The Subdominant Curvaton

[arXiv:0906.3126] in collaboration with Kari Enqvist, Gerasimos Rigopoulos, Sami Nurmi and Tomo Takahashi.

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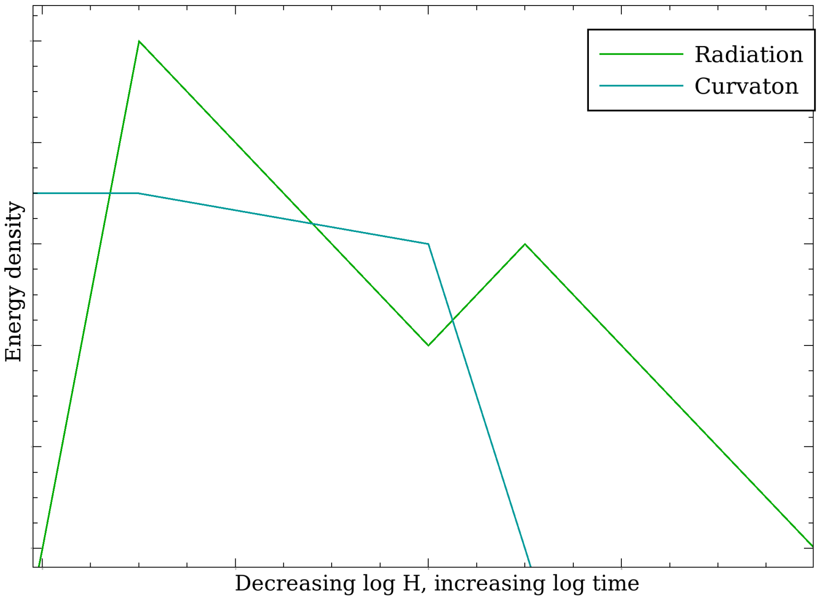
In typical inflationary theory...

- Inflation solves the horizon, the flatness and the relic problems.
- At the same time, the inflaton acquires scalar perturbations that become the primordial perturbations that we observe.
- However, *any* scalar field in a de Sitter -background will acquire classical perturbations for modes that have exited the horizon.

The curvaton scenario

- Background is given by your favourite inflationary scenario, inflationary scale H_*.
- Add another scalar field, the curvaton σ, which acquires Gaussian scalar perturbations for superhorizon modes of the magnitude $\sim H_*$.
- Long after inflation has ended, the curvaton decays into radiation producing the observed primordial perturbations.

A rough sketch of the standard curvaton scenario



Lesson 0:

Instead of the inflaton, an additional scalar field might just as well source the primordial perturbations.

The Subdominant Curvaton

We are interested in the final value of perturbations. This depends roughly on two factors:

- the initial amplitude of the perturbations, H_*/σ_*
- the efficiency of converting the curvaton perturbations to curvature perturbations

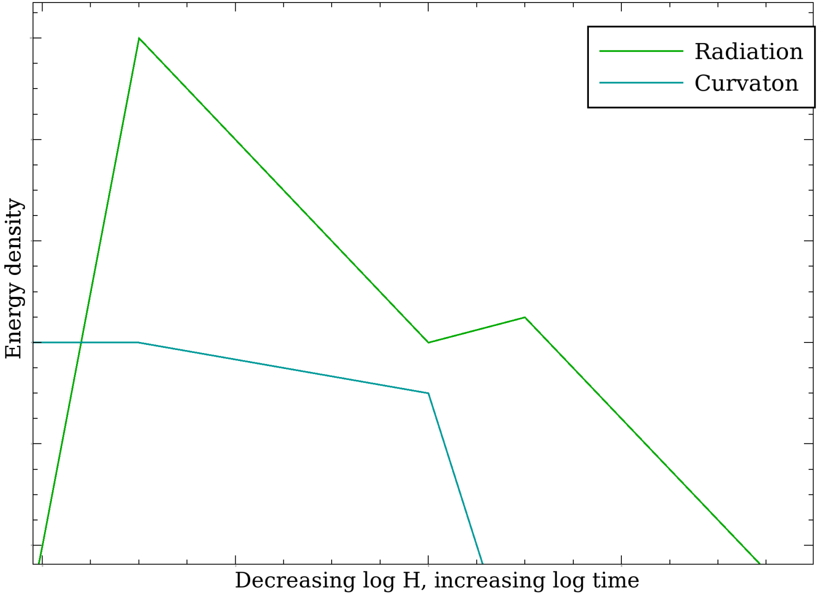
First order approximation for the efficiency factor is $\sim r = \frac{\rho_\sigma}{\rho_\mathrm{r}}$.

This means that we require

$\frac{H_*}{\sigma_*}\frac{\rho_\sigma}{\rho_\mathrm{r}+\rho_\sigma}\Big|_\mathrm{decay} \sim 10^{-5}$

If we increase H_*/σ_* sufficiently, we can decrease $r_\mathrm{decay} \equiv \frac{\rho_\sigma}{\rho_\mathrm{r}+\rho_\sigma}\Big|_\mathrm{decay}$ similarly.

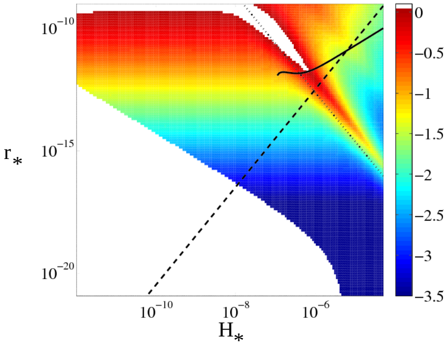
A rough sketch of the subdominant curvaton scenario



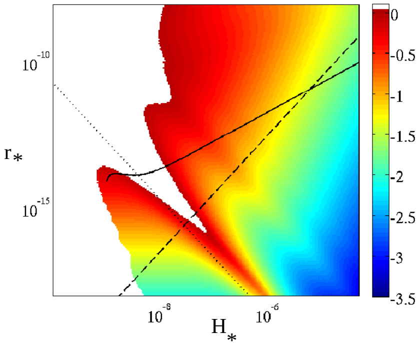
To study this in a more quantitative manner...

- We study a scalar field with a canonical kinetic term and a potential
$$V(\sigma) = \frac{1}{2}m^2\sigma^2 + \lambda\frac{\sigma^{n+4}}{M^4}\,.$$
- In addition, to model the curvaton's interactions with other matter fields (it needs to decay!), we add an effective decay coupling Γ to the EOM's.
- We ask how small can r_{decay} be and still produce the observed perturbations?
- For simplicity put $\lambda = 1$ and $M = M_{\text{Pl}}$, use ΔN-formalism, and solve the EOM's numerically.

Numerical results for r_{decay}



$n = 2, m = 10^{-8}$



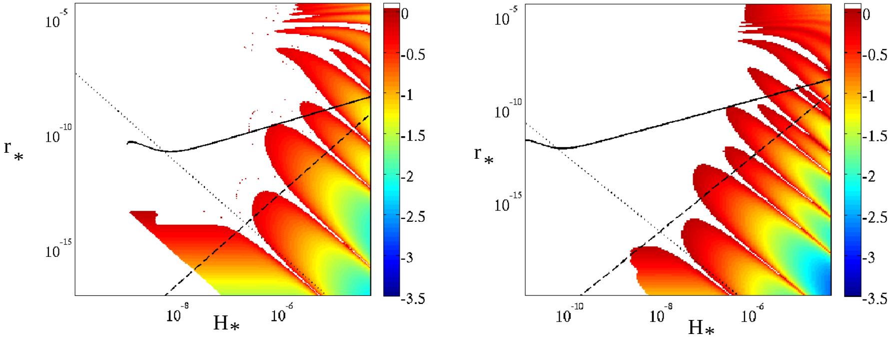
$n = 2, m = 10^{-10}$

$\log_{10} r_{\mathrm{decay}}$

Lesson 1:

The curvaton can be very subdominant, e.g. 0.1%, and still produce the observed perturbations.

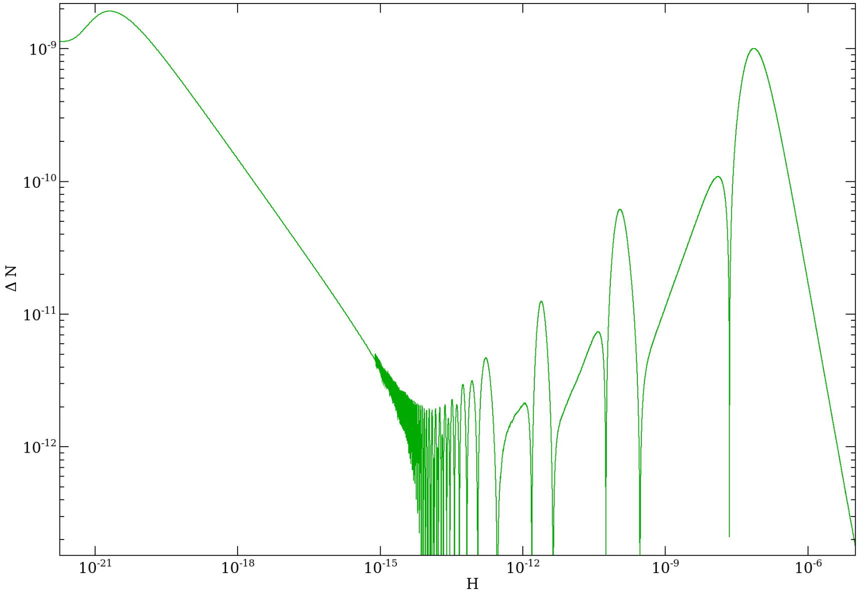
When the quadratic term of the potential dominates, everything behaves smoothly.



$n = 4, m = 10^{-10}$ $\qquad$ $n = 4, m = 10^{-12}$

$\log_{10} r_{\mathrm{decay}}$

But when the non-quadratic term dominates, the final value of ΔN (or r_{decay}) depends strongly (almost chaotically) on the initial conditions!



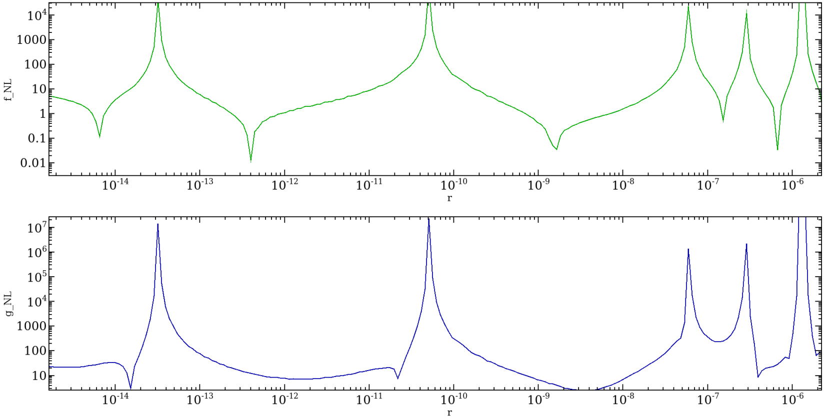
This happens since ΔN oscillates very differently in the quadratic and the non-quadratic regime.

Lesson 2:

The addition of self-interactions
produces interesting non-trivial dynamics.

What about non-Gaussianity? (Work in progress)

- If ΔN oscillates, so do N' and higher derivatives.
- Hence we get oscillations in the non-Gaussianity parameters $f_{\rm NL} \propto N''/N'^2$ and $g_{\rm NL} \propto N'''/N'^3$.



Conclusions

- The curvaton model can create the primordial perturbations successfully, and it is independent of the details of the inflaton model.
- The curvaton can be very subdominant, e.g. less than 0.1% when it decays, and still produce the observed amplitude of primordial perturbations.
- Adding a self-interaction term to the potential produces non-trivial dynamics, ie. oscillations in the parameter space.
- The oscillation translate to oscillations in non-Gaussianity parameters, and hence the curvaton model can produce signals detectable with Planck.

Backup

Motivation for inflation

- Horizon problem
- Flatness problem
- Lack of relics (monopoles etc.)
- Origin of primordial perturbations.

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The primordial perturbations

The observed primordial perturbations appear to

- be extremely Gaussian
- be very scale invariant
- have an absolute magnitude of 10^{-5}
- be nearly adiabatic, ie. no isocurvature has been observed

Since the perturbations are adiabatic, they can be described by a single variable. The two obvious choices are

- density perturbations $\delta\rho_{\mathrm{r}}/\rho_{\mathrm{r}}$ in spacially flat slices.
- curvature perturbations ζ in spacial slices with constant energy density.

These are physically equivalent and are just different gauge choices.

How to get perturbations during inflation?

Quantize a scalar field

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - V(\phi)$$

in a de Sitter -background

$$g = \mathrm{diag}(-1, a(t), a(t), a(t)),\; a(t) \propto e^{Ht}$$

and for the superhorizon Fourier modes, ie. for $k \ll aH$, perturbations will get a classical expectation value

$$|\delta\phi_k| = \frac{H_{\mathrm{exit}}}{\sqrt{2k^3}}\left(\frac{k}{aH}\right)^{\frac{m_\phi^2}{3H^2}}$$

The simplest single field inflaton model

ϕ is the inflaton and sits in a saddle point of its potential, slowly rolling down the potential. $H_*^2 \sim V(\phi_*)$ and the universe is quasi- de Sitter.

1. $V(\phi_*) \sim H_*^2$ gives a de Sitter -background.
2. Universe expands and modes that exit horizon acquire classical perturbations.
3. Once inflation ends, the universe reheats and the perturbations of the inflaton convert to perturbations in the radiation density (or equivalently curvature).

To satisfy the observations of primordial perturbations

- m_ϕ needs to be very small.
- H_*/ϕ during inflation needs to be $\sim 10^{-5}$.
- ϕ needs to decay into radiation before DM decouples.

But why does it need to be the inflaton?

The argument that a scalar field in a de Sitter background obtains Gaussian perturbations is not limited to the inflaton, but applies to any scalar field. So, one can source the perturbations from another scalar field.

What is the motivation for another scalar field?

- Either this can be used to lessen the constraints on the inflaton model (but not much!)
- More convincing: If there is a one scalar field around during inflation, there probably are others. Thus one needs to investigate what the effects of another field are!

The curvaton scenario

Add another scalar field σ which is subdominant during inflation.

1. Inflaton $\phi_* \neq 0$, $V(\phi)$ acts as Λ, universe is quasi- de Sitter.
2. During inflation the curvaton, σ, also acquires perturbations, but is subdominant, $V_\phi(\phi_*) \gg V_\sigma(\sigma_*)$.
3. Inflation ends and the inflaton decays reheating the universe.
4. In the radiation dominated universe, the curvaton starts to oscillate scaling faster than the background.
5. The curvaton decays into radiation, transferring the perturbations in the curvaton to perturbations in the radiation density.

Equations of motion

Consider a model with a Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu\sigma\,\partial^\mu\sigma - V(\sigma)$$

The equations of motion are

$$\begin{aligned} \ddot{\sigma} + (3H + \Gamma)\dot{\sigma} + V'(\sigma) &= 0 \\ \dot{\rho}_{\rm r} &= -4H\rho_{\rm r} + \Gamma\dot{\sigma}^2 \\ H^2 &= \frac{8\pi G}{3}(\rho_{\rm r} + \rho_\sigma) \end{aligned}$$

The curvaton needs to decay: Add an additional parameter Γ.

The Subdominant Curvaton

Previously simple curvaton models have been considered, primarily ones which

- have a quadratic potential: $V(\sigma) = \frac{1}{2}m^2\sigma^2$
- are completely dominant when they decay

We considered more general models, ie.

- a bit more general form of the potential, $V = \frac{1}{2}m^2\sigma^2 + \lambda\frac{\sigma^{n+4}}{M^n}$
- place no constraints on the dominance of the curvaton at its decay

The Subdominant Curvaton

Two initial conditions required

- H_* and $r_* = r|_{\text{reheating}}$

and five free parameters

- m, n, Γ, λ and M (Actually just four since only the combination $\frac{\lambda}{M^n}$ appears.)

For simplicity put $M = M_{\text{Pl}}$ and $\lambda = 1$, and choose $M_{\text{Pl}} = 1$.

EOMs explicitly are

$$
\begin{aligned}
\ddot{\sigma} + (3H + \Gamma)\dot{\sigma} + m^2\sigma + (n+4)\sigma^{n+3} &= 0 \\
\dot{\rho}_{\text{r}} &= -4H\rho_{\text{r}} + \Gamma\dot{\sigma}^2 \\
3H^2 &= \rho_{\text{r}} + \rho_\sigma
\end{aligned}
$$

ζ: A rough estimate

We are interested in the final value of perturbations. This depends roughly on two factors:

- The initial amplitude of the perturbations, H_*/σ_*.
- The efficiency of converting the curvaton perturbations to curvature perturbations.

Efficiency factor is roughly $\sim r = \frac{\rho_\sigma}{\rho_{\rm r}}$.

This means that we require

$\frac{H_*}{\sigma_*}\frac{\rho_\sigma}{\rho_{\rm r}+\rho_\sigma}\Big|_{\rm decay} \sim 10^{-5}$

A sidenote: actually solving this

The naïve analytical approach: write the EOM also for a perturbation of the curvaton and the radiation background, resulting into the group of equations

$$\begin{aligned} \ddot{\sigma} + (3H + \Gamma)\dot{\sigma} + V'(\sigma) &= 0 \\ \ddot{\delta\sigma} + (3H + \Gamma)\dot{\delta\sigma} + V''(\sigma)\delta\sigma &= 0 \\ \ldots \end{aligned}$$

From this try to calculate the final value of $\delta\rho_{\mathrm{r}}/\delta\rho_{\mathrm{r}}$. Solving these equations turns out to be impossible analytically for $n > 0$. For $n = 0$ the solution can be written in terms of parabolic cylinder functions, ie. it gets very dirty very quickly.

Few word on numerics

- Instead of calculating the evolution of σ and $\delta\sigma$ individually, use the ΔN -formalism which is more suited to numerics.
- Time is unphysical. Always compare quantities not with fixed time, but with fixed H.
- Solving the full EOM's becomes increasingly slow as the curvaton oscillates faster and faster in the quadratic regime. Hence one has to revert to approximate EOM's for ρ_σ at some point.

Qualitative behviour of the solutions

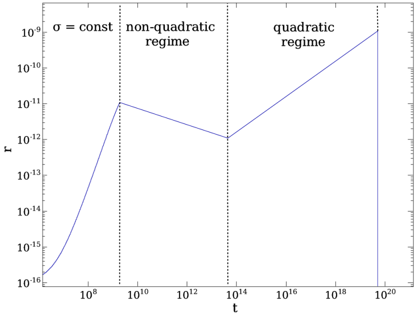
A scaling solution for monomial potentials

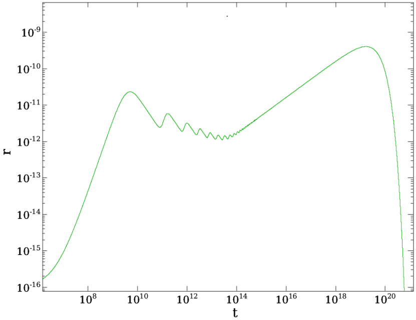
A field oscillating in a monomial potential $V \propto \sigma^{n+4}$ scales as $\rho_\sigma \propto a^{-6\frac{n+4}{n+6}}$. If $n > 4$, there are however no oscillating solutions.

The evolution of the curvaton hence has four distinict phases:

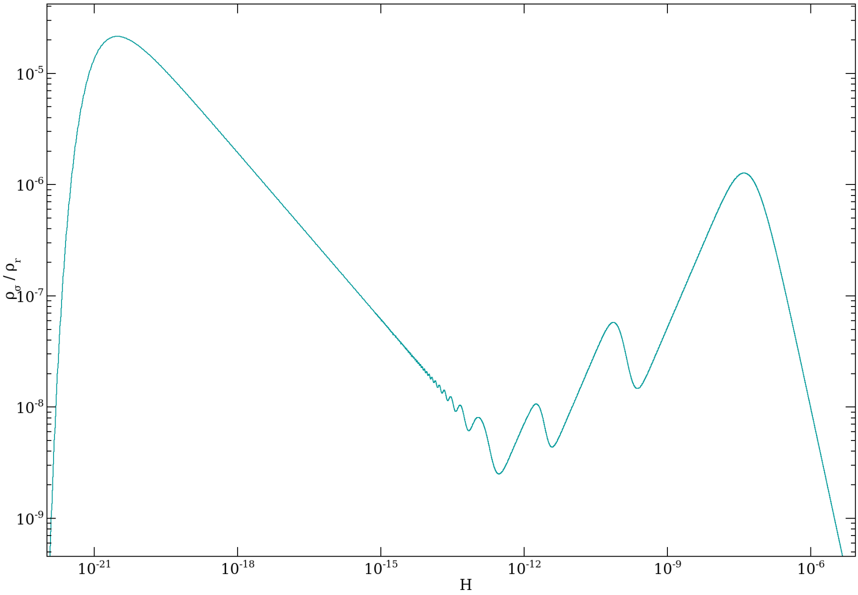
1. Slow-roll, $\sigma \sim \sigma_*$.
2. Non-quadratic regime, $\rho_\sigma \propto a^{-6\frac{n+4}{n+6}}$.
3. Quadratic regime, $\rho_\sigma \propto a^{-3}$.
4. Decay when $H \sim \Gamma$.

Compare qualitative description with numerics

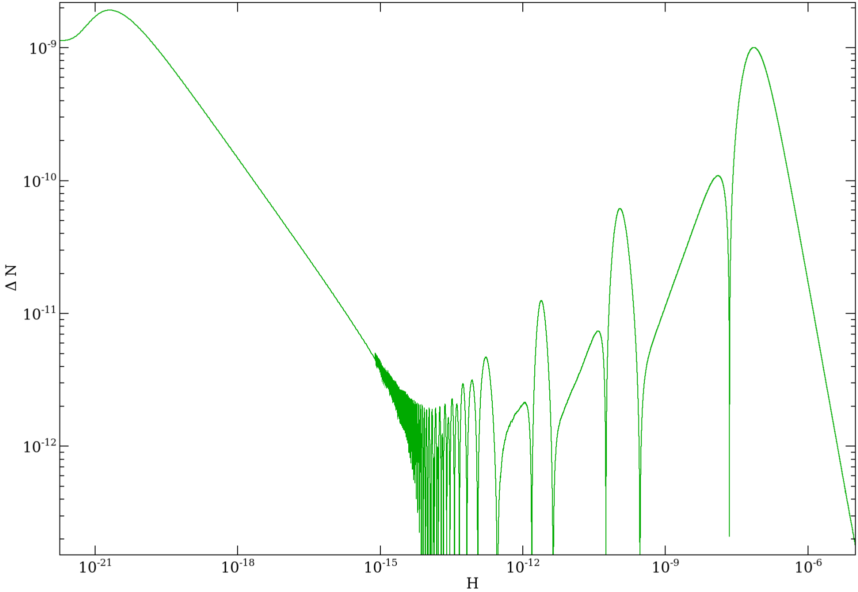




$$H_* = 10^{-7}, m = 10^{-13}, \Gamma = 10^{-20}, n = 2, r_* = 10^{-16}$$



$$H_* = 1e-5, m = 5e-13, \Gamma = 1e-21, n = 4$$



$H_* = 1e - 5, m = 5e - 13, \Gamma = 1e - 21, n = 4$

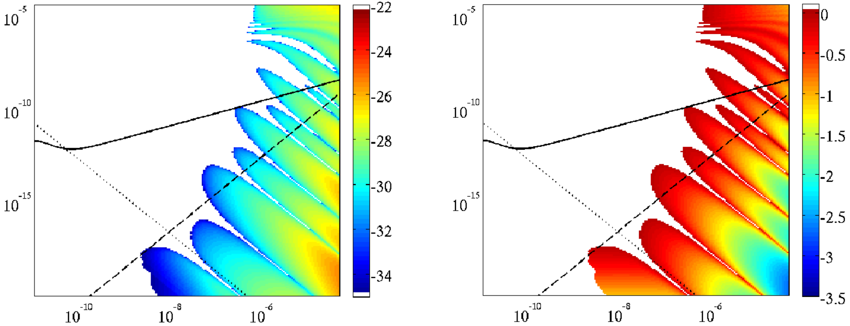
What questions should we ask from the code?

- The model has lots of free parameters. The code assigns final value of the perturbation amplitude for each set of parameters.
- We are interested only in those points where $\zeta \sim 10^{-5}$.

How to enumerate these points?

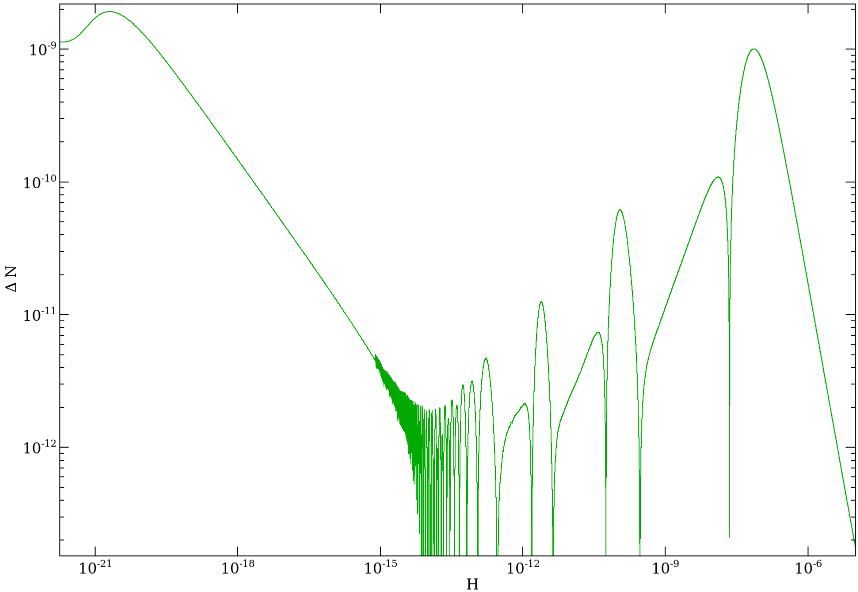
Fix r_*, H_*, n and m, and tune Γ so that final $\Delta N \sim 10^{-5}$. Scan through all possible values of r_*, H_*, n and m.

Whoa! Where do those oscillations come from?



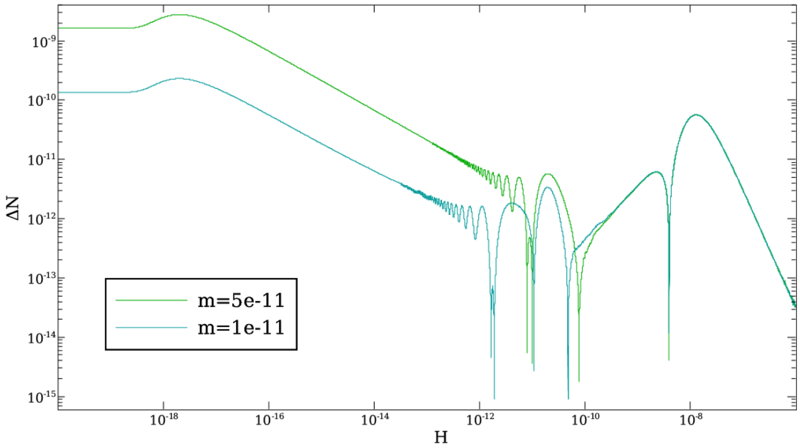
$\log_{10} \Gamma$ $\qquad$ $\log_{10} r_{\text{decay}}$

Horizontal axis is H_*, vertical axis r_*, $n = 4$, $m = 10^{-12}$.



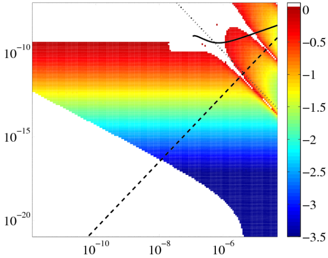
$$H_* = 1e - 5, m = 5e - 13, \Gamma = 1e - 21, n = 4$$

Let's have a more closer look to ΔN

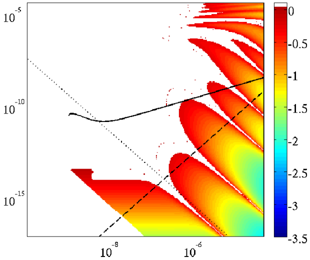


$H_* = 10^{-6}, r_* = 10^{-10}, \Gamma = 10^{-18}, n = 4$

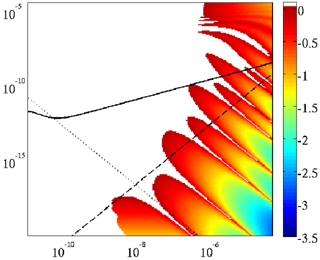
$\log_{10} r_{\rm decay}$ for $n = 4$



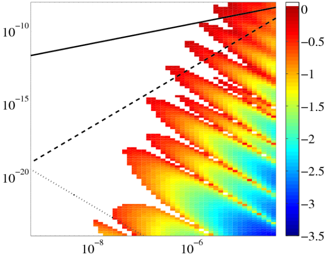
$m = 10^{-8}$



$m = 10^{-10}$

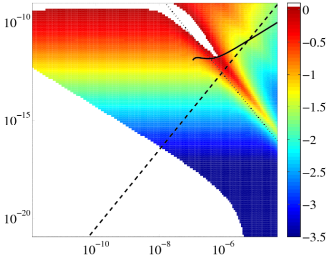


$m = 10^{-12}$

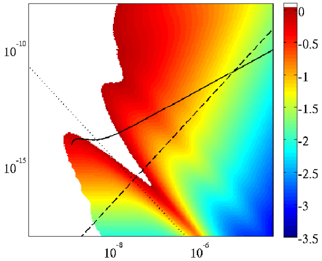


$m = 10^{-14}$

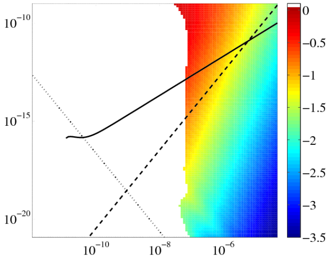
$\log_{10} r_{\text{decay}}$ for $n = 2$



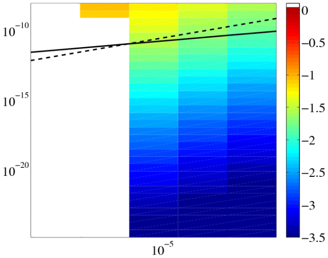
$m = 10^{-8}$



$m = 10^{-10}$

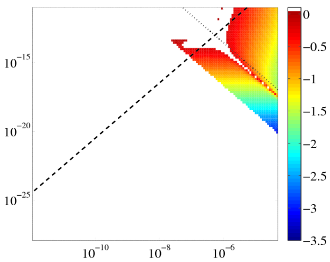


$m = 10^{-12}$

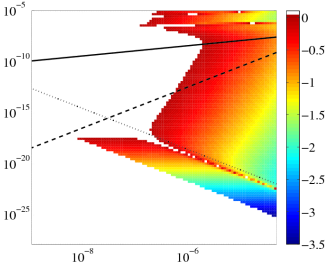


$m = 10^{-14}$

$\log_{10} r_{\mathrm{decay}}$ for $n = 6$



$m = 10^{-10}$



$m = 10^{-12}$

A quick word for f_{NL} and g_{NL}

The observed perturbations are very close to Gaussian, and hence the non-Gaussianity can be described by coefficients in the expansion

$$\zeta = \zeta_1 + \frac{3}{5} + f_{\mathrm{NL}}\zeta_1^2 + \frac{9}{25} g_{\mathrm{NL}}\zeta_1^3 + \mathcal{O}(\zeta_1^4)$$

These f_{NL} and g_{NL} can be calculated in practice from the derivatives of N w.r.t. to the intial conditions, ie.

$$f_{\mathrm{NL}} \propto \frac{N''}{N'^2}, \quad g_{\mathrm{NL}} \propto \frac{N'''}{N'^3}$$

From the plots one can easily see that since $N_1 - N_2$ has lots of features, so do the derivatives of N.

Conclusions

- The curvaton model can create the primordial perturbations successfully, and it is independent of the details of the inflaton model.
- The curvaton does not need to be dominant when it decays, 0.1% is enough.
- A non-trivial potential causes the results to oscillate non-trivially.

Conclusions

- The curvaton model can create the primordial perturbations successfully, and it is independent of the details of the inflaton model.
- The curvaton does not need to be dominant when it decays, 0.1% is enough.
- A non-trivial potential causes the results to oscillate non-trivially.
- The curvaton model creates non-Gaussianity, possibly observable with Planck.
- The non-Gaussianity parameters oscillate as well, and there are (large?) areas where they fit well within current observations.