k-essence models of unified

Inflation, Dark matter & Dark energy

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Refs:

1. N. Bose and A. S. Majumdar, *A k-essence model of inflation, dark matter and dark energy,* Phys. Rev. D **79**, 103517 (2009) [arXiv: 0812.4131].

2. N. Bose and A. S. Majumdar, *Unified model of k-inflation, dark matter and dark energy,* [arXiv: 0907.2330].

Introduction & Motivations

- Inflationary model (kinetic energy driven) originally motivated from string theoretic (Born-Infeld) action. *[c.f. Armendariz-Picon, Damour, Mukhanov, (1999)]*

- Subsequently, models for late time acceleration of the universe driven by scalar field kinetic energy. *[c.f. Chiba et. al (1999), Steinhardt et. al. (2001), Chimento (2004)….]*

- Search for a single field (or similar mechanism) to generate the early and present era acceleration of the universe. *[c.f. quintessential inflation: Peebles (1999), Copeland (2000), Majumdar (2001), Sahni (2004)……]*

- Nature of both dark matter and dark energy are unknown. Could these be manifestations of the same entity ? *[c.f. unified models: ……Liddle et. al. (2008)]*

- How does *k-essence* fare in such schemes ?

K-essence Models

- Type-I:

$$L = F(x) - V(\varphi)$$

$$L = F(x)V(\varphi)$$

- Type-II:

$$x = \dot{\varphi}^2$$

Purely kinetic *k-essence*

- Lagrangian:

$$L = F(x)$$

- Equation of motion:

$$(2xF_{xx} + F_x)\dot{x} + 6HF_x x = 0$$

- Solution [*Scherrer (2004)*]:

$$\sqrt{x}\,F_x = \frac{k}{a^3}$$

- Energy density:

$$\rho = 2xF_x - F$$

- Energy density (ansatz):

$$\rho = \lambda + c_1/a^3 \equiv \lambda + c_1\sqrt{x}\,F_x/k$$

- Consistency:

$$a \propto constt.$$

- *DM & DE with purely kinetic k-essence not compatible with expanding universe.*

Model-I

$$L = F(x)V(\varphi) \qquad F(x) = Kx - m_{pl}^2 L\sqrt{x} + m_{pl}^4 M \qquad V(\varphi) = 1 + e^{-\varphi/\varphi_c}$$

- *Inflationary era: (V ~ constant, or varies slowly)*

Field equation: $(2xF_{xx} + F_x)\dot{x} + 6HF_x x = 0 \qquad \Rightarrow \sqrt{x}F_x = k/a^3$

Energy conservation: $\Rightarrow \dot{\rho} = -3H(\rho + p) = -6HF_x xV$

Fixed points of the eq. ($x = x_0 \equiv \left(0, m_{pl}^4 \frac{L^2}{4K^2}\right)$ *)correspond to extrema of F.*

$p = -\rho$ *is an attractor leading to exponential inflation.*

- *Exit from the inflationary era:* x slowly moves away from x_0

Inflation ends when $\frac{\delta x}{x_0} \approx 1$

Model-II

$$L = F(x) - V(\varphi) \qquad F(x) = Kx - m_{pl}^2 L\sqrt{x} + m_{pl}^4 M \qquad V(\varphi) = \frac{1}{2} m^2 \varphi^2$$

- *Inflationary era dynamics:* $$V(\varphi) >> 2xF_x - F$$

Field eqn.: $$3H F_x \dot{\varphi} + \frac{dV}{d\varphi} \approx 0$$

Slow roll parameters: $$\varepsilon = \frac{1}{16\pi G}\left(\frac{V'}{V}\right)^2 \frac{1}{F_x} \qquad \eta = \frac{1}{8\pi G}\left(\frac{V''}{V}\right)\frac{1}{F_x^2}$$

$F_x \sim O(1)$ identifies with the standard scenario.

Computation of inflationary parameters, e.g., no. of e-folds, spectral index, tensor-to-scalar index, etc..

Post-inflationary evolution

- Stage of kinetic domination after inflation

  F.E.: $$(2xF_{xx}+F_x)\ddot{\varphi}+3HF_x\dot{\varphi}=0 \qquad F(x)=B(1-2A\sqrt{x})^2-C$$

  $$x=\frac{1}{16A^4B^2}\left(2AB+\frac{k}{a^3}\right)$$

  recovering back effectively, kinetic *k-essence.*

- Energy density: $\rho=C+\dfrac{k}{Aa^3}+\dfrac{k^2}{4A^2Ba^6}$

- Eq. of state: $$w=\frac{\dfrac{k^2}{4A^2Ba^6}-C}{C+\dfrac{k}{Aa^3}+\dfrac{k^2}{4A^2Ba^6}}$$

  post-inflation (before radiation domination): $w\approx 1$

  matter domination: $w\approx 0$

- *late time evolution (* $a\rightarrow\infty$ *):* $w\rightarrow -1$

Constraints on model parameters

Observational requirements:

- Inflationary era: amplitude of density perturbations $\delta_H = 2\times10^{-5}$; e-foldings $N > 60$
- Intermediate era: crossover from kinetic to radiation domination before nucleosynthesis; & matter domination subsequently
- Present era: transition to accelerated expansion after structure formation

Impose the constraints: $250\,(GeV)^{-2} \le A \le 10^{10}\,(GeV)^{-2}$ $\quad 10^{-22}\,(GeV)^4 \le B \le 10^{-6}\,(GeV)^4$

with the tuning $C = 10^{-48}\,(GeV)^4$ leading to:

- Scale of inflation $m = 10^{13}\,GeV$ — Slow roll parameters $\varepsilon, \eta \approx 10^{-3}$
- tensor-to-scalar ratio $r \cong 0.12$ — spectral index $n_s \cong 0.95$
- Transition to acceleration $Z_T \cong 0.81$ — present value of eq. state parameter $w_0 \cong -0.75$

Summary & Conclusions

(N. Bose and A. S. Majumdar, Phys. Rev. D **79**, 103517 (2009) [arXiv:0812.4131]; arXiv:0907.2330)

- We consider *k-essence* models with the interplay of kinetic and potential terms.
- Our aim is to obtain inflation, dark matter & dark energy within a unified framework.
- We show that purely kinetic *k-essence* is unable to achieve such a unification.
- Dynamics of models: inflationary potential (slow roll), exit leading to kinetic domination.
- Reheating by gravitational particle production (*problematic with k-domination, low reheat temperatures).*
- Later evolution reproducing dark matter and dark energy at appropriate stages.
- Constraints on model parameters from phenomenological considerations *(tuning of one parameter)*
- Viability of inflationary perturbations, and predictions of z_T and w_0 with upcoming probes.