$\alpha_{\rm QED}(M_Z)$ and future prospects with low energy e^+e^- collider data

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Outline of Talk:

- Motivation
- The Role of $\alpha_{\rm QED,eff}$ in Precision Physics
- Evaluation of $\alpha_{\rm QED}(M_Z^2)$
- Reducing uncertainties via the Euclidean split trick:
  Adler function controlled pQCD
- Prospects for future improvements
- Addendum 1: Using τ–decay spectra + isospin breakings
- Addendum 2: The coupling α_2, M_W and $\sin^2\Theta_f$

1. Motivation

The Parameters of the Standard Model

– in four fermion and vector boson processes –

in addition QCD coupling α_s, y_t vs. M_t, λ_H vs. M_H etc.
unlike in QED and QCD in SM (SBGT)
parameter interdependence

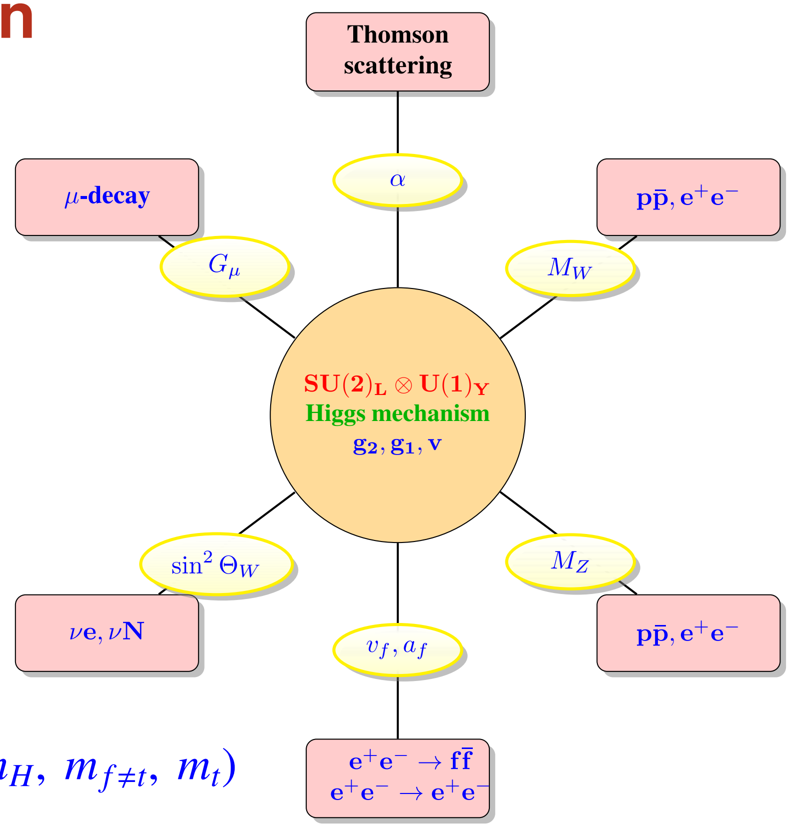
⇛

only 3 independent quantities
(besides fermion masses and mixing parameters)

α, G_μ, M_Z $\Rightarrow$ $\alpha_{\rm eff}(M_Z^2)$ $\Rightarrow$**large hadronic correction**

$\Downarrow$

$$\sin^2\Theta_i\,\cos^2\Theta_i = \frac{\pi\alpha}{\sqrt{2}\,G_\mu\,M_Z^2}\frac{1}{1-\Delta r_i}\quad;\quad \Delta r_i = \Delta r_i(\alpha,\ G_\mu,\ M_Z, m_H,\ m_{f\neq t},\ m_t)$$



parameter relationships between very precisely measurable quantities
precision tests, possible sign of new physics
non-perturbative $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ is limiting precision predictions

Note: 30 SD disagreement between SM prediction and experiment when subleading correctins are dropped!

Gauge coupling unification?

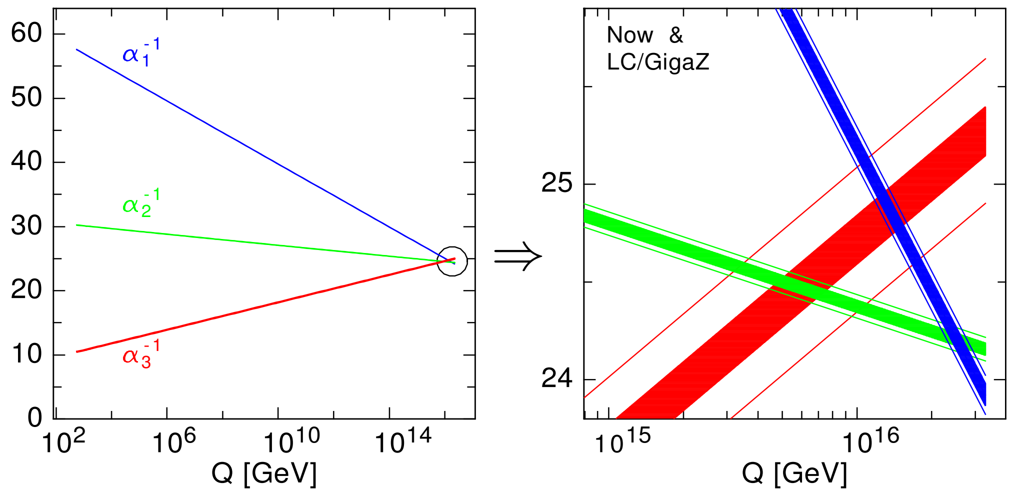
Precise SM predictions require to determine the $U(1)_Y \otimes SU(2)_L \otimes SU(3)_c$ SM gauge couplings $\alpha_{\rm em}$, α_2 and $\alpha_s \equiv \alpha_3$ (QCD) as accurately as possible

**** a theory can not be better than its input parameters ****

$\Rightarrow$ precision limitations due to non-perturbative hadronic contributions $\Leftarrow$

❖beyond SM physics?

from Zerwas

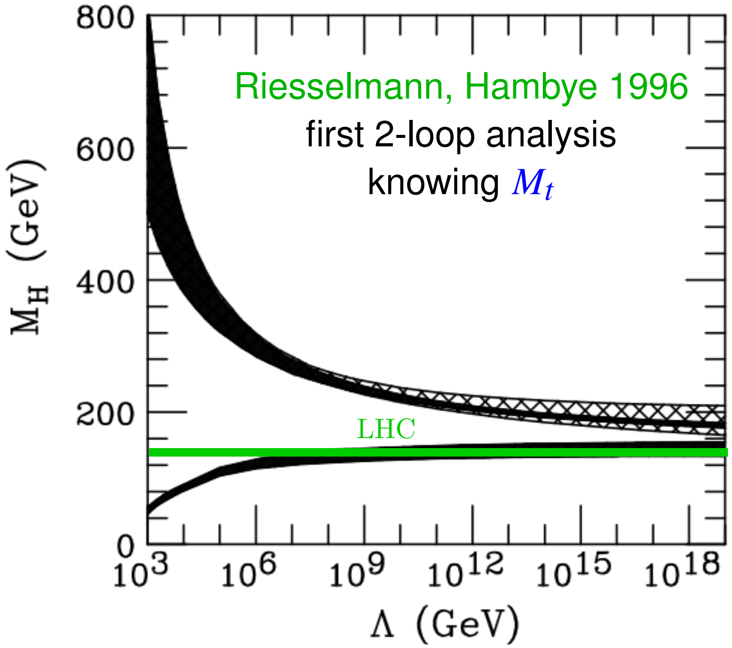


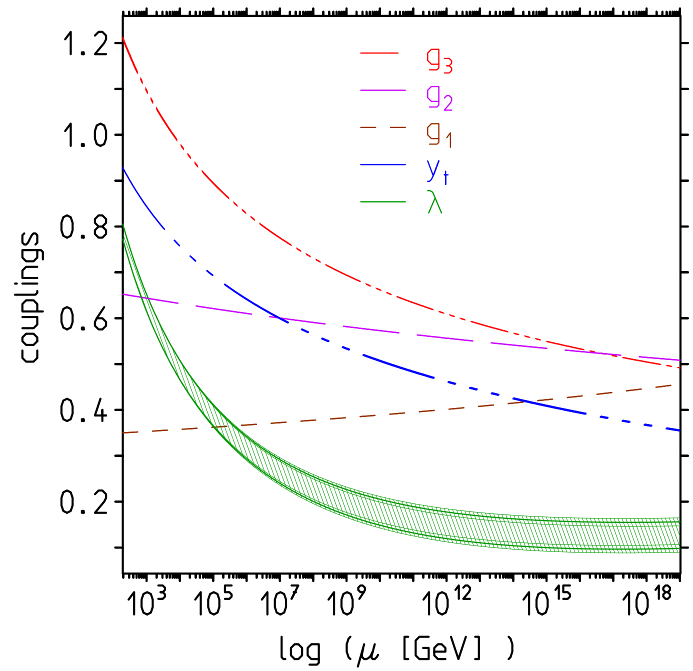
$\alpha_s = 0.1183 \pm 0.0027$ vs ± 0.0009

SM extrapolation up to Planck scale?

After Higgs discovery: Higgs vacuum stability issue!

⇒Need very precise SM parameters: g', g, g_s, y_t, λ





The SM dimensionless couplings in the $\overline{\mathrm{MS}}$ scheme as a function of the renormalization scale for $M_H = 124 - 127$ GeV.

- perturbation expansion works up to the Planck scale!
  no Landau pole or other singularities, Higgs potential likely remains stable!

- $U(1)_Y$ screening (IR free), $SU(2)_L$, $SU(3)_c$ antiscreening (UV free): g_1, g_2, g_3

as expected (standard wisdom)

- Top Yukawa y_t and Higgs λ: screening if standalone (IR free, like QED)

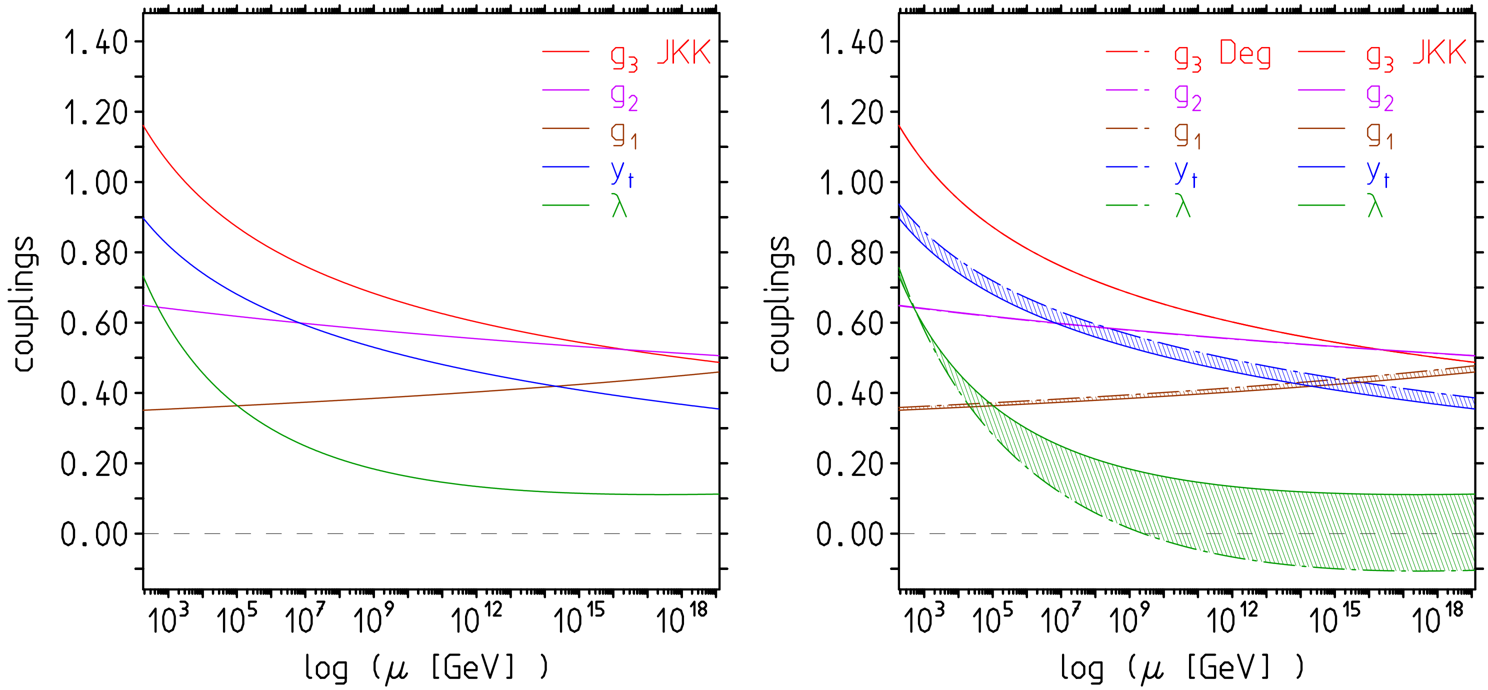
as part of SM, transmutation from IR free to UV free

As SM couplings are as they are: QCD dominance in top Yukawa RG requires $g_3 > \frac{3}{4} y_t$, top Yukawa dominance in Higgs RG requires $\lambda < \frac{3(\sqrt{5}-1)}{2} y_t^2$ in the gaugeless ($g_1, g_2 = 0$) limit.

In the focus:

- does Higgs self-coupling stay positive $\lambda > 0$ up to $\Lambda_{\rm Pl}$?
- the key question/problem concerns the size of the top Yukawa coupling y_t
  decides about stability of our world! — [$\lambda = 0$ would be essential singularity!]

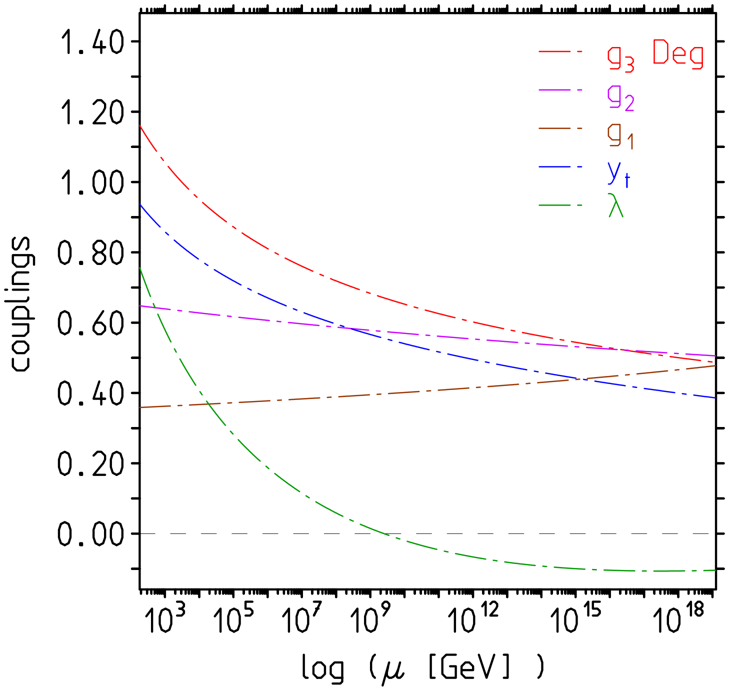
Will be decided by:
- more precise input parameters
- better established EW matching conditions

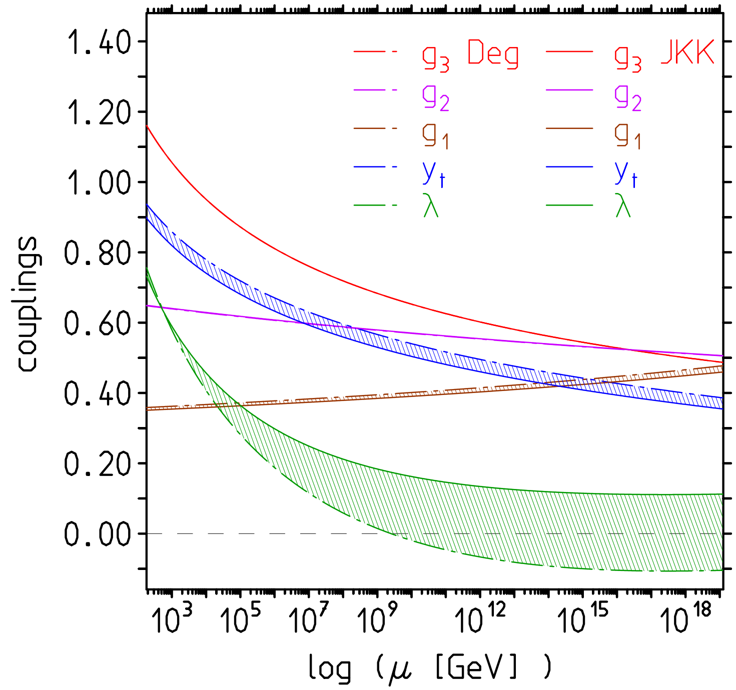


JKK On-Shell vs $\overline{\mathrm{MS}}$ parameter matching

❖ the big issue is the very delicate conspiracy between SM couplings: precision determination of parameters more important than ever $\Rightarrow$ the challenge for LHC and ILC/FCC: precision values for λ, y_t and α_s, and for low energy hadron facilities: more precise hadronic cross sections to reduce hadronic uncertainties in $\alpha(M_Z)$ and $\alpha_2(M_Z)$

New gate to precision cosmology of the early universe!





Shaposnikov et al., Degrassi et al. matching

- the big issue is the very delicate conspiracy between SM couplings: precision determination of parameters more important than ever ⇒ the challenge for LHC and ILC/FCC: precision values for λ, y_t and α_s, and for low energy hadron facilities: more precise hadronic cross sections to reduce hadronic uncertainties in $\alpha(M_Z)$ and $\alpha_2(M_Z)$

New gate to precision cosmology of the early universe!

2. $\alpha(M_Z^2)$ in precision physics (precision physics limitations)

Uncertainties of hadronic contributions to effective α are a problem for electroweak precision physics:

α , G_μ, M_Z **most precise input parameters** $\Rightarrow$ **precision predictions** $\sin^2\Theta_f, v_f, a_f, M_W, \Gamma_Z, \Gamma_W, \cdots$

50% non-perturbative

$\alpha(M_Z), G_\mu, M_Z$ **best effective input parameters for VB physics (Z,W) etc.**

$$\frac{\delta\alpha}{\alpha} \sim 3.6 \times 10^{-9}$$
$$\frac{\delta G_\mu}{G_\mu} \sim 8.6 \times 10^{-6}$$
$$\frac{\delta M_Z}{M_Z} \sim 2.4 \times 10^{-5}$$
$$\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} \sim 0.9 \div 1.6 \times 10^{-4} \quad \text{(present : lost } 10^5 \text{ in precision!)}$$
$$\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} \sim 5.3 \times 10^{-5} \quad \text{(ILC requirement)}$$

LEP/SLD: $\sin^2\Theta_{\rm eff} = (1 - g_{Vl}/g_{Al})/4 = 0.23148 \pm 0.00017$

$$\delta\Delta\alpha(M_Z) = 0.00020 \quad \Rightarrow \quad \delta\sin^2\Theta_{\rm eff} = 0.00007$$

affects Higgs mass bounds, precision tests and new physics searches!!!

For pQCD contributions very crucial: precise QCD parameters $\alpha_s,\ m_c,\ m_b,\ m_t \Rightarrow$ Lattice-QCD

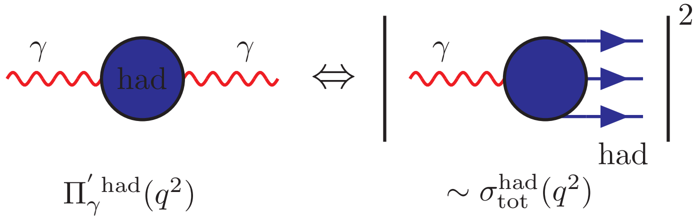
3. Evaluation of $\alpha(M_Z^2)$

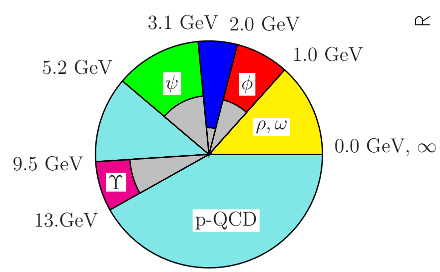
Non-perturbative hadronic contributions $\Delta\alpha^{(5)}_{\rm had}(s) = -\left(\Pi'_\gamma(s) - \Pi'_\gamma(0)\right)$ **can be evaluated in terms of** $\sigma(e^+e^- \to {\rm hadrons})$ **data via dispersion integral:**

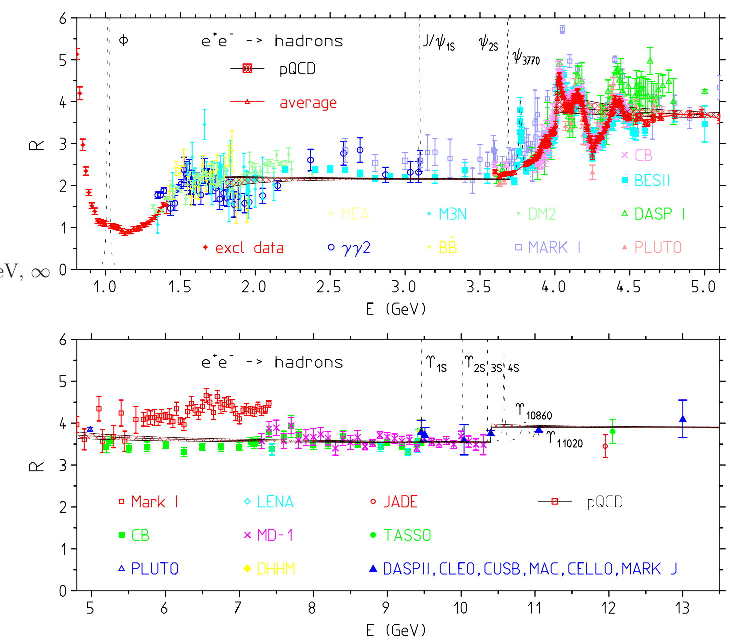
$$\Delta\alpha^{(5)}_{\rm had}(s) = -\frac{\alpha s}{3\pi}\left(\;\oint\limits_{4m_\pi^2}^{E^2_{\rm cut}} ds' \frac{R^{\rm data}_\gamma(s')}{s'(s'-s)} + \oint\limits_{E^2_{\rm cut}}^{\infty} ds' \frac{R^{\rm pQCD}_\gamma(s')}{s'(s'-s)} \right)$$

Compilation: FJ 15
Theory = pQCD: Gorishny et al. 91, Chetyrkin et al. 97...09

where $R_\gamma(s) \equiv \frac{\sigma^{(0)}(e^+e^-\to\gamma^*\to{\rm hadrons})}{\frac{4\pi\alpha^2}{3s}}$





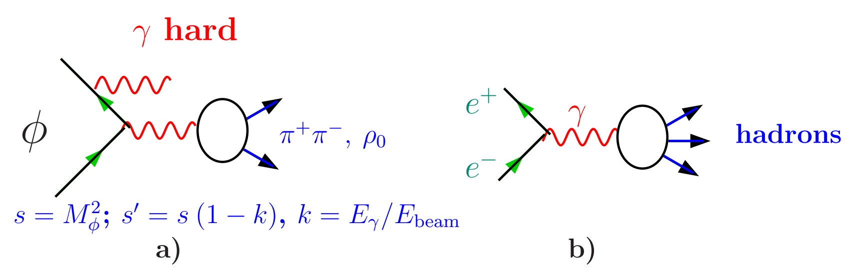


hadronic vacuum polarization

$$\alpha(s) = \frac{\alpha}{1-\Delta\alpha(s)}\;;\quad \Delta\alpha(s) = \Delta\alpha_{\rm lep}(s) + \Delta\alpha^{(5)}_{\rm had}(s) + \Delta\alpha_{\rm top}(s)$$

Present situation: (after KLOE, BaBar and first BESIII results)

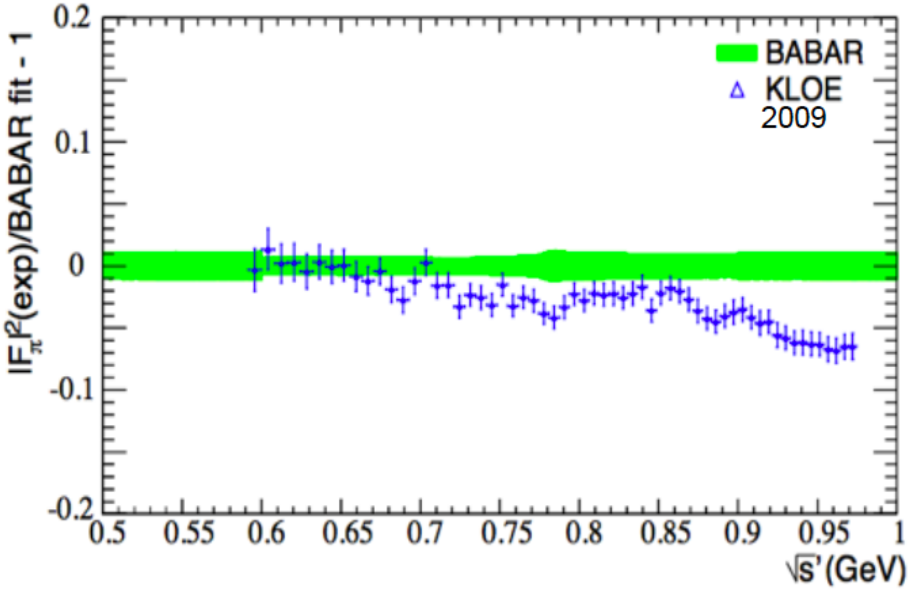
$\Delta\alpha^{(5)}_{\rm hadrons}(M_Z^2)$	$=$	0.027648 ± 0.000176	
		0.027504 ± 0.000118	Adler
$\alpha^{-1}(M_Z^2)$	$=$	128.942 ± 0.021	
		128.961 ± 0.011	Adler

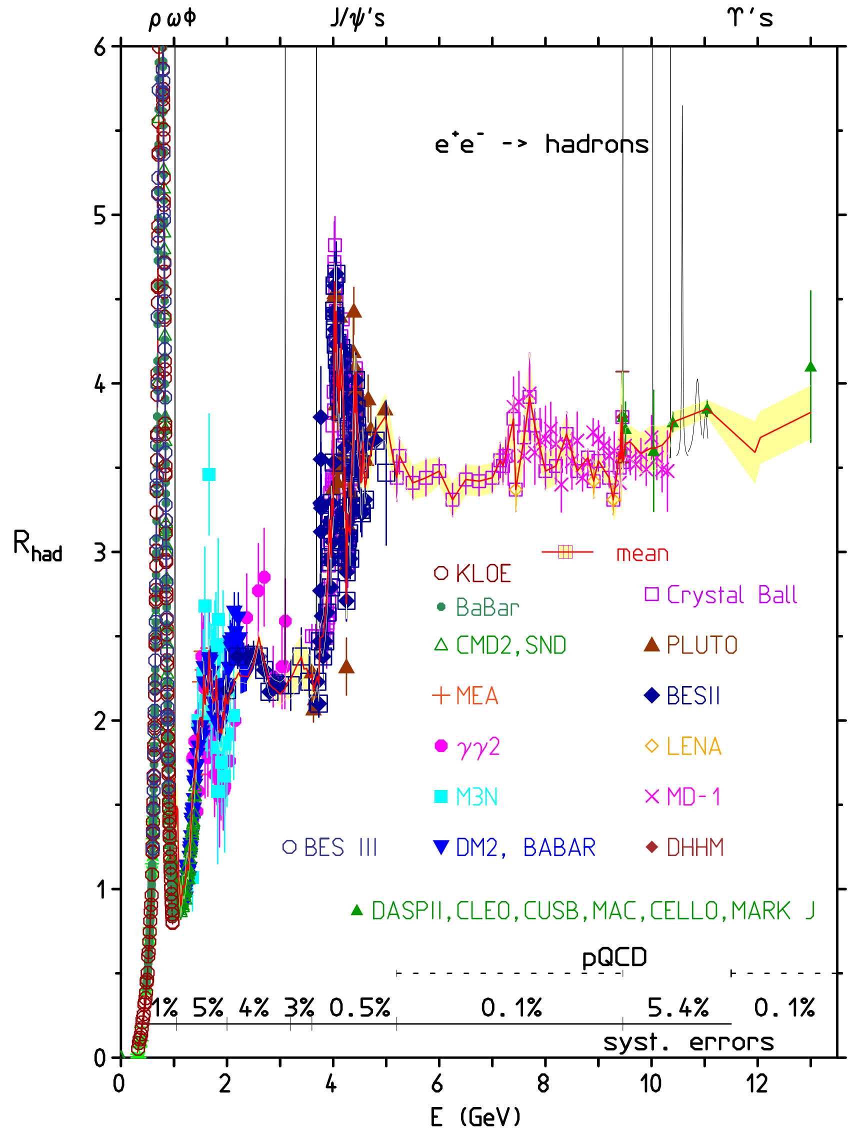


a) Initial state radiation (ISR), b) Standard energy scan.

SCAN: CMD-2, SND (NSK); ISR: KLEO, BaBar, BESIII

New experimental input for HVP: BESIII-ISR,VEPP-2000



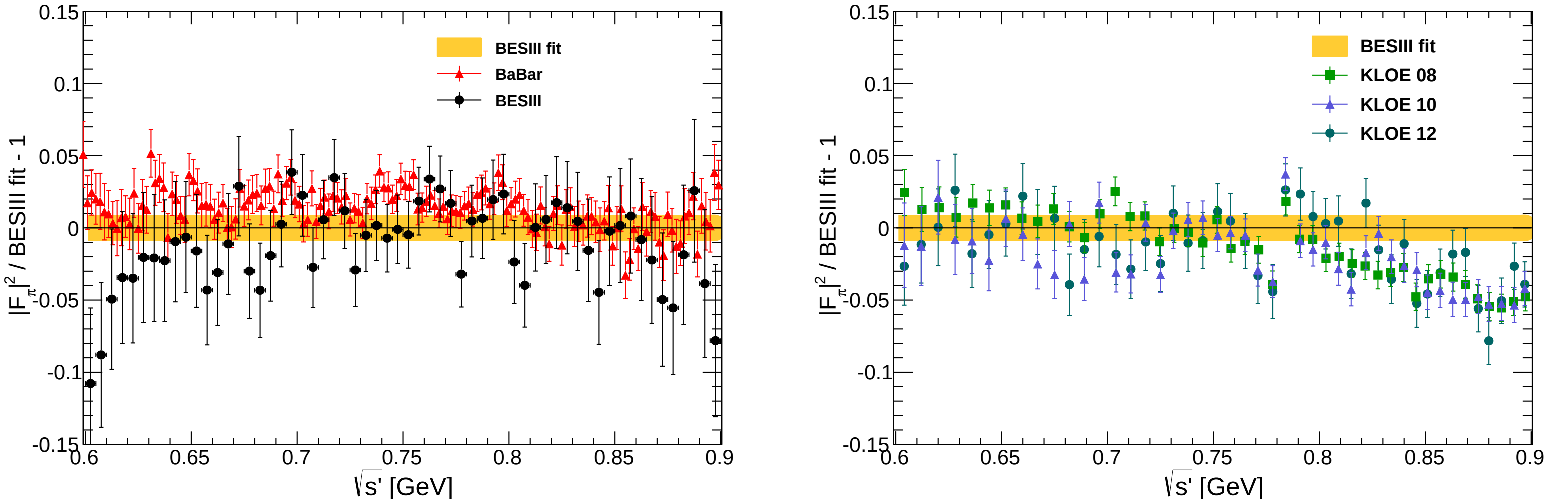


Still: most precise ISR measurements in conflict. BESIII steps to resolve this

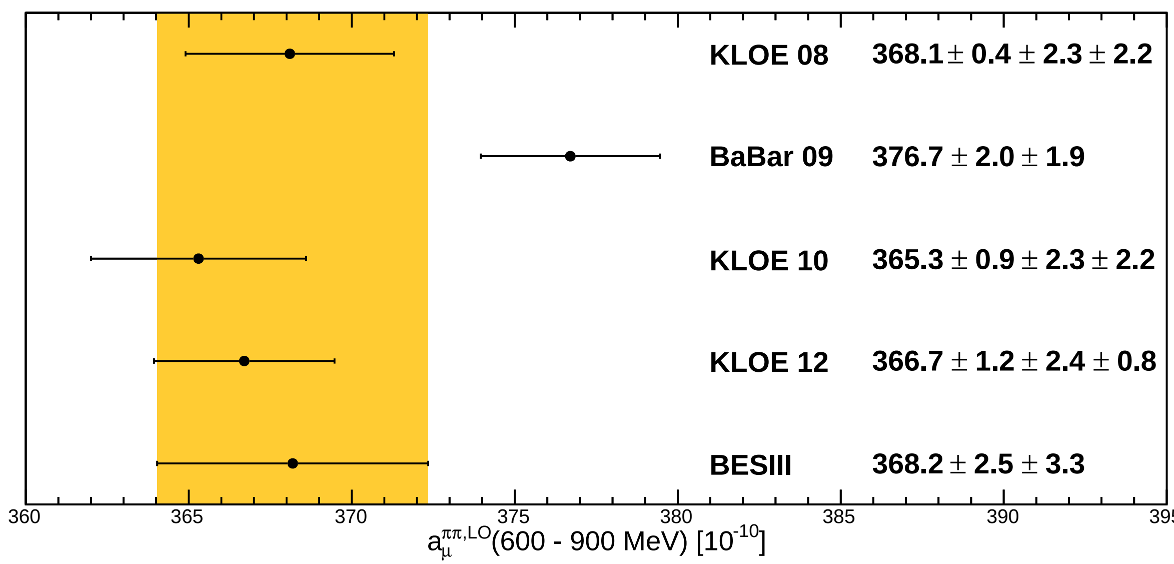
Recent/preliminary results:

- $e^+e^- \to \pi^+\pi^-$ from CMD-3
- $e^+e^- \to \pi^+\pi^-\pi^0$ from Belle
- $e^+e^- \to K^+K^-$ from CMD-3
- $e^+e^- \to K^+K^-$ from SND
- $e^+e^- \to \omega\pi^0 \to \pi^0\pi^0\gamma$ from SND
- $e^+e^- \to \pi^+\pi^-$ from BESIII recent, most important
- R_{uds} and R from 3.21 GeV to 3.72 GeV from KEDR

New from BESIII



BESIII vs BaBar and KLOE



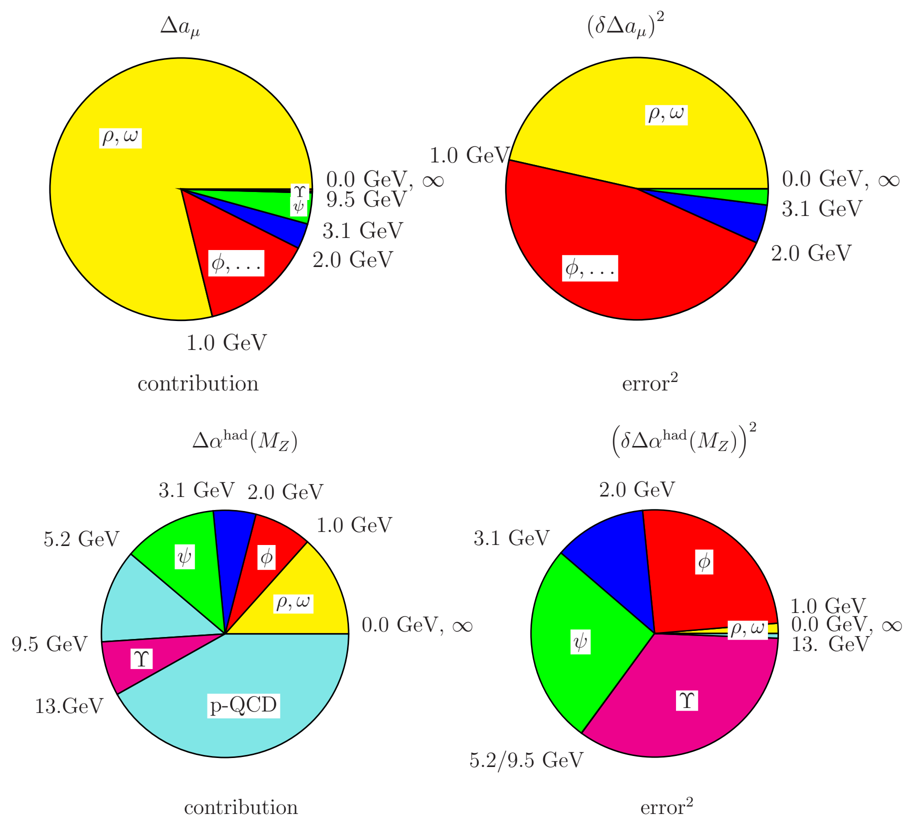
BESIII Collab. arXiv:1507.08188v3: 1.9 σ below BaBar in agreement with KLOE

$\Delta\alpha_{\rm had}(M_Z^2)$ results from ranges:

for $M_Z = 91.1876$ GeV in units 10^{-4}. 2015 update in terms of e^+e^--data and pQCD. 46% data, 54% perturbative QCD. pQCD is used between 5.2 GeV and 9.5 GeV and above 13.0 GeV.

final state	range (GeV)	$\Delta\alpha^{(5)}_{\rm had} \times 10^4$ (stat) (syst) [tot]	rel	abs
ρ	(0.28, 1.05)	33.91 (0.05) (0.18)[0.18]	0.5%	1.1%
ω	(0.42, 0.81)	3.10 (0.04) (0.08)[0.09]	3.0%	0.3%
ϕ	(1.00, 1.04)	4.76 (0.07) (0.11)[0.13]	2.7%	0.5%
J/ψ		12.38 (0.60) (0.67)[0.90]	7.2%	25.5%
Υ		1.30 (0.05) (0.07)[0.09]	6.9%	0.3%
had	(1.05, 2.00)	16.22 (0.07) (0.89)[0.89]	5.5%	25.1%
had	(2.00, 3.10)	15.34 (0.08) (0.61)[0.62]	4.0%	12.1%
had	(3.10, 3.60)	4.93 (0.03) (0.13)[0.14]	2.8%	0.6%
had	(3.60, 5.20)	16.62 (0.11) (0.05)[0.12]	0.3%	0.6%
pQCD	(5.20, 9.46)	33.84 (0.00) (0.03)[0.03]	0.0%	0.1%
had	(9.46,13.00)	18.32 (0.24) (1.01)[1.04]	5.7%	34.0%
pQCD	(13.0,∞)	115.73 (0.00) (0.04)[0.04]	0.0%	0.1%
data	(0.28,13.00)	126.86 (0.67) (1.64)[1.78]	1.4%	100.0%
total		276.43 (0.67) (1.64)[1.78]	0.6%	100.0%

Correlation between different contributions to $a_\mu^{\rm had}$ and $\Delta\alpha^{\rm had\,(5)}$



Contributions from e^+e^- data ranges and form pQCD to $a_\mu^{\rm had}$ and $\Delta\alpha^{\rm had\,(5)}$.

4. Reducing uncertainties via the Euclidean split trick: Adler function controlled pQCD

- experiment side: new more precise measurements of $R(s)$
- future direct measurements Talk Patrick Janot
- theory side: $\alpha_{\rm em}(M_Z^2)$ by the "Adler function controlled" approach

$$\alpha(M_Z^2) \quad = \quad \alpha^{\rm data}(-s_0) + \left[\alpha(-M_Z^2) - \alpha(-s_0)\right]^{\rm pQCD} + \left[\alpha(M_Z^2) - \alpha(-M_Z^2)\right]^{\rm pQCD}$$

where the space-like $-s_0$ is chosen such that pQCD is well under control for $-s < -s_0$. The monitor to control the applicability of pQCD is the Adler function

$$D(Q^2 = -s) = \frac{3\pi}{\alpha} s \frac{d}{{\rm d}s} \Delta\alpha_{\rm had}(s) = -(12\pi^2)\, s\, \frac{{\rm d}\Pi'_\gamma(s)}{{\rm d}s} = Q^2 \int_{4m_\pi^2}^{\infty} \frac{R(s)}{(s+Q^2)^2}$$

which also is determined by $R(s)$ and can be evaluated in terms of experimental e^+e^-–data. Perturbative QCD tail: $D(Q^2) \to N_c \sum_f Q_f^2\, (1 + O(\alpha_s))$ as $Q^2 \to \infty$.

$\Delta\alpha^{\rm had}$ Adler function controlled

✓ use old idea: Adler function: Monitor for comparing theory and data

$$D(-s) \doteq \frac{3\pi}{\alpha}\, s\, \frac{d}{ds}\Delta\alpha_{\rm had}(s) = -\left(12\pi^2\right)\, s\, \frac{d\Pi'_\gamma(s)}{ds}$$

$$\Rightarrow \quad D(Q^2) = Q^2 \left(\int_{4m_\pi^2}^{E^2_{\rm cut}} {\rm d}s\, \frac{R(s)^{\rm data}}{(s+Q^2)^2} + \int_{E^2_{\rm cut}}^{\infty} \frac{R^{\rm pQCD}(s)}{(s+Q^2)^2}\, {\rm d}s \right) .$$

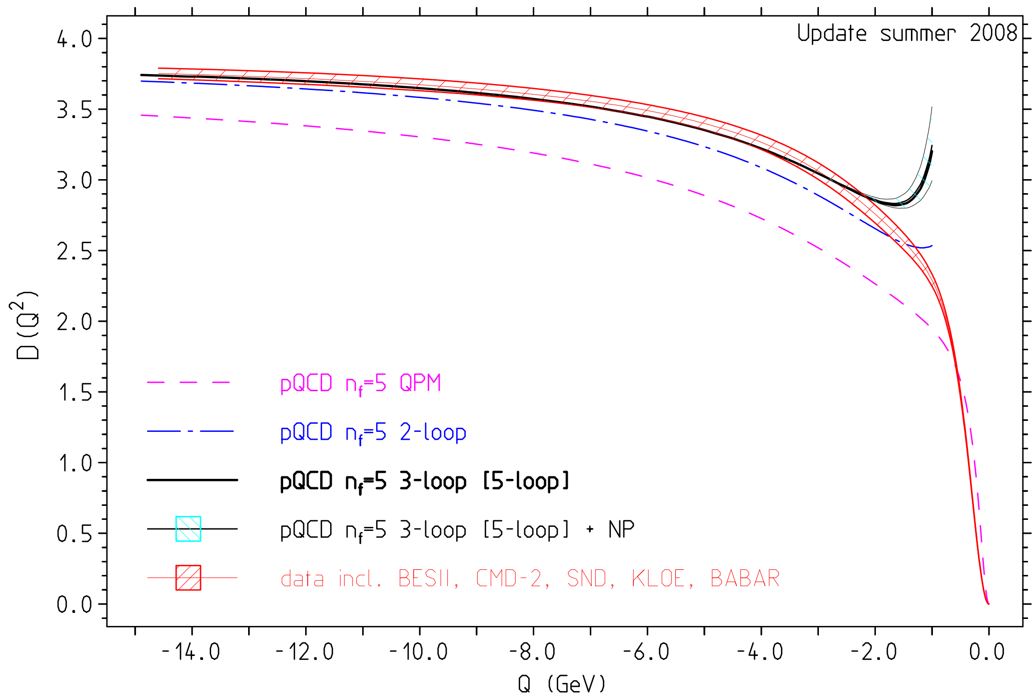
pQCD $\leftrightarrow R(s)$	pQCD $\leftrightarrow D(Q^2)$
very difficult to obtain in theory	smooth simple function in Euclidean region

Conclusion:

- time-like approach: pQCD works well in "perturbative windows" 3.00 - 3.73 GeV, 5.00 - 10.52 GeV and 11.50 - ∞ Kühn,Steinhauser
- space-like approach: pQCD works well for $\sqrt{Q^2=-q^2} > 2.0$ GeV (see plot)

“Experimental” Adler–function versus theory (pQCD + NP)

Error includes statistical + systematic here (in contrast to most R-plots showing statistical errors only)!



(Eidelman, F.J., Kataev, Veretin 98, FJ 08 update)
theory based on results by Chetyrkin, Kühn et al.

⇒ **pQCD works well to predict $D(Q^2)$ down to $s_0 = (2.0\,\mathbf{GeV})^2$; use this to calculate**

$$\Delta\alpha_{\rm had}(-Q^2) \sim \frac{\alpha}{3\pi}\int {\rm d}Q'^2 \frac{D(Q'^2)}{Q'^2}$$

$$\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) = \left[\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) - \Delta\alpha^{(5)}_{\rm had}(-s_0)\right]^{\rm pQCD} + \Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data}$$

and obtain, for $s_0 = (2.0\,\mathbf{GeV})^2$: **(FJ 98/15)**

$$\begin{aligned}
\Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data} &= 0.006392 \pm 0.000064 \\
\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) &= 0.027466 \pm 0.000118 \\
\Delta\alpha^{(5)}_{\rm had}(M_Z^2) &= 0.027504 \pm 0.000119
\end{aligned}$$

❖**shift** $+0.000008$ **from the 5-loop contribution**
❖**error** ± 0.000100 **added in quadrature form perturbative part**
QCD parameters: ● $\alpha_s(M_Z) = 0.1189(20)$**,**
● $m_c(m_c) = 1.286(13)\ [M_c = 1.666(17)]$ **GeV** , ● $m_b(m_c) = 4.164(25)\ [M_b = 4.800(29)]$ **GeV**

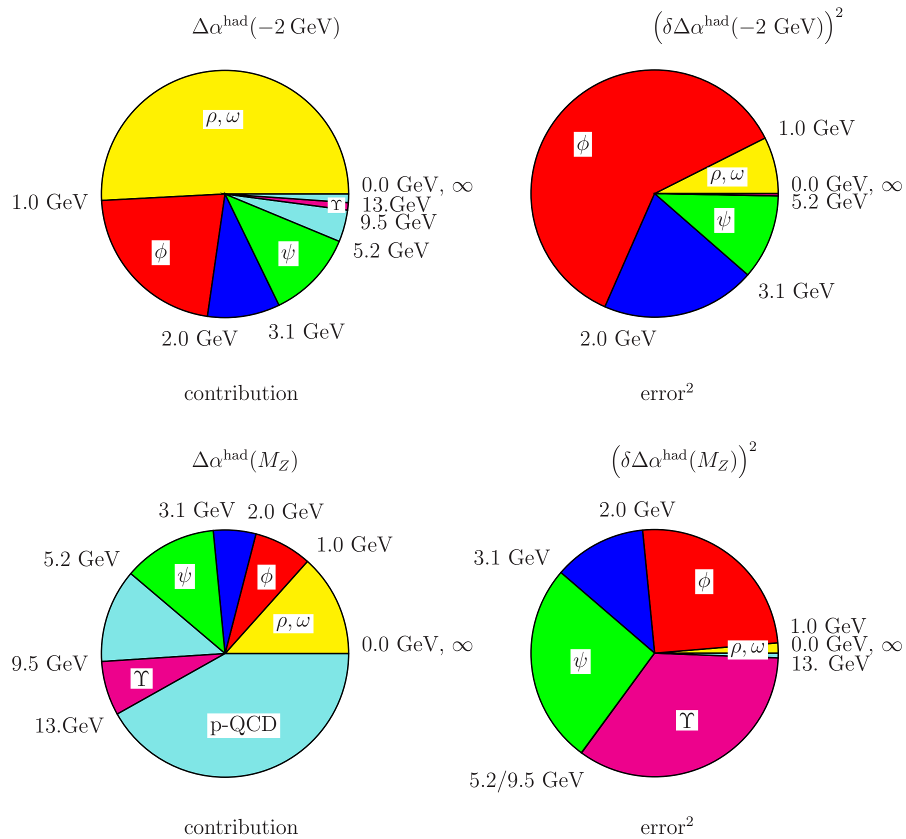
based on a complete 3–loop massive QCD analysis Kühn et al 2007

$\Delta\alpha_{\rm had}(-M_0^2)$ results from ranges:

for $M_0 = 2$ GeV in units 10^{-4}. 2015 update in terms of e^+e^--data and pQCD. 95% data, 5% perturbative QCD. pQCD is used between 5.2 GeV and 9.5 GeV and above 13.0 GeV.

final state	range (GeV)	$\Delta\alpha^{(5)}_{\rm had}(-M_0^2) \times 10^4$ (stat) (syst) [tot]	rel	abs
ρ	(0.28, 1.05)	29.77 (0.04) (0.15)[0.16]	0.5%	5.8%
ω	(0.42, 0.81)	2.69 (0.03) (0.07)[0.08]	3.0%	1.5%
ϕ	(1.00, 1.04)	3.78 (0.05) (0.09)[0.10]	2.7%	2.4%
J/ψ		3.21 (0.15) (0.15)[0.21]	6.7%	10.6%
Υ		0.05 (0.00) (0.00)[0.00]	6.8%	0.0%
had	(1.05, 2.50)	13.76 (0.05) (0.56)[0.56]	4.1%	72.0%
had	(2.50, 3.10)	2.49 (0.01) (0.18)[0.18]	7.2%	7.4%
had	(3.10, 3.60)	1.30 (0.01) (0.03)[0.04]	2.8%	0.3%
had	(3.60, 5.20)	2.87 (0.02) (0.00)[0.02]	0.0%	0.1%
pQCD	(5.20, 9.46)	2.66 (0.00) (0.00)[0.00]	0.0%	0.0%
had	(9.46,13.00)	0.57 (0.01) (0.03)[0.03]	5.5%	0.2%
pQCD	(13.00, 0.00)	0.70 (0.00) (0.00)[0.00]	0.0%	0.0%
data	(0.28,13.00)	60.49 (0.18) (0.63)[0.66]	1.0%	0.0%
total		63.85 (0.18) (0.63)[0.66]	1.0%	100.0%

Of $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ 22% data, 78% pQCD!



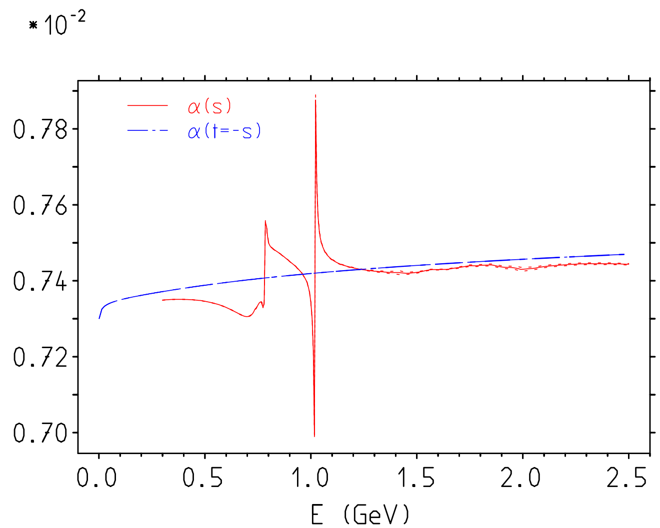
Contributions from e^+e^- data ranges and form pQCD to $\Delta\alpha^{(5)}_{\rm had}(-M_0^2)$ vs. $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$.

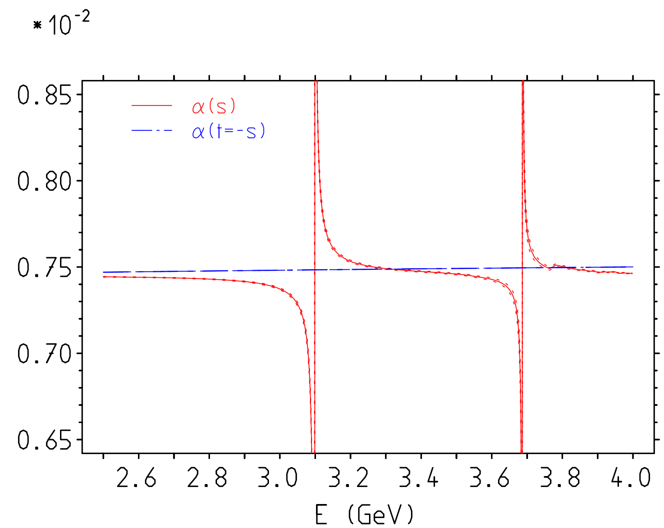
How much pQCD?

Method	range [GeV]	pQCD		
Standard approach:	5.2 - 9.5	33.84(0.03)		
My choice	13.0 - ∞	115.73(0.04)	$\rightarrow$	149.57 (0.05)
Standard approach:	2.0 - 9.5	72.09(0.07)		
Davier et al.	11.5 - ∞	123.24(0.05)	$\rightarrow$	195.33 (0.09)
Adler function controlled:	5.2 - 9.5	2.66(0.00)		
	13.0 - ∞	0.70(0.00)		
	$-\infty - -2.0$	210.68(0.99)		
	$-M_Z \rightarrow M_Z$	0.38(0.00)	$\rightarrow$	214.42 (0.99)

Note: the Adler function monitored Euclidean data vs pQCD split approach is only moderately more pQCD-driven, than the time-like approach adopted by Davier et al. and others.

The time-like vs space-like effective charge



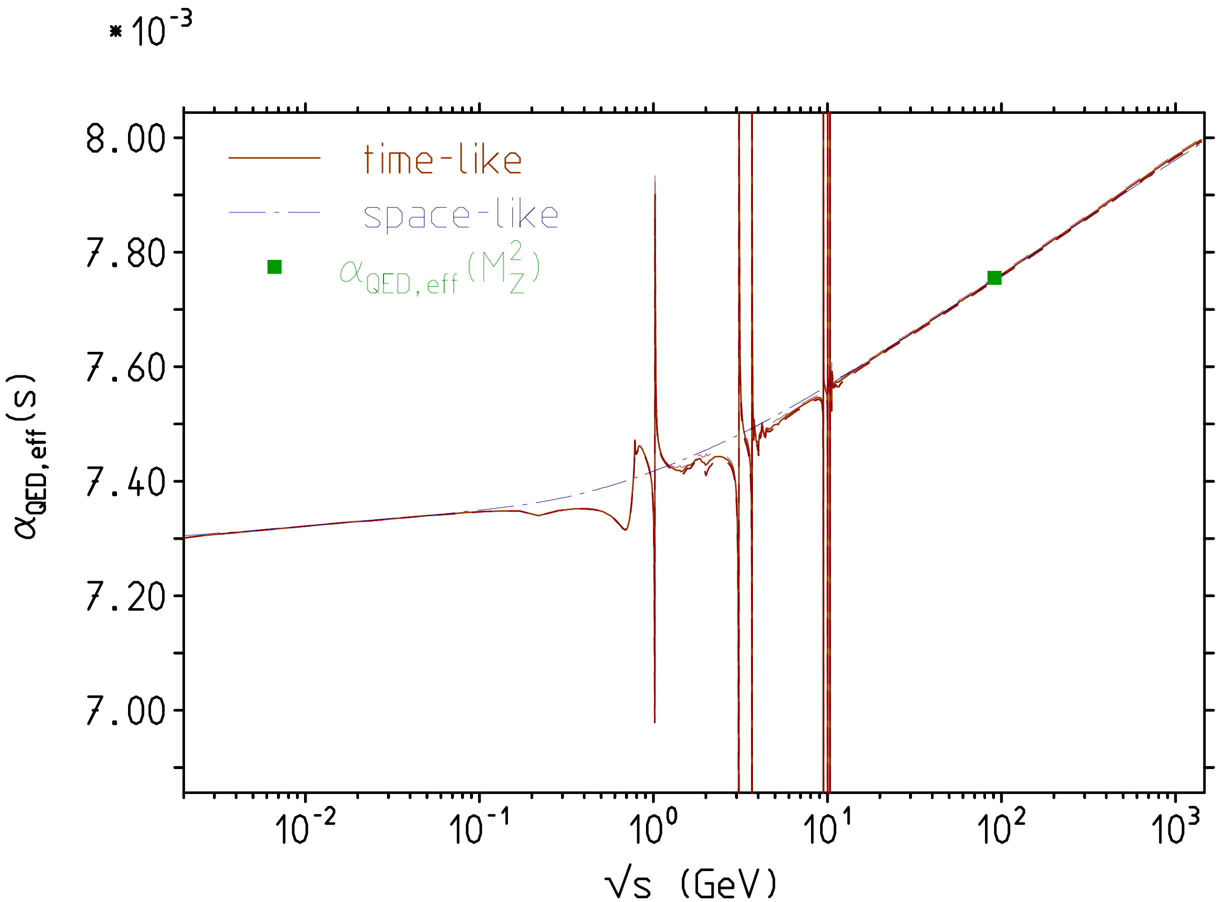


Note that the smooth space-like effective charge agrees rather well with the non-resonant "background" above the Φ (kind of duality)

No proof that this cannot produce non-negligible shifts!

Time-like VP-subtraction cannot be implemented locally near OZI suppressed resonances: $J/\psi, \psi'$ and $\Upsilon_1, \Upsilon_2, \Upsilon_3$

$\alpha_{\rm QED,eff}$: time-like vs. space-like



$\alpha_{\rm QED,eff}$ duality: $\alpha_{\rm QED,eff}(s)$ is varying dramatically near resonances, but agrees quite well in average with space-like version

5. Prospects for future improvements

Mandatory pQCD improvements required are:

- **4–loop massive pQCD calculation of Adler function;**
  required are a number of terms in the low and high momentum series expansions which allow for the appropriate Padé improvements
  [essentially equivalent to a massive 4–loop calculation of $R(s)$];
- m_c, m_b **improvements by sum rule and/or lattice QCD evaluations;**
- **improved α_s in low Q^2 region.**

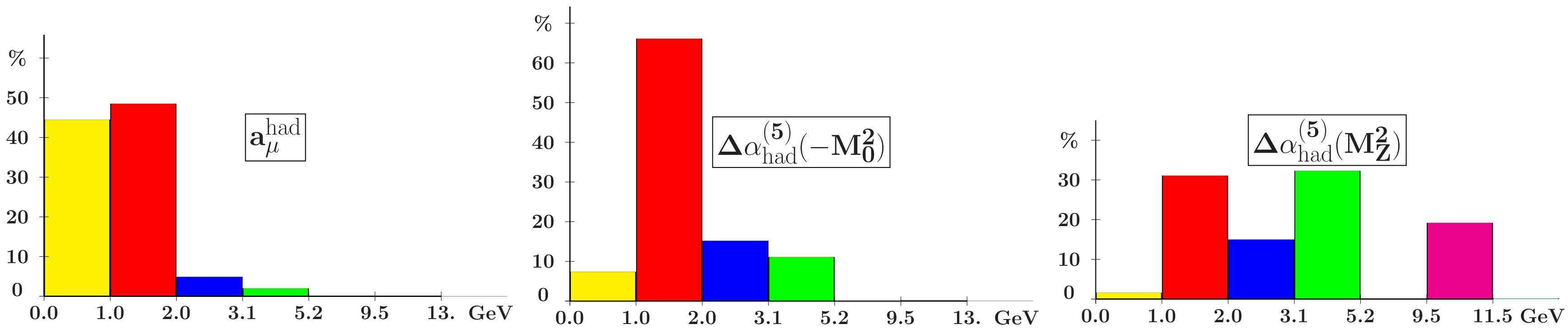
Theory: (QCD parameters) has to improve by factor 10 ! $\rightarrow \pm 0.20$

Requirement may be realistic:

- **pin down experimental errors to 1% level in all non-perturbative regions up to 2.5 GeV**
- **switch to Euclidean approach, monitored by the Adler function**
- **improve on QCD parameters, mainly on m_c and m_b**

> Settling the HVP issue for a_μ settles it largely for $\Delta\alpha(-M_0^2)$

Error profiles:



Contributions to the total error from different energy regions to the hadronic lowest order vacuum polarization contribution to a_μ, $\Delta\alpha(M_Z^2)$ and $\Delta\alpha(-M_0^2)$ for $M_0 = 2$ GeV in percent. These errors are to be added in quadrature to get the total uncertainty. The graph illustrates where experimental effort is needed in order to get a better precision.

The virtues of Adler function approach are obvious:

- **no problems with physical threshold and resonances**
- **pQCD is used only where we can check it to work (Euclidean, $Q^2 \gtrsim$ 2.0 GeV).**
- **no manipulation of data, no assumptions about global or local duality.**
- **non–perturbative "remainder" $\Delta\alpha^{(5)}_{\rm had}(-s_0)$ is mainly sensitive to low energy data !!!**

Future: ILC/FCC-ee requirement: improve by factor 10 in accuracy

- **direct integration of data: 46% from data 54% p-QCD**

$$\Delta\alpha^{(5)\,\mathrm{data}}_{\mathrm{had}} \times 10^4 = 126.86 \pm 1.78 \text{ (1.4\%)}$$

  1% overall accuracy ± 1.27

  1% accuracy for each region (divided up as in table)

  added in quadrature: ± 0.40

  Data: [1.78] vs. [0.40] ⇒ improvement factor 4.5

$$\Delta\alpha^{(5)\,\mathrm{pQCD}}_{\mathrm{had}} \times 10^4 = 149.57 \pm 0.05 \text{ (0.0\%)}$$

  Theory: no improvement needed !

- **integration via Adler function: 22% from data 78% p-QCD**

$$\Delta\alpha^{(5)\,\mathrm{data}}_{\mathrm{had}} \times 10^4 = 060.49 \pm 0.66 \text{ (1.1\%)}$$

  1% overall accuracy ± 0.60

  1% accuracy in region 1.0 to 2.5 GeV

  added in quadrature: ± 0.28

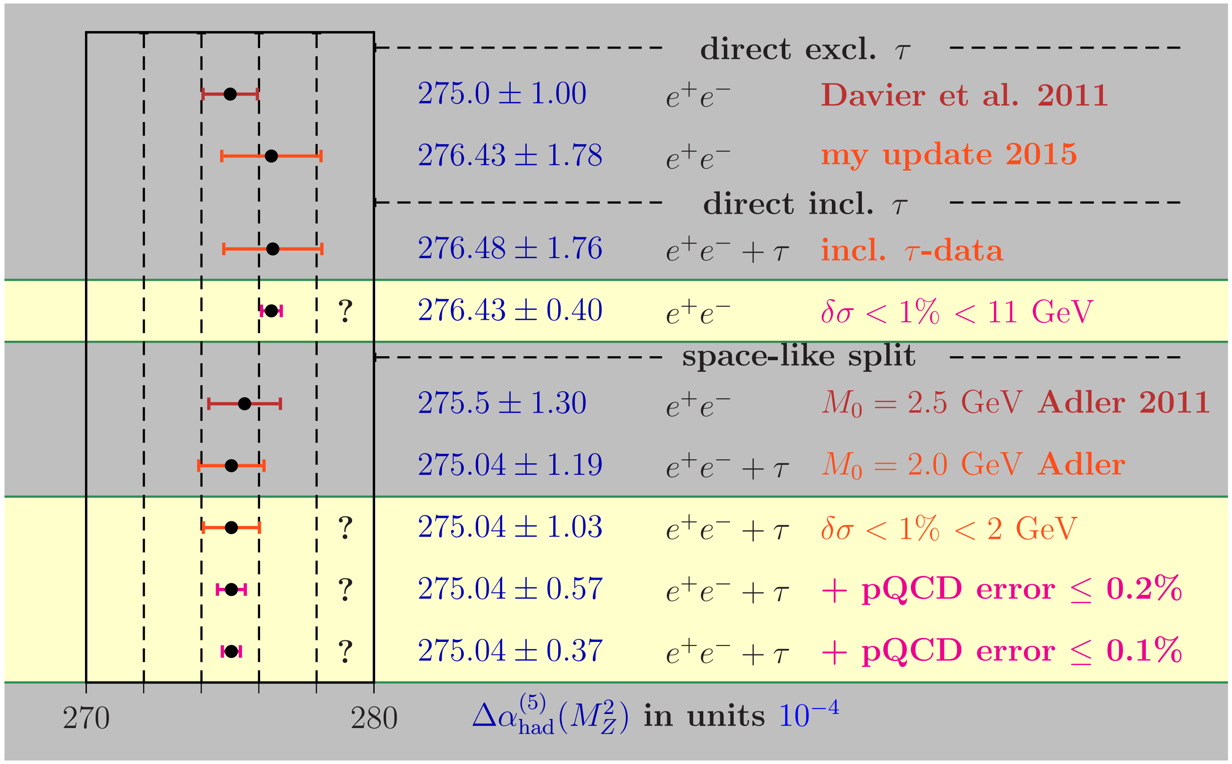
  Data: [1.19] vs. [1.03,0.57,0.37] ⇒ improvement factor 2.1-3.2 (Adler vs Adler)

  [1.78] vs. [1.03,0.57,0.37] ⇒ improvement factor 3.1-4.8 (Standard vs Adler)

$$\Delta\alpha^{(5)\,\mathrm{pQCD}}_{\mathrm{had}} \times 10^4 = 214.48 \pm 1.00 \text{ (0.05\%)}$$

  Theory: massive 4-loop needed and more accurate m_c, m_b and α_s!

- **direct measurement (near/off Z peak)** — **Patrick Janot's talk**



Davier et al. 2011: use pQCD above 1.8 GeV

- **no improvement by remeasuring cross sections above 1.8 GeV**
- **no proof that pQCD works at 0.04% precision as adopted**

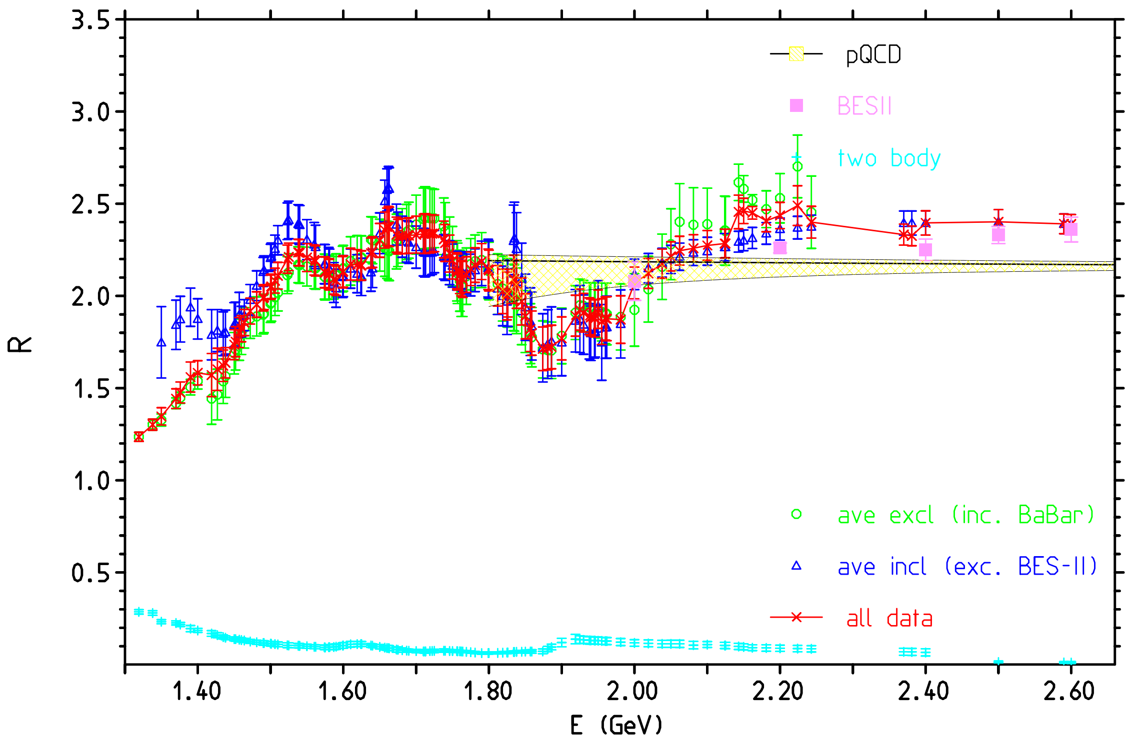
My analysis is data driven: pQCD $5.2 - 9.5$ and > 11.5 GeV

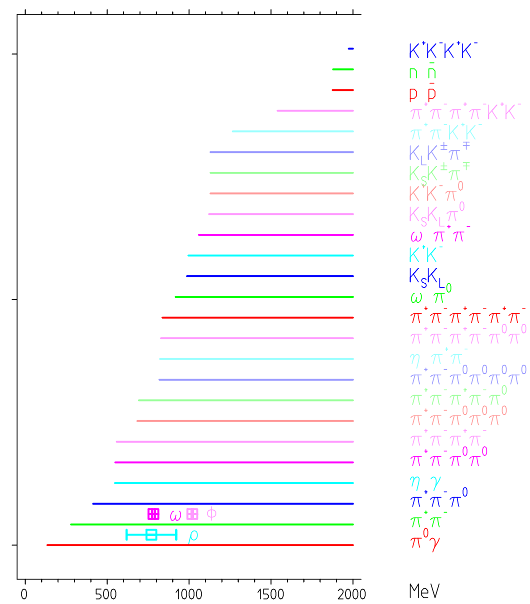
- **pQCD at 0.2% Adler function: pQCD error = ½ × present error**
- **pQCD at 0.1% Adler function: pQCD error = data error ± 0.28**

Note: theory-driven analyses using pQCD above 1.8 GeV cannot be improved by improved cross-section measurements above 2 GeV !!!

Better data needed in any case for range 1.2 - 2.4 GeV

- one of the main issue in HVP is $R_\gamma(s)$ from 1.2 GeV to 2.4 GeV
- has been improved dramatically by the exclusive channel measurements by BaBar
- 20 out of more than 30 channels are measured, many known at the 10 to 15% level
- now exclusive channel data much better quality than the very old inclusive data from Frascati





ILC/FCC-ee community should actively support these activities as integral part of e^+e^--collider precision physics!!!

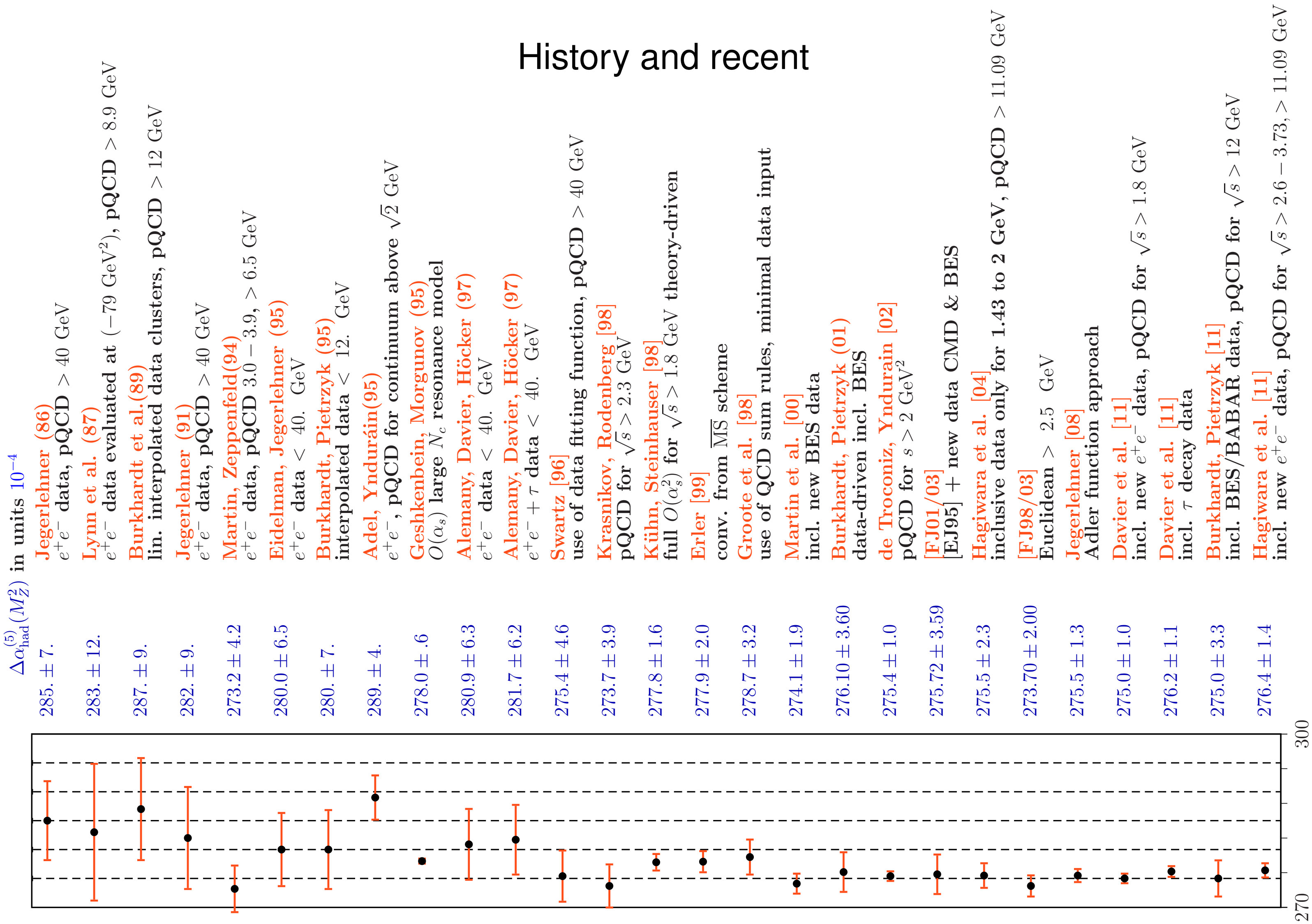
Remember: tremendous progress since middle of 90's

- Novosibirsk VEPP-2M: MD-1, CMD2, SND, KEDR; VEPP-2000: CMD3,SND
- Beijing BEPC: BES II, BESIII
- Cornell CESR: CLEO
- Frascati DAFNE: KLOE
- Stanford SLAC PEP-II: BaBar; KEK Tsukuba: Belle

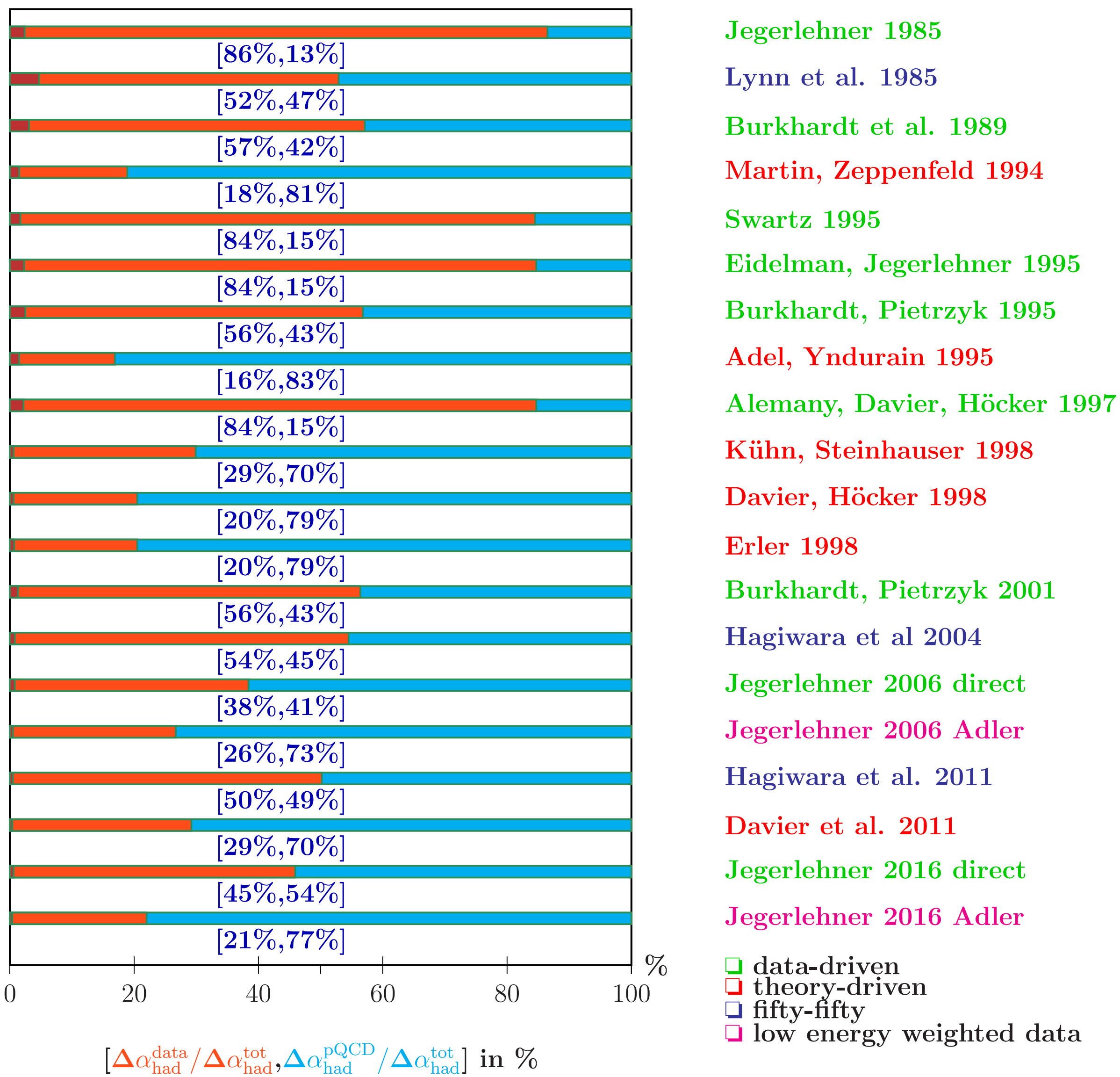
Many analyzes exploiting these results: Davier et al, Hagiwara et al., Burkhardt, Pietrzyk, Yndurain et al....

Indispensable for Muon $g-2$, indirect vs direct LEP Higgs mass etc. and future precision test at ILC/FCC-ee and new physics signals in precision observables. Impact for cosmology!

History and recent



- big progress in data CMD-2, SND, BESII, KLOE, BaBar, $\cdots$
- progress in pQCD [..., Chetyrkin, Kühn et al. ...], by far more progressive use of pQCD

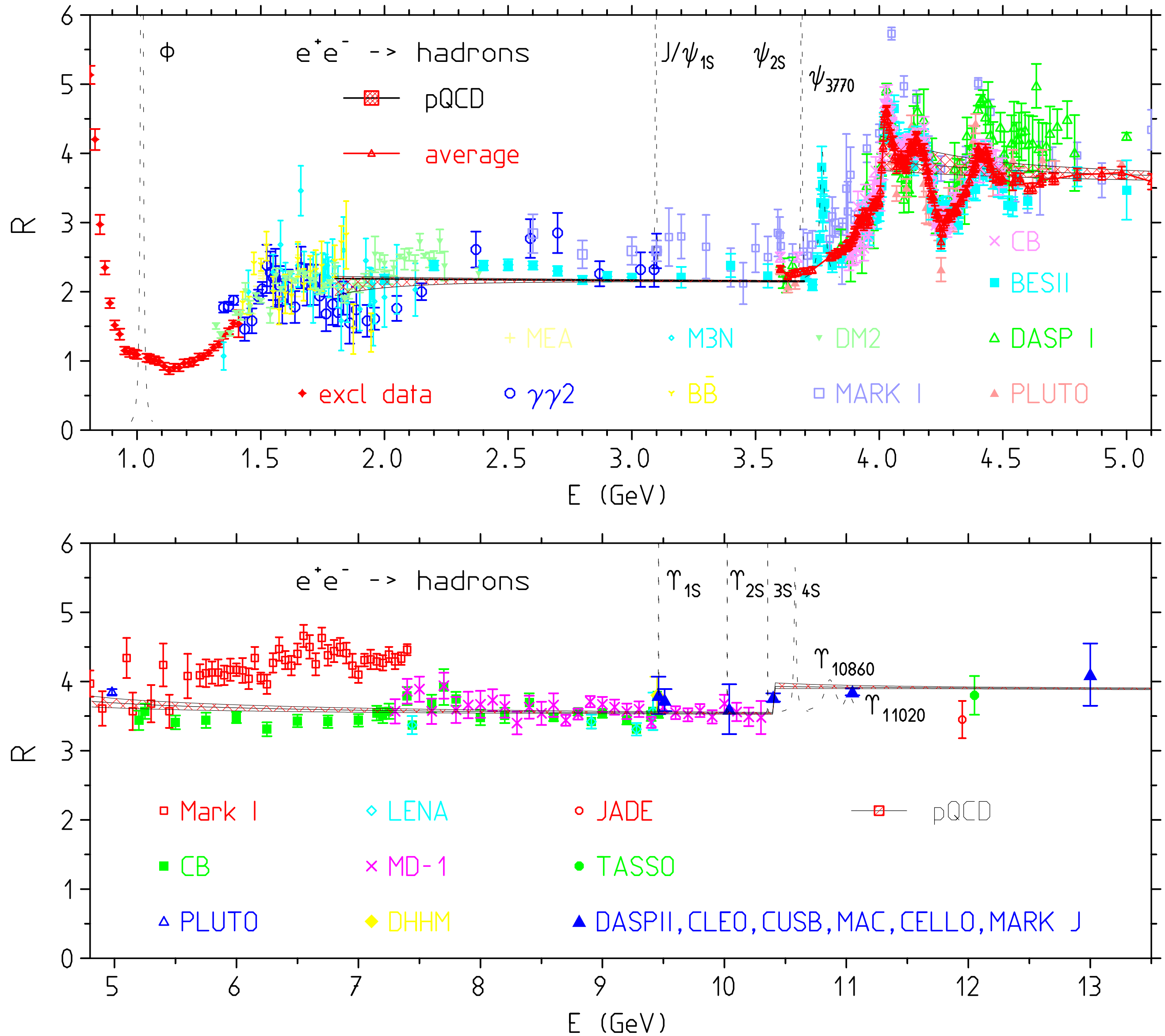


How much pQCD?

Authors	data	pQCD	sum
Jegerlehner 1985:	247.37± 7.	38.63±0.37	286. ± 7.
Lynn et al. 1985:	145. ±13.	129. ±1.	274. ±13.
Burkhardt et al. 1989:	164.32± 8.2	123.68±3.71	288. ± 9.
Martin, Zeppenfeld 1994:	51.5 ± 1.1	221.7 ±4.1	273.2 ± 4.2
Swartz 1995:	232.56± 4.6	42.64±0.10	275.2 ± 4.6
Eidelman, Jegerlehner 1995:	237.55± 6.43	42.82±0.10	280.37± 6.43
Burkhardt, Pietrzyk 1995:	159. ± 7.	121. ±0.2	280. ± 7.
Adel, Yndurain 1995:	45.99± 0.85	226.6 ±4.0	272.59± 4.09
Alemany, Davier, Höcker 1997:	238.01± 6.3	42.82±0.10	280.9 ± 6.3
Kühn, Steinhauser 1998:	82.9 ± 1.40	194.45±0.96	277.43± 1.70
Davier, Höcker 1998:	56.53± 0.83	219.77±1.40	276.3 ± 1.6
Erler 1998 :	56.9 ± 1.1	220.8 ±1.5	277.7 ± 1.9
Burkhardt, Pietrzyk 2001:	155.8 ± 3.6	120.3 ±0.2	276.1 ± 3.6
Hagiwara et al 2004:	150.18± 2.3	125.32±0.15	275.5 ± 2.3
Jegerlehner 2006 direct:	106.07± 2.24	115.66±0.11	276.07± 2.25
Jegerlehner 2006 Adler :	73.69± 0.98	201.83±1.03	275.52± 1.42
Hagiwara et al 2011:	138.70± 1.37	137.56±0.16	276.26± 1.38
Davier et al 2011:	80.57± 1.00	195.33±0.09	275.90± 1.00
Jegerlehner 2016 direct:	126.86± 1.78	149.57±0.05	276.43± 1.78
Jegerlehner 2016 Adler:	60.49± 0.66	214.48±1.00	275.04± 1.19

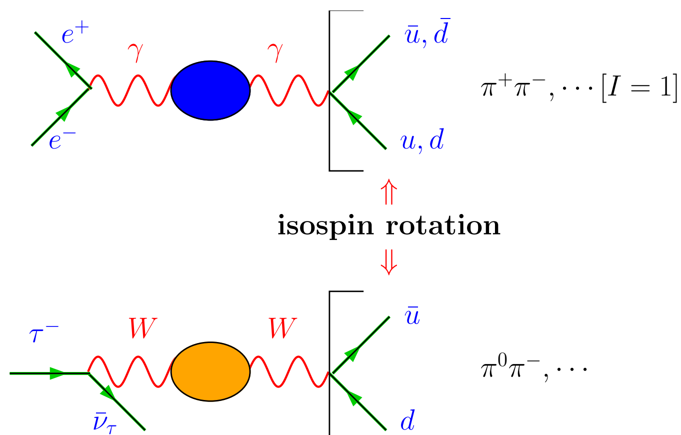
- red theory-driven still can be improved by more precise cross sections below 2 GeV
- green data-driven require better cross section up to above the Υ region
- magenta Adler function based requires 1% cross section from 1.4 to 2.5 GeV and progress in pQCD [massive PQCD to 4-loops, Euclidean regime]
- advantage of euclidean approach can be supplemented directly by lattice QCD data

Data vs pQCD



e+e- -> hadrons
pQCD
average
φ
J/ψ1S
ψ2S
ψ3770
CB
BESII
MEA
M3N
DM2
DASP I
excl data
γγ2
BB̄
MARK I
PLUTO
R
E (GeV)
e+e- -> hadrons
ϒ1S
ϒ2S
3S
4S
ϒ10860
ϒ11020
Mark I
LENA
JADE
pQCD
CB
MD-1
TASSO
PLUTO
DHHM
DASPII,CLEO,CUSB,MAC,CELLO,MARK J
R
E (GeV)

Addendum 1: Using τ–decay spectra + isospin breakings



$e^+e^- \to \pi^+\pi^-, \cdots$ vs $\tau^- \to \bar{\nu}_\tau \pi^0\pi^-, \cdots$

τ–spectra: ALEPH, OPAL, CLEO, Belle

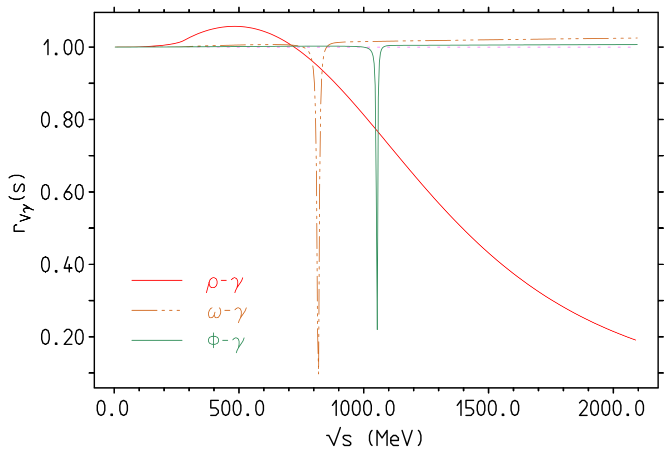
a_μ^{had} incl. $I=1$ $\tau \to \pi\pi\nu_\tau$ in range [0.63-0.96] GeV:

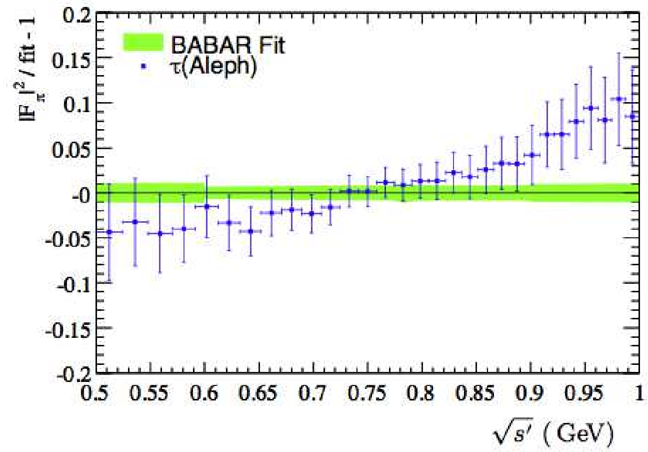
e^+e^-	:	353.82(0.88)(2.17)[2.34]
τ	:	354.25(1.24)(0.61)[1.38]
$e^+e^- + \tau$	:	354.14(0.82)(0.86)[1.19]

The τ vs e^+e^- puzzle of $\pi^+\pi^-$ data: $\rho^0 - \gamma$ mixing

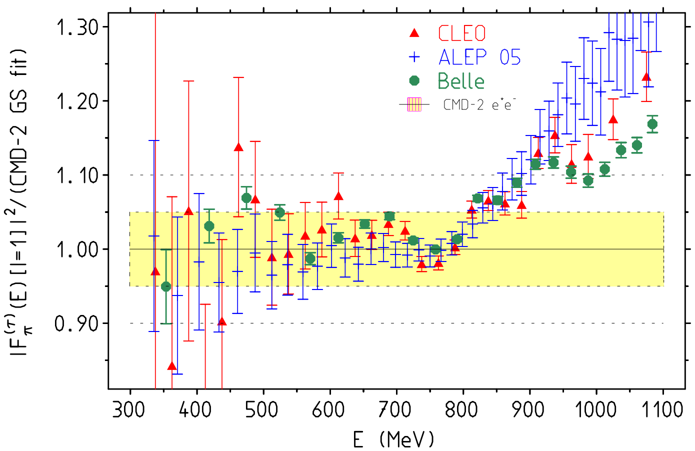
$-\mathrm{i}\,\Pi^{\mu\nu\,(\pi)}_{\gamma\rho}(q) =$  .

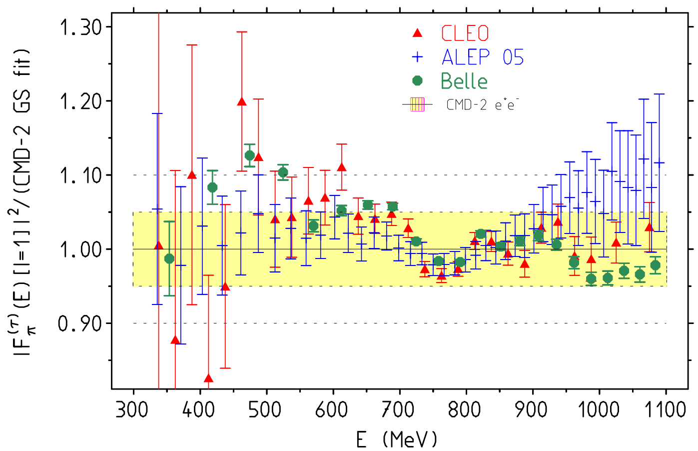
absent in charged (τ) channel



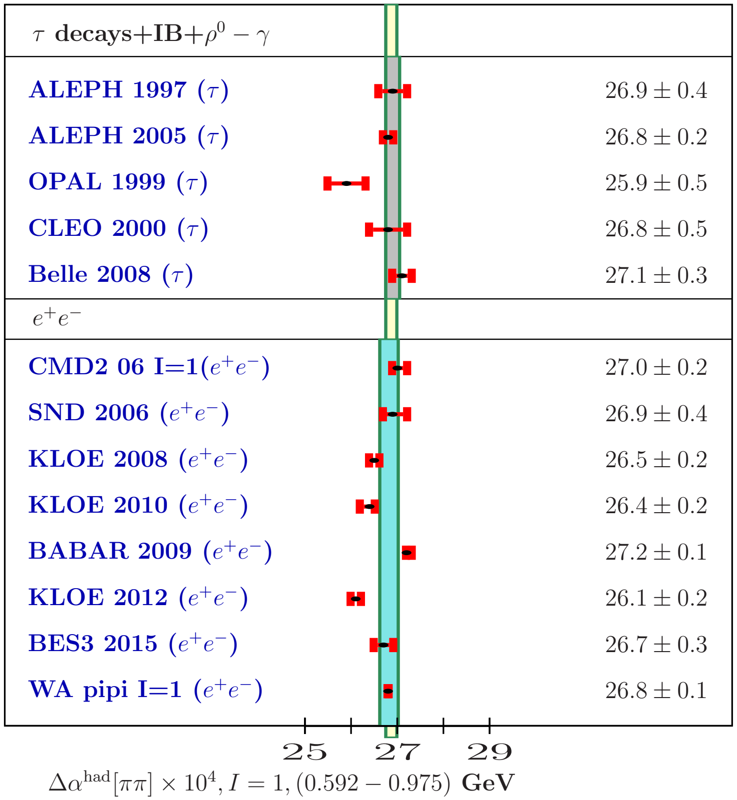


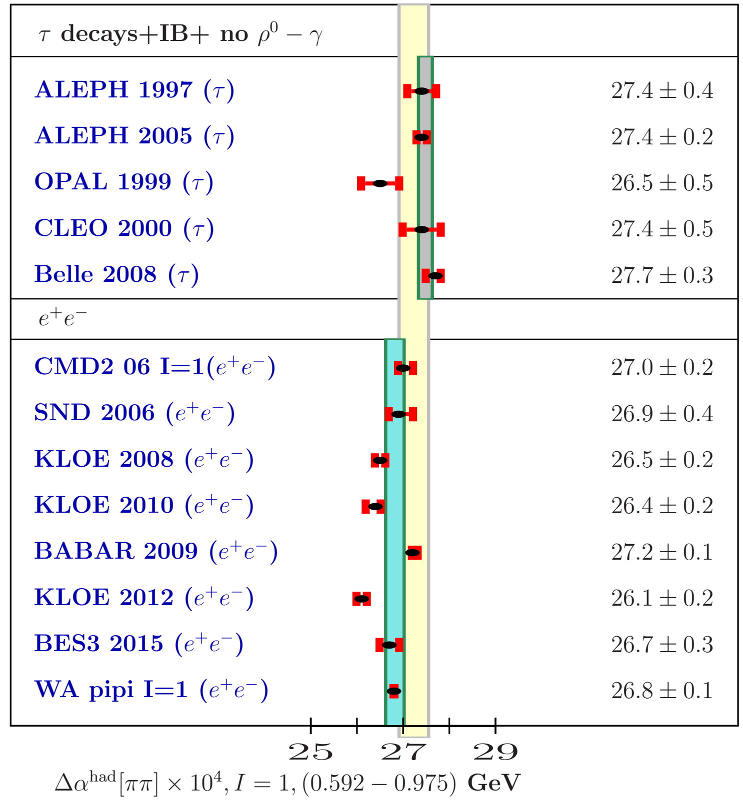
$\rho^0-\gamma$ mixing correction to be applied to τ data [l], ALEPH versus BaBar plot from Davier et al.[r]





$|F_\pi(E)|^2$ in units of e^+e^- I=1 (CMD-2 GS fit): left no mixing correction, right: after mixing correction





$\Delta\alpha(M_Z)$ contributions form I=1 $\pi\pi$ channel in range [0.63, 0.96] GeV in units 10^{-4}. τ data corrected for isospin breaking and [left] for missing $\rho^0-\gamma$ mixing [right] $\rho^0-\gamma$ mixing is not applied.

$\Delta\alpha(M_Z^2)$ contributions form I=1 $\pi\pi$ channel in range [0.63, 0.96] GeV in units 10^{-4}

data	incl. $\rho^0 - \gamma$ mixing correction	
τ data, IB corrected	26.90 (0.06) (0.14)	27.48 (0.06) (0.14)
e^+e^- data, I=1 $\pi\pi$	26.82 (0.08) (0.18)	26.82 (0.08) (0.18)
Combined	26.87 (0.05) (0.11)	27.23 (0.12) (0.30)
$\Delta\alpha^{\rm had}[ee < 2\ \rm GeV] \times 10^4$	57.99(0.12)(0.92)[0.93]	
$\Delta\alpha^{\rm had}[ee + \tau < 2\ \rm GeV] \times 10^4$	58.04(0.12)(0.91)[0.92]	

data	incl. $\rho^0 - \gamma$ mixing correction
$\Delta\alpha^{\rm had}[ee]$	$(276.43 \pm 0.67(\rm stat) \pm 1.64(\rm syst)[\pm 1.78(\rm tot)]) \times 10^{-4}$
$\Delta\alpha^{\rm had}[ee + \tau]$	$(276.48 \pm 0.67(\rm stat) \pm 1,63(\rm syst)[\pm 1.76(\rm tot)]) \times 10^{-4}$

Central values of pQCD contributions to $\Delta\alpha^{\rm had\,(5)}$:

range	$\alpha_s = 0.1184$	$\alpha_s = 0.12$
5.2 GeV - 9.46 GeV	33.84	33.91
13 GeV -∞	115.73	115.82

$\Delta\alpha(-M_0^2)$ contributions form I=1 $\pi\pi$ channel in range [0.63, 0.96] GeV in units 10^{-4}

data	incl. $\rho^0 - \gamma$ mixing correction
τ data, IB corrected	23.46 (0.05) (0.12)
e^+e^- data, I=1 $\pi\pi$	23.39 (0.07) (0.15)
Combined	23.43 (0.04) (0.09)
$\Delta\alpha^{\rm had}[ee < 2\ {\rm GeV}] \times 10^4$	46.43(0.08)(0.56)[0.57]
$\Delta\alpha^{\rm had}[ee + \tau < 2\ {\rm GeV}] \times 10^4$	46.50(0.06)(0.55)[0.56]

data	incl. $\rho^0 - \gamma$ mixing correction	
$\Delta\alpha^{\rm had}[ee]$	$(63.85 \pm 0.18({\rm stat}) \pm 0.63({\rm syst})[\pm 0.66({\rm tot})]) \times 10^{-4}$	Central
$\Delta\alpha^{\rm had}[ee + \tau]$	$(63.92 \pm 0.17({\rm stat}) \pm 0.62({\rm syst})[\pm 0.64({\rm tot})]) \times 10^{-4}$	

values of pQCD contributions to $\Delta\alpha^{\rm had\,(5)}$:

range	$\alpha_s = 0.1184$	$\alpha_s = 0.12$
-2 GeV - -∞	115.73	115.82

In both cases for $\Delta\alpha_{\mathrm{had}}(-M_0^2)$ and $\Delta\alpha_{\mathrm{had}}(M_Z^2)$ improvement marginal because $\pi\pi$ channel alone does not give the dominant contribution!

Addendum 2: The coupling α_2, M_W and $\sin^2\Theta_f$

How to measure α_2:

❖charged current channel M_W ($g \equiv g_2$):

$$M_W^2 = \frac{g^2\, v^2}{4} = \frac{\pi\, \alpha_2}{\sqrt{2}\, G_\mu}$$

❖neutral current channel $\sin^2\Theta_f$

In fact here running $\sin^2\Theta_f(E)$: LEP scale $\Longleftrightarrow$ low energy $\nu_e e$ scattering

$$\sin^2\Theta_e = \left\{ \frac{1-\Delta\alpha_2}{1-\Delta\alpha} + \Delta_{\nu_\mu e,\mathrm{vertex+box}} + \Delta\kappa_{e,\mathrm{vertex}} \right\} \sin^2\Theta_{\nu_\mu e}$$

The first correction from the running coupling ratio is largely compensated by the ν_μ charge radius which dominates the second term. The ratio $\sin^2\Theta_{\nu_\mu e}/\sin^2\Theta_e$ is close to 1.002, independent of top and Higgs mass. Note that errors in the ratio $\frac{1-\Delta\alpha_2}{1-\Delta\alpha}$ can be taken to be 100% correlated and thus largely cancel.

Above result allow us to calculate non-perturbative hadronic correction in $\gamma\gamma$, γZ, ZZ and WW self energies, as

$$
\begin{aligned}
\Pi^{\gamma\gamma} &= e^2 \quad \hat{\Pi}^{\gamma\gamma} \\
\Pi^{Z\gamma} &= \frac{eg}{c_\Theta} \quad \hat{\Pi}_V^{3\gamma} \quad - \quad \frac{e^2 s_\Theta}{c_\Theta} \quad \hat{\Pi}_V^{\gamma\gamma} \\
\Pi^{ZZ} &= \frac{g^2}{c_\Theta^2} \quad \hat{\Pi}_{V-A}^{33} \quad - \quad 2\,\frac{e^2}{c_\Theta^2} \quad \hat{\Pi}_V^{3\gamma} \quad + \quad \frac{e^2 s_\Theta^2}{c_\Theta^2} \quad \hat{\Pi}_V^{\gamma\gamma} \\
\Pi^{WW} &= g^2 \quad \hat{\Pi}_{V-A}^{+-}
\end{aligned}
$$

with $\hat{\Pi}(s) = \hat{\Pi}(0) + s\hat{\pi}(s)$.
Leading hadronic contributions:

$$
\begin{aligned}
\Delta\alpha_{\rm had}^{(5)}(s) &= -e^2 \left[\mathrm{Re}\,\hat{\pi}^{\gamma\gamma}(s) - \hat{\pi}^{\gamma\gamma}(0)\right] \\
\Delta\alpha_{2\,\rm had}^{(5)}(s) &= -\frac{e^2}{s_\Theta^2} \left[\mathrm{Re}\,\hat{\pi}^{3\gamma}(s) - \hat{\pi}^{3\gamma}(0)\right]
\end{aligned}
$$

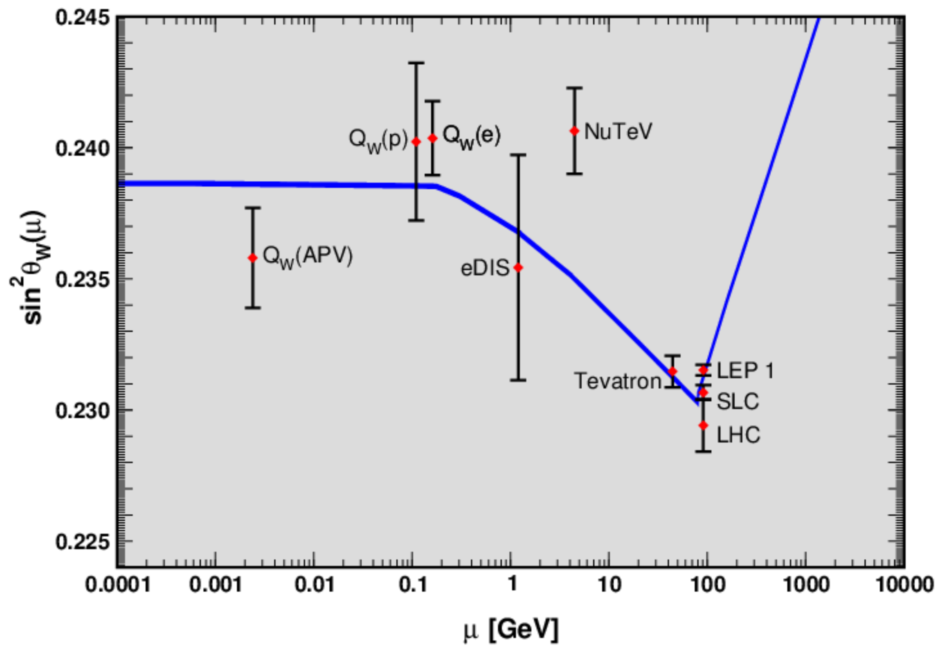
which exhibit the leading hadronic non-perturbative parts, i.e. the ones involving the photon field via mixing. $\Delta\alpha^{(5)}_{\rm had}(s)$ and $\Delta\alpha^{(5)}_{2\,\rm had}(s)$ via e^+e^--data and isospin arguments [(u,d), s flavor separation]:

$$\Pi^{3\gamma}_{ud} = \frac{1}{2}\,\Pi^{\gamma\gamma}_{ud}\;;\quad \Pi^{3\gamma}_{s} = \frac{3}{4}\,\Pi^{\gamma\gamma}_{s}$$

$$\Pi^{\gamma\gamma} = \Pi^{(\rho)} + \Pi^{(\omega)} + \Pi^{(\phi)} + \cdots \quad \Rightarrow \quad \Pi^{3\gamma} = \tfrac{1}{2}\,\Pi^{(\rho)} + \tfrac{3}{4}\,\Pi^{(\phi)} + \cdots$$

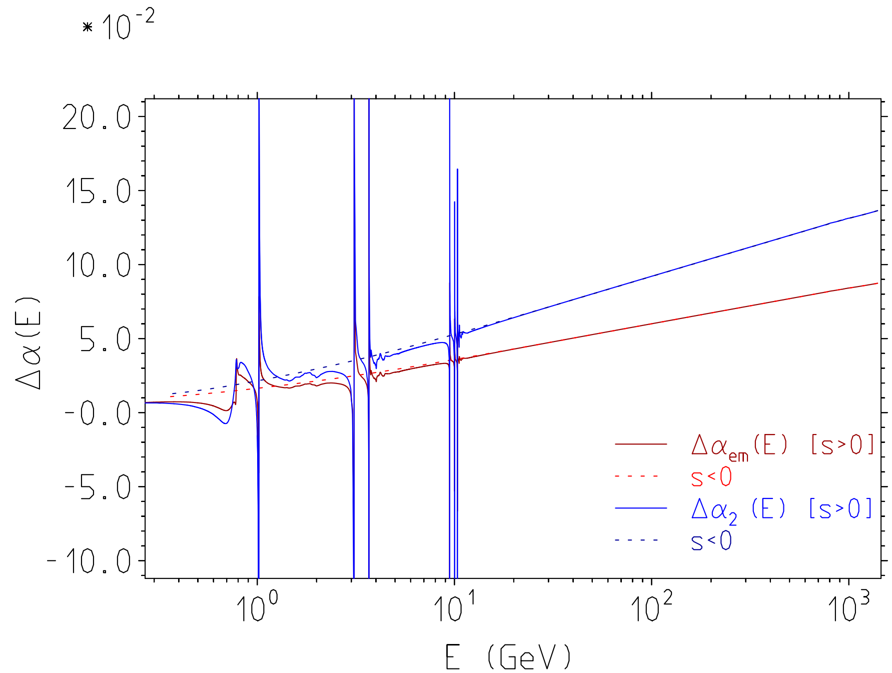
Note: gauge boson SE potentially very sensitive to New Physics (oblique corrections)

➠new physics may be obscured by non-perturbative hadronic effects; need to fix this!

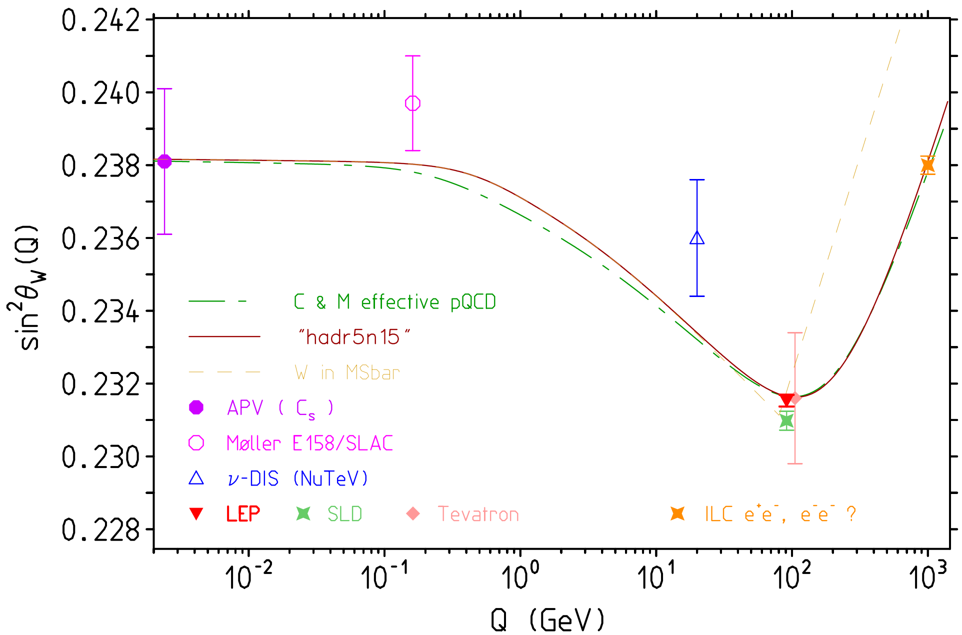


0.245
0.240
0.235
0.230
0.225
$\sin^2\theta_W(\mu)$
0.0001
0.001
0.01
0.1
1
10
100
1000
10000
μ [GeV]
Q_W(p)
Q_W(e)
NuTeV
Q_W(APV)
eDIS
Tevatron
LEP 1
SLC
LHC

PDG version

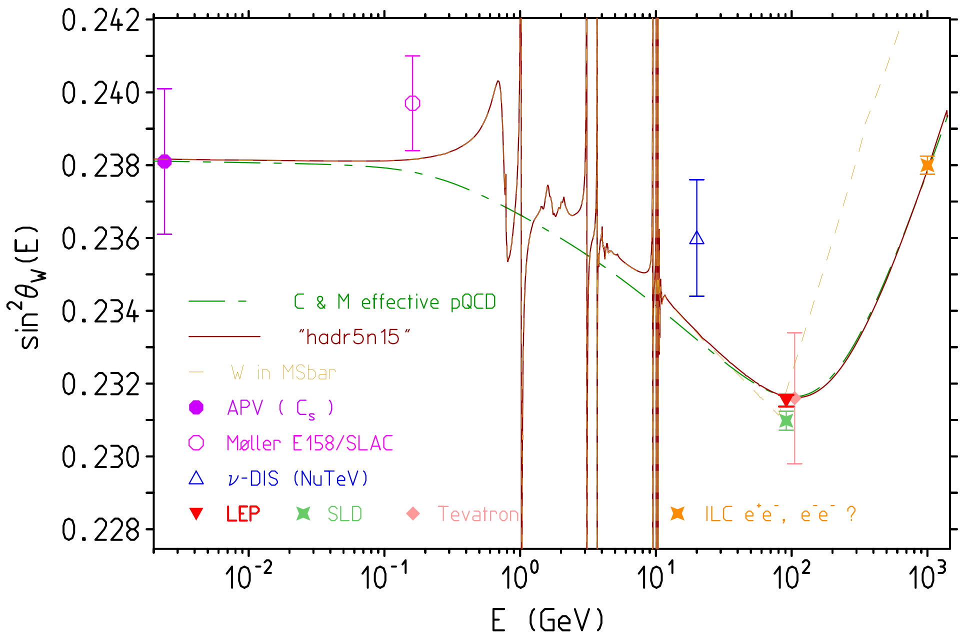


$\Delta\alpha_{\rm em}(E)$ and $\Delta\alpha_2(E)$ as functions of energy E in the time-like and space-like domain. The smooth space-like correction (dashed line) agrees rather well with the non-resonant "background" above the ϕ-resonance (kind of duality). In resonance regions as expected "agreement" is observed in the mean, with huge local deviations.



$\sin^2\Theta_W(Q)$ as a function of Q in the space-like region. Hadronic uncertainties are included but barely visible. Uncertainties from the input parameter $\sin^2\theta_W(0) = 0.23822(100)$ or $\sin^2\theta_W(M_Z^2) = 0.23156(21)$ are not shown. Future ILC/FCC measurements at 1 TeV would be sensitive to Z', H^{--} etc.

Except from the LEP and SLD points (which deviate by 1.8 σ), all existing measurements are of rather limited accuracy unfortunately!



$\sin^2\Theta_W(E)$ as a function of E in the time-like region.

$\sin^2\Theta_{\mathrm{eff}}$

exhibiting a specific dependence on the gauge boson SEs

is an excellent monitor for New Physics