Tools for precision electroweak calculations

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1. **Status report of WG2 (Precision Electroweak Calculations)**
2. **Open questions**
3. **Tools for electroweak precision observables**

Status report of WG2 (Precision Electroweak Calculations

Phenomenology WG2, conveners: S. Heinemeyer and A. Freitas

Main topics

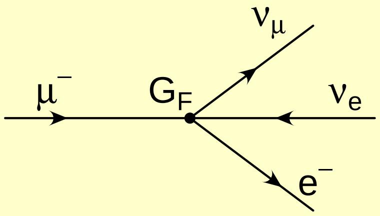
- SM corrections to electroweak precision observables and related processes
  - Fixed-order multi-loop calculations
  - QED/QCD radiation and soft/collinear photon resummation
  - Monte-Carlo tools
  - Evaluation of theory uncertainties

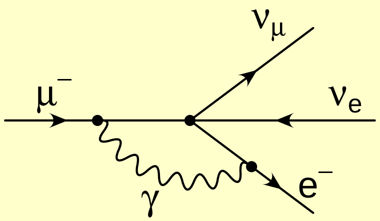
- SM input parameters
  - Robust theoretical definition (in particular for m_{t})
  - Prospects for experimental determination of m_t, m_b, α_s, $\Delta\alpha_{\mathrm{had}}$, ...
  - Theory uncertainties in parameter extraction

- Implications for BSM physics
  - (N)MSSM
  - “Beautiful” mirrors
  - **...**
  - Effective field theory

W mass

μ decay in Fermi Model



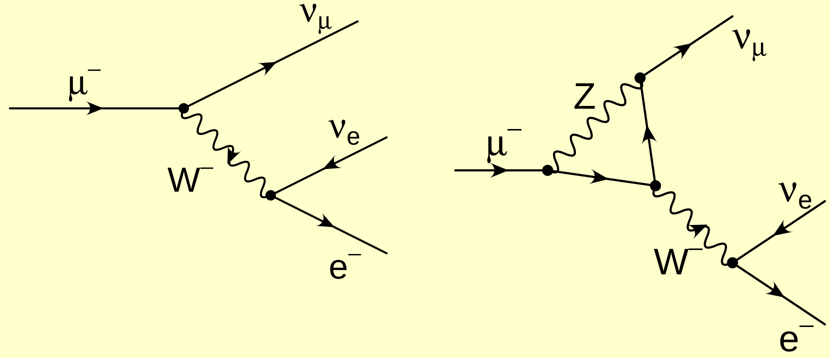


← QED corr. (2-loop)

$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} F\left(\frac{m_e^2}{m_\mu^2}\right)(1 + \Delta q)$$

Ritbergen, Stuart '98
Pak, Czarnecki '08

μ decay in Standard Model



$$\frac{G_F^2}{\sqrt{2}} = \frac{e^2}{8 s_W^2 M_W^2}(1 + \Delta r)$$

electroweak corrections

Z-pole observables

- Deconvolution of initial-state QED radiation:

$$\sigma[e^+e^- \to f\bar{f}] = \mathcal{R}_{\text{ini}}(s,s') \otimes \sigma_{\text{hard}}(s')$$

- Subtraction of γ-exchange, γ–Z interference, box contributions:

$$\sigma_{\text{hard}} = \sigma_Z + \sigma_\gamma + \sigma_{\gamma Z} + \sigma_{\text{box}}$$

- Z-pole contribution:

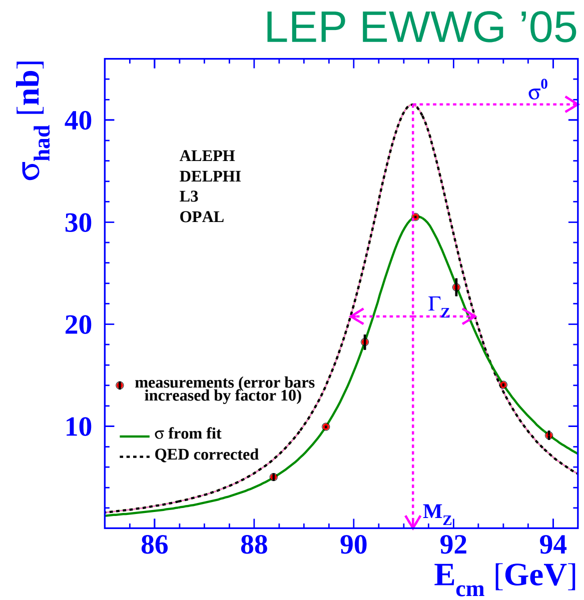
$$\sigma_Z = \frac{R}{(s-\overline{M}_Z^2)^2 + \overline{M}_Z^2\overline{\Gamma}_Z^2} + \sigma_{\text{non-res}}$$

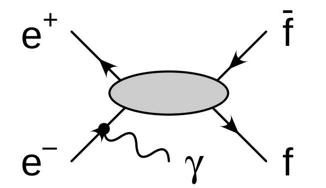
- In experimental analyses:

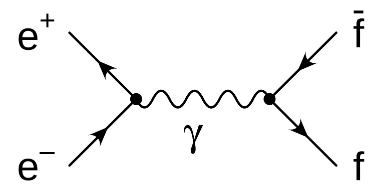
$$\sigma \sim \frac{1}{(s-M_Z^2)^2 + s^2\Gamma_Z^2/M_Z^2}$$

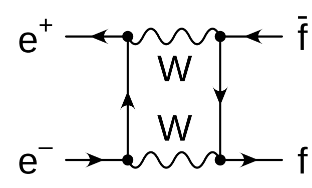
$$\overline{M}_Z = M_Z \Big/ \sqrt{1+\Gamma_Z^2/M_Z^2} \approx M_Z - 34 \text{ MeV}$$

$$\overline{\Gamma}_Z = \Gamma_Z \Big/ \sqrt{1+\Gamma_Z^2/M_Z^2} \approx \Gamma_Z - 0.9 \text{ MeV}$$









Z-pole observables

Effective weak mixing angle:

Z-pole asymmetries:

$$A_{\mathrm{FB}}^{f} \equiv \frac{\sigma(\theta < \frac{\pi}{2}) - \sigma(\theta > \frac{\pi}{2})}{\sigma(\theta < \frac{\pi}{2}) + \sigma(\theta > \frac{\pi}{2})} = \frac{3}{4}\mathcal{A}_e \mathcal{A}_f$$

$$A_{\mathrm{LR}} \equiv \frac{\sigma(\mathcal{P}_e > 0) - \sigma(\mathcal{P}_e < 0)}{\sigma(\mathcal{P}_e > 0) + \sigma(\mathcal{P}_e < 0)} = \mathcal{A}_e$$

$$\mathcal{A}_f = 2\frac{g_{Vf}/g_{Af}}{1 + (g_{Vf}/g_{Af})^2} = \frac{1 - 4|Q_f|\sin^2\theta_{\mathrm{eff}}^{f}}{1 - 4|Q_f|\sin^2\theta_{\mathrm{eff}}^{f} + 8(|Q_f|\sin^2\theta_{\mathrm{eff}}^{f})^2}$$

Most precisely measured for $f = \ell$ (also $f = b, c$)

Current uncertainties for EWPOs

	Experiment	Theory error	Main source
M_W	80.385 ± 0.015 MeV	4 MeV	α^3, $\alpha^2\alpha_\text{s}$
Γ_Z	2495.2 ± 2.3 MeV	0.5 MeV	α^2_bos, α^3, $\alpha^2\alpha_\text{s}$, $\alpha\alpha^2_\text{s}$
σ^0_had	41540 ± 37 pb	6 pb	α^2_bos, α^3, $\alpha^2\alpha_\text{s}$
$R_b \equiv \Gamma^b_\text{Z}/\Gamma^\text{had}_\text{Z}$	0.21629 ± 0.00066	0.00015	α^2_bos, α^3, $\alpha^2\alpha_\text{s}$
$\sin^2\theta^\ell_\text{eff}$	0.23153 ± 0.00016	4.5×10^{-5}	α^3, $\alpha^2\alpha_\text{s}$

Methods for theory error estimates:

- Parametric factors, $i.e.$ factors of α, N_c, N_f, ...
- Geometric progression, $e.g.$ $\frac{\mathcal{O}(\alpha^3)}{\mathcal{O}(\alpha^2)} \sim \frac{\mathcal{O}(\alpha^2)}{\mathcal{O}(\alpha)}$
- Renormalization scale dependence (often underestimates error)
- Renormalization scheme dependence (may underestimate error)

Future projections

	ILC	FCC-ee	perturb. error with 3-loop†	Param. error ILC*	Param. error FCC-ee**
$M_{\rm W}$ [MeV]	3–5	~ 1	1	2.6	1
Γ_Z [MeV]	~ 1	~ 0.1	$\lesssim 0.2$	0.5	0.06
R_b $[10^{-5}]$	15	$\lesssim 5$	5–10	< 1	< 1
$\sin^2\theta^{\ell}_{\rm eff}$ $[10^{-5}]$	1.3	0.6	1.5	2	2

† **Theory scenario:** $\mathcal{O}(\alpha\alpha_{\rm s}^2)$, $\mathcal{O}(N_f\alpha^2\alpha_{\rm s})$, $\mathcal{O}(N_f^2\alpha^2\alpha_{\rm s})$

(N_f^n = at least n closed fermion loops)

Parametric inputs:

*** ILC:** $\delta m_t = 100$ MeV, $\delta\alpha_{\rm s} = 0.001$, $\delta M_{\rm Z} = 2.1$ MeV

****FCC-ee:** $\delta m_t = 50$ MeV, $\delta\alpha_{\rm s} = 0.0001$, $\delta M_{\rm Z} = 0.1$ MeV

also: $\delta(\Delta\alpha) \sim 5 \times 10^{-5}$

SM input parameters

- m_t: Several sources of theory error
  ($e^+e^- \to t\bar{t}$ cross-section, scheme conversion, α_s)
  $\longrightarrow$ Projected future precision: $\delta_\mathrm{th} m_\mathrm{t} \sim 50$ MeV   Hoang '15

- M_W: From threshold scan of $e^+e^- \to W^{(*)}W^{(*)}$
  Current state of art: full 1-loop for $e^+e^- \to WW \to 4f$ and higher-order contribution from effective field theory
  $\longrightarrow \delta_\mathrm{th} M_\mathrm{W} \sim 3$ MeV   Actis, Beneke, Falgari, Schwinn '08
  With 2-loop and resummed higher-order terms for $e^+e^- \to WW$ and $W \to ff'$ expect $\delta_\mathrm{th} M_\mathrm{W} \sim 1$ MeV

- α_s:
  $\delta_\mathrm{exp}\alpha_s \sim 0.0001$ from measuring R_ℓ at FCC-ee

  Theory error (with fermionic electroweak 3-loop corrections):
  $\delta_\mathrm{th} R_\ell \sim 0.0015 \quad \Rightarrow \quad \delta\alpha_s^\mathrm{th} \sim 0.0002$

SM input parameters

- $\Delta\alpha_{\text{had}}$: Could be limiting factor
  a) From $e^+e^- \to$ had. using dispersion relation
  Current: $\delta(\Delta\alpha_{\text{had}}) \sim 10^{-4}$
  Improvement to $\delta(\Delta\alpha_{\text{had}}) \sim 5 \times 10^{-5}$ likely
  b) Direct determination at FCC-ee from $e^+e^- \to \mu^+\mu^-$ off the Z peak
  (i.e. $A_{\text{FB}}^{\mu\mu}$ at $\sqrt{s} \sim 88$ GeV and $\sqrt{s} \sim 95$ GeV)
  $\longrightarrow \delta_{\text{th}}(\Delta\alpha_{\text{had}}) \sim 3 \times 10^{-5}$ Janot '15

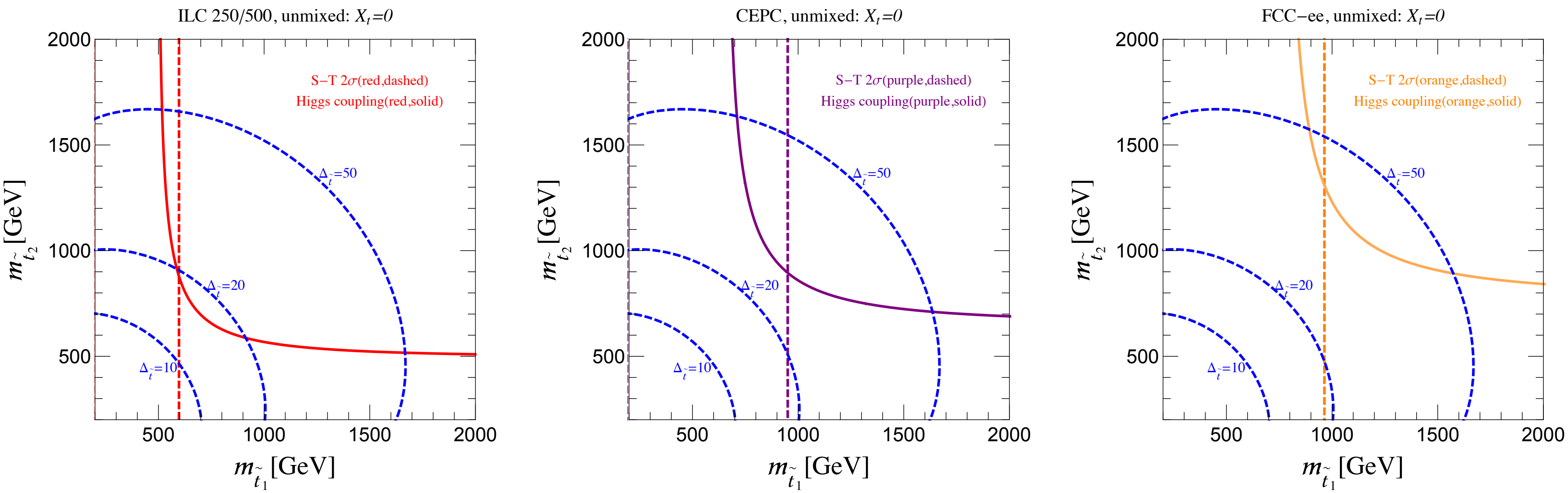
  Requires high-precision theory prediction for $e^+e^- \to \mu^+\mu^-$ including γ-exchange and box contributions:
  $\longrightarrow$ $\delta_{\text{th}}(\Delta\alpha_{\text{had}}) \sim$ few $\times 10^{-4}$ (existing results)
  $\delta_{\text{th}}(\Delta\alpha_{\text{had}}) \sim 10^{-4}$ (available methods)
  $\delta_{\text{th}}(\Delta\alpha_{\text{had}}) \sim$ few $\times 10^{-5}$ (reasonable future developments)

Implications for BSM physics

Indirect constraints on MSSM:

Fan, Reece, Wang ’14

Constraints on stop masses, no mixing ($\Delta_{\tilde{t}}$ = fine-tuning measure)

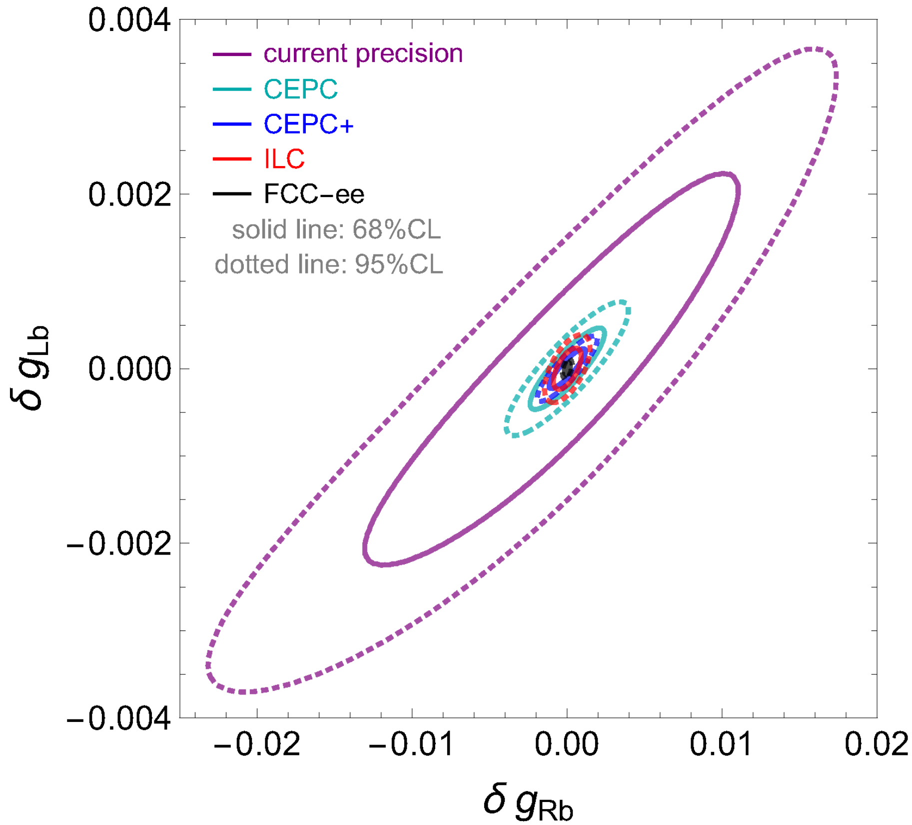


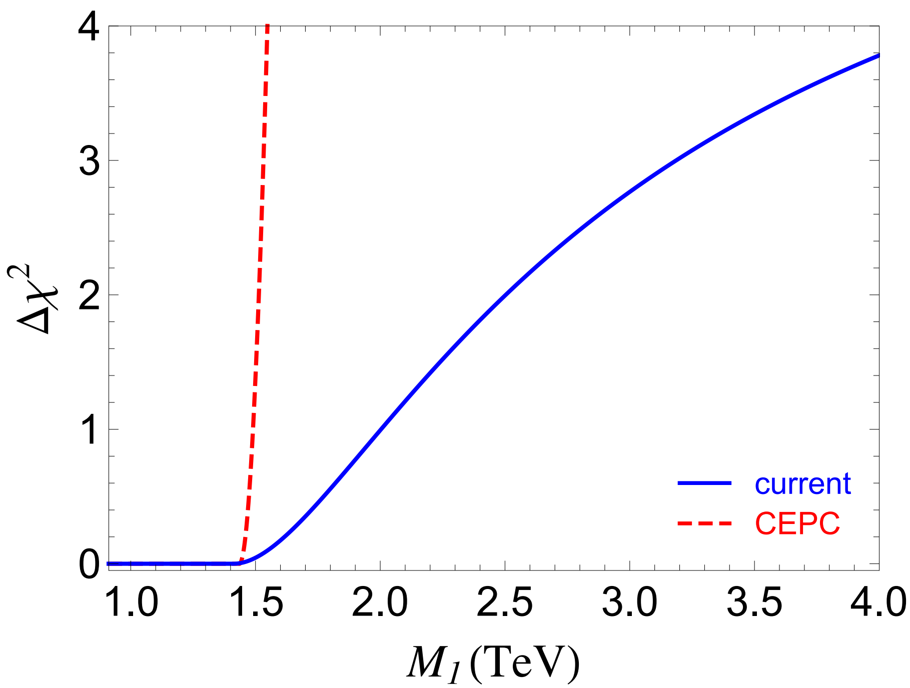
Indirect constraints on beautiful mirrors:

Modified $Zb\bar{b}$ couplings due to vector-like b partners

Choudhury, Tait, Wagner '01

Gori, Gu, Wang '15





Implications for BSM physics

Effective field theory: $\mathcal{L} = \sum_i \frac{c_i}{\Lambda^2}\mathcal{O}_i + \mathcal{O}(\Lambda^{-3})$ $(\Lambda \gg M_Z)$

$$\mathcal{O}_{\phi 1} = (D_\mu \Phi)^\dagger \Phi \, \Phi^\dagger (D^\mu \Phi) \qquad \alpha \Delta T = -\frac{v^2}{2}\frac{c_{\phi 1}}{\Lambda^2}$$

$$\mathcal{O}_{BW} = \Phi^\dagger B_{\mu\nu} W^{\mu\nu} \Phi \qquad \alpha \Delta S = -e^2 v^2 \frac{c_{BW}}{\Lambda^2}$$

$$O_{LL}^{(3)e} = (\bar{L}_L^e \sigma^a \gamma_\mu L_L^e)(\bar{L}_L^e \sigma^a \gamma^\mu L_L^e) \qquad \Delta G_F = -\sqrt{2}\frac{c_{LL}^{(3)e}}{\Lambda^2}$$

$$O_R^f = i(\Phi^\dagger \overleftrightarrow{D}_\mu \Phi)(\bar{f}_R \gamma^\mu f_R) \qquad f = e, \mu\, \tau, b, lq$$

$$O_L^F = i(\Phi^\dagger \overleftrightarrow{D}_\mu \Phi)(\bar{F}_L \gamma^\mu F_L) \qquad F = \binom{\nu_e}{e}, \binom{\nu_\mu}{\mu}, \binom{\nu_\tau}{\tau}, \binom{u, c}{d, s}, \binom{t}{b}$$

$$O_L^{(3)F} = i(\Phi^\dagger \overleftrightarrow{D}_\mu^a \Phi)(\bar{F}_L \sigma_a \gamma^\mu F_L)$$

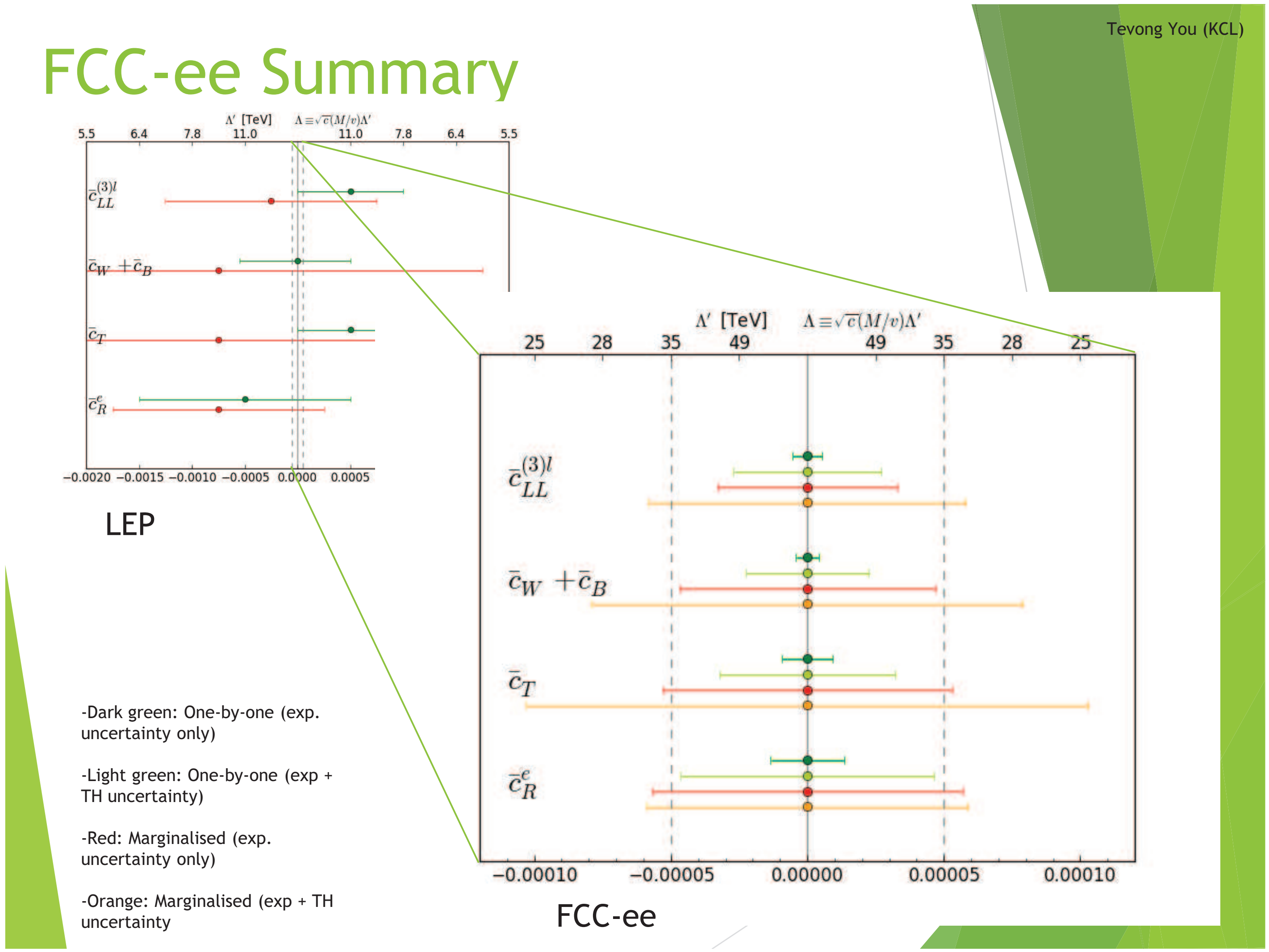
More operators than EWPOs

$\longrightarrow$ Some can be constrained by $W \to \ell\nu$, had., $e^+e^- \to W^+W^-$

Implications for BSM physics

EFT description (dimension-6 operators):

Model-independent, but more general than S/T/U parameters



Ellis, You '15

Open questions

- Many methods for evaluating **theory error** and no clear statistical interpretation
  $\longrightarrow$ Ideally we want $\Delta_{\text{th}} \ll \Delta_{\text{exp}}$
  But: Theory error may be limiting factor for ILC/FCC-ee precision

- Order of calculations required for **EWPO**
  (electroweak 3-loop + leading 4-loop/5-loop?)

- Order of calc. required for **other processes** used to extract M_{W}, $\Delta\alpha_{\text{had}}$, etc.
  (electroweak 2-loop + leading 3-loop)

- Need consistent use of **short-distance definition** of m_{t} across all sectors

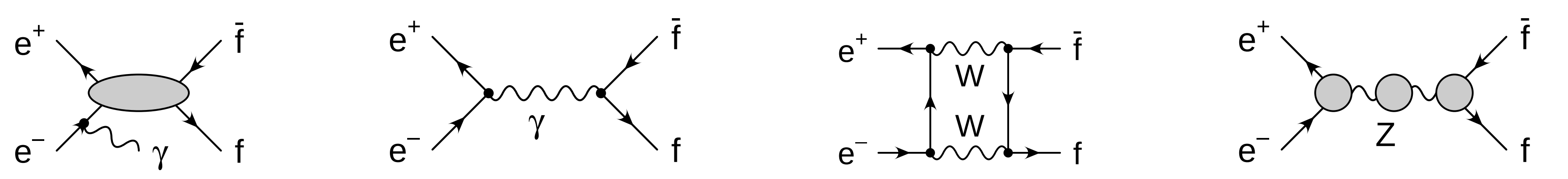
- **MC Tools:** Control of QED effects to sub-permille level is challenge,
  but progress is underway

- Evaluation of **new physics reach** including all theory and parametric errors

Tools for electroweak precision observables

“Analytical” tools for $e^+e^- \to f\bar{f}$

- State of the art: `Zfitter 6.42` — Bardin et al. ’99, Arbuzov et al. ’05
  Older code: `TOPAZ0` — Montagna, Nicrosini, Passarino, Piccinini ’98,01
- Describes true observables ($\sigma_{e^+e^-\to f\bar{f}}(s)$, etc.)
  and pseudo-observables (Γ_Z, σ^0_{had}, $\mathcal{A}_f$, etc.)
- Final-state QED and QCD corrections at $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha\alpha_s)$, $\mathcal{O}(\alpha_s^3)$
- Deconvolution of initial-state and initial-final QED radiation
  at $\mathcal{O}(\alpha)$, $\mathcal{O}(\alpha^2 L)$ and $\mathcal{O}(\alpha^3 L^2)$ ($L \equiv \log(s/m_e^2)$)
- Full NLO electroweak corrections for $e^+e^- \to f\bar{f}$
- Partial $\mathcal{O}(\alpha^2)$ and higher-order electroweak corrections



"Analytical" tools for $e^+e^- \to f\bar{f}$

Drawbacks:

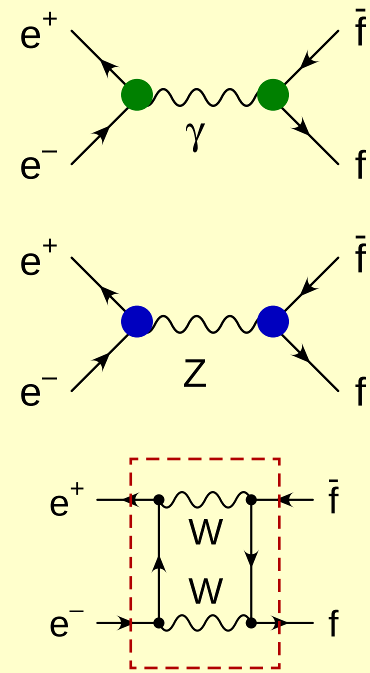
- Not all available NNLO and higher-order corrections implemented (code structure makes implementation difficult)
- For consistent treatment beyond NLO, need expansion of $\mathcal{A}[e^+e^- \to \mu^+\mu^-]$ about $s_0 = M_Z^2 - iM_Z\Gamma_Z$:

$$\mathcal{A}[e^+e^- \to f\bar{f}] = \frac{R}{s-s_0} + S + (s-s_0)T + \ldots$$

$$R = g_Z^e(s_0)\,g_Z^f(s_0)$$

$$S = \left[\frac{1}{M_Z^2} g_\gamma^e g_\gamma^f + g_Z^e g_Z^{f\,\prime} + g_Z^{e\,\prime} g_Z^f + S_{\rm box}\right]_{s=s_0}$$

$g_V^f(s)$: effective $Vf\bar{f}$ couplings



At NNLO: Need R at $\mathcal{O}(\alpha^2)$, S at $\mathcal{O}(\alpha)$, etc.

Monte-Carlo tools for $e^+e^- \to f\bar{f}$

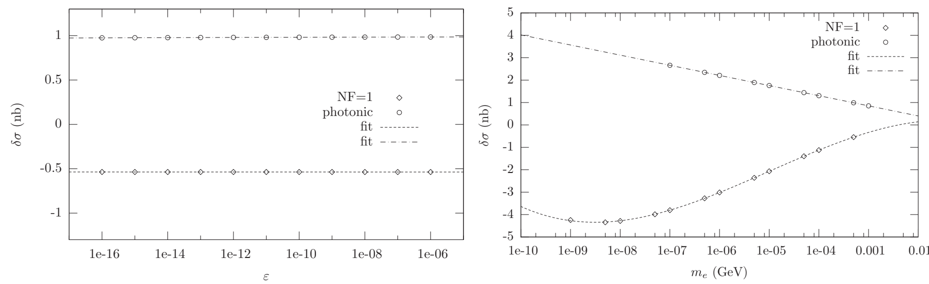
- State of the art: `KKMC`, `BabaYaga` — Jadach, Ward, Wąs '13; Carloni Calame et al. '12
- YFS exponentiation for QED radiation, approximate NNLO QED
- currently $\mathcal{O}(0.1\%)$ precision, $\mathcal{O}(0.01\%)$ feasible in (near) future, but more may be needed for FCC-ee

Comparison with (a subset of) NNLO

G. Balossini et al., NPB758 (2006) 227

Comparison of $\sigma_{SV}^{\alpha^2}$ calculation of BabaYaga@NLO with

- Penin (photonic): switching off the vacuum polarisation contribution in BabaYaga@NLO, as a function of the logarithm of the soft photon cut–off (left plot) and of a fictitious electron mass (right plot)



- ★ differences are infrared safe, as expected
- ★ $\delta\sigma(\text{photonic})/\sigma_0 \propto \alpha^2 L$, as expected
- Numerically, for various selection criteria at the Φ and B factories

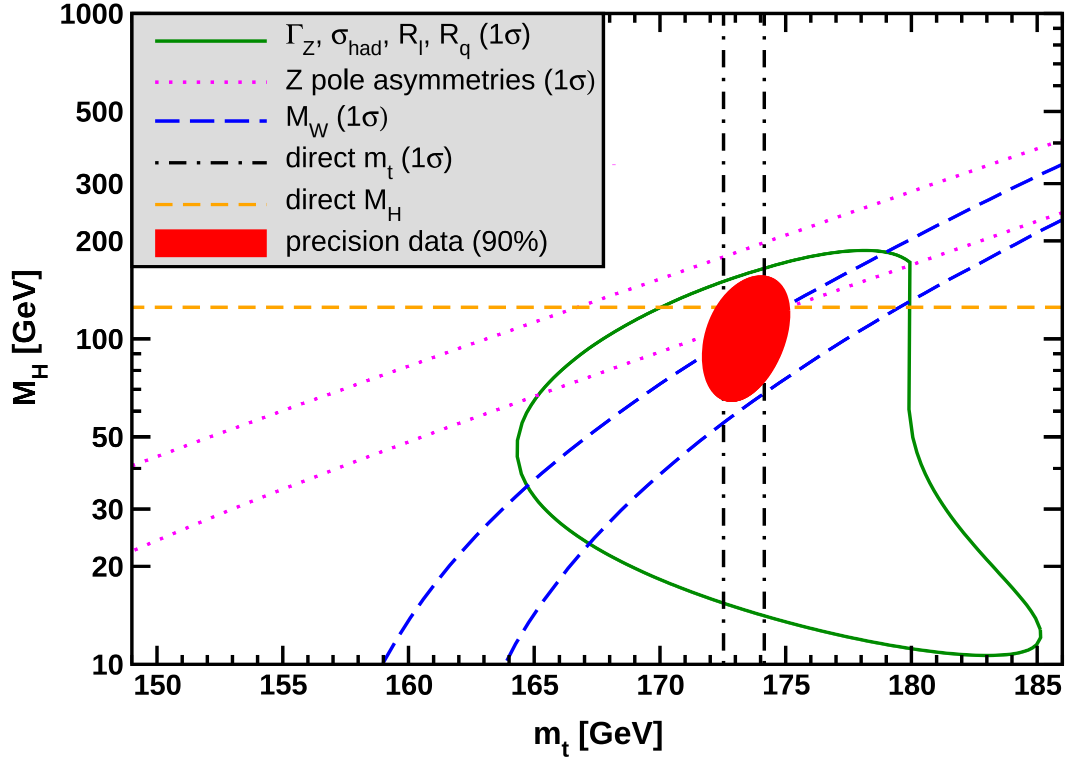
$$\sigma_{SV}^{\alpha^2}(\text{Penin}) - \sigma_{SV}^{\alpha^2}(\text{BabaYaga@NLO}) < 0.02\% \times \sigma_0$$

F. Piccinini

- State of the art: `GAPP`, `Gfitter`, Ciuchini et al.

  Erler '00,11
  Baak et al. '14
  Ciuchini, Franco, Mishima, Silvestrini '13

- Based on pseudo-observables (Γ_Z, $\sigma^0_{\rm had}$, $\mathcal{A}_f$, etc.)
- Include all available NNLO and higher-order corrections
- Constraints on various types of new physics and/or of d=6 operators



Erler '16

Conclusions

EWPOs:

- FCC-ee will reduce exp. error by factor $\gtrsim 10$ compared to LEP/SLC
  $\longrightarrow$ Current SM theory calculations not sufficient
  $\longrightarrow$ 3-loop and partial 4-loop (5-loop?) corrections needed!

- Good control over input parameters $m_{\rm t}$, $M_{\rm W}$, $\alpha_{\rm s}$ and $\Delta\alpha_{\rm had}$ is crucial
  $\longrightarrow$ New ideas may be helpful
  $\longrightarrow$ Probably limited by theory uncertainties!

- Need for new/improved computer tools:
  $\longrightarrow$ Monte-Carlo methods for multiple photon corrections
  $\longrightarrow$ Consistent complex pole expansion for electroweak part