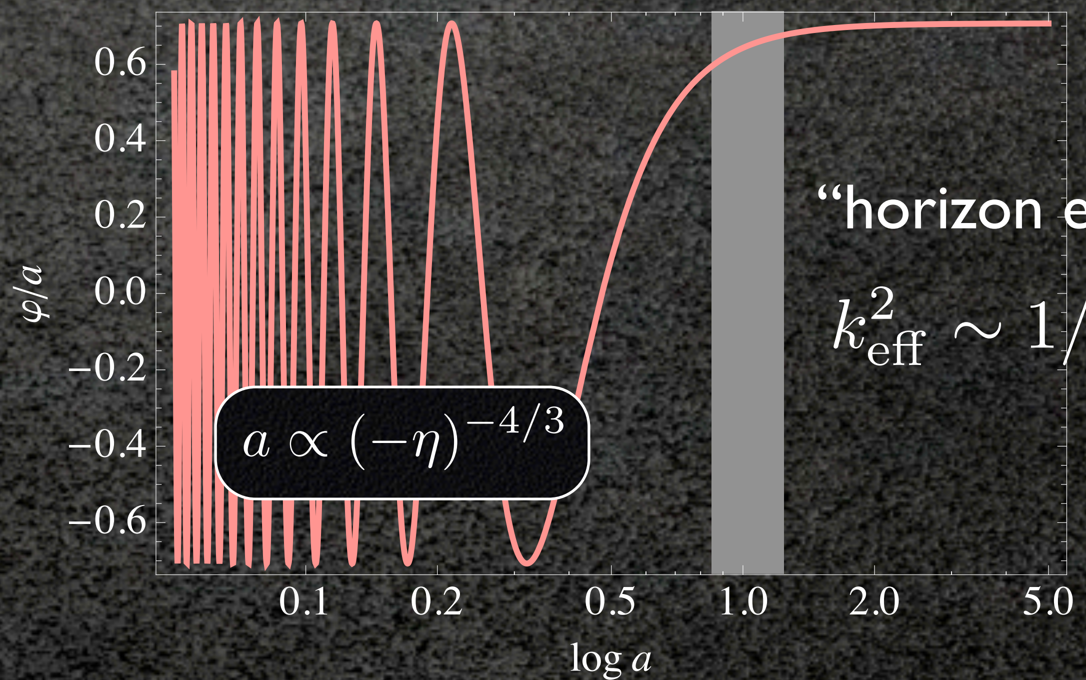

"horizon exit" at $k_{\text{eff}}^2 \sim 1/(-\eta)^2$

Independent of the cosmic expansion rate, H

Exactly scale-invariant even for non-de Sitter background

How to break scale invariance

- Standard scenario: Slightly tilted spectrum from slow-roll inflation

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z=3 UV　　　　z=1 IR

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z=3 UV z=1 IR

Spectral tilt

- k^5: parity violating term for GWs Takahashi and Soda, 2009

Non-standard dispersion relation implied by Horava's quantum gravity

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This work:

Evaluate the spectral index using the **uniform approximation**

K. Yamamoto, **T.K.**, and G. Nakamura
Phys. Rev. D, to appear; **arXiv: 0907.1549**

Uniform approximation

Olver, 1997;Martin, Schwarz, 2003; Habib *et al.* 2002; 2004

- Based essentially on the WKB approximation
- Can fit the exact behavior of the solution accurately
- Suited to calculate the *k*-dependence of the solution
  - w/o slow-roll assumptions

Uniform approximation

Olver, 1997;Martin, Schwarz, 2003; Habib *et al.* 2002; 2004

- Mode equation $\varphi_k'' = \left[g(\eta) - \frac{1}{4\eta^2}\right]\varphi_k \quad g(\eta) := -k_{\mathrm{eff}}^2(\eta) + \frac{a''}{a} + \frac{1}{4\eta^2}$

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  - admits the approximate solution in terms of Airy functions

$$\varphi_k = \sqrt{\frac{\pi}{2}}\left[\frac{\zeta(\eta)}{g(\eta)}\right]^{1/4}[\mathrm{Ai}(\zeta) - i\,\mathrm{Bi}(\zeta)]$$

- where $\zeta(\eta) := \pm\left[\pm\frac{3}{2}\int_{\bar\eta}^{\eta}\sqrt{\pm g(\eta')}d\eta'\right]^{2/3} \qquad \begin{array}{ll} + & \text{for} \quad \eta > \bar\eta \\ - & \text{for} \quad \eta < \bar\eta \end{array}$

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- The solution satisfies the Wronskian normalization condition and has the asymptotic early-time behavior:

$$\varphi_k \approx \exp[-i\int_{\bar\eta}^{\eta}k_{\rm eff}(\eta')d\eta']/\sqrt{2k_{\rm eff}}$$

Uniform approximation contd.

- Asymptotic late-time behavior: $\varphi_k \approx -\frac{i}{\sqrt{2}}[g(\eta)]^{-1/4}\exp\left[\int_{\bar{\eta}}^{\eta}\sqrt{g(\eta')}d\eta'\right]$
- Uniform approximation formula for the spectral index:

$$n_T = 3 + \lim_{\eta\to 0^-} 2\frac{d}{d\ln k}\int_{\bar{\eta}(k)}^{\eta}\sqrt{g(\eta')}d\eta'$$

Uniform approximation contd.

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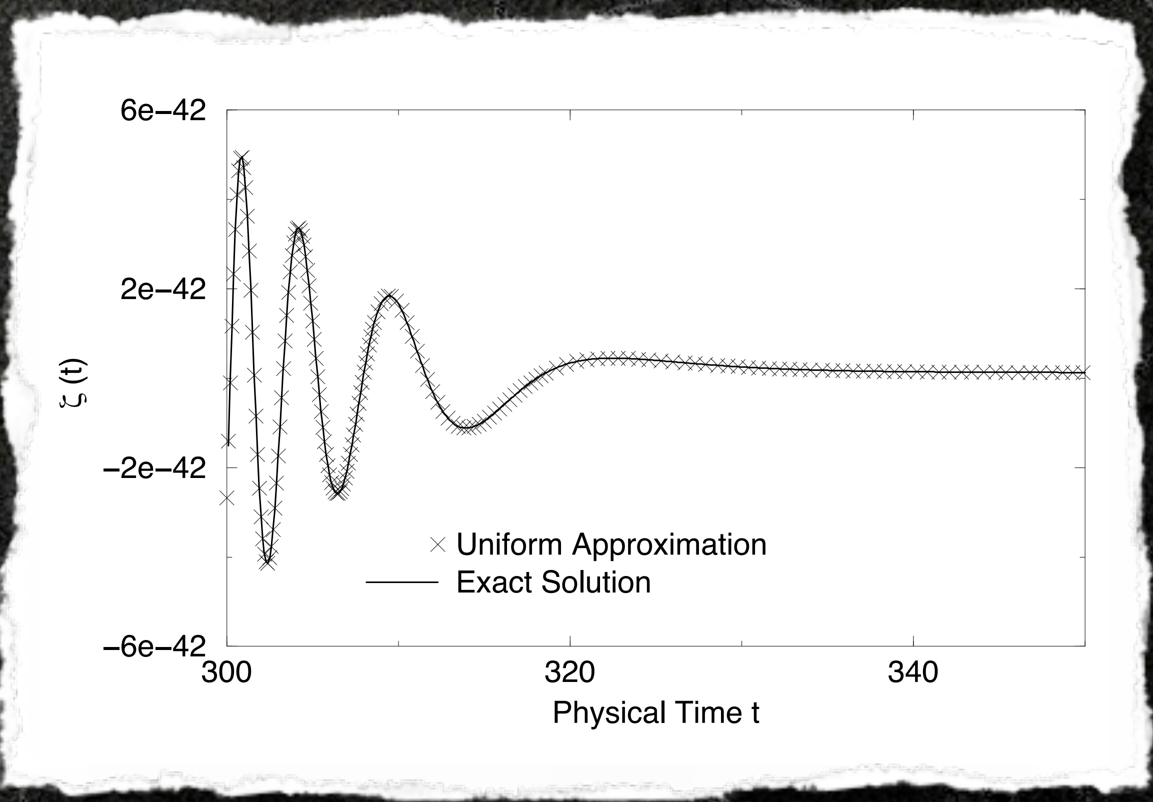
$$n_T = 3 + \lim_{\eta \to 0^-} 2 \frac{d}{d \ln k} \int_{\bar{\eta}(k)}^{\eta} \sqrt{g(\eta')}d\eta'$$

- So far, the uniform approximation has been used only for inflation models with the standard dispersion relation, $\omega^2 = k^2/a^2$

Habib *et al.* 2002



Application I

- Power-law inflation $a \propto t^{1+n}$ with $k_{\rm eff}^2 = k^2 \left(\frac{k^2}{M^2 a^2}\right)^{\mu}$

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- Can be compared with the exact solution
- *k*-independent small correction to the amplitude: $|1 - \mathcal{C}| = \mathcal{O}(0.1)$

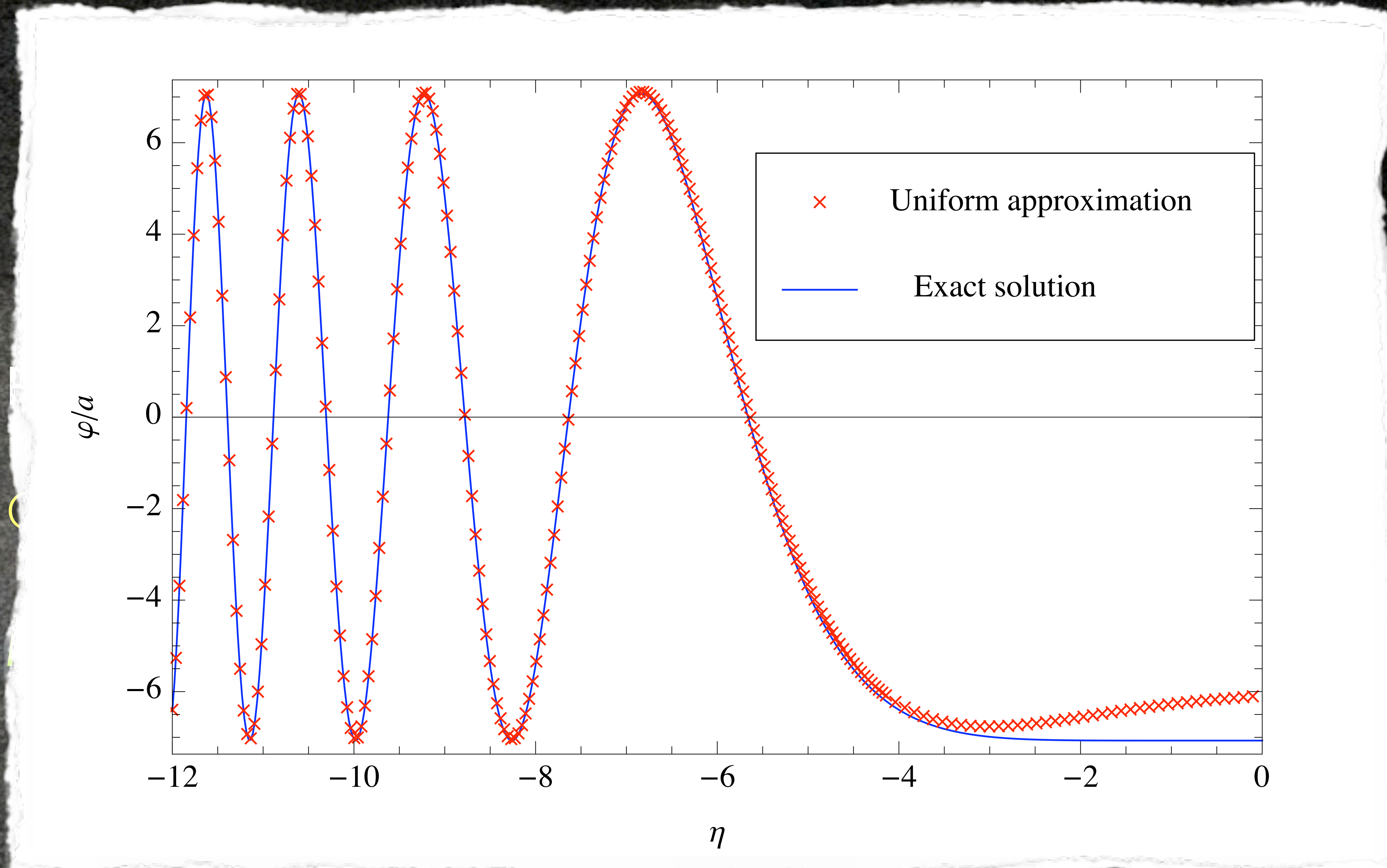
$$|\varphi_{\mathrm{uniform}}| = \mathcal{C} |\varphi_{\mathrm{exact}}|$$

$$\mathcal{C} := \sqrt{2\pi} e^{-\nu} \nu^{\nu - 1/2} / \Gamma(\nu)$$

$$\nu := \frac{1 + 3n/2}{\mu + n(1+\mu)}$$

- **Exact result reproduced for the spectral index**

$$n_T = \frac{\mu - 2}{\mu + n(1+\mu)}$$



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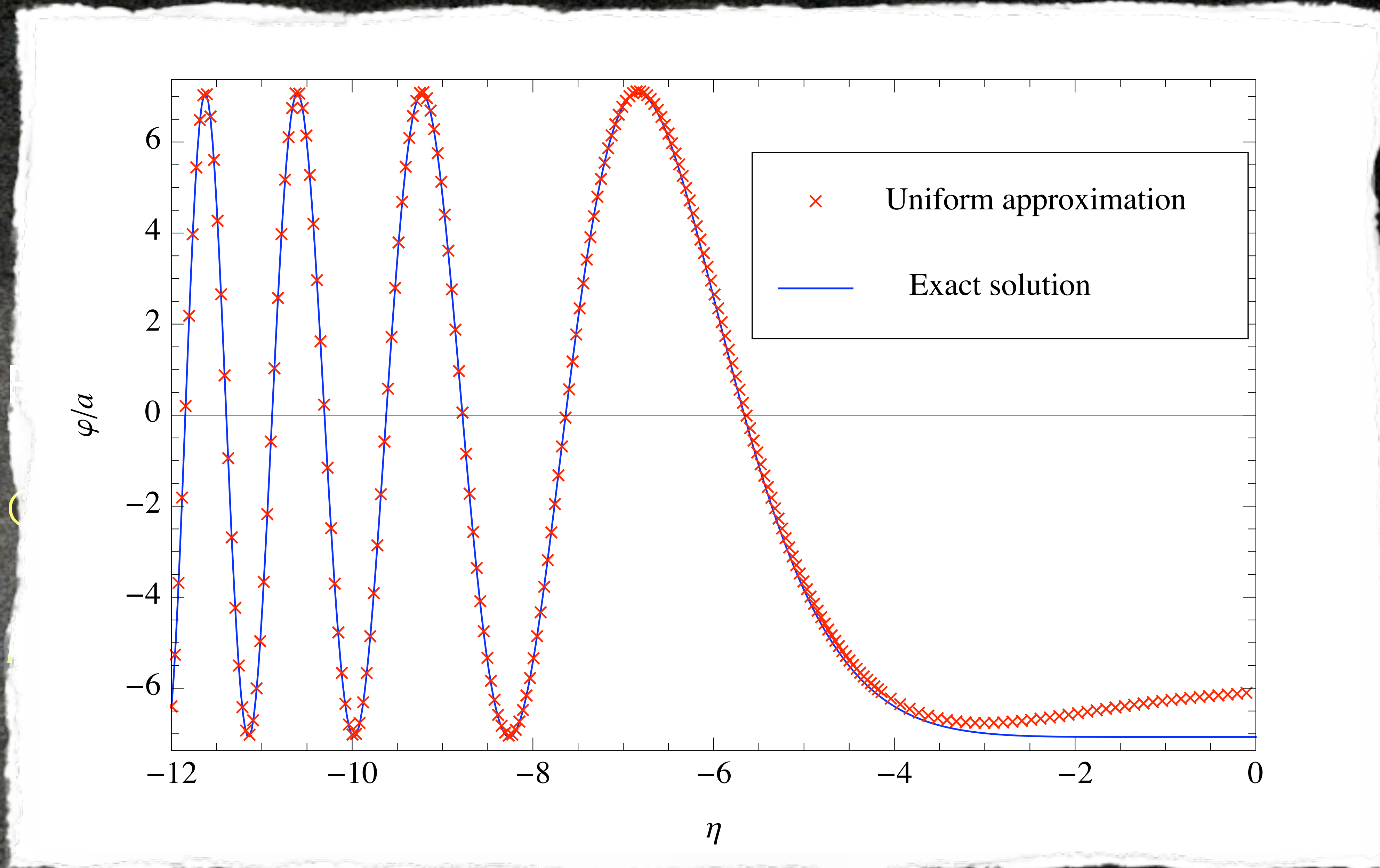


- **Exact result reproduced for the spectral index**

$$n_T = \frac{\mu - 2}{\mu + n(1+\mu)}$$

Scale-invariant for

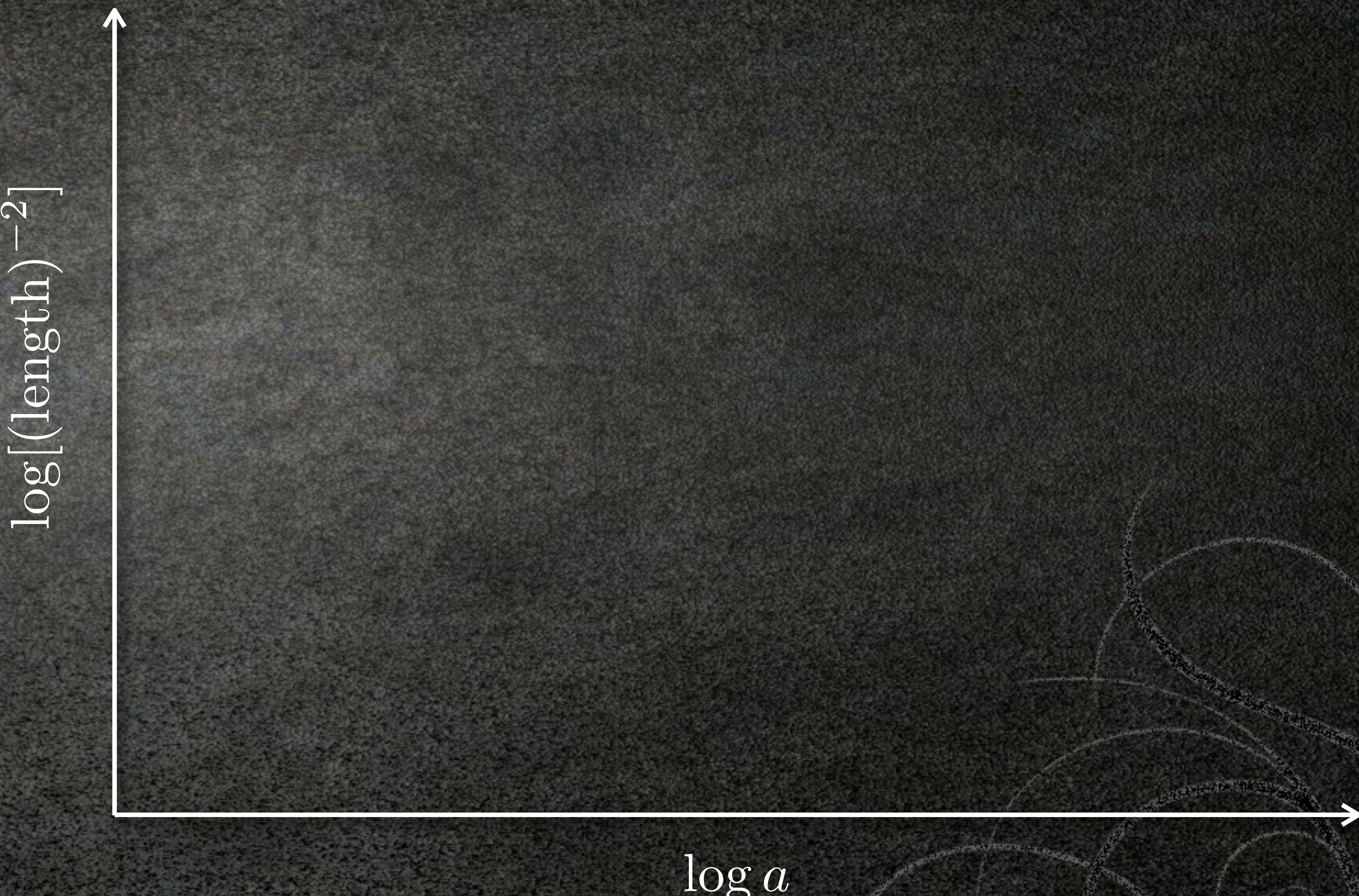
$\mu = 2 \quad (k_{\rm eff}^2 \propto k^6)$

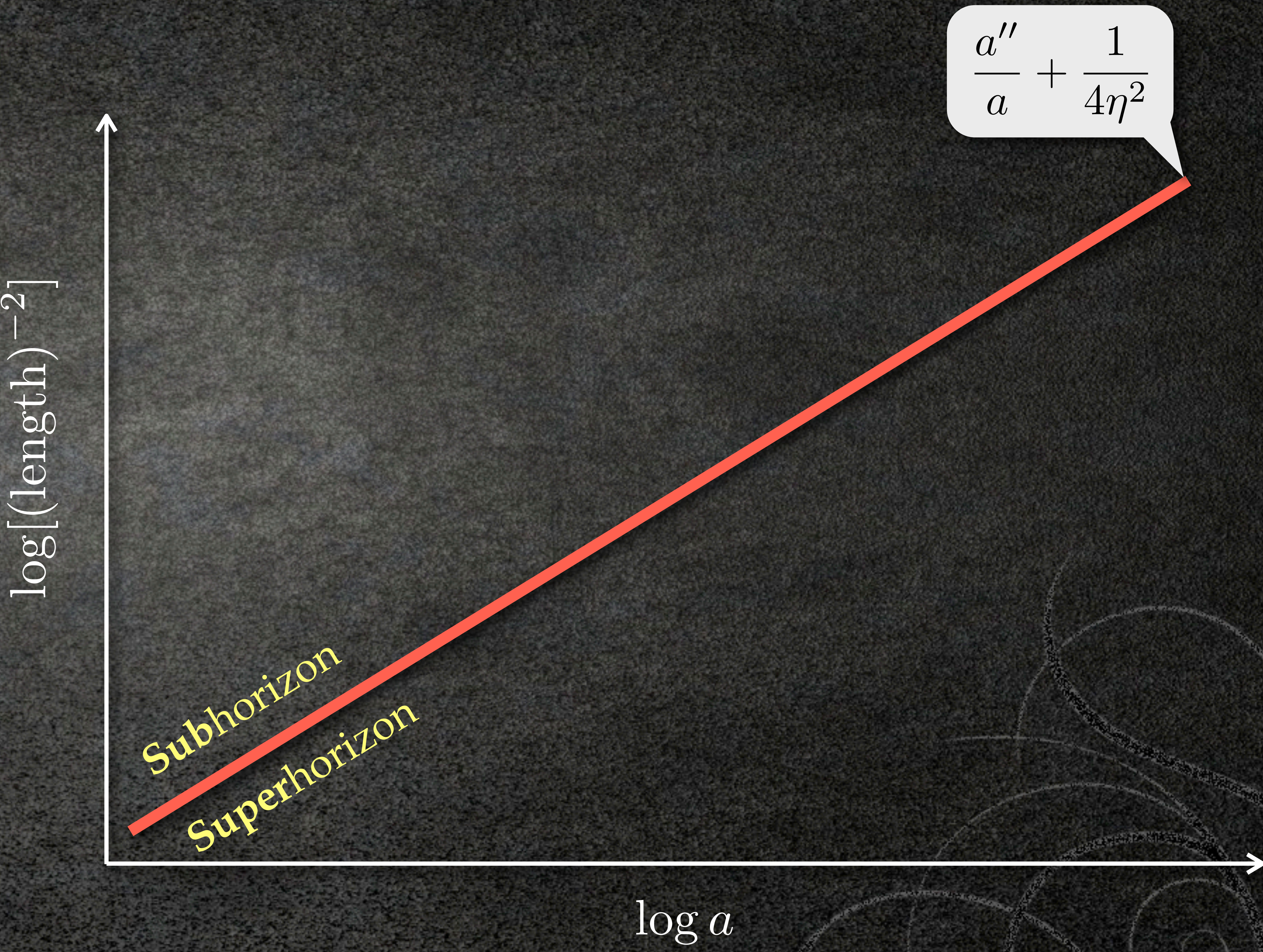
Consistent with Mukohyama 2009; Cai and Zhang, 2009

Application 2

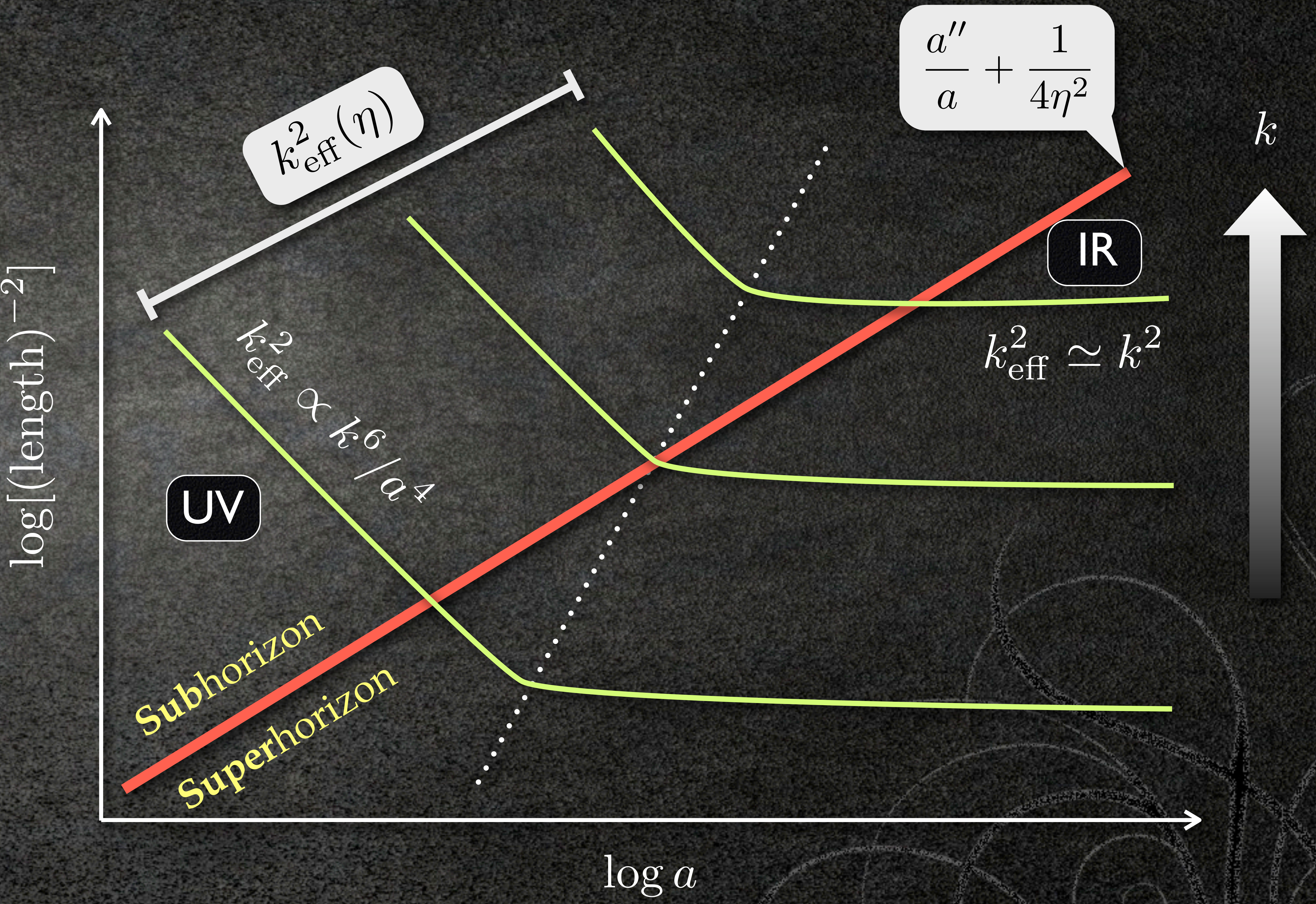
- Horava-Lifshitz cosmology, power-law inflation $a \propto t^{1+n}$

$$k_{\text{eff}}^2(\eta) = k^2\left(\frac{k^4}{M^4a^4} + \alpha_1\frac{k^3}{M^3a^3} + \alpha_2\frac{k^2}{M^2a^2} + 1\right)$$


$\log[(\text{length})^{-2}]$
$\log a$



$\frac{a''}{a} + \frac{1}{4\eta^2}$
$\log[(\mathrm{length})^{-2}]$
Subhorizon
Superhorizon
$\log a$



$\frac{a''}{a} + \frac{1}{4\eta^2}$
k
$k_{\text{eff}}^2(\eta)$
IR
$k_{\text{eff}}^2 \simeq k^2$
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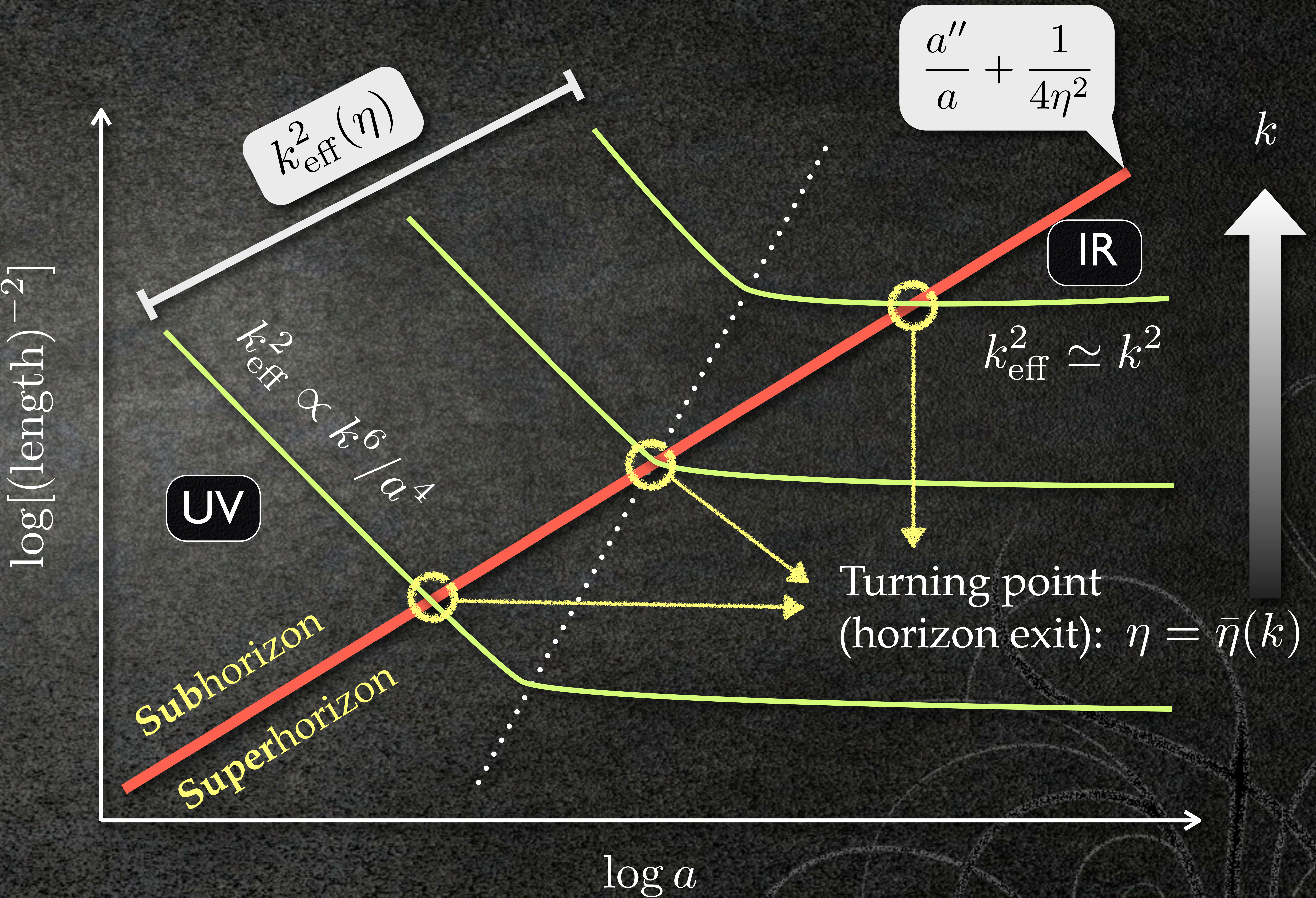

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(horizon exit): $\eta = \bar{\eta}(k)$
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- Analytic approximate results for limiting cases:
  - Horizon exit in the UV limit (large q)

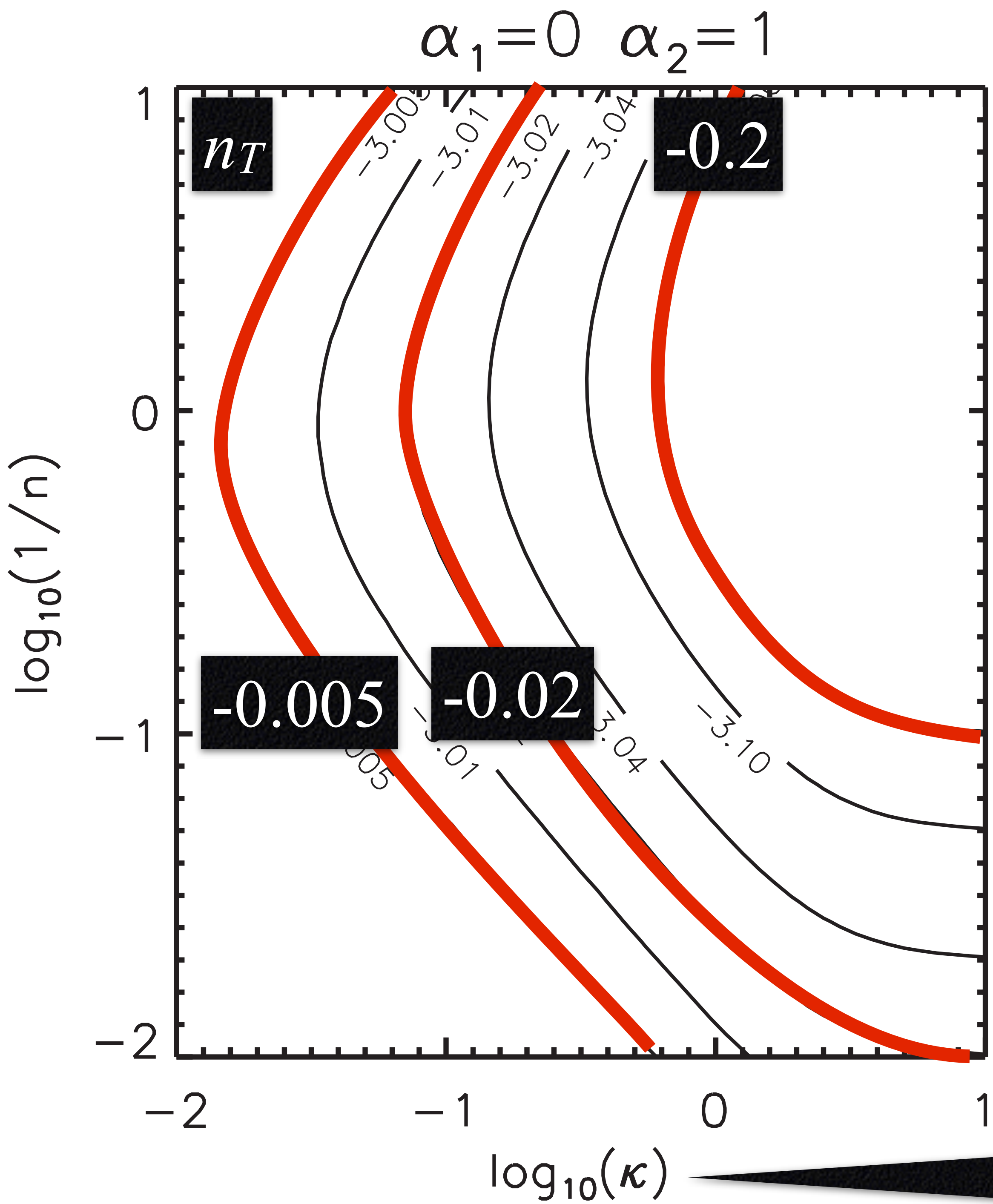
$$n_T \simeq - \left(\frac{\alpha_1}{q} - \frac{\alpha_1^2 - 2\alpha_2}{2q^2} \right) \frac{1}{2+3n}$$

  - Horizon exit in the IR limit (small q)

$$n_T \simeq -\frac{2}{n} + \left(\alpha_2 q^2 + \frac{3}{2}\alpha_1 q^3 \right) \frac{2+3n}{n^2} \qquad \text{where} \quad q := \frac{k}{M a(\bar{\eta})}$$

- Generic cases: numerical evaluation of uniform approximation formula for n_T

Numerical example



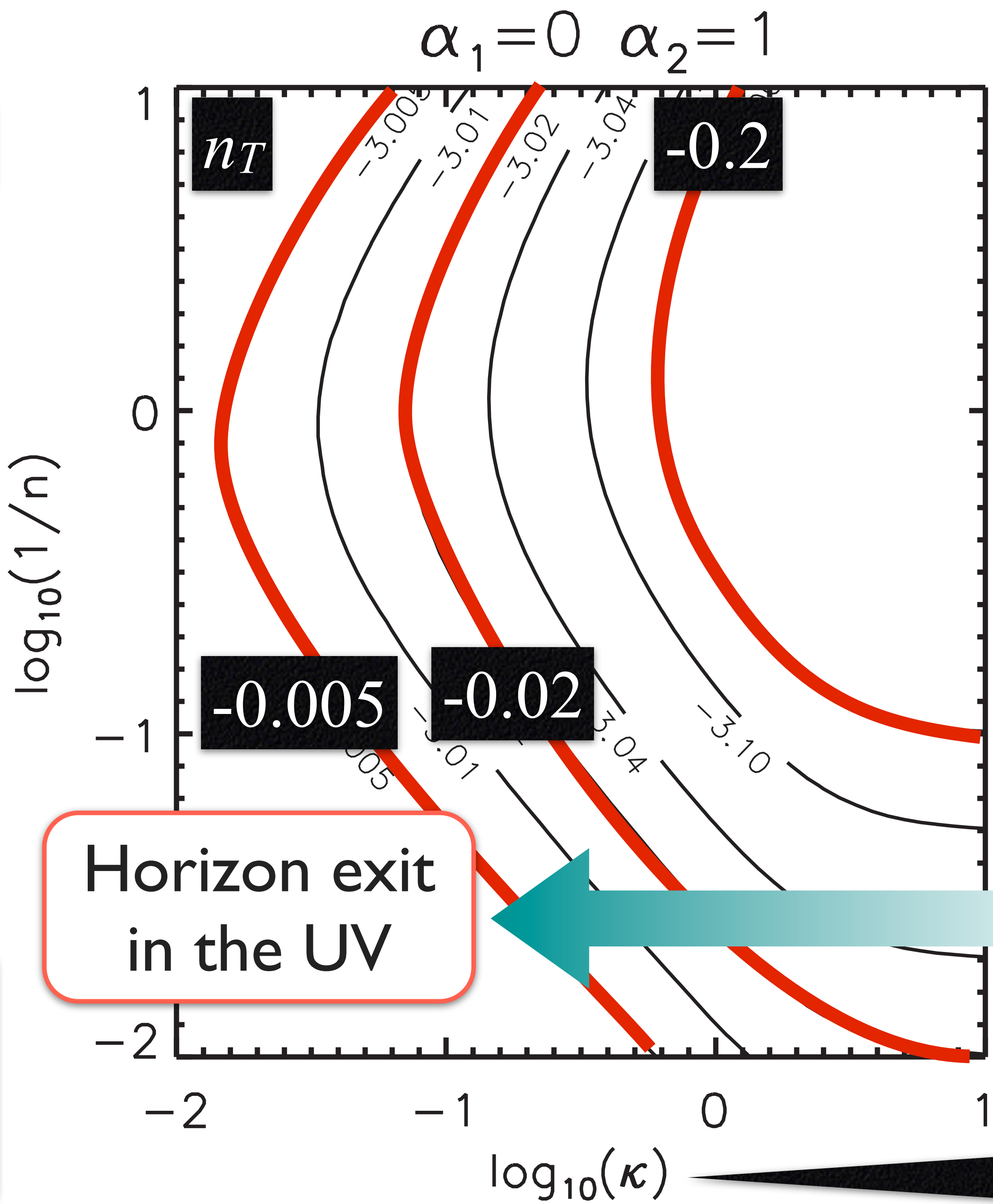
$\kappa \propto k^{1/(1+n)}$

Numerical example

Non-slow-roll

Slow-roll limit

$$-\dot{H}/H^2 \ll 1$$



Horizon exit in the UV

Horizon exit in the IR

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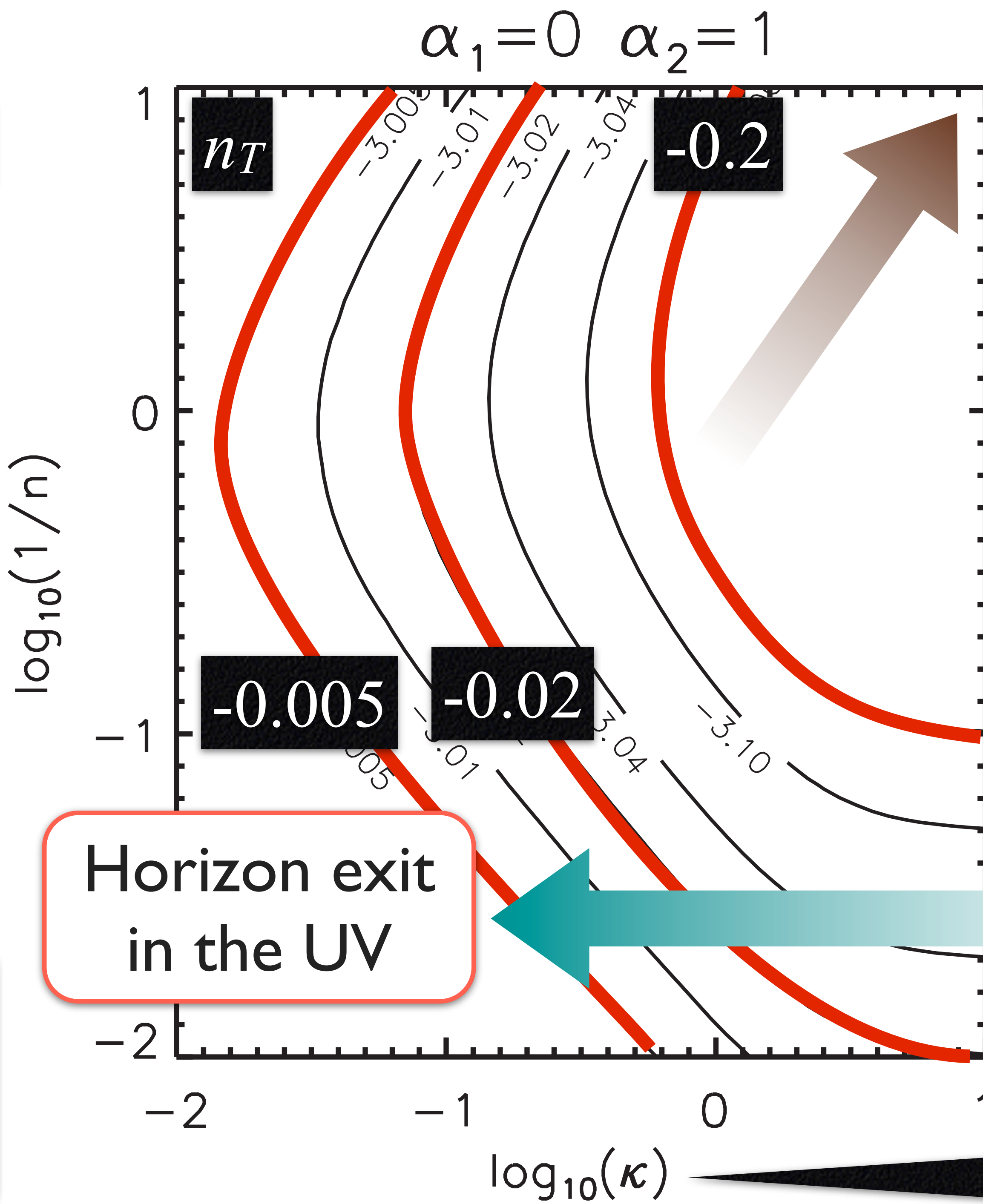

Horizon exit in the UV

Horizon exit in the IR

Large deviation from $n_T = 0$

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Conclusions

- In Horava's gravity, anisotropic scaling in the UV implies modified dispersion relation in the early universe
- New mechanism to flatten the primordial spectrum:

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- Subdominant terms in the modified dispersion relation give rise to small **tilt** of the spectrum
- Spectral index can be computed accurately using the **uniform approximation**

Thank you!

Appendix

What is neglected in uniform approximation

- The following quantity is neglected as a first approximation:

$$\psi(\zeta) := [4g(\eta)g''(\eta) - 5g'^2(\eta)]\frac{\zeta}{16g^3(\eta)} + \frac{\zeta q(\eta)}{g(\eta)} + \frac{5}{16\zeta^2}$$

where $g(\eta) := -k_{\rm eff}^2 + \frac{a''}{a} + \frac{1}{4\eta^2}$ $\qquad q(\eta) := -\frac{1}{4\eta^2}$

$$\zeta(\eta) := \pm\left[\pm\frac{3}{2}\int_{\bar{\eta}}^{\eta}\sqrt{\pm g(\eta')}d\eta'\right]^{2/3}$$

- Next-order expression is more involved.

Olver, 1997;Martin, Schwarz, 2003;
Habib *et al.* 2002; 2004