

Gravitational waves from phase transitions

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CC, R. Durrer and G. Servant, 0909.0622

CC, R. Durrer, T. Konstandin and G. Servant, 0901.1661

CC, R. Durrer and G. Servant, 0711.2593

Gravitational waves

- Once emitted, propagate without interaction: direct probe of physical processes in the early universe
- First order phase transitions are sources of GW
- Primordial sources: stochastic background of GW
- Temperature of the phase transition : characteristic frequency
- Strength of the phase transition : amplitude
- Signal potentially interesting for LISA, PTA, advanced LIGO
- Analytical evaluation of the GW signal in terms of free parameters

Stochastic background of GW

$$ds^2 = a^2(t)(dt^2 - (\delta_{ij} + 2h_{ij})dx^i dx^j) \quad G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$\ddot{h}_{ij} + \frac{2}{t}\dot{h}_{ij} + k^2 h_{ij} = 8\pi G a^2 \Pi_{ij}$$

Source: $\Pi_{ij}(\mathbf{k}, t)$ tensor anisotropic stress

- energy density of GW $\Omega_{GW} = \frac{\langle \dot{h}_{ij} \dot{h}^{ij} \rangle}{G\rho_c} = \int \frac{dk}{k} \frac{d\Omega_{GW}}{d \log k}$
- characteristic frequency of GW produced at time t_*

$$k_* \geq \mathcal{H}_*$$

$$k_{100\text{GeV}} \gtrsim 10^{-5} \text{Hz} \quad k_{100\text{MeV}} \gtrsim 10^{-8} \text{Hz}$$

GW from phase transitions : frequency

FIRST ORDER

- ⦿ Collision of bubbles walls
- ⦿ Turbulent motions in the primordial plasma
- ⦿ Magnetic fields

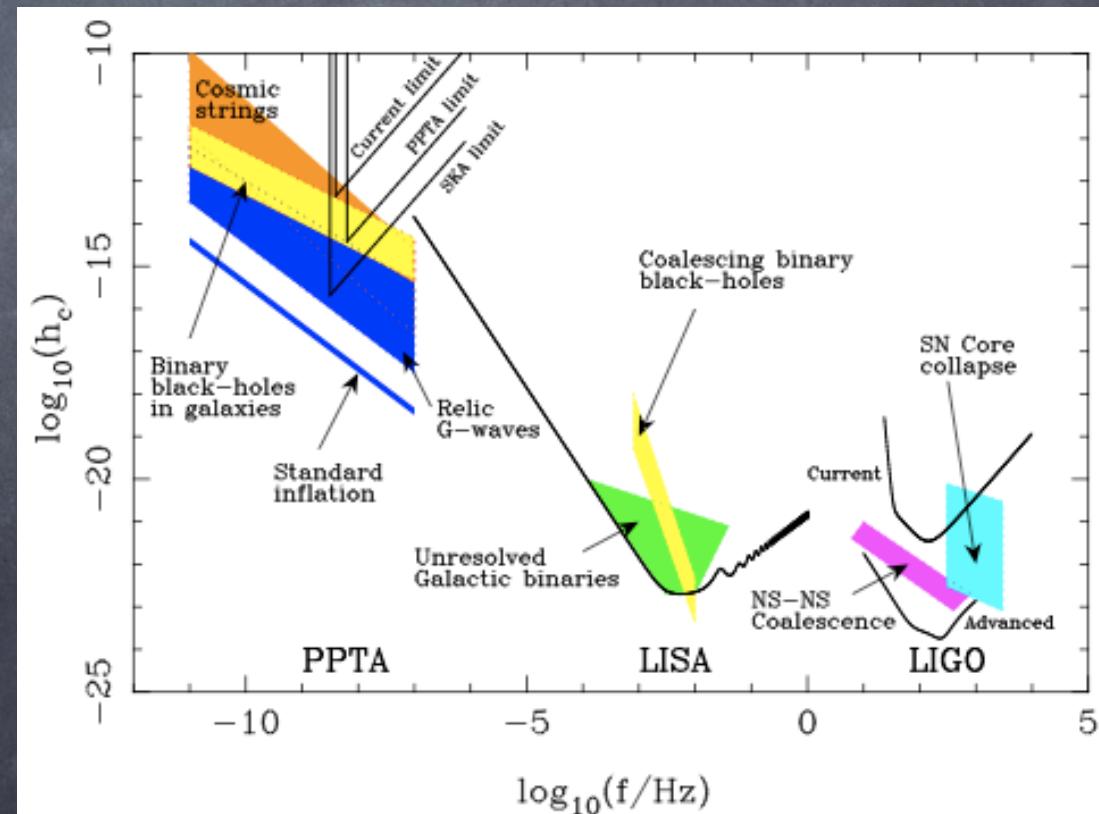
β^{-1} duration

$R \simeq v_b \beta^{-1}$ size at collision

v_b speed of the wall

$$k_* \simeq \beta, R^{-1}$$

$$k_* \simeq 10^{-2} \frac{\beta}{\mathcal{H}_*} \frac{T_*}{100 \text{ GeV}} \text{ mHz}$$



GW from phase transitions : amplitude

$$\Omega_{GW} \sim \Omega_{\text{rad}} \left(\frac{\mathcal{H}_*}{\beta} \right)^2 \left(\frac{\Omega_S^*}{\Omega_{\text{rad}}^*} \right)^2$$

duration of the
source with respect
to Hubble time

energy density of the
source with respect to
radiation energy density

example: turbulence

$$T_{ij} = (\rho + p)v_i v_j \quad \frac{\Omega_T^*}{\Omega_{\text{rad}}^*} = \frac{2}{3} \langle v^2 \rangle$$

$$\langle v^2 \rangle = \frac{1}{3}$$



Jouguet
detonation

$$\alpha = \frac{\rho_{\text{vac}}}{\rho_{\text{rad}}} = \frac{1}{3}$$

$$v_b = 0.87$$

To determine the GW signal :

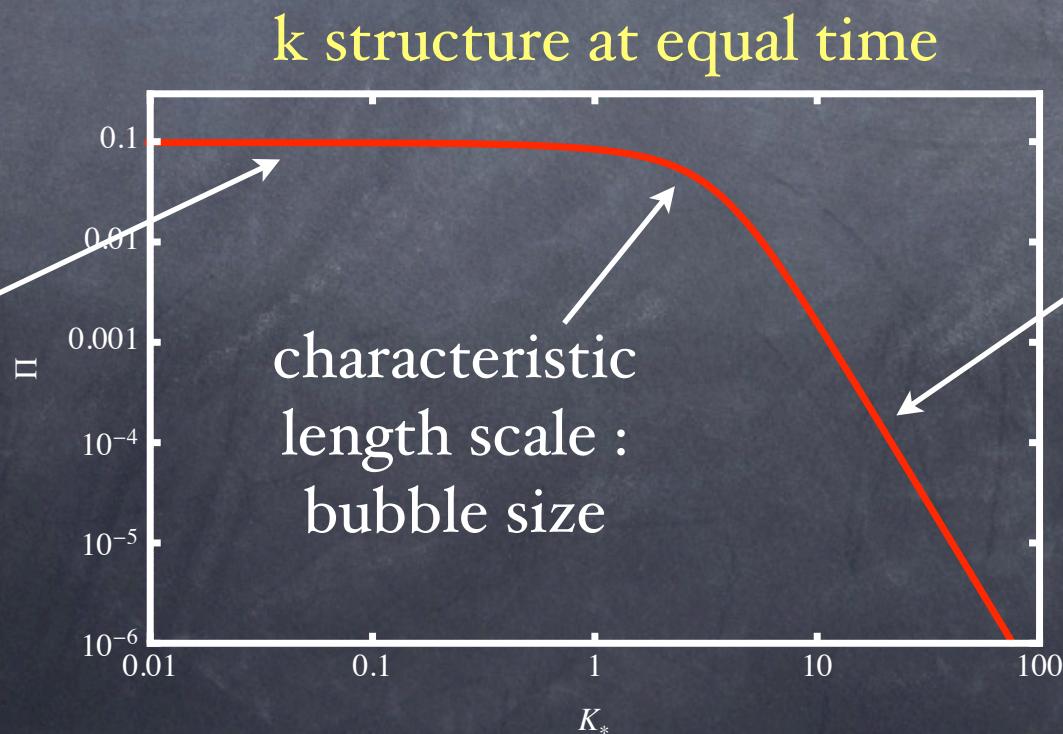
GW power spectrum

$$\frac{d\Omega_{GW}}{d \log k} \propto k^3 \int_{t_{in}}^{t_{fin}} \frac{dt_1}{t_1} \int_{t_{in}}^{t_{fin}} \frac{dt_2}{t_2} \cos[k(t_1 - t_2)] \Pi(k, t_1, t_2)$$

Anisotropic stress
power spectrum

$$\langle \Pi_{ij}(\mathbf{k}, t_1) \Pi_{ij}^*(\mathbf{q}, t_2) \rangle = \delta(\mathbf{k} - \mathbf{q}) \Pi(k, t_1, t_2)$$

flat: spatially
uncorrelated,
causality



slope
depending on
source power
spectrum :
 $k^{-11/3}$

Kolmogorov
turbulence

Time correlation of the anisotropic stress

BUBBLES :

- different collision events are uncorrelated
- single collision event is coherent

Completely coherent

$$\Pi(k, t_1, t_2) = \sqrt{\Pi(k, t_1)} \sqrt{\Pi(k, t_2)}$$

MHD

TURBULENCE :

- motions decorrelate with eddy turnover time
- decorrelation time depends on eddy size

Top hat decorrelation

correlated for $|t_1 - t_2| < \frac{1}{k}$

$$\Pi(k, t_1, t_2) = \{\Pi(k, t_1)\Theta[t_1 - t_2]\Theta[1 - k(t_1 - t_2)] + t_1 \leftrightarrow t_2\}$$

This affects the peak and the high frequency slope of the GW spectrum

General form of the GW power spectrum

peak position :

coherent source $k_* \simeq \beta$

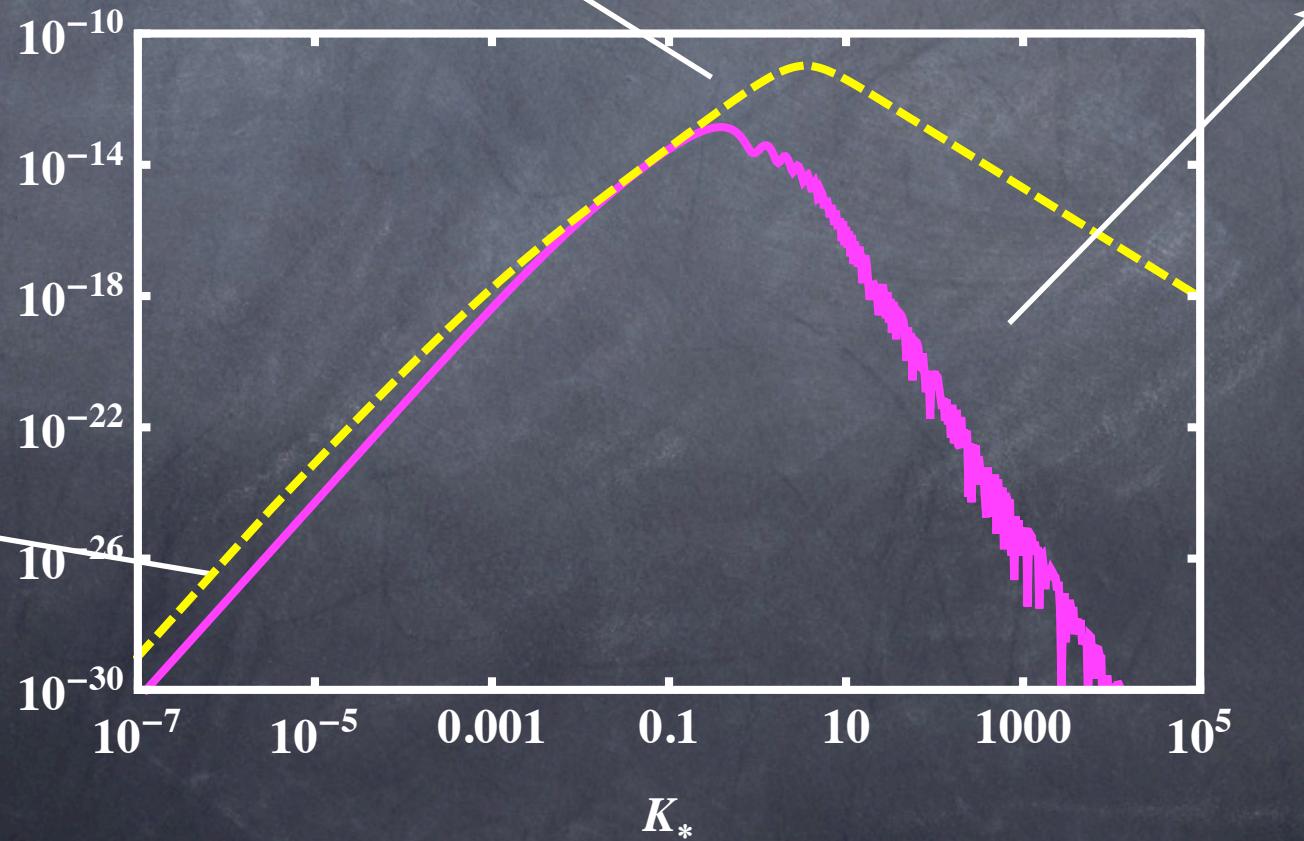
decorrelating source $k_* \simeq R^{-1}$

$$R \simeq v_b / \beta$$

high frequency tail : depends on both power spectrum and time correlation

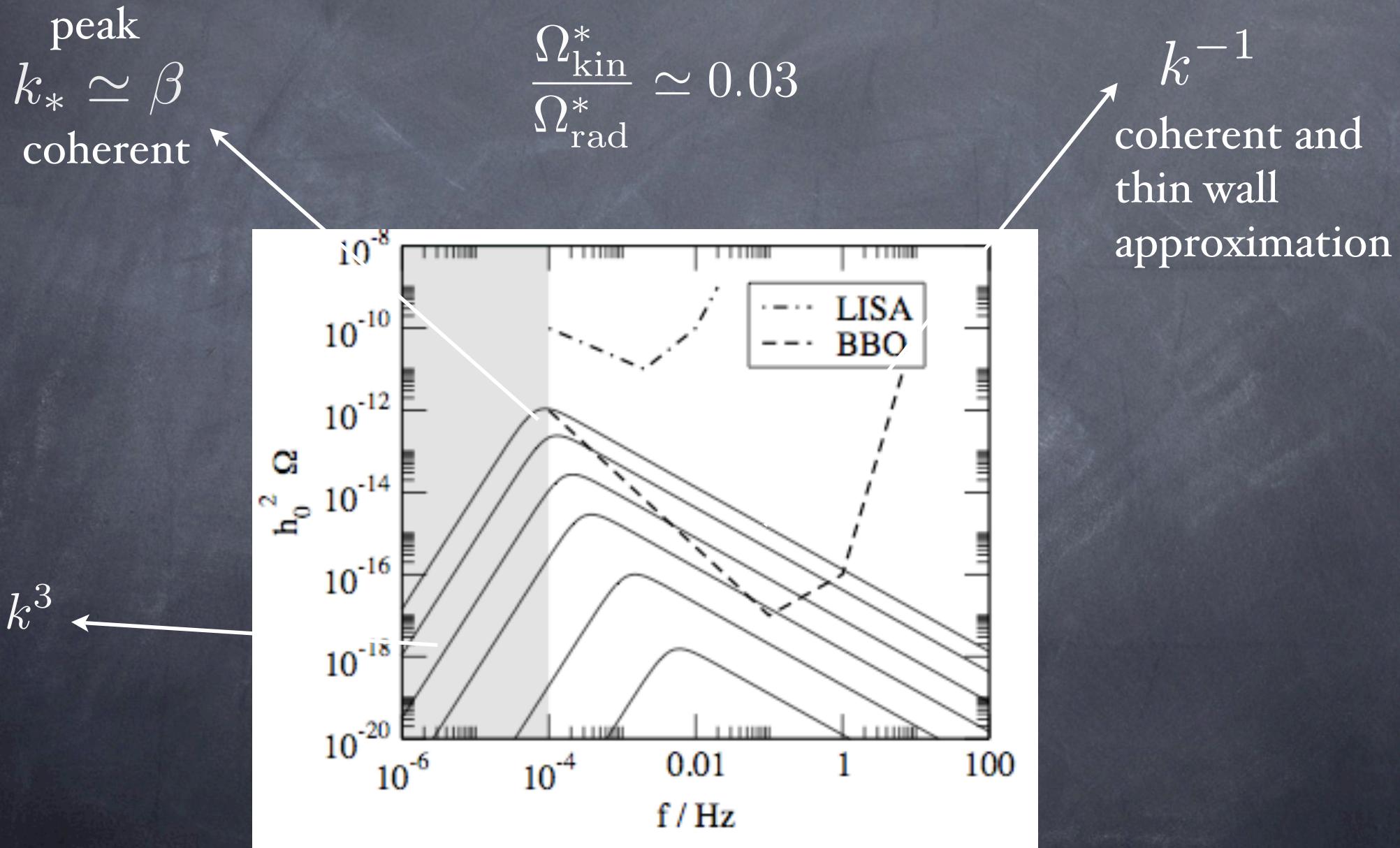
low frequency tail : causality of the source

$$k^3$$

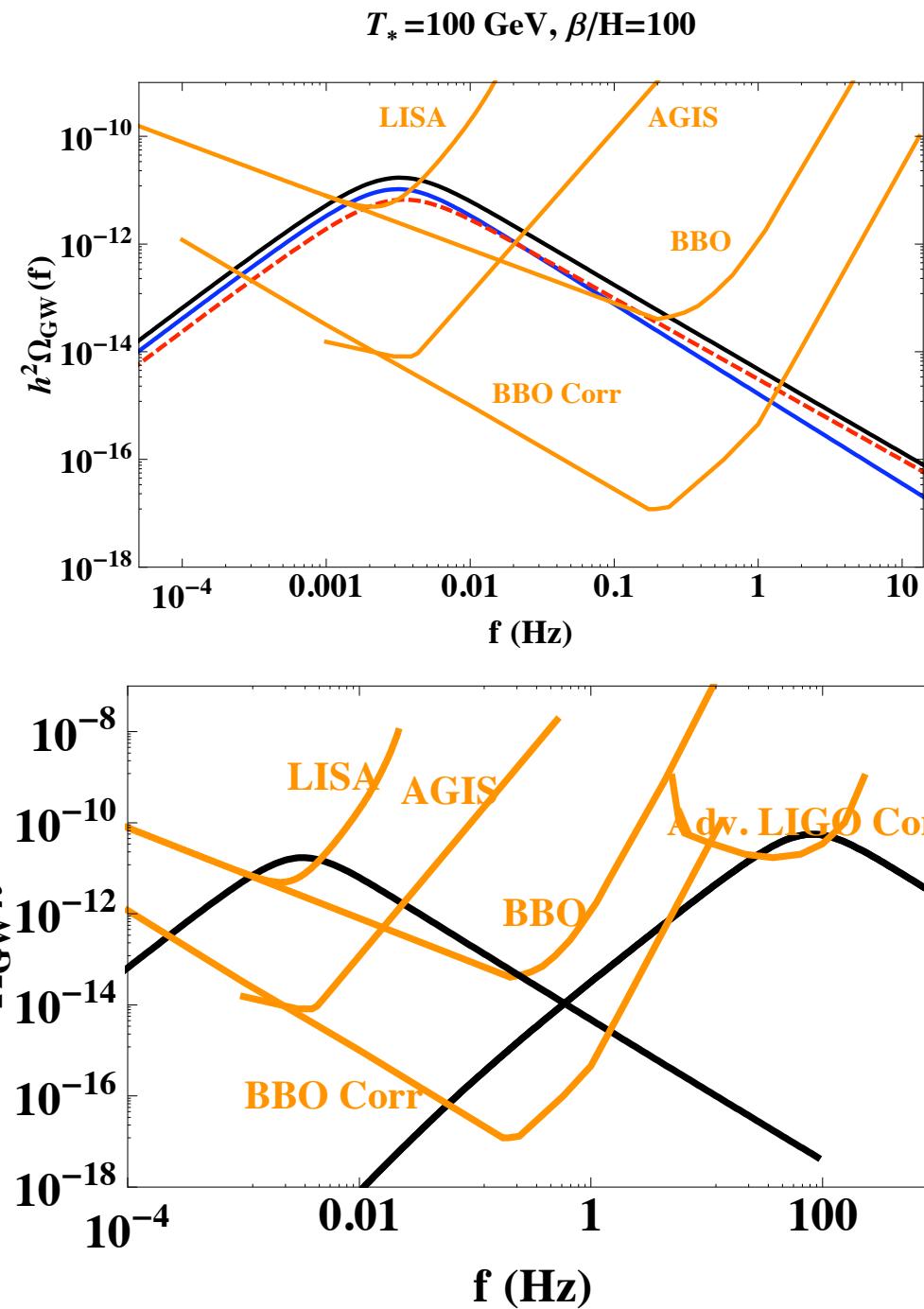


GW spectrum from bubbles

Simulations by Huber and Konstandin, o8o6.1828



GW spectrum from MHD turbulence



$$\frac{\Omega_{\text{MHD}}^*}{\Omega_{\text{rad}}^*} \simeq 0.2$$

peak $k_* \simeq R^{-1}$
(decorrelating source)

high frequency slope:

$k^{-5/3}$ Kolmogorov

$k^{-3/2}$ Iroshnikov Kraichnan

$$T_* = 5 \cdot 10^6 \text{ GeV}$$

$$\beta/\mathcal{H}_* = 50$$