

# Precision measurements of the top mass: theory vs experiment

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1. Introduction
2. Top mass definitions and determination at hadron colliders
3. Progress in theory and Monte Carlo implementation
4. Top mass definitions and determination at lepton colliders
5. Progress for top production at threshold and in the continuum
6. Conclusions

See also:

Frascati workshop on 'Top mass: challenges in definition and determination', May 2015  
<https://agenda.infn.it/conferenceDisplay.py?confId=9202>

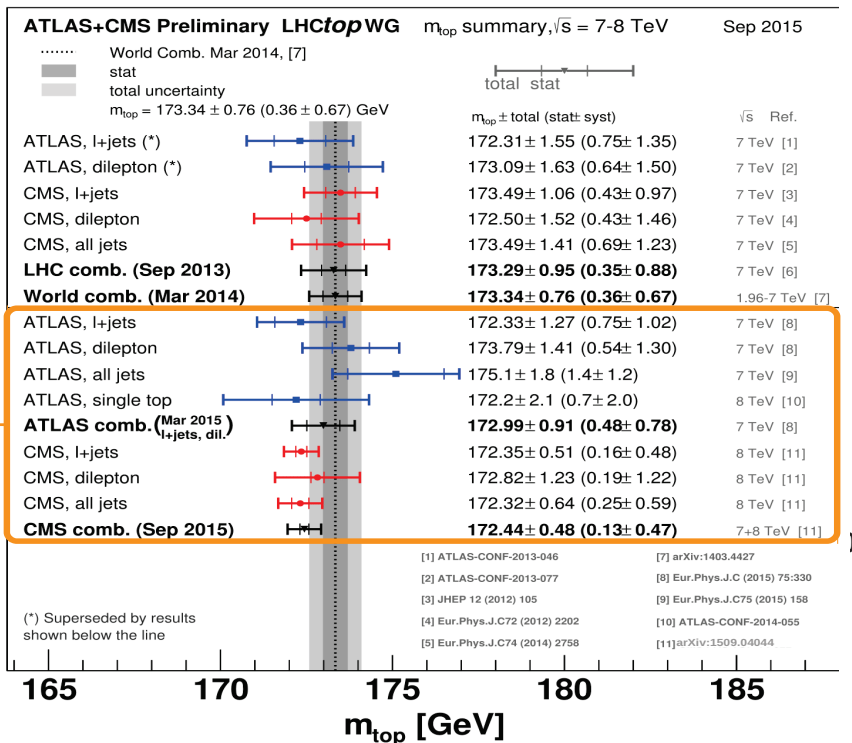
Top 2015 workshop: <http://top2015.infn.it/>

Overlaps with talks by M. Vos, J. Reuter and M. Steinhauser

The top quark is the heaviest elementary particle and plays a crucial role in the electroweak symmetry breaking, constraining the Higgs mass even before its discovery

Processes with tops are backgrounds for many BSM searches, involving heavy particles

Stability of the SM vacuum depends on top and Higgs masses (Degrassi et al, JHEP'12)



All values in GeV	CDF	D0	ATLAS	CMS	Tevatron	LHC	WA
$m_{\text{top}}$	173.19	174.85	172.65	173.58	173.58	173.28	173.34
Stat	0.52	0.78	0.31	0.29	0.44	0.22	0.27
iJES	0.44	0.48	0.41	0.28	0.36	0.26	0.24
stdJES	0.30	0.62	0.78	0.33	0.27	0.31	0.20
flavourJES	0.08	0.27	0.21	0.19	0.09	0.16	0.12
bJES	0.15	0.08	0.35	0.57	0.13	0.44	0.25
MC	0.56	0.62	0.48	0.19	0.57	0.25	0.38
Rad	0.09	0.26	0.42	0.28	0.13	0.32	0.21
CR	0.21	0.31	0.31	0.48	0.23	0.43	0.31
PDF	0.09	0.22	0.15	0.07	0.12	0.09	0.09
DetMod	<0.01	0.37	0.22	0.25	0.09	0.20	0.10
b-tag	0.04	0.09	0.66	0.11	0.04	0.22	0.11
LepPt	<0.01	0.20	0.07	<0.01	0.05	0.01	0.02
BGMC	0.10	0.16	0.06	0.11	0.11	0.08	0.10
BGData	0.15	0.19	0.06	0.03	0.12	0.04	0.07
Meth	0.07	0.15	0.08	0.07	0.06	0.06	0.05
MHI	0.08	0.05	0.02	0.06	0.06	0.05	0.04
Total Syst	0.85	1.25	1.40	0.99	0.82	0.92	0.71
Total	1.00	1.48	1.44	1.03	0.94	0.94	0.76
$\chi^2/\text{ndf}$	1.09 / 3	0.13 / 1	0.34 / 1	1.15 / 2	2.45 / 5	1.81 / 4	4.33 / 10
$\chi^2$ probability [%]	78	72	56	56	78	77	93



Theoretical uncertainties on the top mass measurement

Monte Carlo systematics (MC); modelling QCD radiation effects (Rad); colour reconnection (CR); parton distribution functions (PDF)

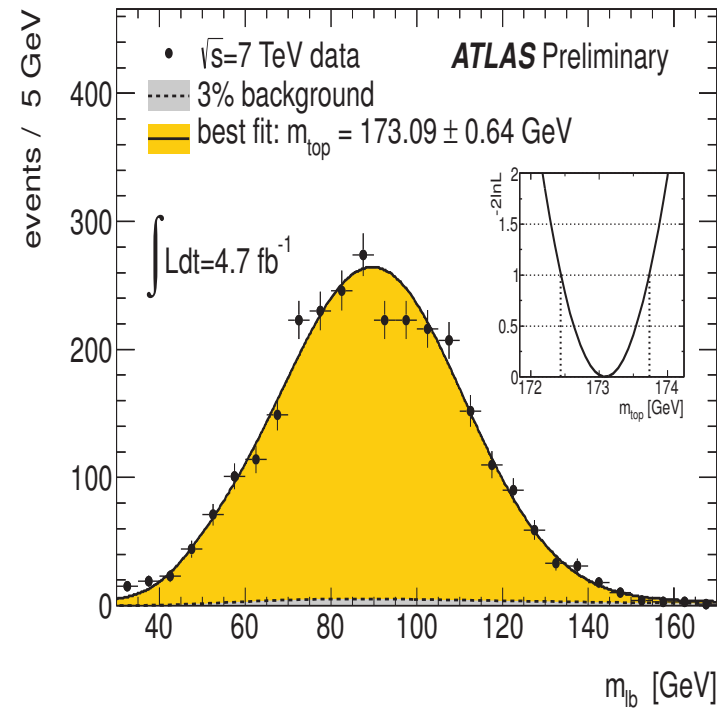
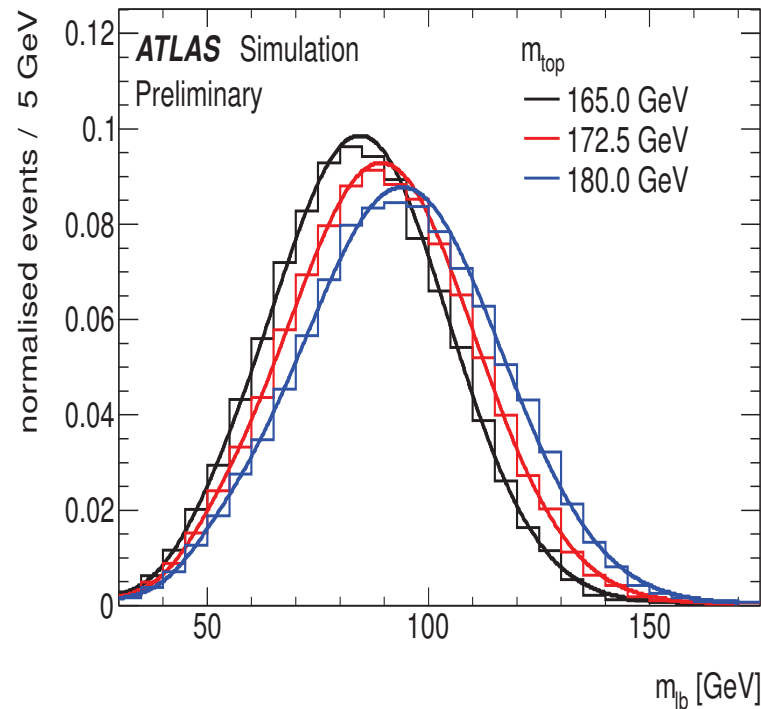
Overall theory error on world average mass:  $\Delta m_t \simeq 0.54$  GeV

Top measurements compare data with theory:  $m_t$  is the parameter in the prediction

Standard reconstruction: template, ideogram and matrix-element methods

Alternatives:  $t\bar{t}(j)$  cross section, endpoint,  $m_{b\ell}$ ,  $J/\psi$ ,  $b$ -jet energy, leptonic observables

Example: data vs MC templates and  $m_t$  ('Monte Carlo' mass) minimizes the  $\chi^2$



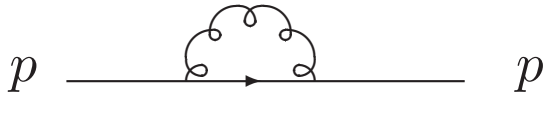
Standard generators: LO hard scattering; (N)LL parton showers; hadronization models

POWHEG and aMC@NLO: NLO  $t\bar{t}$  and single top production

aMC@NLO includes off-shell and non-resonant effects, not yet NLO decays

Improvement in POWHEG: NLO top decays and approximate treatment of top width

Top-mass definitions depend on subtraction of the UV divergences in self energy  $\Sigma(p)$

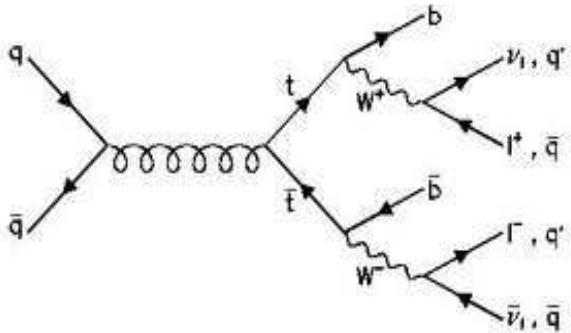


$$\Sigma = \left[ \frac{1}{\epsilon} - \gamma + \ln 4\pi - \ln \frac{m_0^2}{\mu^2} + A \right] \not{p} - \left[ 4 \left( \frac{1}{\epsilon} - \gamma + \ln 4\pi - \ln \frac{m_0^2}{\mu^2} \right) + B \right] m_0$$

$$S^R(p) = - \frac{i}{\not{p} - m_0 + \Sigma^R(p, m_0, \mu)} \Rightarrow S_{o.s.}^R(p) \sim \frac{i}{\not{p} - m_{pole}} ; S_{\overline{MS}}^R \sim \frac{i}{\not{p} - m_{\overline{MS}} - (A - B)m_{\overline{MS}}}$$

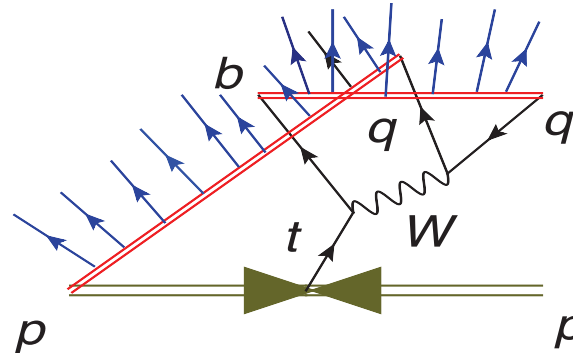
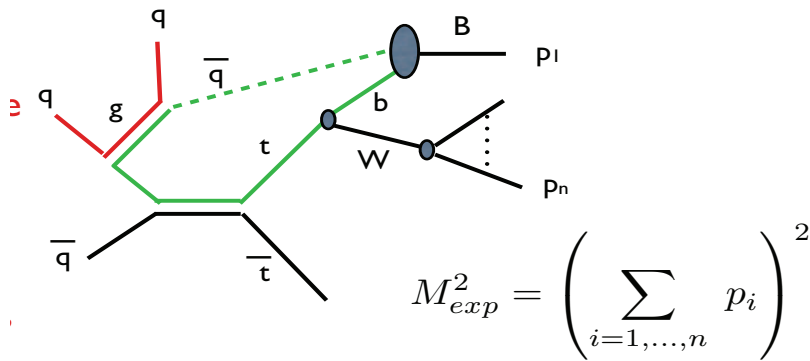
Pole mass is the pole of the propagator;  $\overline{MS}$  mass is quite far from the pole

Measurements relying on (on-shell) top decays must yield a mass close to  $m_{pole}$



$$\frac{1}{[(p_W + p_b)^2 - m_t^2]^2 + m_t^2 \Gamma_t^2} \sim \frac{1}{\pi} \delta[(p_W + p_b)^2 - m_t^2] \text{ for } \Gamma_t \ll m_t$$

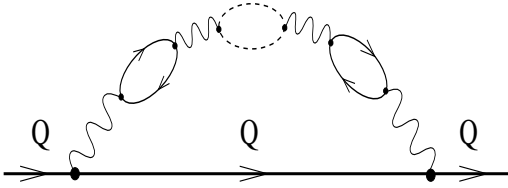
Width and colour-reconnection effects can spoil this picture



Left: M.L.Mangano, TOP 2013 workshop,

Right: S.Argyropoulos, LNF'15 workshop

## Higher-order corrections to the self energy: renormalon ambiguity



$$\Sigma(m, m) \sim m \sum_n \alpha_S^n (2b_0)^n n!$$

$$\delta m_{\text{pole}} \approx \mathcal{O}(\Lambda) \approx \mathcal{O}(100 \text{ MeV})$$

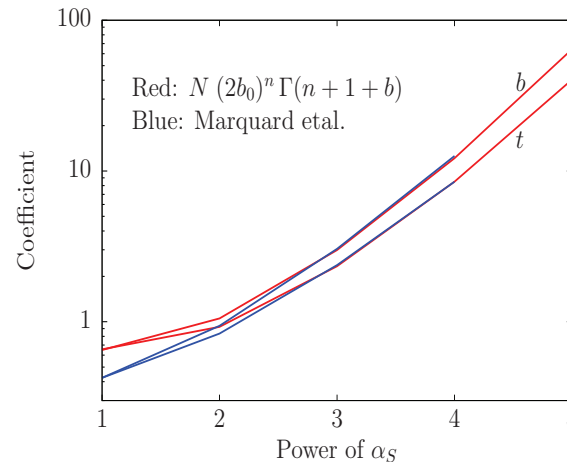
Pole vs  $\overline{\text{MS}}$  mass at 4 loops [ $\bar{m} = \bar{m}(\bar{m})$ ] (P.Marquard et al, PRL'15, talk by M.Steinhauser)

$$m_{\text{pole}} = \bar{m} [1 + 0.42 \alpha_S + 0.83 \alpha_S^2 + 2.38 \alpha_S^3 + (8.49 \pm 0.25) \alpha_S^4]; \Delta m_{\text{pole,MS}} \simeq 195 \text{ MeV}$$

Renormalon calculation (Beneke, '94) - large- $n$  expansion:

$$m_{\text{pole}} = \bar{m} \times \left( 1 + \sum_{n=0}^{\infty} r_n \alpha_S^{n+1} \right); r_n \rightarrow N (2b_0)^n \Gamma \left( n + 1 + \frac{b_1}{b_0^2} \right) \left( 1 + \sum_{k=1}^{\infty} \frac{s_k}{n^k} \right)$$

Fitting  $N$ :  $N \simeq 0.726$



Can be used to predict higher-order terms:

$$m_{\text{pole}} = (163.63 + 7.56 + 1.62 + 0.50 + 0.19 + 0.10 + \dots) \text{ GeV}$$

Minimum for  $n \sim 8-9$ :  $\Delta m \approx |r_8 \alpha_S^8 - r_9 \alpha_S^9| \approx 68 \text{ MeV}$  (P.Nason, summary talk at TOP 2015)

Attempt to address the MC mass using the SCET formalism  $Q \gg m_t \gg \Gamma_t \gg \Lambda_{\text{QCD}}$

MSR masses in terms of an infrared scale  $R$ , e.g.  $\mu_F$ ,  $\bar{m}(\mu)$ , etc. (A.Hoang et al)

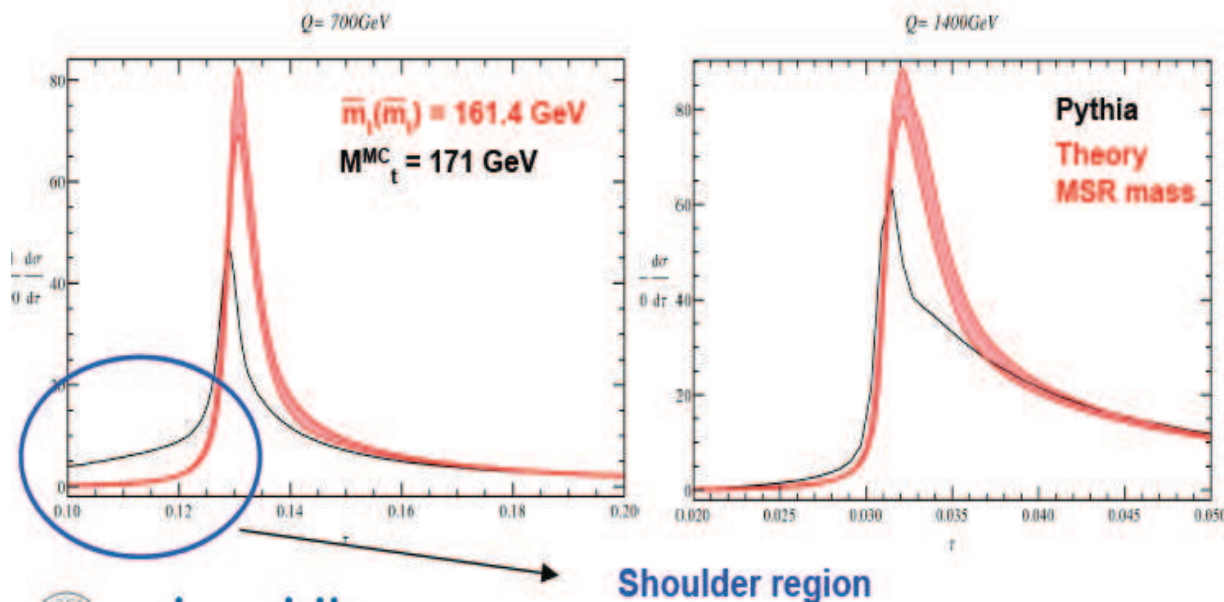
$$m^{\text{MSR}}(R) = m^{\text{pole}} - \Sigma^{\text{fin}}(R, \mu) \quad \text{for } R < m$$

Jet mass: MSR mass with  $R \sim \Gamma_t$  ;  $Q_0 \sim 1$  GeV shower cutoff

$$m_{\text{pole}} = m_J(\mu) + e^{\gamma_E} \Gamma_t \frac{\alpha_S(\mu) C_F}{\pi} \left( \ln \frac{\mu}{\Gamma_t} + \frac{1}{2} \right) + \mathcal{O}(\alpha_S^2)$$

$Q_0 \sim 1\text{-}2$  GeV  $\Rightarrow m_{\text{pole}} - m_J(Q_0) \simeq 150\text{-}200$  MeV

$m_t^{\text{MC}} = m_t^{\text{MSR}}(R) + \Delta_t(R) \Rightarrow$  fix PYTHIA mass and tune  $m_t^{\text{MSR}}(R)$



SCET at NLO+(N)NLL+ power corrections: prospects to extend to hadron colliders

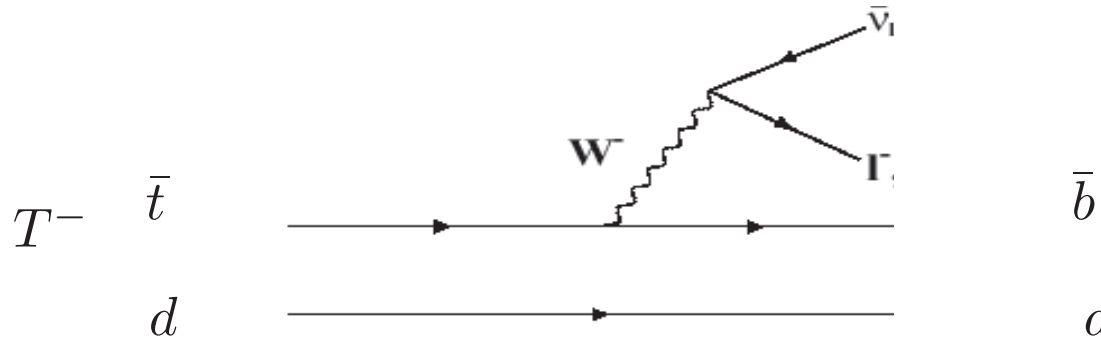
Other attempts: run HERWIG with top-hadron states (G.C., R.Chierici and M.Mangano):

Pretend that top quarks hadronize and decay via the spectator model

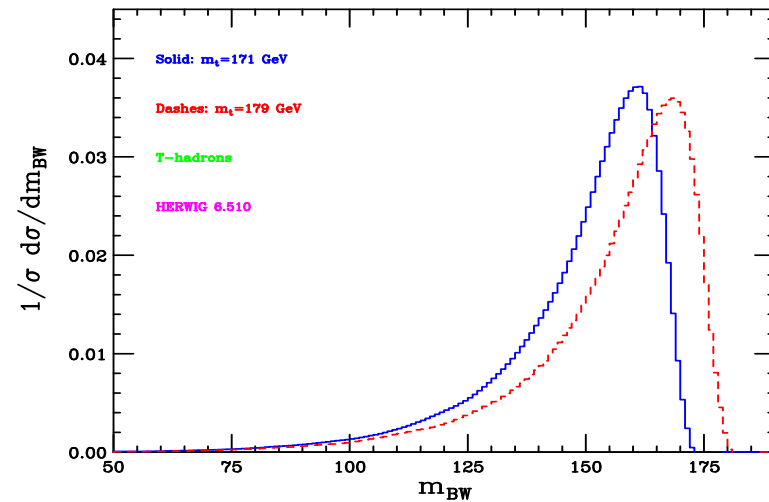
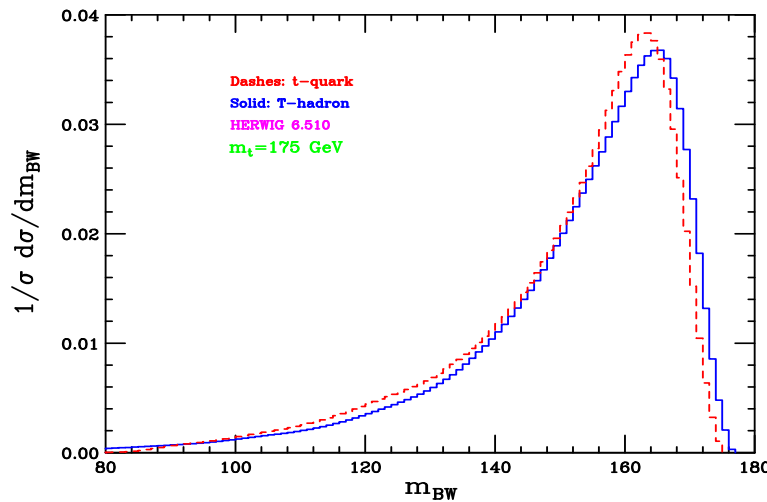
From a given observable  $R$  extract the Monte Carlo mass  $m_t^{T,MC}$

Study the same observable  $R$  with standard top samples:  $m_t^{T,MC} = m_t^{t,MC} + \Delta m_{t,T}$

Relate the  $T$ -meson mass to the pole or  $\overline{MS}$  masses by using lattice, NRQCD, etc.

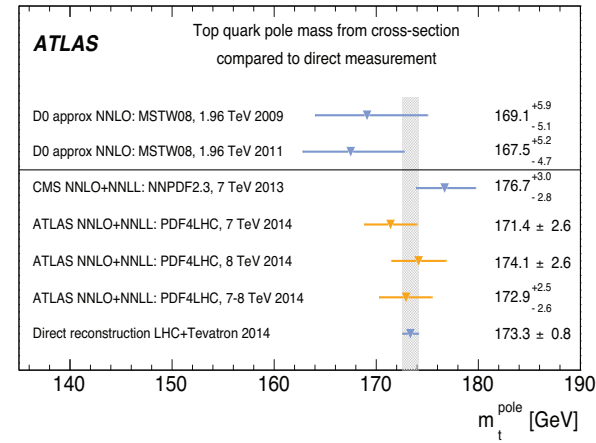
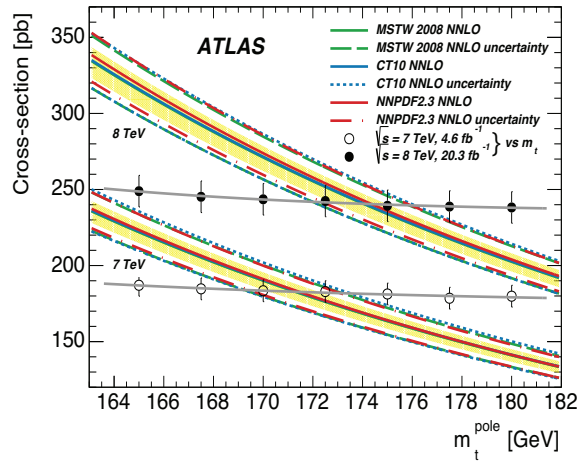


$e^+e^- \rightarrow T\bar{T} \rightarrow (bW^+\bar{q})(\bar{b}W^-q')$  collisions at  $\sqrt{s} = 1$  TeV (G.C., TOP2015 procs)



In progress: relating shift in  $\Delta m_{BW}$  to  $\Delta m_{t,T}$  and impact of colour reconnection

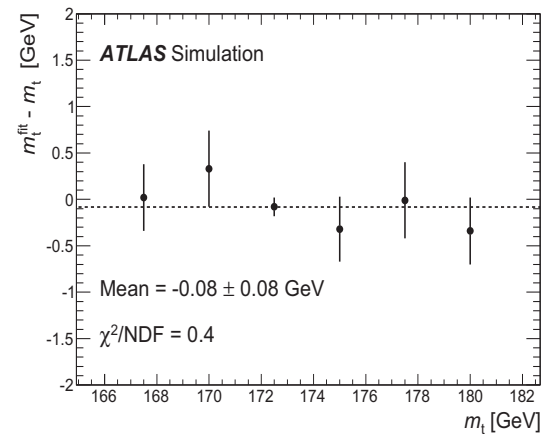
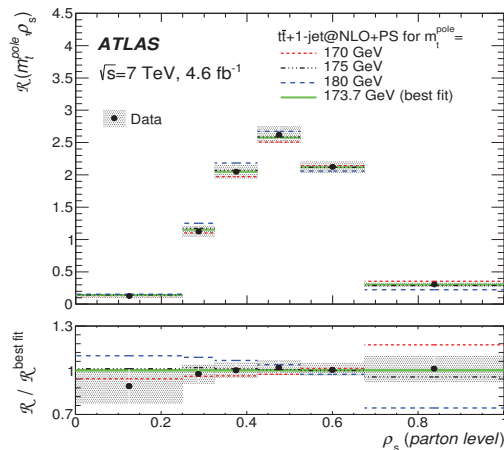
# Pole mass from total NNLO+NNLL $t\bar{t}$ cross section (Czakon, Fielder and Mitov, '13):



Recent extension to NNLO differential distributions (M.Czakon, D.Heymes and A.Mitov, 1511.00549)

NLO calculation of  $t\bar{t}$ +jet cross section with the pole mass (S.Alioli et al., '13)

POWHEG+PYTHIA, unfolding shower, hadronization and detector to recover  $t\bar{t}j$



$$m_t^{\text{pole}} = [173.1 \pm 1.5(\text{stat}) \pm 1.4(\text{syst})_{-0.5}^{+1.0}(\text{theo})] \text{ GeV}$$

Errors are expected to become smaller ( $\sim 1 \text{ GeV}$ ) thanks to higher statistics

# Perspectives on top mass measurement at the LHC (Snowmass, EPJ'14, to be updated)

## Conventional methods:

$\Delta m_t \simeq 600\text{-}700 \text{ MeV (0.3-0.4\%)}$

	Ref.[70]	Projections				
CM Energy	7 TeV	14 TeV				
Cross Section	167 pb	951 pb				
Luminosity	$5fb^{-1}$	$100fb^{-1}$		$300fb^{-1}$		$3000fb^{-1}$
Pileup	9.3	19	30	19	30	95
Syst. (GeV)	0.95	0.7	0.7	0.6	0.6	0.6
Stat. (GeV)	0.43	0.04	0.04	0.03	0.03	0.01
<b>Total</b>	<b>1.04</b>	<b>0.7</b>	<b>0.7</b>	<b>0.6</b>	<b>0.6</b>	<b>0.6</b>
Total (%)	0.6	0.4	0.4	0.3	0.3	0.3

## Endpoint method:

$\Delta m_t \simeq 500\text{-}1000 \text{ MeV (0.3-0.6\%)}$

	Ref.[76]	Projections		
CM Energy	7 TeV	14 TeV		
Cross Section	167 pb	951 pb		
Luminosity	$5fb^{-1}$	$100fb^{-1}$	$300fb^{-1}$	$3000fb^{-1}$
Syst. (GeV)	1.8	1.0	0.7	0.5
Stat. (GeV)	0.90	0.10	0.05	0.02
<b>Total</b>	<b>2.0</b>	<b>1.0</b>	<b>0.7</b>	<b>0.5</b>
Total (%)	1.2	0.6	0.4	0.3

## $J/\psi$ method ( $m_{J/\psi\ell}$ or $m_{3\ell}$ )

$\Delta m_t \simeq 600\text{-}2300 \text{ MeV (0.4-1.3\%)}$

	Ref. analysis	Projections				
CM Energy	8 TeV	14 TeV			33 TeV	100 TeV
Cross Section	240 pb	951 pb			5522 pb	25562 pb
Luminosity	$20fb^{-1}$	$100fb^{-1}$	$300fb^{-1}$	$3000fb^{-1}$	$3000fb^{-1}$	$3000fb^{-1}$
Theory (GeV)	-	1.5	1.5	1.0	1.0	0.6
Stat. (GeV)	7.00	1.8	1.0	0.3	0.1	0.1
<b>Total</b>	-	<b>2.3</b>	<b>1.8</b>	<b>1.1</b>	<b>1.0</b>	<b>0.6</b>
<b>Total (%)</b>	-	<b>1.3</b>	<b>1.0</b>	<b>0.6</b>	<b>0.6</b>	<b>0.4</b>

Lepton colliders: top production at threshold in NRQCD:  $e^+e^- \rightarrow t\bar{t}$ ,  $\sqrt{s} \sim 2m_t$

Strongly-ordered scales (hard, soft, ultrasoft):  $mv^2 \ll mv \ll m, v \ll 1$

$$R = \frac{\sigma_{t\bar{t}}}{\sigma_{\mu^+\mu^-}} = v \sum_k \left(\frac{\alpha_S}{v}\right)^k \sum_i (\alpha_S \ln v)^i \{1(\text{LL}); \alpha_S, v(\text{NLL}); \alpha_S^2, \alpha_S v, v^2(\text{NNLL})\}$$

Fixed-order N<sup>k</sup>LO ( $k = 0, 1, 2, \dots$ ) resums terms  $\alpha_S^m v^n$ , with  $m + n = 1, \dots, k + 1$

Resummation of  $\alpha_S^k \ln^k v$  (LL)  $\alpha_S^{k+1} \ln^k v, v\alpha_S^k \ln^k v$  (NLL) ...  $v \sim \alpha_S \ll 1, \alpha_S \ln v \sim 1$

State of the art NNNLO (Beneke et al,'15) and NNLL (Hoang, Stahlhofen,'14)

Pole mass is not adequate; 1S and potential-subtracted are suitable threshold masses

$$m_{1S} = \frac{1}{2} \{m [\Upsilon(1S)_{t\bar{t}}]\} ; m_{\text{PS}}(\mu_F) = m_{\text{pole}} - \frac{1}{2} \int_{|q| < \mu_F} \frac{d^3q}{(2\pi)^3} \tilde{V}(q)$$

Relating 1S and PS masses to  $\overline{\text{MS}}$  mass:

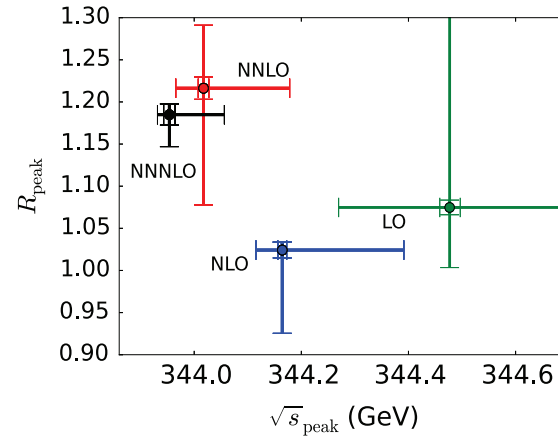
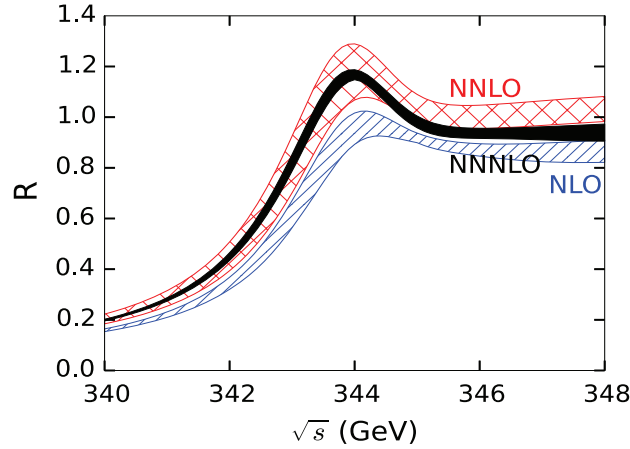
4-loop impact: 44 MeV (PS), 8 MeV (1S)

Overall uncertainty on  $\overline{\text{MS}}$  mass conversion:

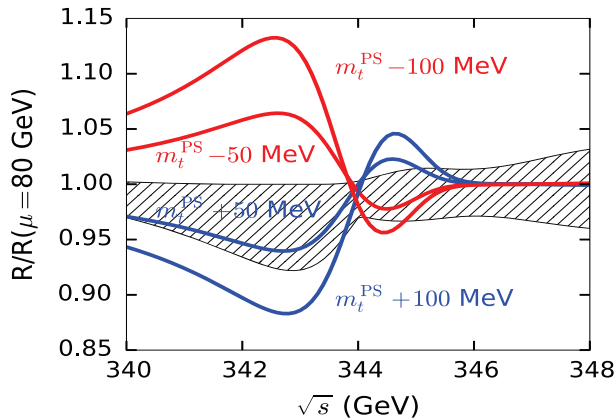
23 MeV (PS); 7 MeV (1S)

input #loops	$m^{\text{PS}} =$	$m^{1\text{S}} =$
	171.792	172.227
1	165.097	165.045
2	163.943	163.861
3	163.687	163.651
4	163.643	163.643
4 ( $\times 1.03$ )	163.637	163.637

# Threshold cross section: NNNLO fixed-order calculations (PS mass)



Peak shift: 310 MeV (LO-NLO); 150 MeV (NLO-NNLO); 64 MeV (NNLO-NNNLO)



$$\Delta m_t^{\text{PS}} = 50 \text{ MeV} \Rightarrow \Delta R/R \simeq 6\%$$

$$\text{Scale variation} \Rightarrow \Delta R/R \simeq \pm 3.8\%$$

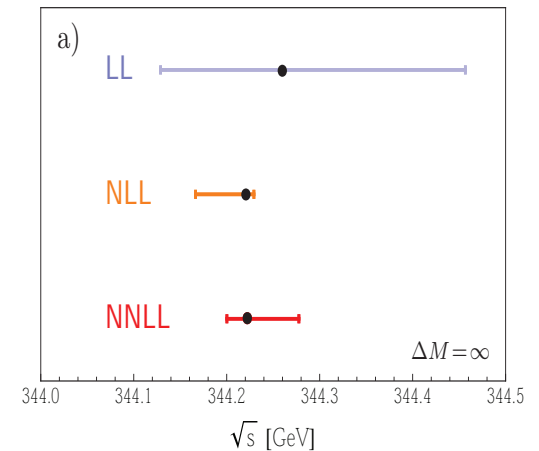
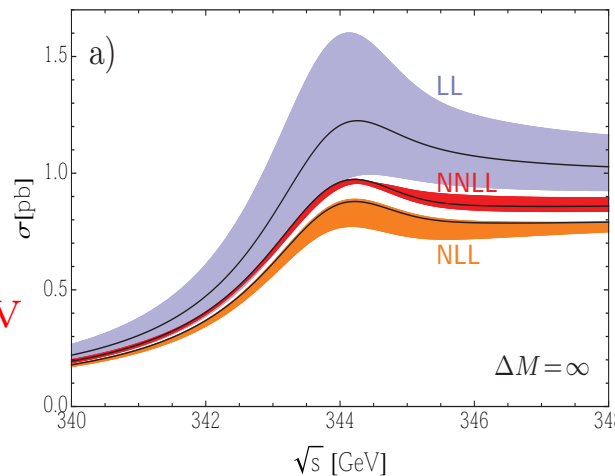
$$\Delta m_t^{\text{PS}} \leq 50 \text{ MeV feasible from } R \text{ measurement}$$

## NNLL resummation (1S mass)

A.Hoang and M.Stahlhofen'14

$$\Delta R_{\text{peak}} \simeq 80 \text{ MeV} \Rightarrow \Delta m_t^{1S} \simeq 40 \text{ MeV}$$

Matching NNNLO and NNLL?



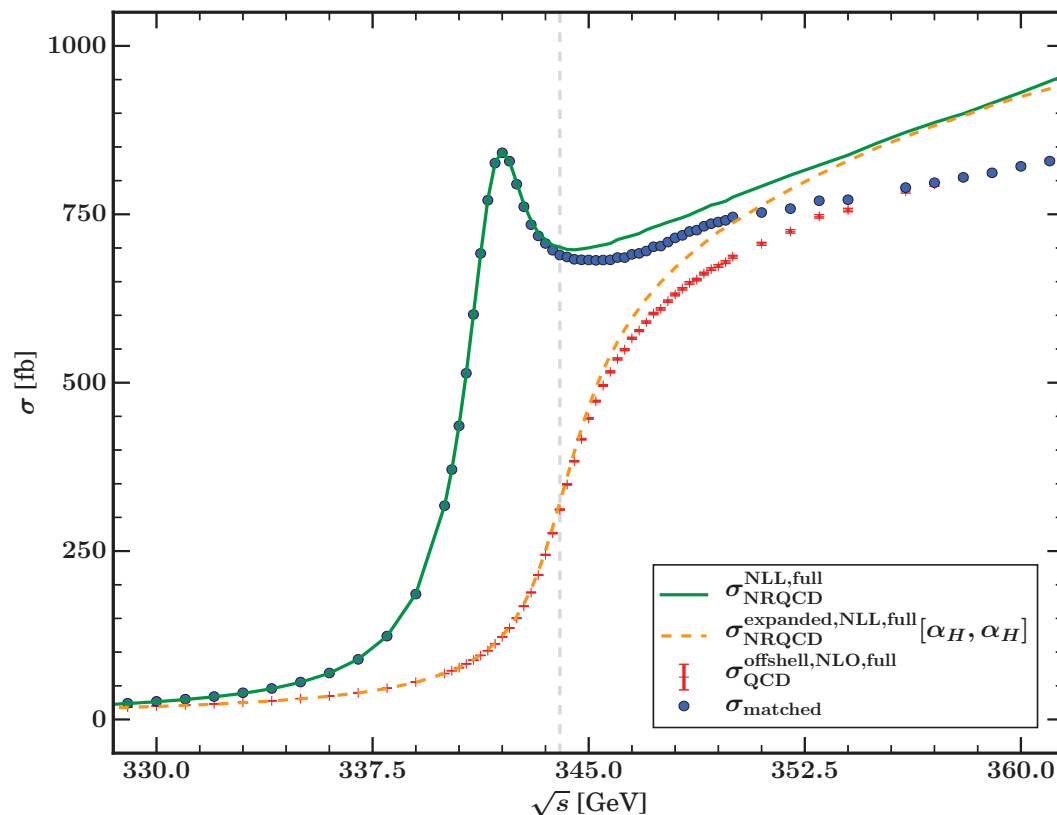
## Top threshold matching in the WHIZARD code (J.Reuter et al, '16)

Monte Carlo implementation of  $e^+e^- \rightarrow t\bar{t}$  at and above threshold at NLO

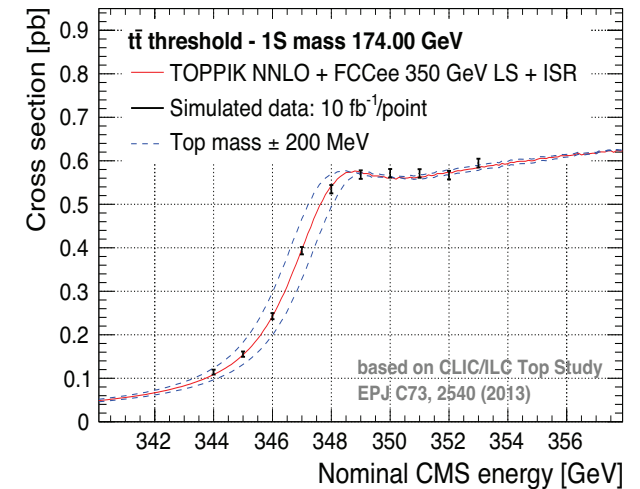
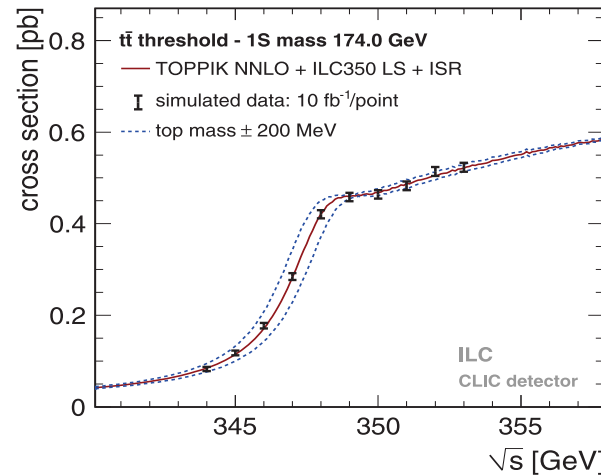
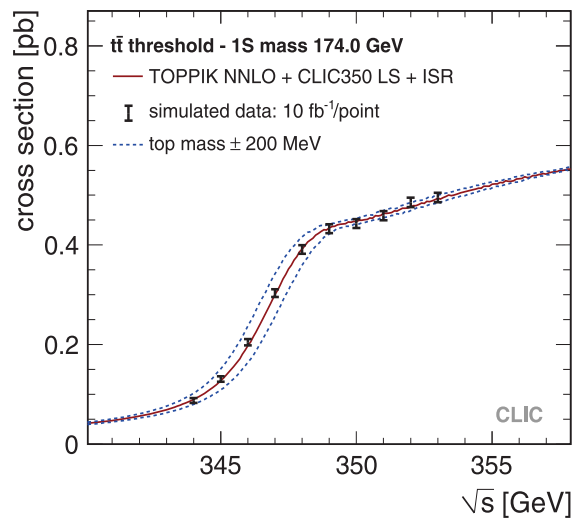
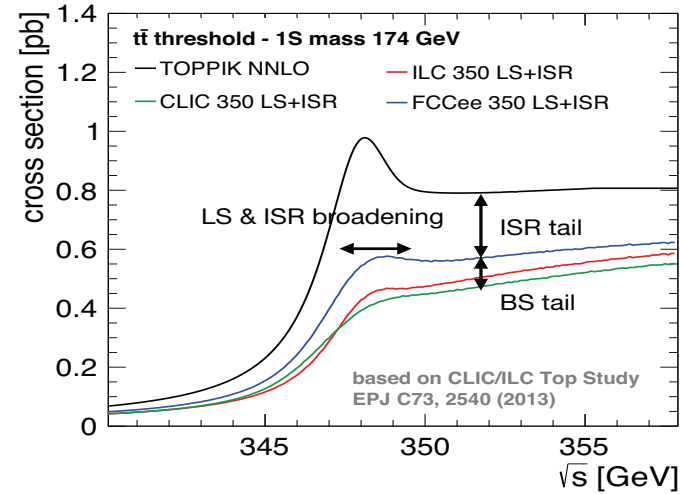
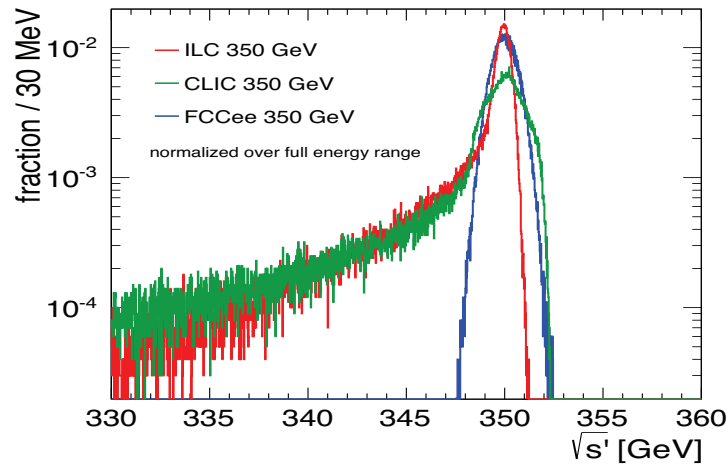
At threshold: TOPPIK code, with the  $t\bar{t}\gamma$  and  $t\bar{t}Z$  vertices given by NR form factors

Beyond threshold: relativistic NLO QCD factorizing top production and decay

Matching: add NLL and NLO and subtract  $\mathcal{O}(\alpha_S)$  from expanded NLL



# Threshold scan at $\mathcal{L}_{\text{int}} = 100 \text{ fb}^{-1}$ (K. Seidel et al, EPJ'13, F.Simon, talks at Fcc-ee/TLEP'14 and LCTOP'15)



Statistical error (1D or 2D fits) and theoretical systematics as 1-3% on normalization

$\Delta m_t^{\text{stat}} \simeq 21\text{-}34 \text{ MeV}$  (CLIC), 18-27 MeV (ILC), 16-22 MeV (Fcc-ee)

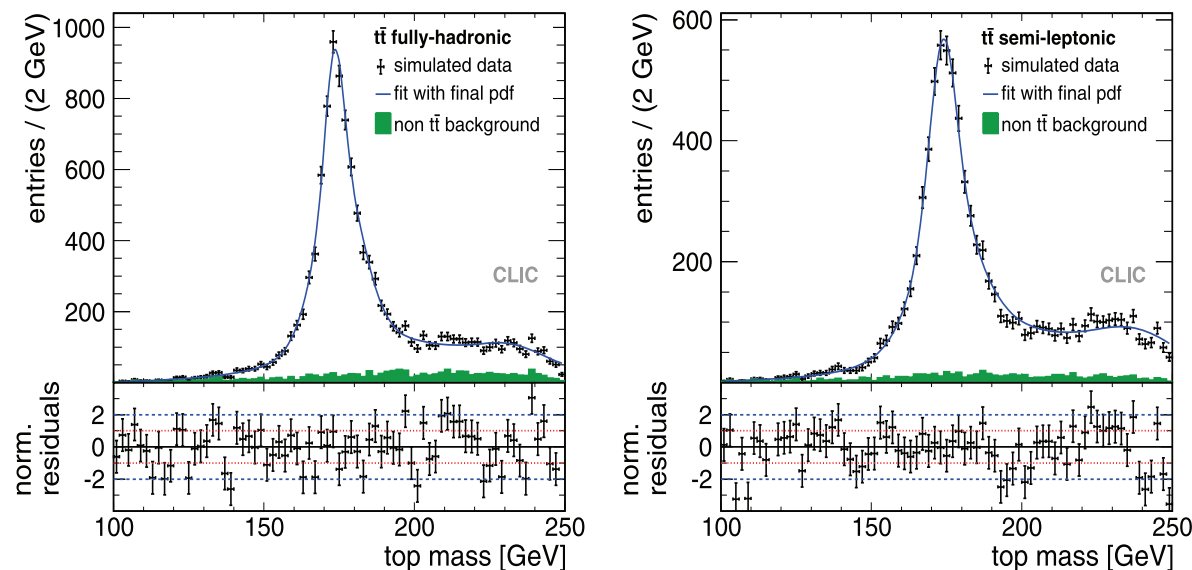
$\Delta m_t^{\text{theo}} \simeq 5/8\text{-}18/56 \text{ MeV}$  (CLIC), 5/9-18/55 MeV (ILC), 8/9-14/41 MeV (Fcc-ee)

Preliminary results with NNNLO at ILC (F.Simon, LCWS'15):  $\Delta m_t^{\text{th, scale}} \simeq 32 \text{ MeV}$

Above threshold:  $m_t$  from invariant mass of decay products (like at hadron colliders)

Analysis at CLIC, for  $\sqrt{s} = 500$  GeV and  $\mathcal{L} = 100 \text{ fb}^{-1}$

PYTHIA for signal and  $e^+e^- \rightarrow WW/ZZ$ ; WHIZARD for other backgrounds



Channel	$m_{\text{top}}$	$\Delta m_{\text{top}}$	$\Gamma_{\text{top}}$	$\Delta \Gamma_{\text{top}}$
fully-hadronic	174.049	0.099	1.47	0.27
semi-leptonic	174.293	0.137	1.70	0.40
combined	174.133	0.080	1.55	0.22

Further systematics: JES ( $\Delta m_t \simeq 50$  MeV) and colour reconnection ( $\Delta m_t \simeq 35$  MeV)

Prospects to use updated NLO+shower generators

## Conclusions

### Top mass at hadron colliders

Present top world average, based on standard reconstruction methods, exhibits an error about 700 MeV, with a recent CMS measurement yielding  $\Delta m \approx 500$  MeV

Measured mass close to pole mass, with ongoing work, based on SCET or  $T$ -hadron, aiming at estimating the theoretical uncertainty (e.g. colour reconnection)

Recent implementation of NLO+showers for top decays

Alternative methods ( $t\bar{t}(j)$  cross section, endpoint,  $J/\psi$ ,  $m_{b\ell}$ , etc.) expected to yield lower uncertainties in 13 and 14 TeV LHC runs

### Top mass at lepton colliders

Great progress in calculations for  $t\bar{t}$  production at threshold: NNNLO fixed-order and NNLL resummation with 1S and PS masses

Current theoretical errors  $\Delta m_t \leq 50$  MeV at threshold; matching between NNNLO and NNLL doable and desirable

Matching between threshold and continuum regime within the WHIZARD code at NLO

Simulation of  $t\bar{t}$  production at CLIC/ILC/Fcc- $ee$ , including background, luminosity and beamsstrahlung:  $\Delta m_t \simeq 50$  MeV (threshold) and 100 MeV (continuum) foreseen