Coupled Currents in Cosmic Strings with N carrier bosonic fields

M. L. , X. Martin, P. Peter, *Phys. Rev.* D (2009)

Marc Lilley

Institut d'Astrophysique de Paris, France

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General considerations

Current carrying strings: microscopic approach

Current carrying strings: worldsheet approach

Mechanical stability for arbitrary N

$N = 2$ case

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General considerations

Generic production of cosmic (super-) strings

In the SB patterns of *most* supersymmetric grand unified theories

e.g., Jeannerot *et al.*, Phys. Rev. D **68** (2003)

Super-strings of cosm. size in brane inflation

e.g., Sarangi and Tye, Phys. Lett. B **536**, (2002); Jones *et al.*, JHEP **0207** (2002); Copeland *et al.*, JHEP **0203** (2004)

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Important to know what kind of strings one is dealing with.

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One or more currents

Microscopic approach

Current-carrying string described by

$$\mathcal{L} = \mathcal{L}_{\mathrm{NG}} - \sum_{i}^{N} \left[\frac{1}{2} \partial_\mu \Phi_i^* \partial^\mu \Phi_i - \frac{g_i}{2} \left(|H|^2 - \eta^2 \right) |\Phi_i|^2 - V(\Phi_i) \right] - \sum_{i \neq j}^{N} \mathcal{V}(\Phi_i, \Phi_j)$$

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Global U(1) complex scalar fields [Peter, 1992]

$$\left.\begin{array}{c}\text{straight,}\\ \text{static}\\ \text{config.}\end{array}\right\} \Phi_i(x^\mu) = \phi_i(r) e^{i\psi_i} \qquad \text{and} \qquad \psi_i = \omega_i t - k_i z$$

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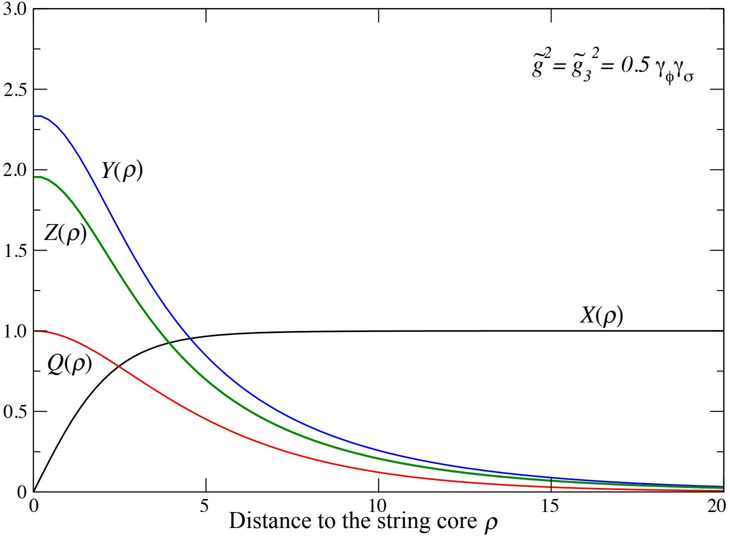
2 topological invariants

$$\text{EM flux} :: \Phi = \int \mathcal{F}_{\mu\nu} \mathrm{d}\sigma^{\mu\nu} = -\frac{2\pi n}{e}$$

$$\left.\begin{array}{c}\text{Phase change}\\ \text{around}\\ \text{closed loop}\\ \text{[Witten, 1985]}\end{array}\right\} \oint \frac{\mathrm{d}\psi_i}{\mathrm{d}\zeta} \mathrm{d}\zeta = 2\pi m_i$$

Field profiles as a function of radial coord. for $N = 2$



$X(\rho)$:: Higgs field

$Q(\rho)$:: Gauge field associated with Higgs field

$Y(\rho)$ and $Z(\rho)$:: global U(1) fields

$\tilde{g}$:: cross-coupling constant

γ_i:: self-coupling constant

Worldsheet approach

Worldsheet Lagrangian [Carter, 1989]

$$\tilde{\mathcal{L}}(w_1, \ldots, w_N) = 2\pi \int \mathcal{L}\, r \, \mathrm{d}r \quad \text{with} \quad w_i = \partial^\mu \varphi_i \partial_\mu \varphi_i = k_i^2 - \omega_i^2$$

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$$U - T = |w|\mathcal{K} \quad \text{with} \quad \begin{cases} w = k^2 - \omega^2 \\ \mathcal{K} = 2\dfrac{\mathrm{d}\tilde{\mathcal{L}}}{\mathrm{d}w} \end{cases}$$

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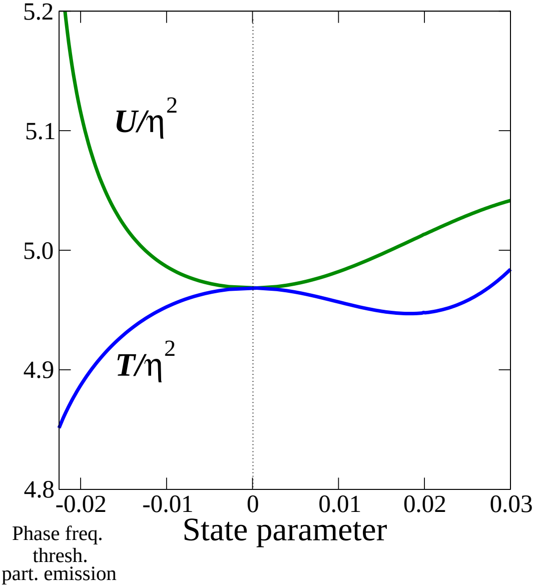
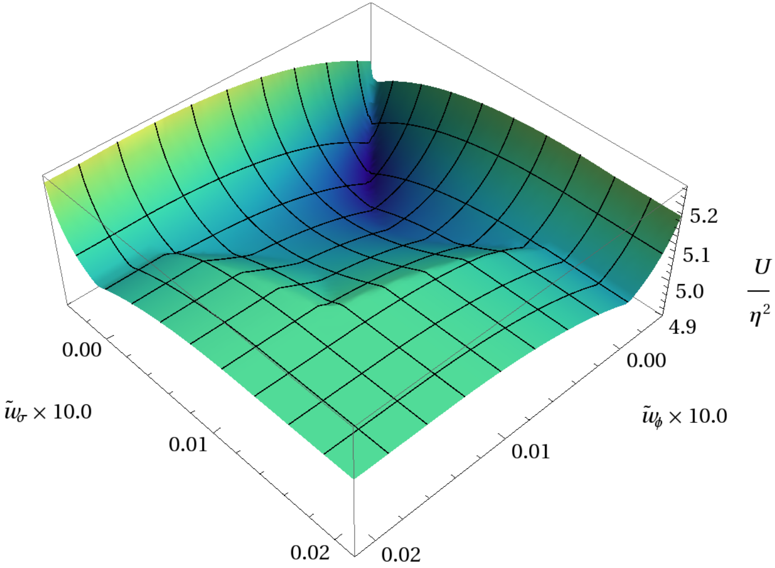
EOS for current-carrying strings: N **arbitrary** [new]

$$U - T = \sqrt{\mathcal{F}\left[(\mathcal{K}^{ii}\chi_{jj})^2, \chi_{ij}^2 \mathcal{K}_{ii}\mathcal{K}_{jj}\right]} \quad \text{with} \quad \begin{cases} \chi_{ij} = k_i k_j - \omega_i \omega_j \\ \mathcal{K}^{ij} = 2\dfrac{\delta\tilde{\mathcal{L}}}{\delta\chi_{ij}} \end{cases}$$

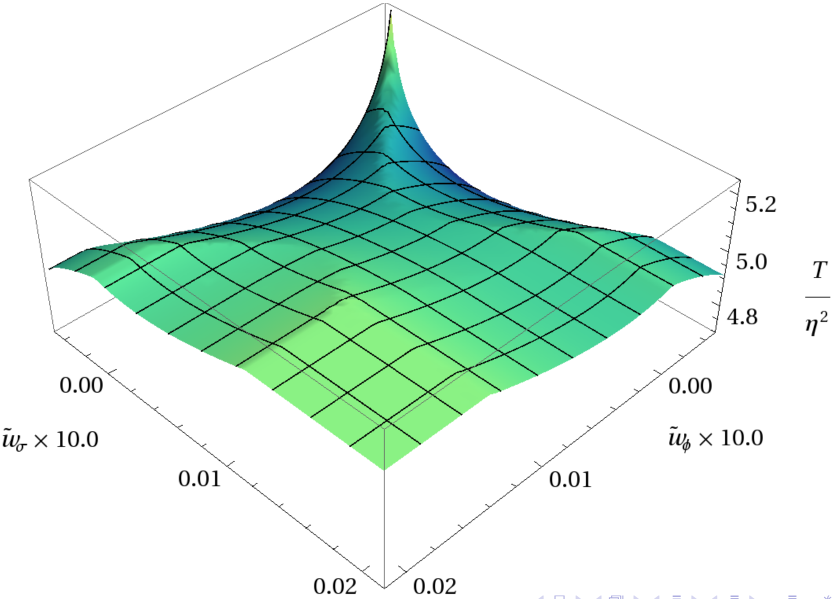
Energy and Tension, $N = 1$



Energy, $N = 2$ and $\chi_{12} = 0$



Tension, $N = 2$ and $\chi_{12} = 0$



Mechanical stability for $N = 1$ [Carter, 1989]

Perturb cons. equations and worldsheet integr. condition

1. Integrability of the worldsheet
2. Conservation of $\tilde{T}^{\mu\nu}$
3. Irrotationality condition of the phase gradients
4. *Current conservation*

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Perturbed quantities $(Q \to \chi_{ij}, \mathcal{K}_{ij} \ldots)$

Trans :: $\delta(Q) = e^{i(\omega_{\rm t} t - k_{\rm t} z)} \varepsilon(Q)$ Long :: $\delta(Q) = e^{i(\omega_\ell t - k_\ell z)} \varepsilon(Q)$

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Transverse stability

disp. rel. $\in \mathbb{R}$ :: $\dfrac{\omega_{\mathrm{t}}^2}{k_{\mathrm{t}}^2} = \dfrac{T}{U} \geq 0$ (*transverse wave*)

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$$\text{disp. rel.} \in \mathbb{R} \quad :: \quad \frac{\omega_\mathrm{t}^2}{k_\mathrm{t}^2} = \frac{T}{U} \geq 0 \qquad (\textit{transverse wave})$$

Longitudinal stability

$$\text{disp. rel.} \in \mathbb{R} \quad :: \quad \frac{\omega_\ell^2}{k_\ell^2} = -\frac{\mathrm{d}T}{\mathrm{d}U} \geq 0 \qquad (\textit{accoustic wave})$$

Mechanical stability for arbitrary N

Transverse stability

$$\frac{\omega_{\mathrm{t}}^2}{k_{\mathrm{t}}^2} = \frac{T}{U} \geq 0$$

Note: transverse perturbations decouple from perturbations in χ_{ij} and $\mathcal{K}_{ij}$

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Longitudinal stability

Homogeneous system of $2N$ coupled equations

$\Rightarrow$ Obtain $\dfrac{\omega_\ell}{k_\ell}$ from $|\ldots| = 0$ and impose $\dfrac{\omega_\ell}{k_\ell} \in \mathbb{R}$

$$\boxed{\text{sets bounds on the } \chi_{ij}\text{'s}}$$

$N = 2$ case

Transverse stability

$$\frac{T}{U} \geq 0 \quad \Rightarrow \quad \chi_{12}^2 \leq \frac{1}{\mathcal{K}_{11}\mathcal{K}_{22}} \left[\mathcal{A} - \left(\frac{1}{2}\chi_{11}\mathcal{K}_{11} - \frac{1}{2}\chi_{22}\mathcal{K}_{22} \right) \right]$$

where $\mathcal{A} = 2\pi \int \mathcal{F}(H, A_\mu, \Phi_i, \ldots) r \, \mathrm{d}r$.

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Longitudinal stability

$$c_0 + c_1\xi + c_2\xi^2 + c_3\xi^3 + c_4\xi^4 = 0 \qquad \text{where} \qquad \begin{cases} \xi = \mathcal{G}\left(\omega_\ell / k_\ell\right) \\ c_i = \mathcal{H}\left(\mathcal{K}_{ij}, \chi_{ij}, \dots\right) \end{cases}$$

Suffices to require $\xi \in \mathbb{R} \Rightarrow$ sets bounds on the χ_{11}, χ_{22} and χ_{12}

$\boxed{\text{compute } U,\ T,\ \mathcal{K}_{ij},\ \xi,\ \textit{etc.} \text{ from field profiles obtained numerically}}$

Analytic approximation [Carter and Peter, 1995]

$$\tilde{\mathcal{L}}_i(w_i) = -m_h^2 - \frac{m_{\Phi_i}^2}{2} \ln\left(1 + \frac{w_i}{m_{\Phi_i}^2}\right) \qquad w_i < 0$$

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Neglects

$$\sum_{i \neq j}^{N} \mathcal{V}(\Phi_i, \Phi_j)$$

Expected to be a good approx. for $\tilde{g}_{ij} \ll 1$

Mechanical stability of approx. model

Transverse stability

$T/U \geq 0$

$$w_1 w_2 \geq 0 \quad :: \quad \chi_{12}^2 \leq w_1 w_2 \frac{(U_1 - m^2 + T_2)(U_2 - m^2 + T_1)}{(U_1 - T_1)(U_2 - T_2)}$$

$$w_1 w_2 \leq 0 \quad :: \quad \chi_{12}^2 \leq |w_1 w_2| \frac{\left(T_1 + T_2 - m^2\right)\left(U_1 + U_2 - m^2\right)}{(U_1 - T_1)(U_2 - T_2)}$$

Longitudinal stability

All perturbation modes decouple $\Rightarrow -dT/dU \geq 0$

$$-m_i^2 \leq w_i \leq \frac{m_i^2}{3}$$

Has been shown to agree with the numerical results when $\tilde{g} \ll 1$

Conclusions

- Cosmic (super-)strings seem to be inevitable
- They come with various attributes (intercommutation probability, mechanical stability, *etc.*)
- Important to develop tools in order to deal with these $\neq$ types of strings

- Focused on mechanical stability of strings
- Extended existing formalism to N current carrier fields
- Checked against fully analytic approximation