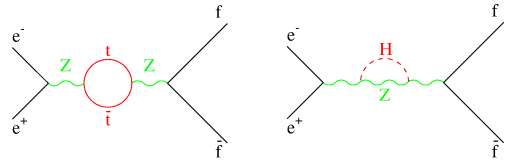
Direct measurement of $\alpha_{QED}(m_Z^2)$ @ FCC-ee

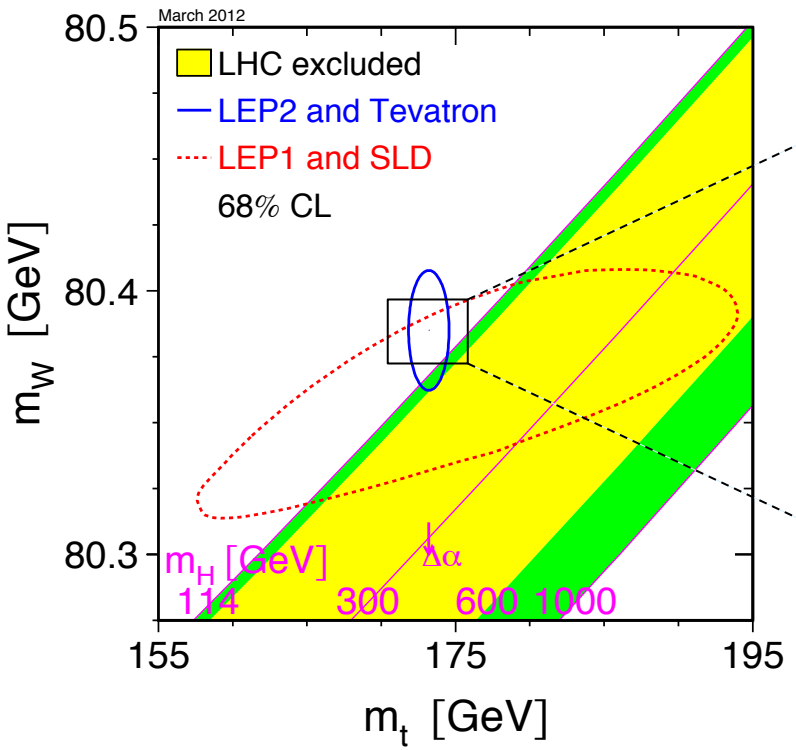
- **Outline**
  - Why measure $\alpha_{QED}(m_Z^2)$?
    - Physics behind precision
    - Impact of $\alpha_{QED}(m_Z^2)$ precision
  - Sensitivity of of $e^+e^- \rightarrow \mu^+\mu^-$ to $\alpha_{QED}(m_Z^2)$
    - Cross section
    - Forward-backward asymmetry
    - Statistical uncertainty and sensitivity
  - Determination of aQED(mZ2)
  - Systematic uncertainties
    - Theory
      - Higher orders
      - ISR, IFI
    - Parametric
    - Experimental
  - Conclusions

See arXiv:1512:05544
(To be published in JHEP)

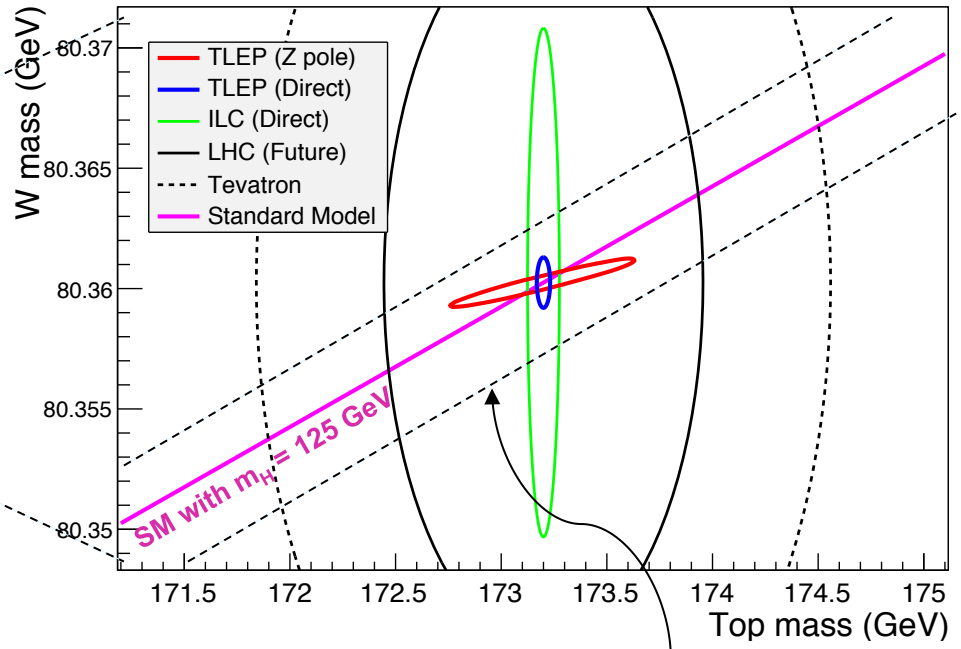
Physics behind precision

- **EWPO measurements allow a prediction of m_{top}, m_W, m_H, $\sin^2\theta_W$ in the SM**
  - **Compare with FCC-ee direct measurements**
    - **The standard model has nowhere to go**





If theory uncertainties match exp'tal uncertainties



Without m_Z@FCC-ee, the SM line would have a 2.2 MeV width

- **Sensitivity to new physics ?**

Impact of $\alpha_{QED}(m_Z^2)$ precision

- **Uncertainty on these predictions are of two origins**
  - **Parametric**
  - **Higher orders**
- **Today's examples (from R. Kogler, Moriond EW 2013)**

$$M_W = 80.3593 \pm 0.0056_{m_t} \pm 0.0026_{M_Z} \pm 0.0018_{\Delta\alpha_{\rm had}} \pm 0.0017_{\alpha_S} \pm 0.0002_{M_H} \pm 0.0040_{\rm theo} = 80.359 \pm 0.011_{\rm tot}$$

Exp: 0.015

$$\sin^2\theta^{\ell}_{\rm eff} = 0.231496 \pm 0.000030_{m_t} \pm 0.000015_{M_Z} \pm 0.000035_{\Delta\alpha_{\rm had}} \pm 0.000010_{\alpha_S} \pm 0.000002_{M_H} \pm 0.000047_{\rm theo} = 0.23150 \pm 0.00010_{\rm tot}$$

Exp: 0.00014

Impact of $\alpha_{QED}(m_Z^2)$ precision

- **Uncertainty on these predictions are of two origins**
  - **Parametric**
  - **Higher orders (QCD, EW, mixed)**
- **Reduced uncertainties at the FCC-ee**

FCC-ee

$$M_W = 80.3593 \pm 0.0001_{m_t} \pm 0.0001_{M_Z} \pm 0.0018_{\Delta\alpha_{\rm had}} \pm 0.0002_{\alpha_S} \pm 0.0000_{M_H} \pm 0.0040_{\rm theo} = 80.359 \pm 0.011_{\rm tot}$$

Exp: 0.0005

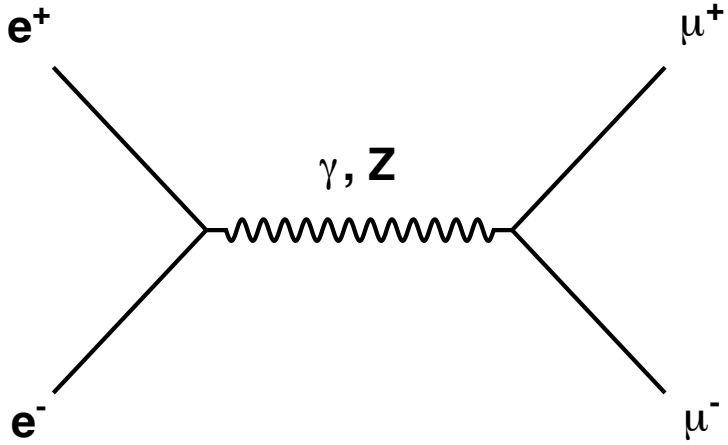
$$\sin^2\theta^{\ell}_{\rm eff} = 0.231496 \pm 0.000001_{m_t} \pm 0.000001_{M_Z} \pm 0.000035_{\Delta\alpha_{\rm had}} \pm 0.000001_{\alpha_S} \pm 0.000000_{M_H} \pm 0.000047_{\rm theo} = 0.23150 \pm 0.00010_{\rm tot}$$

Exp: 0.000006

- **Must reduce current uncertainty on $\alpha_{QED}(m_Z^2)$ by a factor ~4-5**
  - **New generation of theoretical calculations is necessary**

Sensitivity of $e^+e^- \to \mu^+\mu^-$ to $\alpha_{QED}(m_Z^2)$

- The $e^+e^- \to \mu^+\mu^-$ cross section



$$\mathcal{G} = \frac{c_\gamma^2}{s},$$

$$\mathcal{Z} = \frac{c_Z^2 (v^2 + a^2)^2 \times s}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2},$$

$$\mathcal{I} = \frac{2 c_\gamma c_Z v^2 \times (s - m_Z^2)}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2},$$

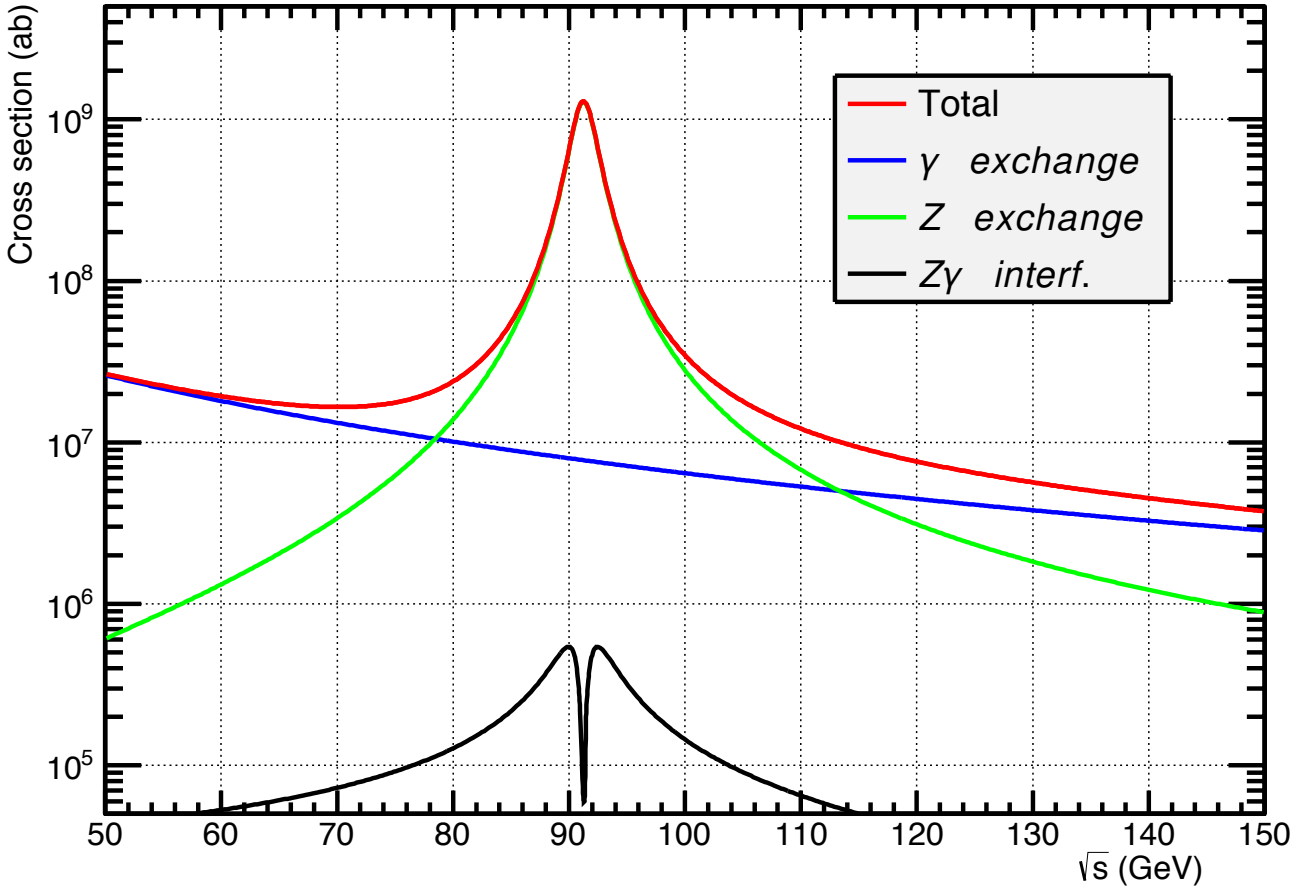
$$c_\gamma = \sqrt{\frac{4\pi}{3}} \alpha_{\mathrm{QED}}(s), \quad c_Z = \sqrt{\frac{4\pi}{3}} \frac{m_Z^2}{2\pi} \frac{G_F}{\sqrt{2}}, \quad a = -\frac{1}{2}, \quad v = a \times (1 - 4 \sin^2 \theta_W),$$

(~ −0.037)

- Photon exchange (G) proportional to $\alpha^2(s)$
- Z exchange (Z) proportional to G_F^2
- Interference term proportional to $\alpha(s)\, G_F$
  - Need to choose $\sqrt{s}$ judiciously to maximize sensitivity to $\alpha(s)$
  - If $\sqrt{s}$ is close to m_Z, the $\sqrt{s} \to m_Z$ extrapolation uncertainty is negligible

Sensitivity of $e^+e^- \rightarrow \mu^+\mu^-$ to $\alpha_{QED}(m_Z^2)$

- The $e^+e^- \rightarrow \mu^+\mu^-$ cross section



Sensitivity of $e^+e^- \to \mu^+\mu^-$ to $\alpha_{QED}(m_Z^2)$

- **The $e^+e^- \to \mu^+\mu^-$ angular distribution**
  - Absolute cross section measurement might be challenging to the required precision
    - Uncertainty of the integrated luminosity determination
  - Rely of a self-normalizing quantity, the forward-backward asymmetry $A_{\rm FB}^{\mu\mu} = \dfrac{\sigma_{\mu\mu}^{\rm F} - \sigma_{\mu\mu}^{\rm B}}{\sigma_{\mu\mu}^{\rm F} + \sigma_{\mu\mu}^{\rm B}},$

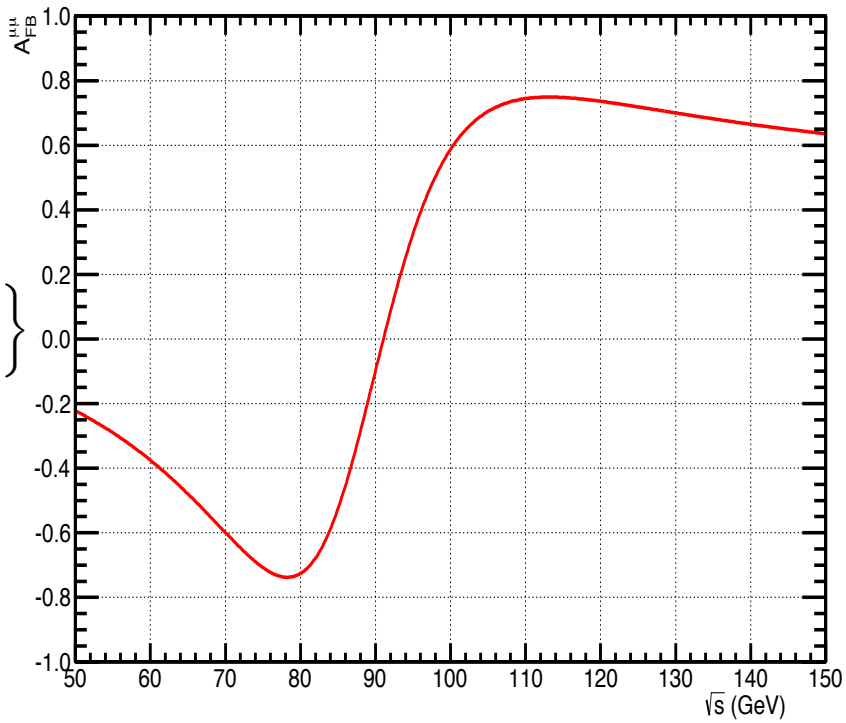
$$\frac{d\sigma_{\mu\mu}}{d\cos\theta}(s) \propto G_1(s) \times (1+\cos^2\theta) + G_3(s) \times 2\cos\theta,$$

$$A_{\rm FB}^{\mu\mu}(s) = \frac{3}{4}\frac{G_3(s)}{G_1(s)}.$$

$$G_1(s) = \mathcal{G} + \mathcal{I} + \mathcal{Z} \quad \text{and} \quad G_3(s) = \frac{a^2}{v^2}\left\{\mathcal{I} + \frac{4v^4/a^4}{(1+v^2/a^2)^2}\mathcal{Z}\right\}$$

$$\boxed{A_{\rm FB}^{\mu\mu} = A_{\rm FB,0}^{\mu\mu} + \frac{3}{4}\frac{a^2}{v^2}\frac{\mathcal{I}}{\mathcal{G}+\mathcal{Z}}.}$$

$$A_{\rm FB,0}^{\mu\mu} = (3/4) \times 4v^2a^2/(a^2+v^2)^2 \simeq 0.016.$$



Sensitivity of $e^+e^- \to \mu^+\mu^-$ to $\alpha_{QED}(m_Z^2)$

- **Variation of A_{FB} with α_{QED}**

$$A_{\rm FB}^{\mu\mu} = A_{\rm FB,0}^{\mu\mu} + \frac{3}{4}\frac{a^2}{v^2}\frac{\mathcal{I}}{\mathcal{G}+\mathcal{Z}}.$$

$dZ/d\alpha = 0$ **$dI/d\alpha = I/\alpha$** **$dG/d\alpha = 2G/\alpha$**

  - **For a small variation $\Delta\alpha$**

$$\Delta A_{\rm FB}^{\mu\mu} = \frac{\Delta\alpha}{\alpha} \times \frac{3}{4}\frac{a^2}{v^2}\frac{\mathcal{I}(\mathcal{Z}-\mathcal{G})}{(\mathcal{G}+\mathcal{Z})^2} = \left(A_{\rm FB}^{\mu\mu} - A_{\rm FB,0}^{\mu\mu}\right) \times \frac{\mathcal{Z}-\mathcal{G}}{\mathcal{Z}+\mathcal{G}} \times \frac{\Delta\alpha}{\alpha}.$$

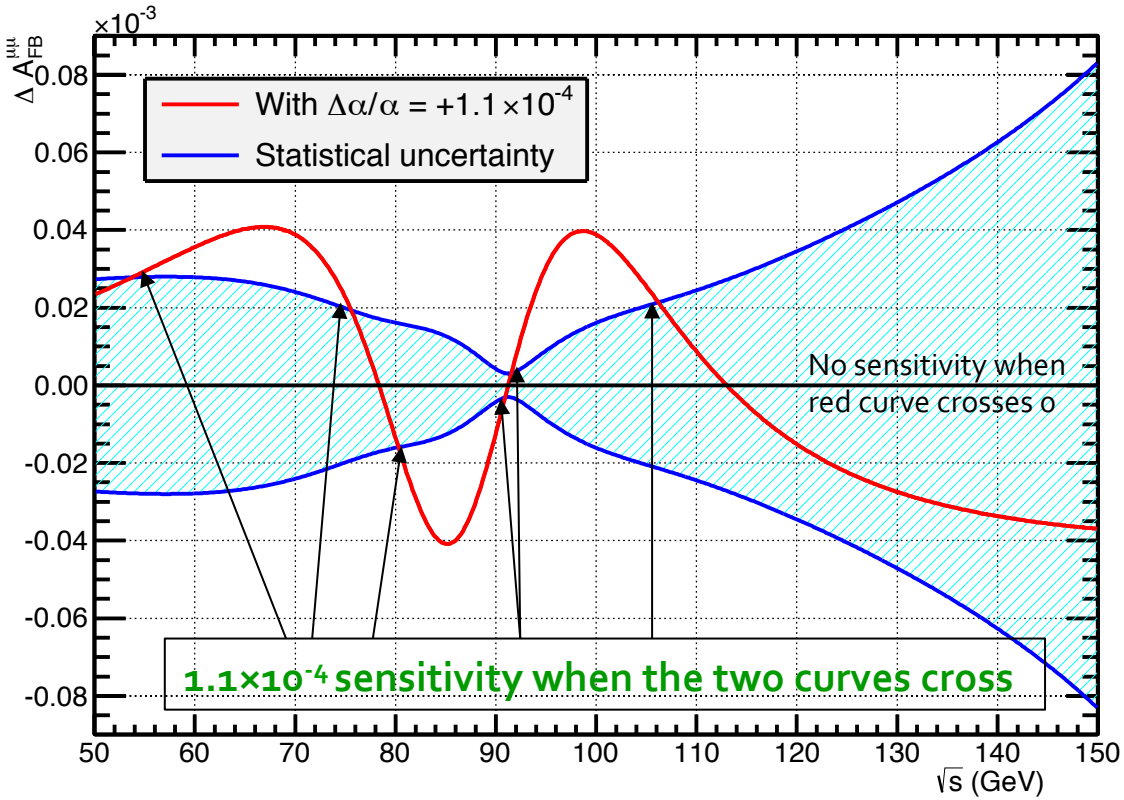
$$\frac{\Delta\alpha}{\alpha} = \frac{\Delta A_{\rm FB}^{\mu\mu}}{A_{\rm FB}^{\mu\mu} - A_{\rm FB,0}^{\mu\mu}} \times \frac{\mathcal{Z}+\mathcal{G}}{\mathcal{Z}-\mathcal{G}} \simeq \frac{\Delta A_{\rm FB}^{\mu\mu}}{A_{\rm FB}^{\mu\mu}} \times \frac{\mathcal{Z}+\mathcal{G}}{\mathcal{Z}-\mathcal{G}},$$

- **Statistical uncertainty on A_{FB}**

$$\sigma\left(A_{\rm FB}^{\mu\mu}\right) = \sqrt{\frac{1 - A_{\rm FB}^{\mu\mu\,2}}{\mathcal{L}\sigma_{\mu\mu}}}.$$

Sensitivity of $e^+e^- \rightarrow \mu^+\mu^-$ to $\alpha_{QED}(m_Z^2)$

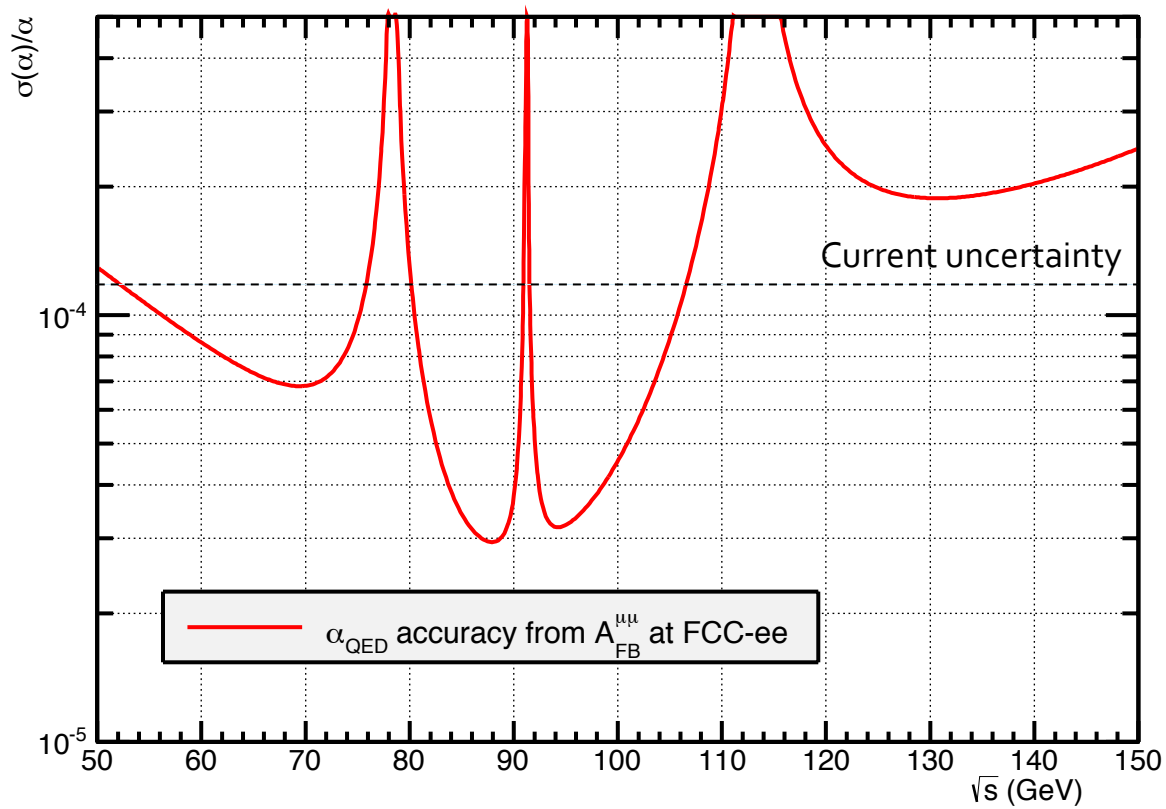
- **Comparison of the sensitivity and the statistical uncertainty**
  - Statistical uncertainty: one year with target luminosities (86 ab^{-1} at the Z pole)
    - Compared with ΔA_{FB} for current α_{QED} uncertainty



  - ΔA_{FB} changes sign for $\sqrt{s} = m_Z$ and when Z = G (78 and 112 GeV): no sensitivity to α_{QED}

Sensitivity of $e^+e^- \rightarrow \mu^+\mu^-$ to $\alpha_{QED}(m_Z^2)$

- **Turning the previous plot in a $\sigma(\alpha)/\alpha$ plot, for a year of running at any $\sqrt{s}$**



- **Optimal centre-of-mass energies for a 3×10^{-5} uncertainty on α_{QED}**
  - One year at $\sqrt{s_-}$ = 87.9 GeV or one year at $\sqrt{s_+}$ = 94.3 GeV
  - Even better: six months at $\sqrt{s_-}$ and six months at $\sqrt{s_+}$

Determination of $\alpha_{QED}(m_Z^2)$

- **Two measurements**
  - Two asymmetries at two $\sqrt{s}$: $A_{FB}(s_-)$ and $A_{FB}(s_+)$

$$\alpha_- \equiv \alpha_{\rm QED}(s_-) \text{ and } \alpha_+ \equiv \alpha_{\rm QED}(s_+),$$

- **Two equations**
  - Running from $\sqrt{s_-}$ to m_Z and running from $\sqrt{s_+}$ to m_Z to obtain $\alpha_0 = \alpha_{QED}(m_Z^2)$

$$\frac{1}{\alpha_0} = \frac{1}{\alpha_\pm} + \beta \log \frac{s_\pm}{m_{\rm Z}^2}$$

- **Solve for $\alpha_0 = \alpha_{QED}(m_Z^2)$**

$$\frac{1}{\alpha_0} = \frac{1}{2}\left(\frac{1-\xi}{\alpha_-} + \frac{1+\xi}{\alpha_+}\right), \quad \text{where} \quad \xi = \frac{\log s_- s_+/m_{\rm Z}^4}{\log s_-/s_+} \simeq 0.045,$$

$$\frac{\Delta\alpha_0}{\alpha_0} \simeq 0.528\frac{\Delta A_{\rm FB}}{A_{\rm FB}}(s_-) \oplus 0.563\frac{\Delta A_{\rm FB}}{A_{\rm FB}}(s_+)$$

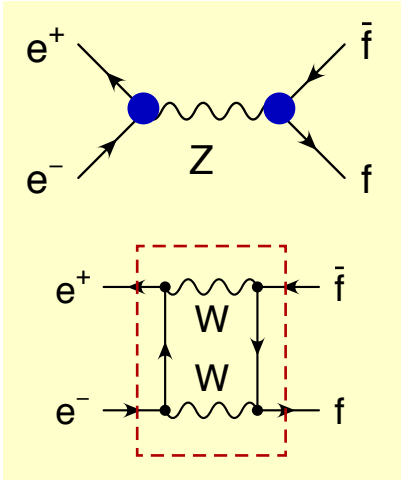
Systematic uncertainties

- **Summary of the study**

Type	Source	Uncertainty
Experimental	E_{beam} calibration	1×10^{-5}
	E_{beam} spread	$< 10^{-7}$
	Acceptance and efficiency	negl.
	Charge inversion	negl.
	Backgrounds	negl.
Parametric	m_{Z} and Γ_{Z}	1×10^{-6}
	$\sin^2\theta_{\text{W}}$	5×10^{-6}
	G_{F}	5×10^{-7}
Theoretical	QED (ISR, FSR, IFI)	$< 10^{-6}$
	Missing EW higher orders	few 10^{-4}
	New physics in the running	0.0
Total (except missing EW higher orders)	Systematics	1.2×10^{-5}
	Statistics	3×10^{-5}

Systematic uncertainties

- **Missing EW higher orders**
  - See presentation from Ayres Freitas
    - Current status: full one-loop calculation available
      - Box EW correction : 8-9 × 10^{-4}
      - Vertex EW correction : ~10^{-3}
      - Uncertainty : few 10^{-4}
    - Higher-order corrections computable with current techniques
      - At the level of a few 10^{-4}
      - Uncertainty : few 10^{-5}
    - New techniques might be needed for 3-loop corrections
      - At the level of a few 10^{-5}
      - Uncertainty : < 10^{-5}



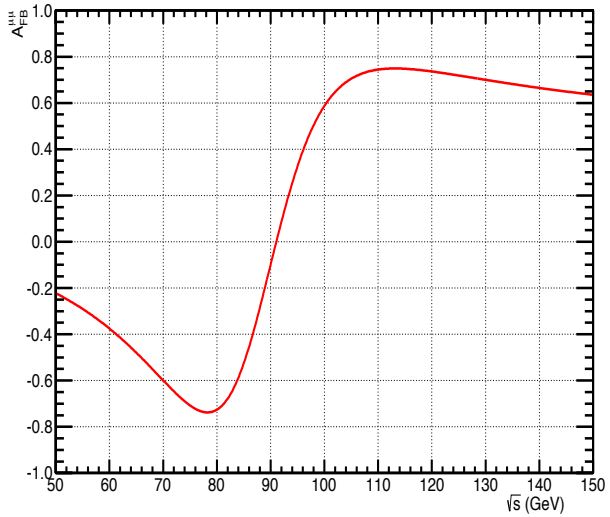
- **Significant precision improvement needed for all EWPO predictions**
  - Need to set up a consistent international effort now
    - To benefit from the experience of our experts
    - To train a new generation of theorists

Systematic uncertainties

- **Pure QED corrections (FSR, ISR, IFI)**
  - Initial state radiation (ISR) changes the muon angular distribution
    - Because of the longitudinal boost
    - Because of the reduction of $\sqrt{s}$ ($\Delta A_{FB} < 0$)
  - Both effect can be dealt with in case of one ISR photon

$$\cos\theta^* = \frac{\sin(\theta^+ - \theta^-)}{\sin\theta^+ + \sin\theta^-}. \quad \text{and} \quad \frac{s'}{s} = \frac{\sin\theta^+ + \sin\theta^- - |\sin(\theta^+ + \theta^-)|}{\sin\theta^+ + \sin\theta^- + |\sin(\theta^+ + \theta^-)|}$$

    - Measure both μ^+ and μ^- angles
    - Determine angular distribution in the c.m. frame
    - Measure asymmetry as a function of $\sqrt{s'}$
    - Compare to the prediction



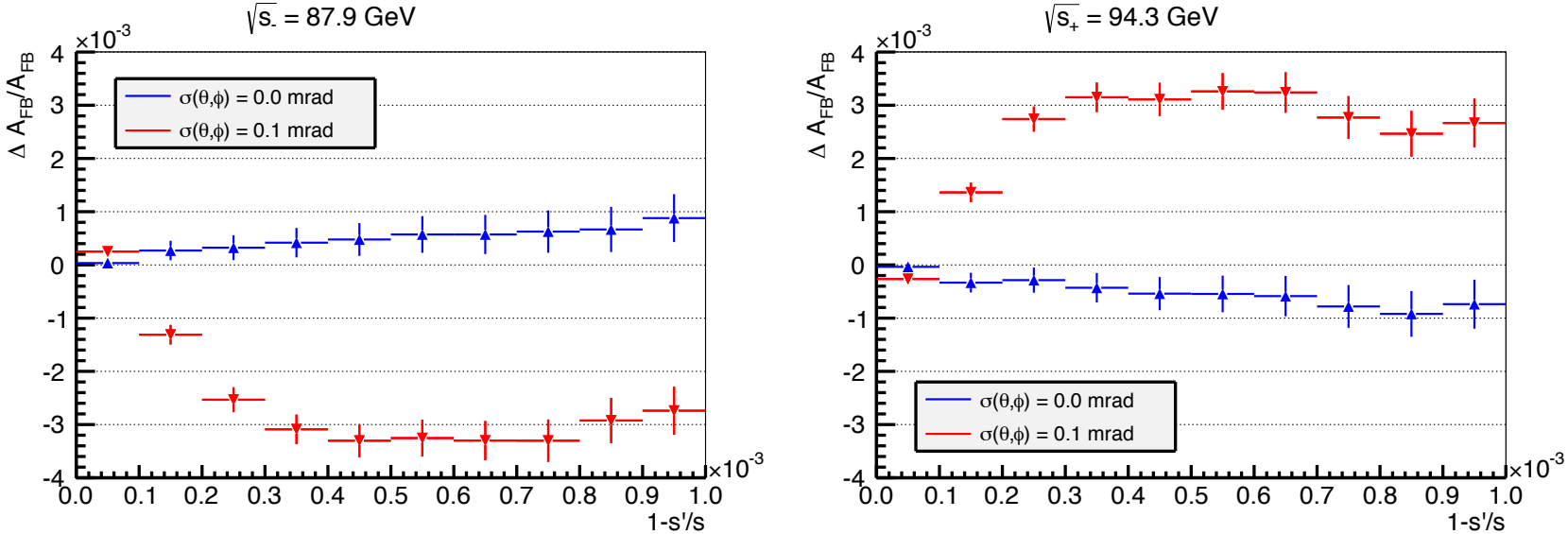
$\Delta A_{FB}/A_{FB}(s_-) > 0$
$\Delta A_{FB}/A_{FB}(s_+) < 0$

(But large effect and not exact cancellation)

  - s' and θ^* modified by
    - The possibility of radiating two photons
    - The angular resolution of the detector
      - ➨ Always reduces the (μ^+,μ^-) angle when back to back

Systematic uncertainties

- **Effect of double ISR and angular resolution**
  - Plot relative bias, $\Delta A_{FB}/A_{FB}$, as a function of $\sqrt{s'}$, for $s'/s > 0.999$ and acop < 0.35 mrad
    - In blue, perfect angular resolution
    - In red, 0.1 mrad angular resolution (consistent with SiD design)

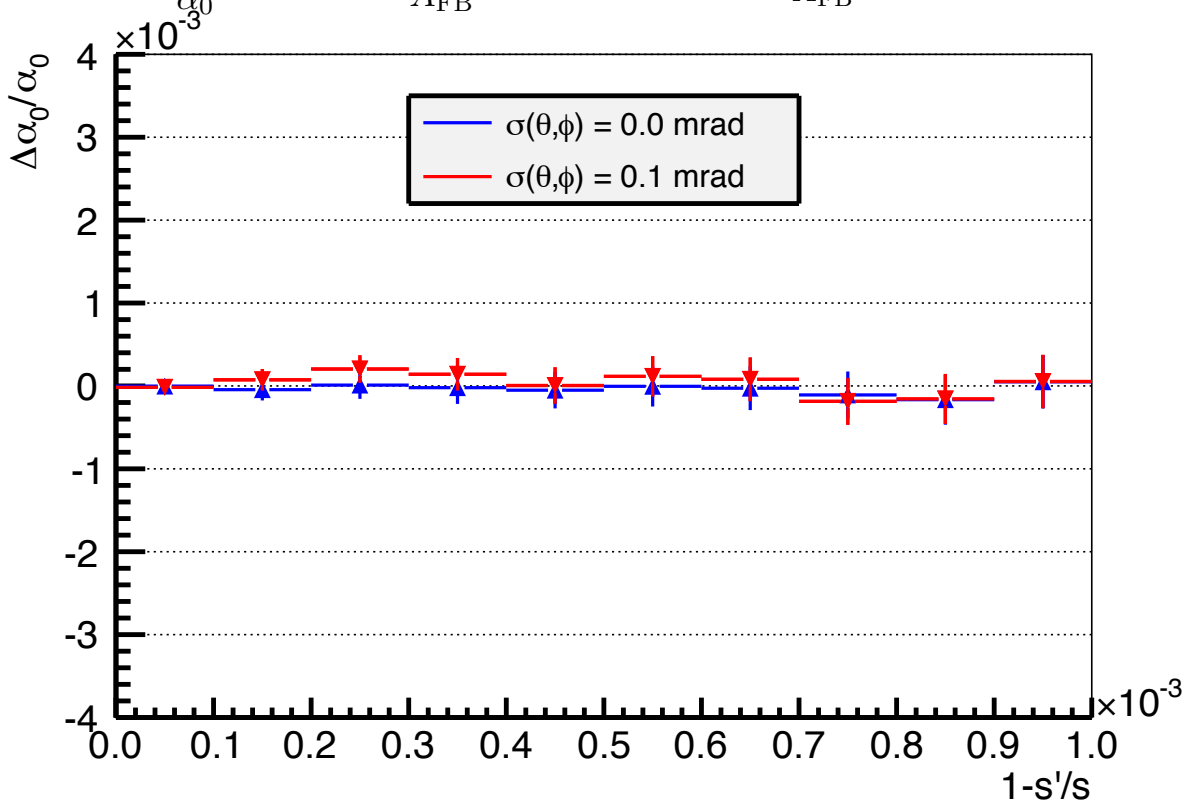


  - Large effect (several 10^{-3}, i.e., two orders of magnitude too large)
    - Effect on A_{FB} is "universal", i.e., ΔA_{FB} almost independent of $\sqrt{s}$
      - ➨ Relative biasses almost cancel when averaging of s_- and s_+

Systematic uncertainties

- **Effect of double ISR and angular resolution (cont'd)**

$$\frac{\Delta\alpha_0}{\alpha_0} \simeq 0.528\frac{\Delta A_{\rm FB}}{A_{\rm FB}}(s_-) + 0.563\frac{\Delta A_{\rm FB}}{A_{\rm FB}}(s_+)$$



- **Total bias on $\alpha_{QED}(m_Z^2)$ of the order of 8×10^{-6}**
  - **And is well predictable by QED soft photon prediction: negligible uncertainty**

Systematic uncertainties

- **Initial-Final State interference (see also Staszek's talk)**
  - Staszek's statements
    - IFI effect on A_{FB} increases when the cut on s'/s becomes tighter
      - ➨ ... up to 4% (compared to 0.4% for ISR?)
    - IFI cancellation between s_+ and s_- might not be perfect
      - ➨ ... but IFI is very well predicted in the soft photon region
  - Patrick's answers (1512.05544)
    - The cut on the acoplanarity angle to 0.35 mrad bound to reduce IFI effect on A_{FB}
      - ➨ It constrains the photon to be along the beam axis
      - ➨ Reduces de facto interference of ISR with FSR photons from outgoing muons (bound to be in the detector acceptance)
    - The muon angular distribution is also affected by IFI. Not only A_{FB} is.

$$\frac{\mathrm{d}\sigma_{\mu\mu}}{\mathrm{d}\cos\theta^*}(s') \propto \left\{1+\cos^2\theta^* + \frac{8}{3}A_{\mathrm{FB}}(s')\cos\theta^*\right\} \times \left\{1 + f\left(\frac{s'}{s}\right)\log\frac{1+\cos\theta^*}{1-\cos\theta^*}\right\},$$

      - ➨ Can be fitted away from the FCC-ee high statistics distribution
  - Patrick-Staszek combination
    - Last word will come from an analysis of Staszek events with IFI by Patrick

Systematic uncertainties

- **Parametric uncertainties: Effective Weinberg angle $\sin^2\theta_W$**
  - **Strong dependence of A_{FB} with $\sin^2\theta_W$:**

$$A_{\rm FB}^{\mu\mu}(s) = \frac{3v^2a^2}{(v^2+a^2)^2} + \frac{c_\gamma}{c_{\rm Z}}\frac{s-m_{\rm Z}^2}{2s}\frac{3a^2}{(v^2+a^2)^2}.$$

    - **Numerically**

$$\frac{\Delta A_{\rm FB}^{\mu\mu}}{A_{\rm FB}^{\mu\mu}}(s_-) \simeq -6.92\Delta\sin^2\theta_{\rm W} \quad \text{and} \quad \frac{\Delta A_{\rm FB}^{\mu\mu}}{A_{\rm FB}^{\mu\mu}}(s_+) \simeq +4.87\Delta\sin^2\theta_{\rm W}.$$

      - **Propagation to α_{QED}:**

$$\frac{\Delta\alpha_0}{\alpha_0} \simeq 0.528\frac{\Delta A_{\rm FB}}{A_{\rm FB}}(s_-) + 0.563\frac{\Delta A_{\rm FB}}{A_{\rm FB}}(s_+) \simeq -0.91\Delta\sin^2\theta_{\rm W}.$$

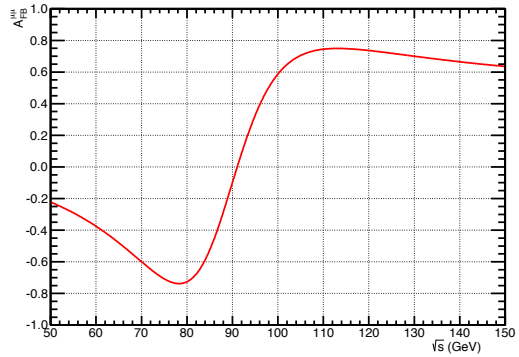
  - **Measurement of $\sin^2\theta_W$ with $A_{FB}(m_Z)$ at the FCC-ee**
    - **$\Delta\sin^2\theta_W = 6 \times 10^{-6}$, error dominated by the uncertainty on $\sqrt{s}$**
      - **Paper in preparation by Alain Blondel**

$$\boxed{\frac{\Delta\alpha_0}{\alpha_0} \simeq \quad 5\times10^{-6}.}$$

Systematic uncertainties

- **Experimental uncertainties: Centre-of-mass energy**
  - **Strong dependence of A_{FB} with $\sqrt{s}$ (and m_Z)**

$$A_{\mathrm{FB}}^{\mu\mu}(s, m_{\mathrm{Z}}) \propto (s - m_{\mathrm{Z}}^2)/(s m_{\mathrm{Z}}^2).$$



  - **Uncertainty of the beam energy measurements**
    - **Total ~ 50 keV**
      - **of which 45 keV correlated and 23 keV uncorrelated between energy points**
    - **Uncorrelated variables**
      - $D = \sqrt{s} - m_{\mathrm{Z}}$ and $\Sigma = (\sqrt{s} + m_{\mathrm{Z}})/2$
      - $\sigma_D = 46\,\mathrm{keV}$ and $\sigma_\Sigma = 94\,\mathrm{keV}$
  - **Propagation to A_{FB}**

$$\frac{\sigma(A_{\mathrm{FB}}^{\mu\mu})}{A_{\mathrm{FB}}^{\mu\mu}} \simeq \frac{1}{\sqrt{s} m_{\mathrm{Z}}} \sqrt{\left(s + m_{\mathrm{Z}}^2 - \sqrt{s} m_{\mathrm{Z}}\right)^2 \frac{\sigma_D^2}{D^2} + \cancel{\left(s + m_{\mathrm{Z}}^2 + \sqrt{s} m_{\mathrm{Z}}\right)^2 \frac{\sigma_\Sigma^2}{\Sigma^2}}},$$

  - **Propagation to α_{QED} :** $\frac{\sigma(\alpha_\pm)}{\alpha_\pm} \simeq \frac{\sigma_{D_\pm}}{D_\pm}, \quad \text{with} \quad D_\pm = \sqrt{s_\pm} - m_{\mathrm{Z}},$ **(uncorrelated)**

$$\boxed{\frac{\sigma(\alpha_0)}{\alpha_0} \simeq \frac{1}{2} \sqrt{(1-\xi)^2 \frac{\sigma^2(\alpha_-)}{\alpha_-^2} + (1+\xi)^2 \frac{\sigma^2(\alpha_+)}{\alpha_+^2}} \simeq 1 \times 10^{-5}}$$

Systematic uncertainties

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	$\sin^2\theta_{\text{W}}$	5×10^{-6}
	G_{F}	5×10^{-7}
(IFI to be numerically checked) Theoretical	QED (ISR, FSR, IFI)	$< 10^{-6}$
	Missing EW higher orders	few 10^{-4}
	New physics in the running	0.0
Total (except missing EW higher orders)	Systematics	1.2×10^{-5}
	Statistics	3×10^{-5}

Conclusions

- **The FCC-ee can deliver a direct measurement of $\alpha_{QED}(m_Z^2)$ to 3×10^{-5}**
  - With an accuracy 4-5 times smaller than current measurements of $\Delta\alpha_{had}$
    - Needed to exploit EWPO measurements at <u>ANY</u> e^+e^- collider

- **The FCC-ee is the only e^+e^- collider able to do so (because it is circular)**
  - First key breakthrough: The large integrated luminosities
    - Target luminosities with 4 IP's allow this measurement to be done in a year
  - Second key breakthrough: The ultra-precise measurement of the beam energy
    - Two orders of magnitude better than at linear colliders
      - ➨ Still the dominant experimental systematic uncertainty at the FCC-ee

- **The only obstacle today (beyond building the collider) is THEORY**
  - Electroweak corrections to A_{FB} need to be computed to higher orders
    - Today, full one-loop calculation available, uncertainty ~ few 10^{-4}
    - Will need full two- and maybe full three-loop calculations
      - ➨ Some of these calculations can be done with today's techniques, some not.
  - A consistent international effort must be setup for all EWPO predictions.