Paolo Creminelli (ICTP, Trieste)

The effective theory of quintessence: the w < -1 side unveiled

arXiv:0811.0827 [astro-ph] (JCAP)

with G. D'Amico, J. Noreña and F. Vernizzi

see also hep-th/0606090 (JHEP)

with M. Luty, A. Nicolis and L. Senatore

The Universe accelerates

In 1998 the Universe started accelerating...

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \qquad w \equiv p/\rho$$

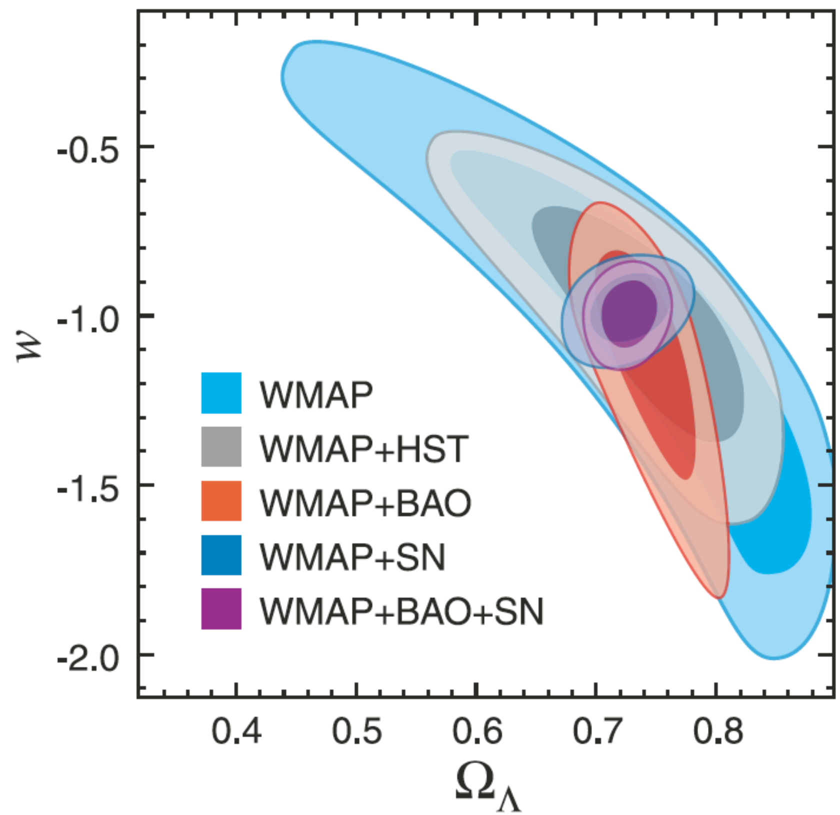
- Data are converging towards w ≈ -1
- Λ is the simplest explanation: w = -1
- **Quintessence**
  (here a general single field dark energy,
  with no direct coupling with matter)

$w_Q(z) \neq -1$

Not spatially homogeneous

Compelling evidence from supernovae + other observations

Komatsu etal 2008



Outline

- Study the most general theory of single field quintessence
- Quintessence **perturbations** with a given background evolution $w_Q(z)$
- Is there life for $\mathbf{w_Q < -1}$? Can we cross $w_Q = -1$ (**phantom divide**)?
- Phenomenology of **$c_s = 0$ models**

Building up the action

K-essence: $$S = \int \mathrm{d}^4 x \sqrt{-g}\, P(\phi, X)\ , \qquad X = -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

Let us expand around: $$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 \qquad \phi = \phi_0(t)$$

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} \qquad \rho_Q = 2X_0 P_X - P_0\ , \qquad p_Q = P_0$$

Action for perturbations, making explicit the **background dependence**

Convenient parametrization: $$\boxed{\phi(t,\vec{x}) = \phi_0(t + \pi(t,\vec{x}))}$$

$$\begin{aligned}
\phi(t,\vec{x}) &= \phi_0 + \dot{\phi}_0\pi + \frac{1}{2}\ddot{\phi}_0\pi^2 + \dots\ , \\
X(t,\vec{x}) &= X_0 + \dot{X}_0\pi + \frac{1}{2}\ddot{X}_0\pi^2 + 2X_0\dot{\pi} + 2\dot{X}_0\pi\dot{\pi} + X_0\left(\dot{\pi}^2 - \frac{(\nabla\pi)^2}{a^2}\right) + \dots
\end{aligned}$$

$$S = \int \mathrm{d}^4x\, a^3 \left[P_0 + \dot{P}_0\pi + \frac{1}{2}\ddot{P}_0\pi^2 + 2P_X X_0\dot{\pi} + 2\left(P_X X_0\right)^{\cdot}\pi\dot{\pi} + P_X X_0\left(\dot{\pi}^2 - \frac{(\nabla\pi)^2}{a^2}\right) + 2P_{XX}X_0^2\dot{\pi}^2\right]$$

The action for perturbations

... integrating by parts + using background EOM

Metric perturbations in synchronous gauge: $\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)(\delta_{ij} + h_{ij})\mathrm{d}x^i\mathrm{d}x^j$

$$\boxed{S = \int \mathrm{d}^4x\, a^3 \left[\frac{1}{2}\left(\rho_Q + p_Q + 4M^4\right)\dot{\pi}^2 - \frac{1}{2}(\rho_Q + p_Q)\frac{(\nabla\pi)^2}{a^2} + \frac{3}{2}\dot{H}(\rho_Q + p_Q)\pi^2 - \frac{1}{2}(\rho_Q + p_Q)\dot{h}\pi\right]}$$

$M^4 \equiv P_{XX}X_0^2$

Perturbations cannot be switched off if $\rho_Q + p_Q \neq 0$

$(\rho_Q + p_Q)(t)$ and $M^4(t)$ are completely unconstrained

One can always find $P(\phi, X)$:

$$P(\phi, X) = \frac{1}{2}(p_Q - \rho_Q)(\phi) + \frac{1}{2}(\rho_Q + p_Q)(\phi)X + \frac{1}{2}M^4(\phi)(X-1)^2$$

$\phi = t$ and the correct $\rho_Q(t)$ and $p_Q(t)$

$\boxed{\text{No field redefinition ambiguities:} \quad \phi \to \tilde{\phi}(\phi)}$

No ghost!

We require a positive definite time kinetic term

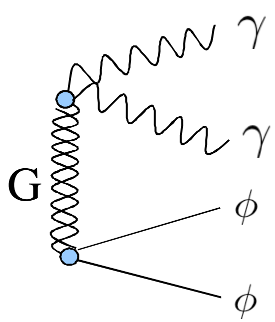
$$\frac{1}{2}(\rho_Q + p_Q + 4M^4)\dot{\pi}^2 > 0$$

E.g. a minimal ghost field: $\mathcal{L} = +\frac{1}{2}(\partial\phi)^2 + V(\phi)$ **$w_Q < -1$!!**

- **Classically**. Hamiltonian not bounded. Possibility of exchanging energy between positive and negative energy sectors.

  No pathology until linear theory remains valid.

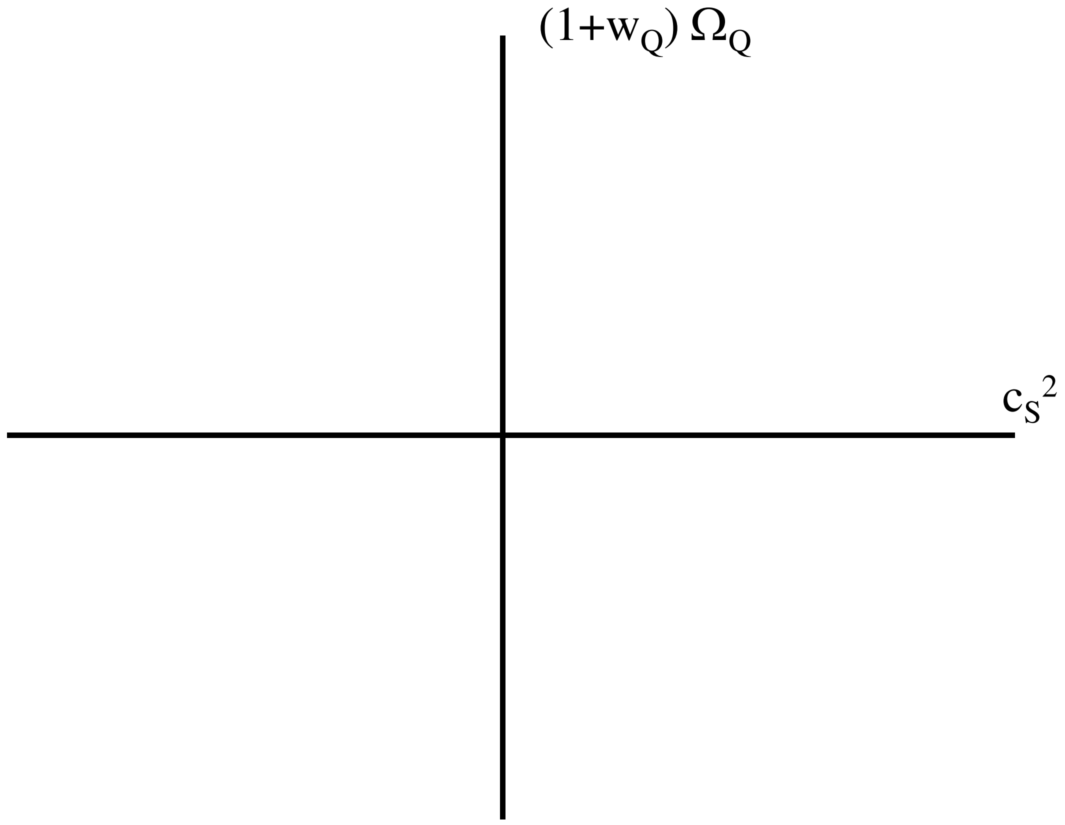
- **Quantum mechanically**. Vacuum is unstable.
  Decay rate is infinite in any Lorentz invariant theory.



$$\Gamma \sim \frac{\Lambda^8}{M_P^4}$$

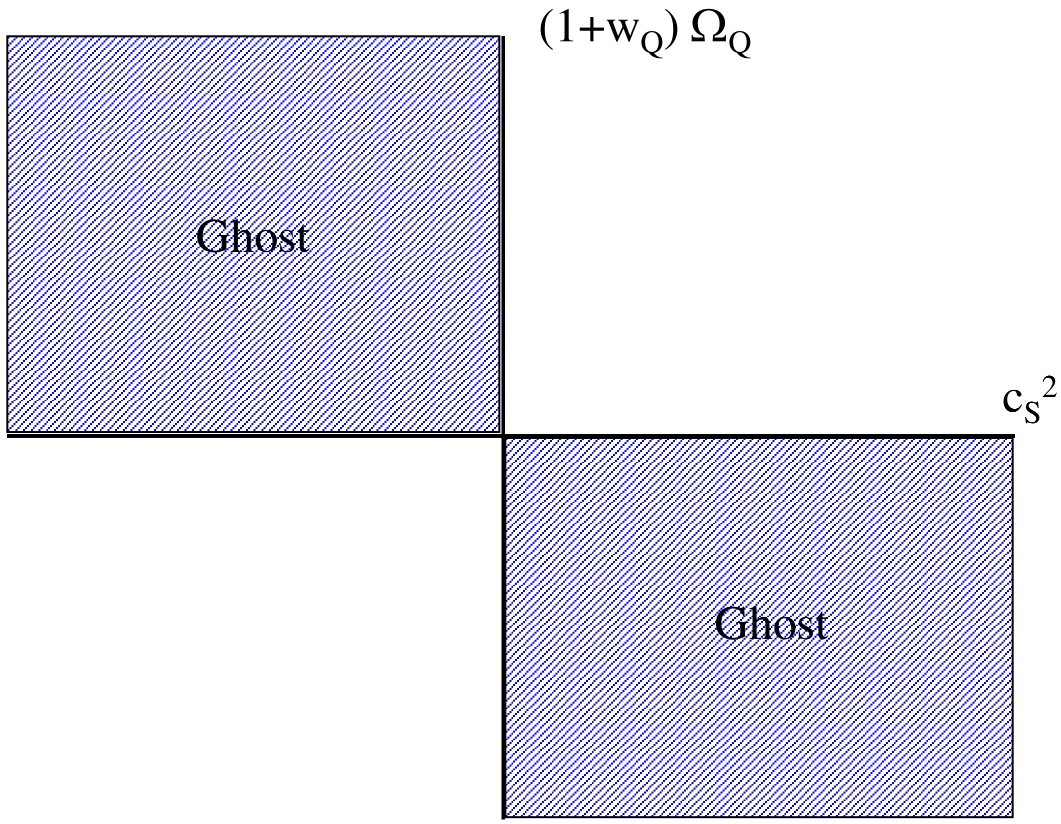
Cline, Jeon, Moore 03

Quintessential plane



Let us study the different theoretical constraints on quintessence

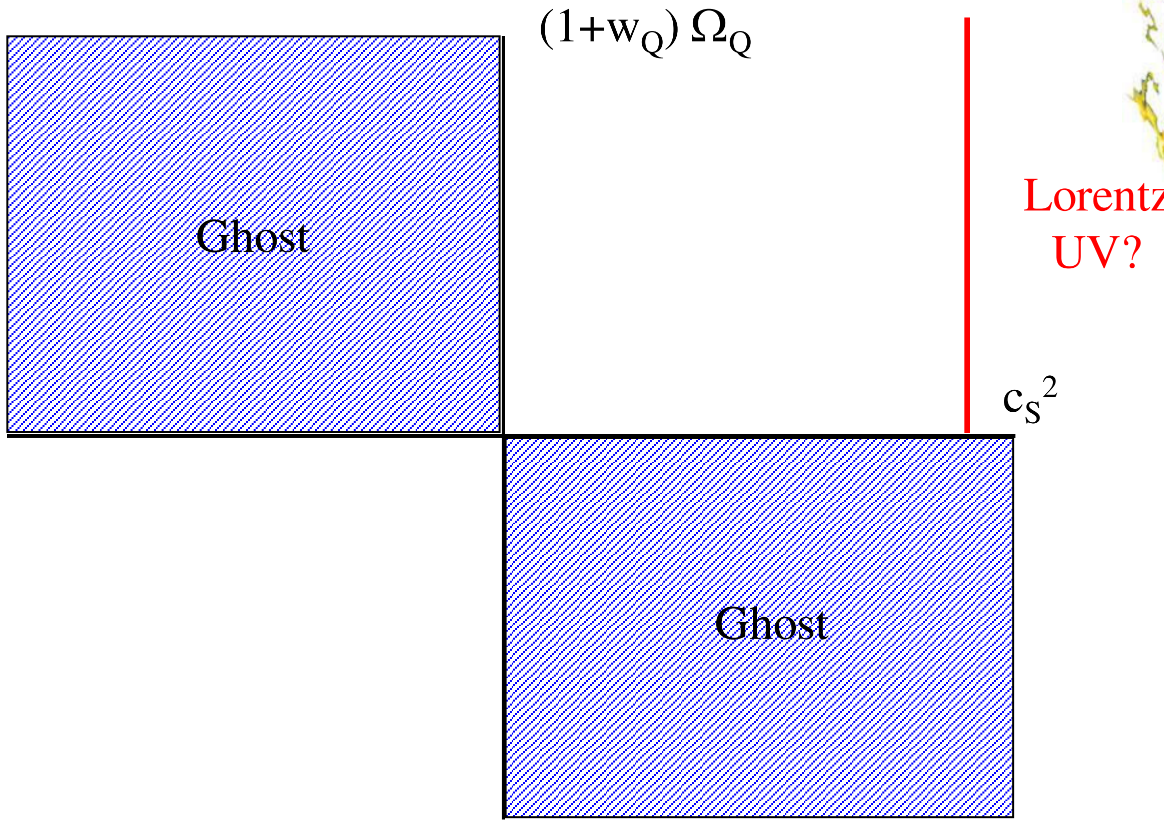
No ghost and c_S^2



$$\frac{1}{2}\left(\rho_Q + p_Q + 4M^4\right)\dot{\pi}^2 - \frac{1}{2}(\rho_Q + p_Q)\frac{(\nabla\pi)^2}{a^2} \longrightarrow c_s^2 = \frac{\rho_Q + p_Q}{\rho_Q + p_Q + 4M^4}$$

c_S^2 has the same sign of $1+w_Q$

Faster than light?



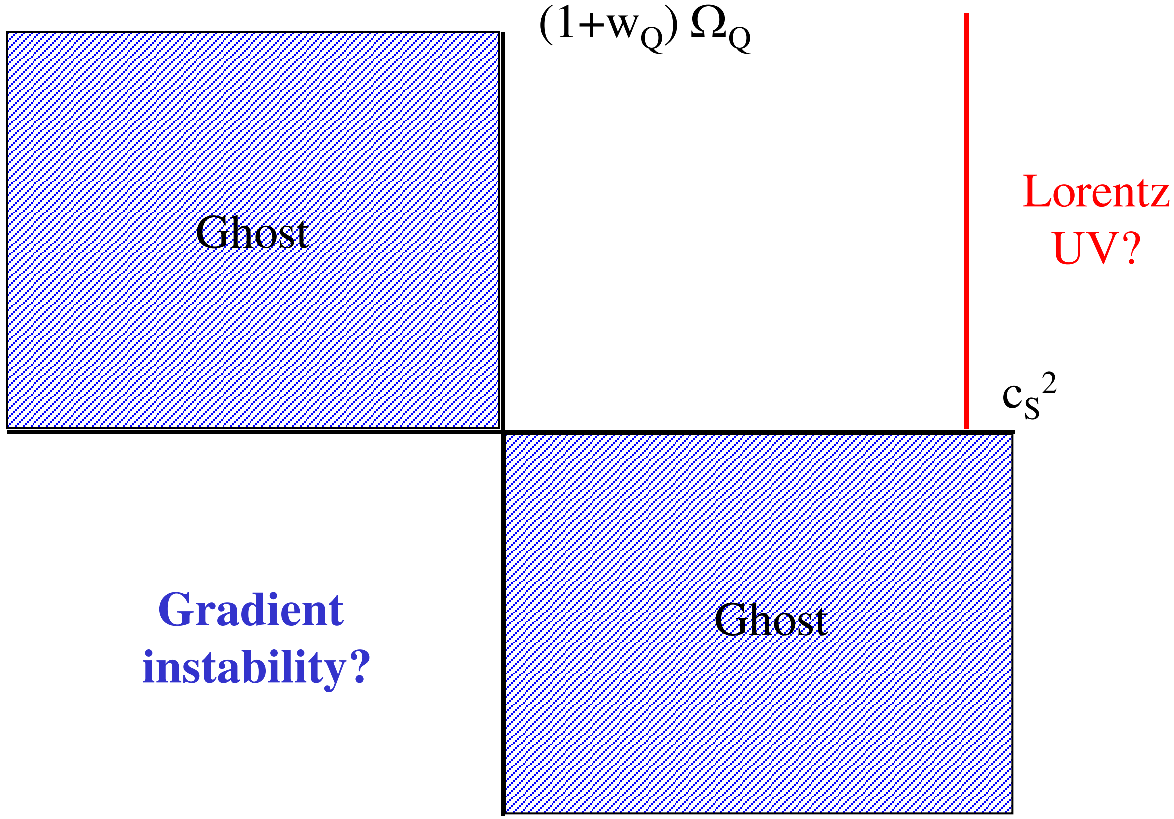
$$\frac{1}{2}\left(\rho_Q + p_Q + 4M^4\right)\dot{\pi}^2 - \frac{1}{2}(\rho_Q + p_Q)\frac{(\nabla\pi)^2}{a^2} \longrightarrow c_s^2 = \frac{\rho_Q + p_Q}{\rho_Q + p_Q + 4M^4}$$

$c_S^2 > 1$ ($M^4 < 0$) implies a non-Lorentz invariant UV completion

Arkani-Hamed etal '06
Babichev etal '07

w < -1 and gradient instabilities

Wise etal 04
Rattazzi etal 05



$$\frac{1}{2}\left(\rho_Q + p_Q + 4M^4\right)\dot{\pi}^2 - \frac{1}{2}(\rho_Q + p_Q)\frac{(\nabla\pi)^2}{a^2} \longrightarrow c_s^2 = \frac{\rho_Q + p_Q}{\rho_Q + p_Q + 4M^4}$$

It is difficult to violate the Null Energy Condition: $T_{\mu\nu}n^\mu n^\nu \geq 0$

Small c_S^2 limit

Instability rate: $\omega = i\, c_s\, k$. What happens when c_S^2 is very small?
Relevant instabilitites, $\omega > H$, only at short scales

$$S = \int d^4x\, a^3 \left[\frac{1}{2} \left(\rho_Q + p_Q + 4M^4\right) \dot{\pi}^2 - \frac{1}{2}(\rho_Q + p_Q)\frac{(\nabla\pi)^2}{a^2} + \frac{3}{2}\dot{H}(\rho_Q + p_Q)\pi^2 - \frac{1}{2}(\rho_Q + p_Q)\dot{h}\pi \right]$$

Consider the limit $\rho_Q + p_Q = 0$, no spatial kinetic term. **Enhanced symmetry**: $\pi \to \pi + c$

Scalar with shift symmetry: $\mathcal{L} = \sqrt{-g}\, M^4 P(X)\ ,\qquad X \equiv -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$

In an expanding Universe, we expect $\dot{\phi} \to 0$.

Another possibility: $\phi = c\, t \qquad P'(c^2) = 0 \qquad (\text{with } P'' > 0)$

$$T_{\mu\nu} = M^4 P(X) g_{\mu\nu} + 2M^4 P'(X)\partial_\mu\phi\partial_\nu\phi$$

Ghost Condensate limit: time evolving scalar in exact de Sitter

Arkani-Hamed etal 03

Higher derivative

We have to consider higher derivative operators

$$\mathcal{L}_{\bar{M}} = -\frac{\bar{M}^2}{2}(\Box\phi + 3H(\phi))^2$$

It does not change the background evolution.
Only perturbations.

$$\mathcal{L}_{\bar{M}} = -\frac{\bar{M}^2}{2}\left(\ddot{\pi} + 3H\dot{\pi} - 3\dot{H}\pi - \frac{\nabla^2\pi}{a^2}\right)^2$$

Leading spatial derivative term

$\ll M^4\dot{\pi}^2$

Higher time derivative terms can be neglected for ω < M
No additional degrees of freedom

In the ghost condensate limit:
$\omega \propto k^2$

The Ghost Condensate is a point of enhanced symmetry.

A small breaking of the shift symmetry (and thus a small c_s^2) is **technically natural**

Stability analysis

Gradient instability: $$\left(\rho_Q + p_Q + 4M^4\right)\omega^2 - \left(\rho_Q + p_Q\right)\frac{k^2}{a^2} - \bar{M}^2\frac{k^4}{a^4} = 0$$

$$\omega^2_{\rm grad} \simeq -\frac{(\rho_Q + p_Q)^2}{\bar{M}^2 M^4} \longrightarrow -\frac{\rho_Q + p_Q}{\bar{M} M^2} \lesssim H$$

Jeans instability: taking into account the mixing with gravity gives rise to a sort of Jeans like instability

$$S = \int d^4x \left[2M^4\dot{\pi}^2 - \frac{\bar{M}^2}{2}\left(\frac{\dot{h}}{2} - \nabla^2\pi\right)^2\right] \longrightarrow \ddot{\pi} + \frac{\bar{M}^2}{4M^4}\nabla^4\pi = \frac{\bar{M}^2}{8M^4}\nabla^2\dot{h}$$

Solving for h: $$\ddot{\pi} + \frac{\bar{M}^2}{4M^4}\nabla^4\pi = -\frac{\bar{M}^2}{2M_{\rm Pl}^2}\nabla^2\pi \longrightarrow \omega^2_{\rm Jeans} \simeq -\left(\frac{\bar{M}M^2}{M_{\rm Pl}^2}\right)^2$$

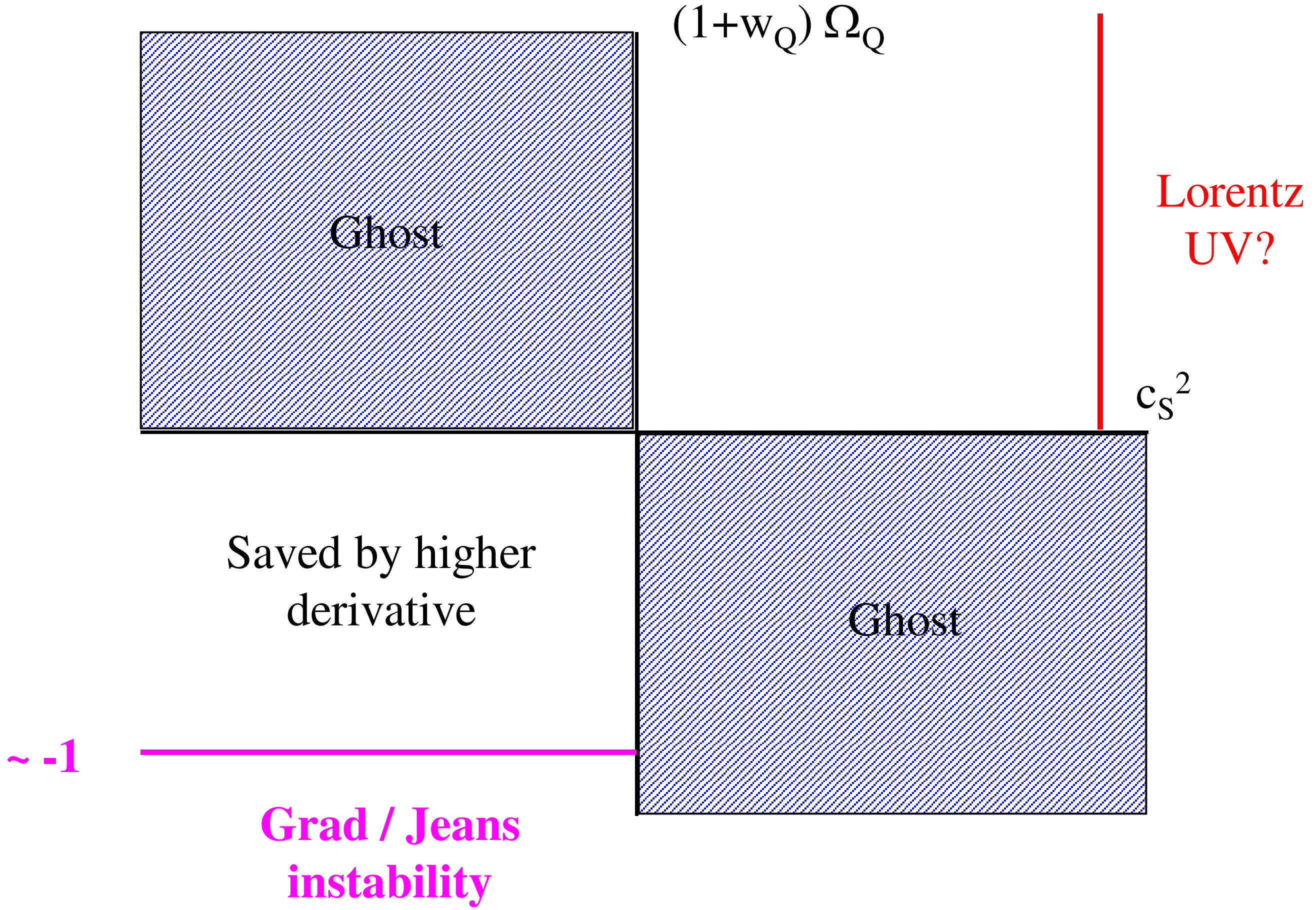
Stability window

$$\boxed{-(1 + w_Q)\Omega_Q \lesssim \frac{\bar{M}M^2}{HM_{\rm Pl}^2} \lesssim 1}$$

Very conservative...

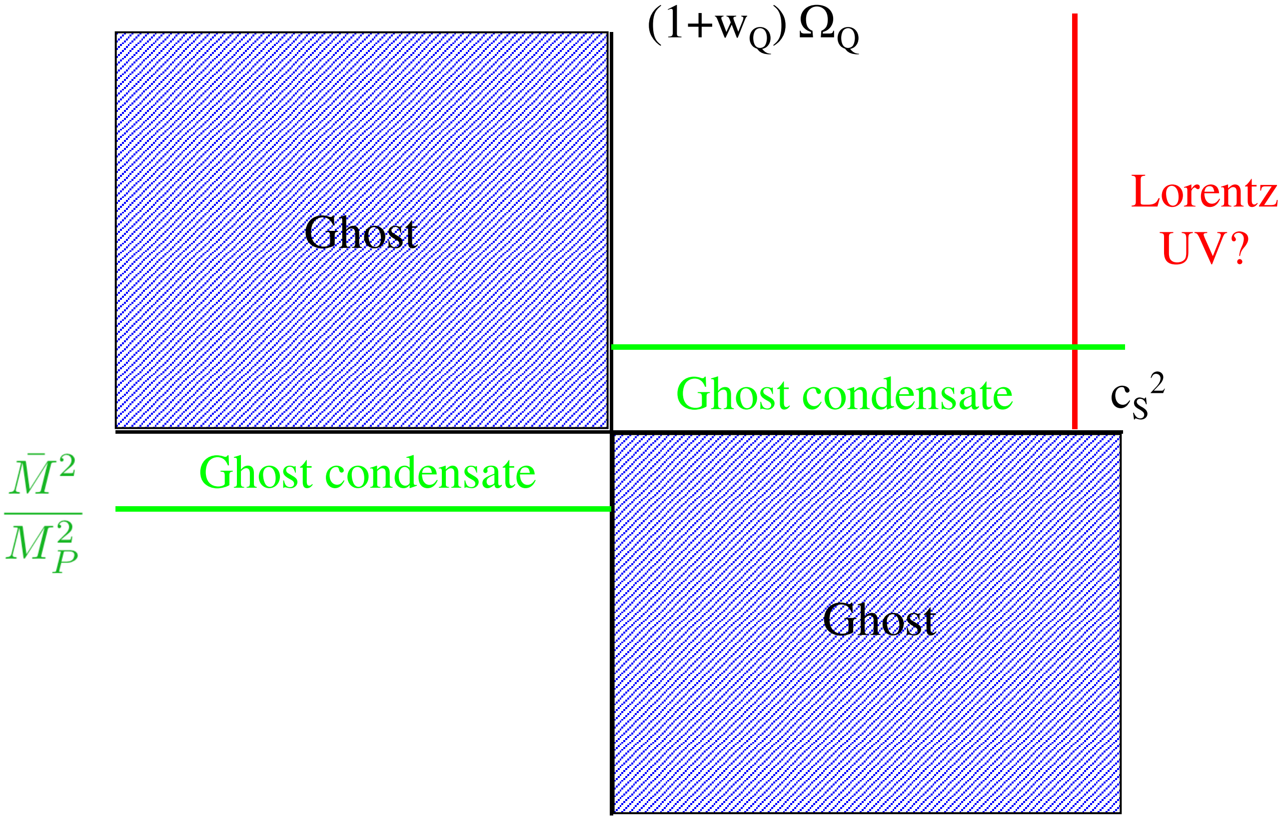
Arkani-Hamed etal 05

Back to the plane



This limit is very conservative and anyway pheno irrelevant

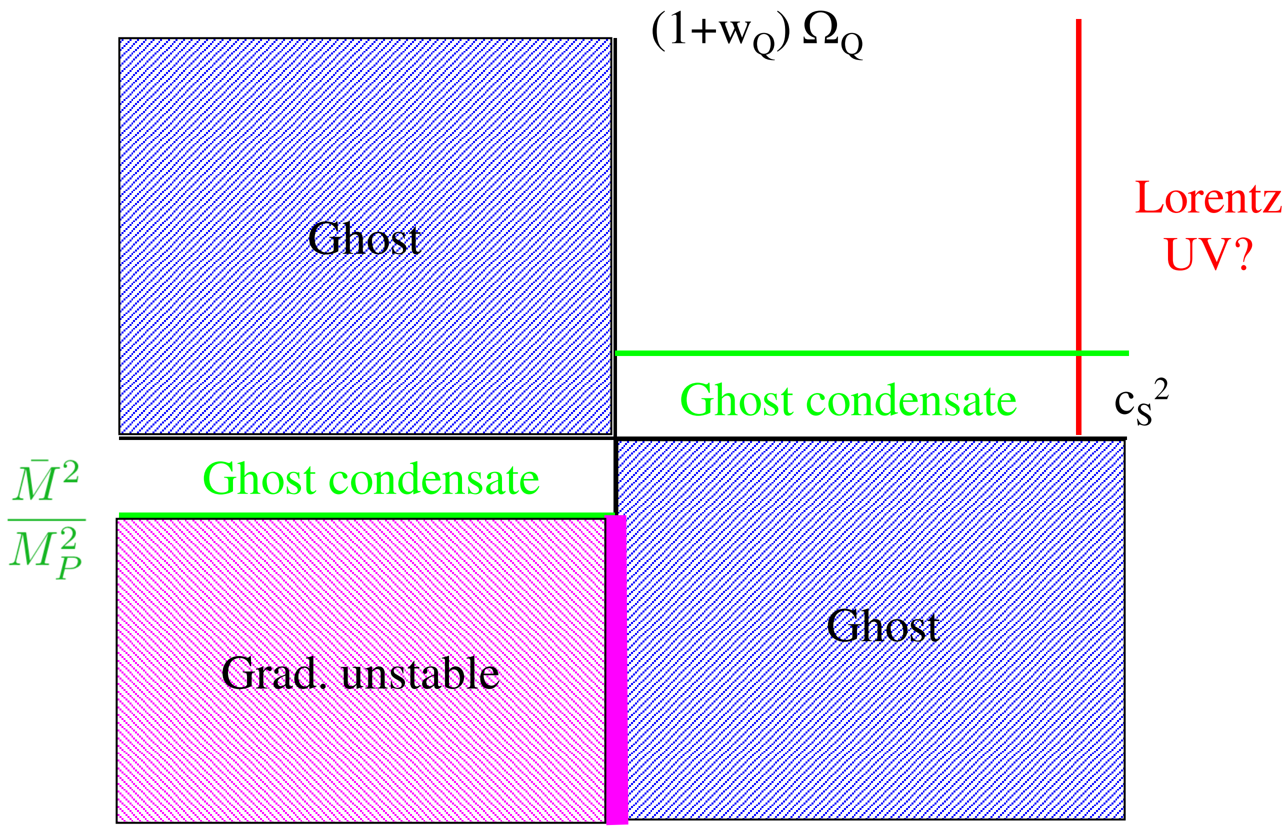
Higher derivative in the codes?



$$S_Q \supset \frac{1}{2}\int d^3x dt\, a^3 \left[-(\rho_Q + p_Q)\frac{(\nabla\pi)^2}{a^2} - \bar{M}^2\left(\frac{\nabla^2\pi}{a^2}\right)^2\right]$$

Cosmo modes k/a~H are dominated by $\omega = c_s\, k$ for: $\left|(1+w_Q)\Omega_Q\right| \gg \frac{\bar{M}^2}{M_P^2}$

Small c_s^2: how small?



$$S_Q \supset \frac{1}{2}\int d^3x dt\, a^3\left[4M^4\dot{\pi}^2 - (\rho_Q + p_Q)\frac{(\nabla\pi)^2}{a^2} - \bar{M}^2\left(\frac{\nabla^2\pi}{a^2}\right)^2\right]$$

$$\omega_{\rm inst.} \simeq (1+w_Q)\Omega_Q \frac{M_P^2 H^2}{M^2\bar{M}}$$

$$\omega_{\rm inst.} \ll H \quad \Rightarrow \quad c_s^2 \ll \frac{H\bar{M}}{M^2}$$

The scales M are the cutoff of my theory

$M > (.1\text{mm})^{-1}$ --> $|c_s^2| < 10^{-30}$!!

The phantom divide

- What happens to perturbations when $w_Q = -1$?

Fluid equations:

e.g. Bean, Doré 03

$$\dot{\delta} = -(1+w)\left\{\left[k^2 + 9\mathcal{H}^2(c_s^2 - c_a^2)\right]\frac{\theta}{k^2} + \frac{\dot{h}}{2}\right\} - 3\mathcal{H}(c_s^2 - w)\delta$$

$$\frac{\dot{\theta}}{k^2} = -\mathcal{H}(1-3c_s^2)\frac{\theta}{k^2} + \frac{c_s^2}{1+w}\delta\,.$$

$$\theta \equiv ik^j v_j \qquad c_a^2 \equiv \dot{p}/\dot{\rho} = w - \frac{1}{3H}\frac{\dot{w}}{1+w}$$

$$c_s^2 \equiv \delta\hat{p}/\delta\hat{\rho} \qquad T_i^0 = 0$$

The one given by scalar kinetic term

- The phantom psychosis:

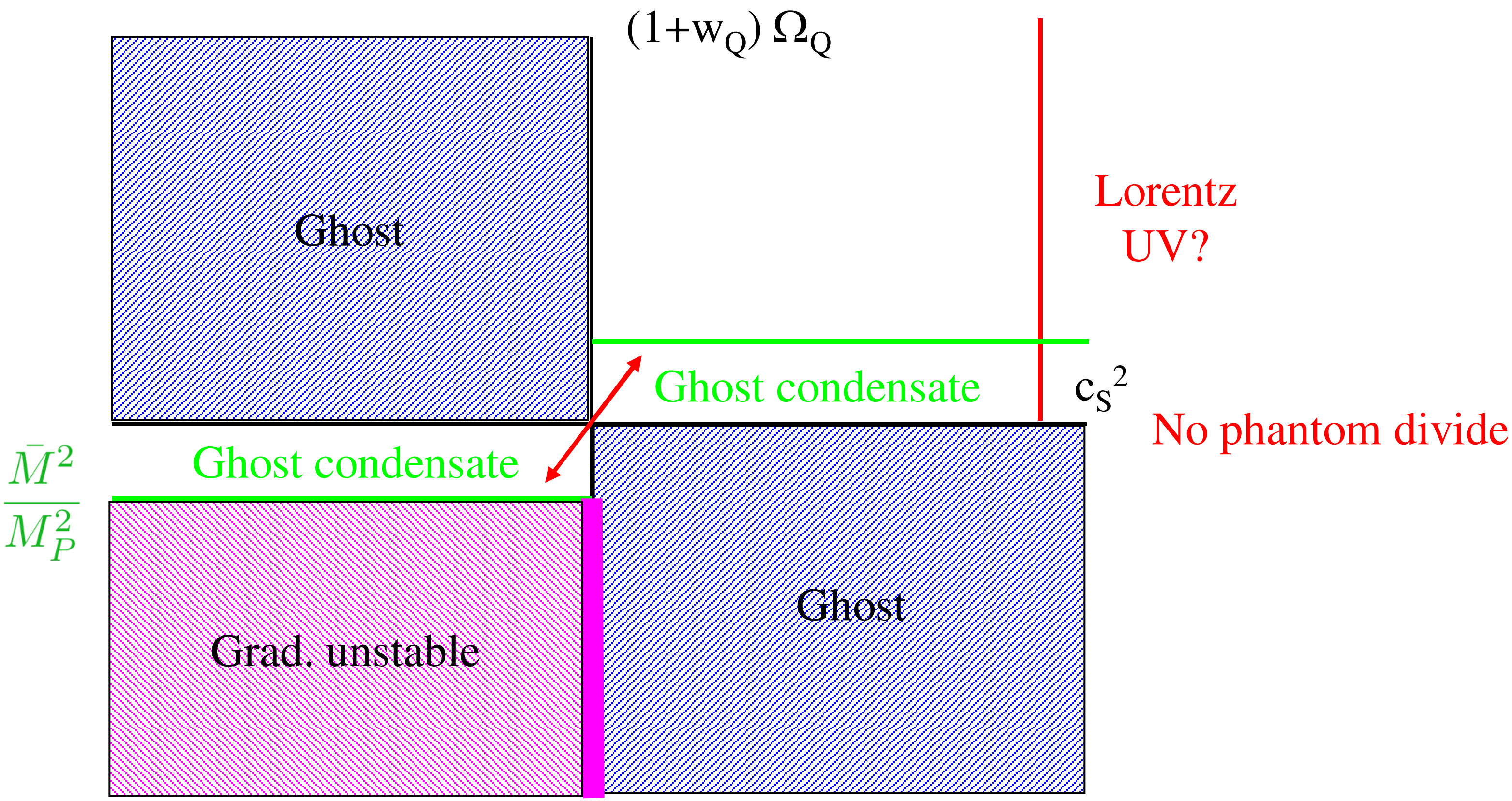
- 1st divergence: $c_a^2 \rightarrow \infty$ [Hu 04] So what?

- 2nd divergence: in θ equation [Caldwell, Doran 05] $c_S{}^2$ ⌫ 0 at the crossing

- Instability: $c_S{}^2$ ⌫ 0 ☑ $c_S{}^2 < 0$ Higher derivative terms

[Vikman 04, Caldwell Doran 05, Kunz Sapone 06]

The phantom divide is ... a phantom



$$S_Q = \frac{1}{2}\int d^3x dt\, a^3 \Big[4M^4\dot{\pi}^2 + \cancel{(\rho_Q + p_Q)}\dot{\pi}^2 - \cancel{(\rho_Q + p_Q)}\frac{(\nabla\pi)^2}{a^2} + 3\dot{H}\cancel{(\rho_Q + p_Q)}\pi^2$$

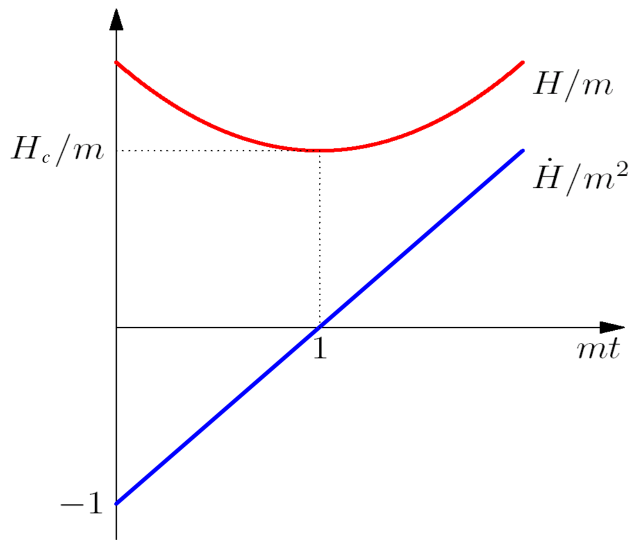
$$-\cancel{(\rho_Q + p_Q)}\dot{h}\pi - \bar{M}^2\Big(\frac{\dot{h}}{2} - \frac{\nabla^2\pi}{a^2}\Big)^2\Big]$$

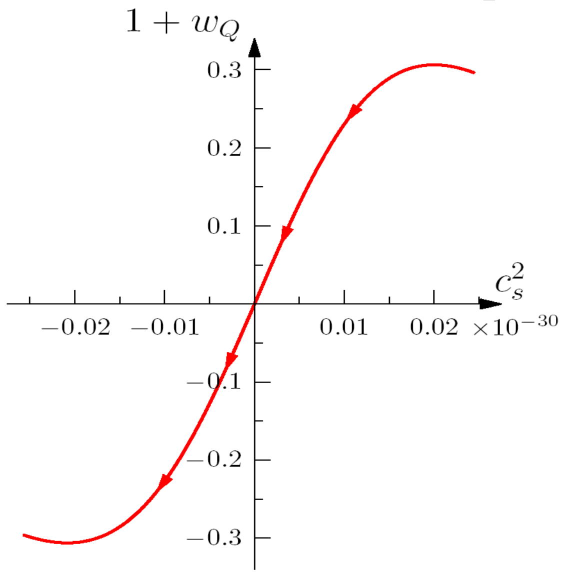
Nothing strange happens when you cross $w_Q = -1$

For example...

$$P(X,\phi) = -3M_{\rm Pl}^2 H^2(\phi) - M_{\rm Pl}^2 \dot{H}(\phi)(X+1) + \frac{1}{2}M^4(\phi)(X-1)^2$$

No other energy components





- The GC strip is very tiny. Effectively **w_Q= -1 is crossed by a k-essence with c_S^2 = 0**
- Numerical recipe. When comparing with data $w_Q(z)$ going through w_Q= -1, set c_S^2 = 0

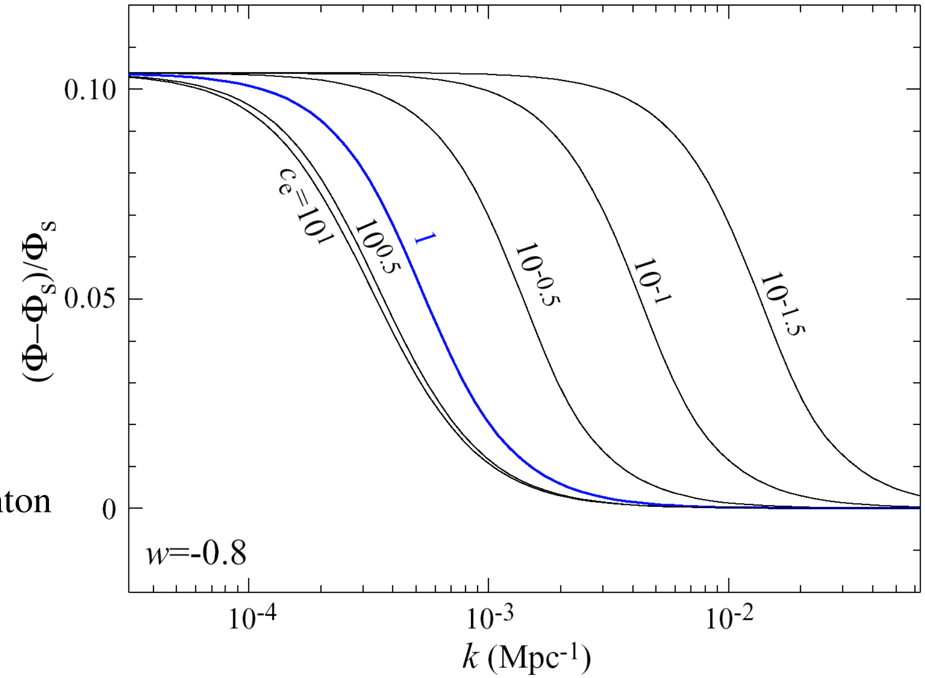
Phenomenology of $c_S = 0$

Theoretical motivation to study $c_s = 0$

Dark energy follow (or escapes!) from dark matter wells: $$\delta_Q \simeq \frac{1+w}{1-3w}\delta_{\rm DM}$$

Clusters on scales large than sound horizon: $$1/k_{\rm DE\ s.h.} = a\int_0^t \frac{c_s}{a}dt \simeq 2c_s H_0^{-1}$$

Fractional difference in Φ between smooth and clustered quintessence

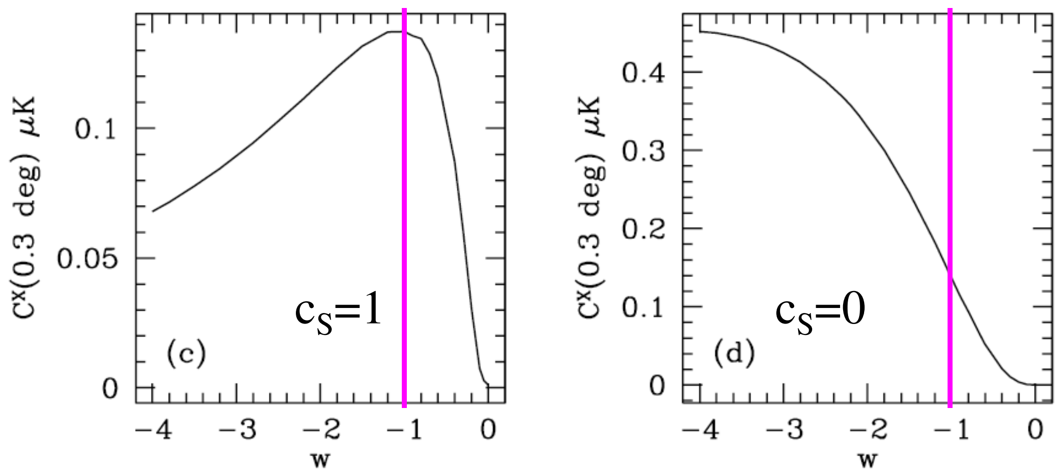


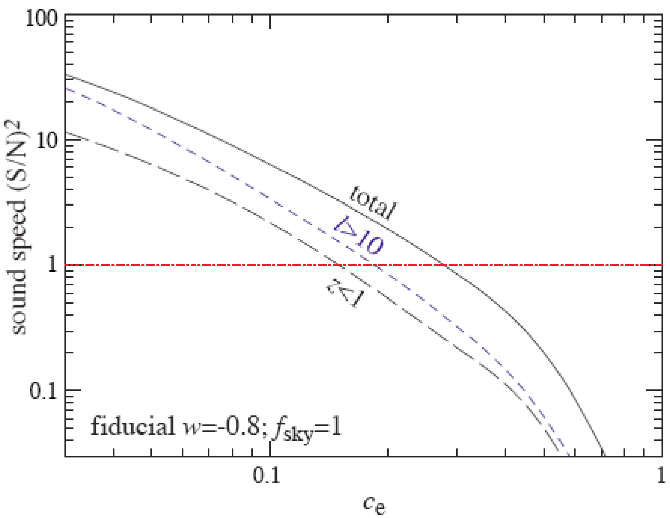
Hu, Scranton 04

ISW- galaxy correlation

Is it possible to exp distinguish c_s=0 from c_s=1? Until which value of $1+w_Q$?

Corasaniti, Giannantonio, Melchiorri 05





Hu, Scranton 04

Distinction possible for $|1 + w_Q| \gtrsim 0.05$?

Forecasts done only for w > -1...

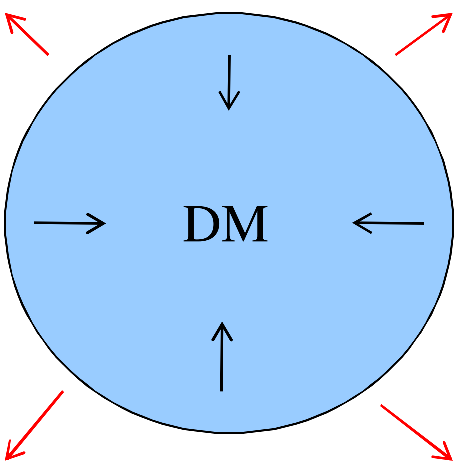
Non-linear clustering

What happens at very short scales? For $c_s^2 = 0$ quintessence clusters at all scales.

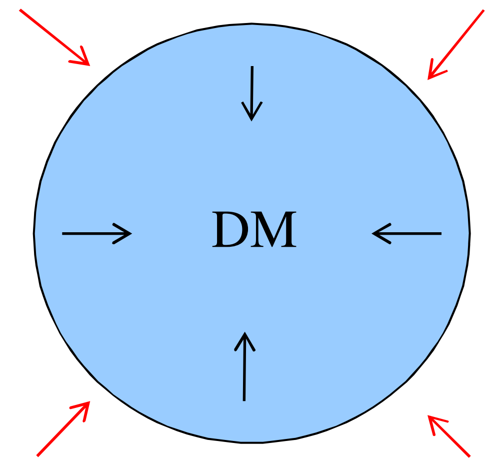
Effect on **non-linear structure formation**

Spherical collapse

$c_s = 1$



$c_s = 0$



$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho_m + \rho_Q + 3\bar{p}_Q)$$

Evolution of the radius

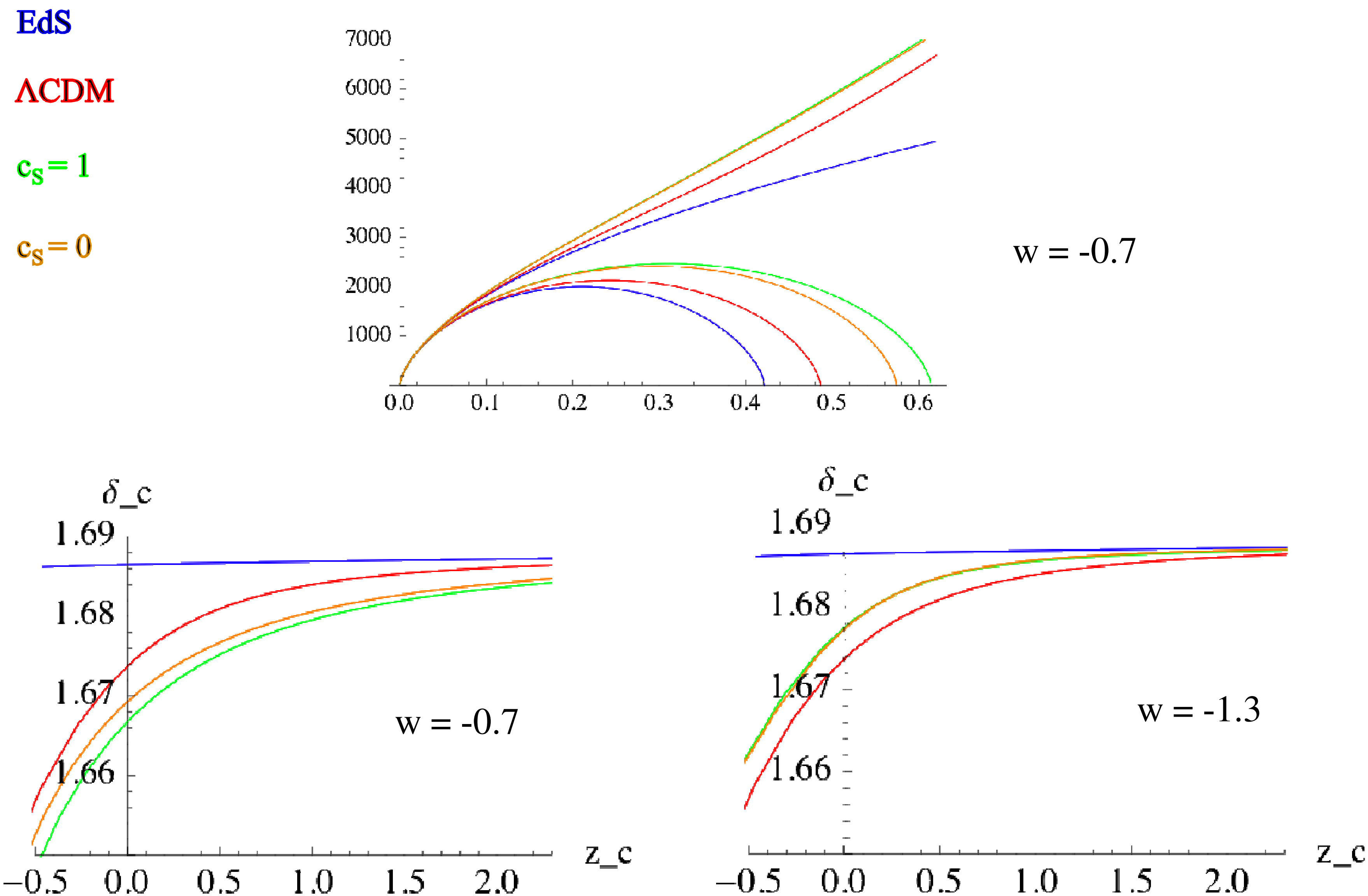
$$\dot{\rho}_m + 3\frac{\dot{R}}{R}\rho_m = 0$$

Evolution of DM

$$\dot{\rho}_Q + 3\frac{\dot{R}}{R}(\rho_Q + \bar{p}_Q) = 0$$

Evolution of quintessence

Spherical collapse

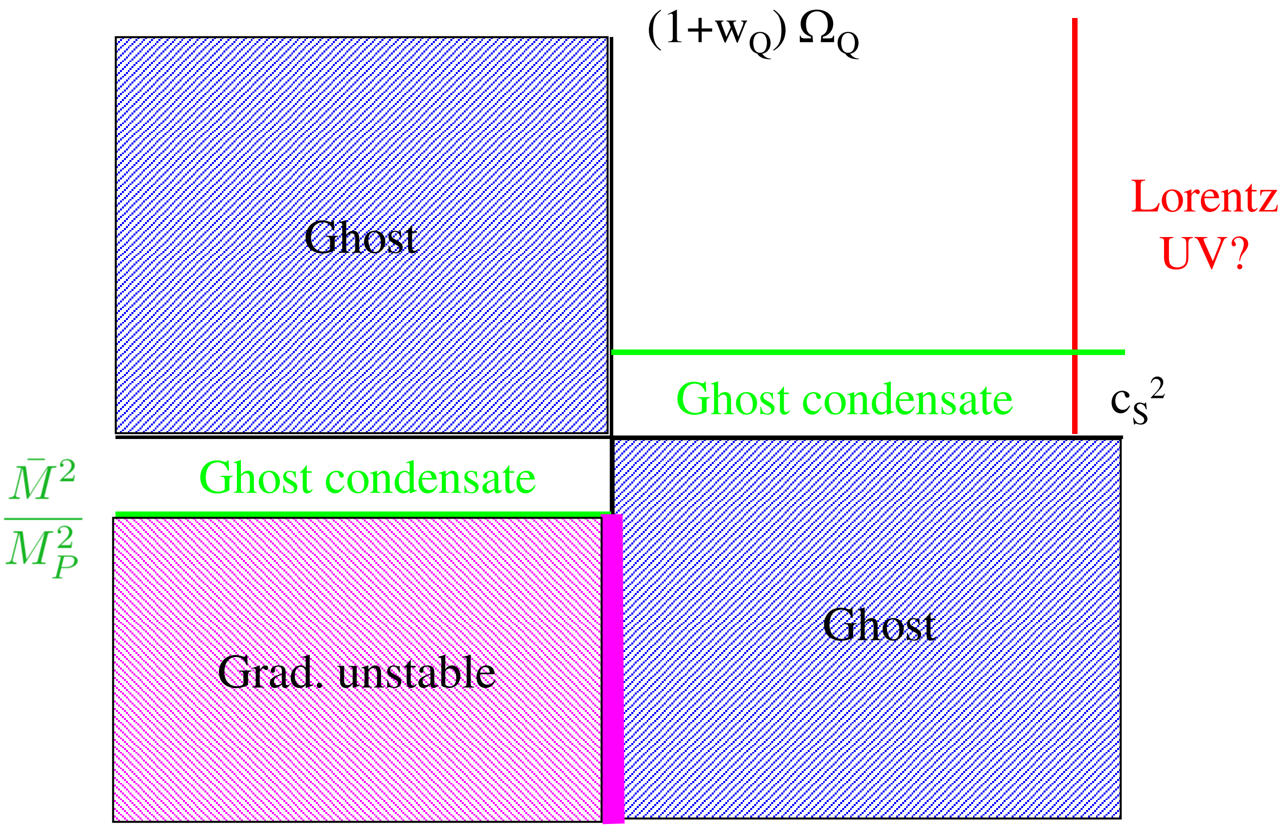


in progress with G.D'Amico, J.Noreña, L.Senatore, F.Vernizzi

Conclusions

- General framework to study single field quintessence models
- Higher derivative terms can stabilize the $w_Q < -1$ region
  Phenomenology is the same of **k-essence models with $c_S^2 = 0$**
- The **phantom divide can be smoothly crossed** if one sets $c_S^2 = 0$
- Phenomenology of models with $c_S^2 = 0$ VS $c_S^2 = 1$ must be further explored
- **Quintessential plane**

Quintessential plane



Quintessence ~ Λ

$\boxed{\text{Ghost condensate limit}}$ For cosmo scales: ω ~ k^2 $(1+w_Q)\Omega_Q \ll \frac{\bar{M}^2}{M_P^2}$

$$S_Q = \frac{1}{2}\int d^3x dt\, a^3 \left[4M^4\dot{\pi}^2 - \bar{M}^2\left(\frac{\dot{h}}{2} - \frac{\nabla^2\pi}{a^2}\right)^2\right]$$

The scalar degree of freedom does not disappear even for 1+w$_Q$=0

$$\ddot{\pi} + 3H\dot{\pi} = -\frac{\bar{M}^2}{12M^4M_P^2}\frac{\nabla^2\delta\rho_{\rm DM}}{Ha^2}$$

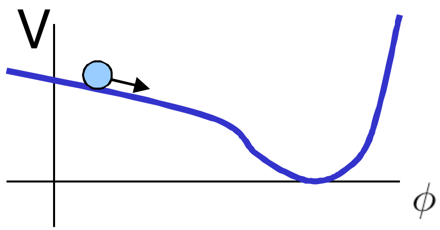
The driving of DM is not suppressed by 1+w$_Q$ in this limit

$$\delta\rho_Q = 4M^4\dot{\pi} \sim \frac{\bar{M}^2}{M_P^2}\delta\rho_{\rm DM} \lll \delta\rho_{\rm DM}$$

No relevant perturbation!

The ghost condensate is a modification of gravity, but only on very short scales
Irrelevant cosmologically

A more general approach



Usual approach to quintessence/inflation:

1. Take a Lagrangian for a scalar $\mathcal{L}(\phi, \partial_\mu\phi, \Box\phi\ldots)$
2. Solve EOM of the scalar + FRW. Find an accelerating solution $\ddot{a} > 0$

$$\phi = \phi_0(t) \qquad ds^2 = -dt^2 + a^2(t)d\vec{x}^2$$

3. Study perturbations around this solution to work out predictions

We want to **focus directly on the theory of perturbations** around the accelerating solution

- Time diffeomorphisms are broken: $t \to t + \xi^0(t, \vec{x}) \qquad \delta\phi \to \delta\phi + \dot{\phi}_0(t)\xi^0$
- In unitary gauge $\phi(t, \vec{x}) = \phi_0(t)$ the scalar mode is eaten by the graviton:

  3 degrees of freedom. Like in a broken gauge theory.

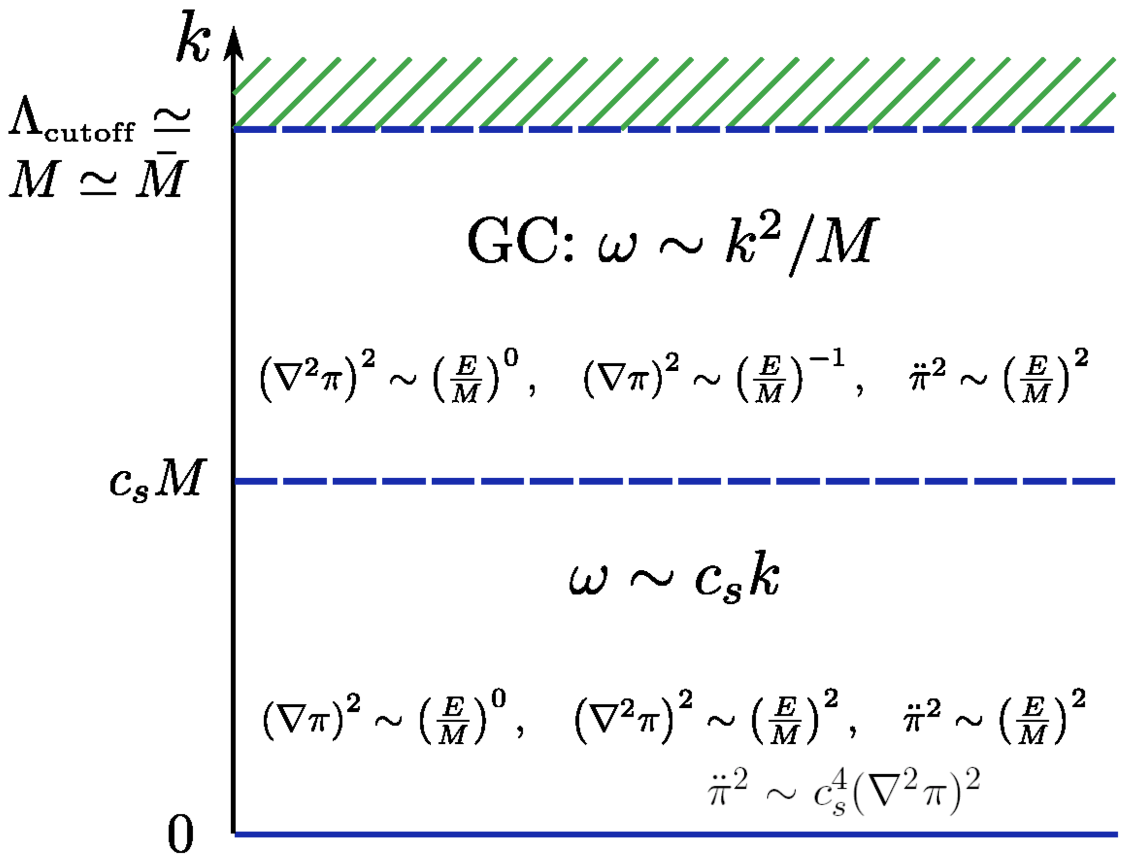
- The most generic action in unitary gauge

$$S = \int \mathrm{d}^4x\,\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R + \mathcal{L}_m + p_Q - \frac{1}{2}(\rho_Q + p_Q)(g^{00}+1) + \frac{M^4(t)}{2}(g^{00}+1)^2 \right.$$
$$\left. - \frac{\bar{M}^2(t)}{2}\delta K^2 - \frac{\hat{M}(t)^3}{2}\delta K(g^{00}+1)\right].$$

Scaling in EFT

Arkani-Hamed et al ‘03, Simon ’91, Weinberg ‘08

$$S = \frac{M^4}{2}\int d^3x\,dt\left[\dot{\pi}^2 - c_s^2(\nabla\pi)^2 - \frac{(\nabla^2\pi)^2}{M^2} + \ldots\right.$$



- scaling transformations:

$$E \to sE\,, \qquad t \to s^{-1}t\,,$$
$$x \to s^{-1/2}x\,, \qquad \pi \to s^{1/4}\pi$$

$$E \to sE\,, \qquad t \to s^{-1}t\,,$$
$$x \to s^{-1}x\,, \qquad \pi \to s^{1}\pi$$