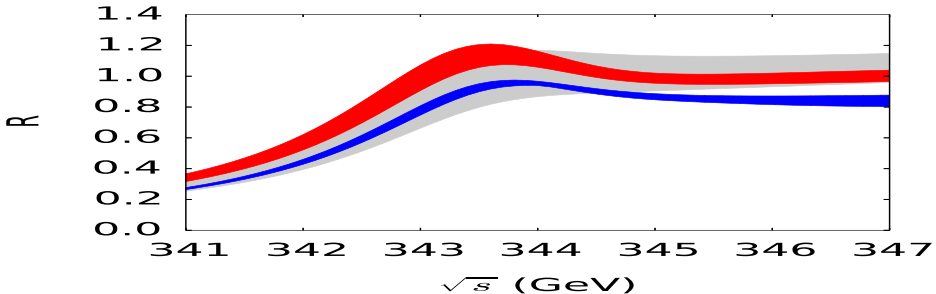


# Top quark threshold production and the top quark mass

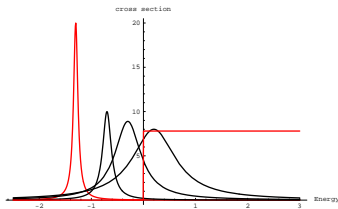
FCC-ee Mini-Workshop “Physics Behind Precision”, CERN, February 2-3, 2016

Matthias Steinhauser | TTP Karlsruhe

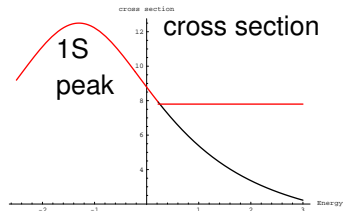


# I. $\sigma(e^+e^- \rightarrow t\bar{t})$ to N<sup>3</sup>LO

$$“ e^+ e^- \rightarrow b \bar{b} ”$$

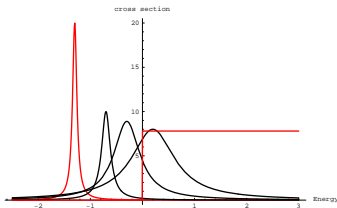


$$“ e^+ e^- \rightarrow t \bar{t} ”$$

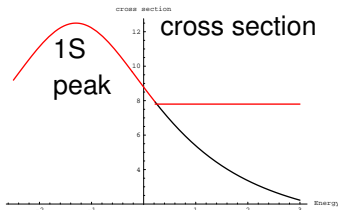


Energy

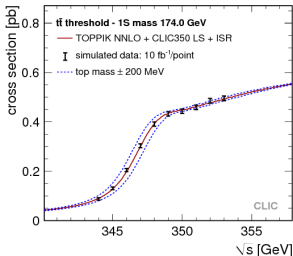
$$“ e^+ e^- \rightarrow b \bar{b} ”$$



$$“ e^+ e^- \rightarrow t \bar{t} ”$$



Energy



$$\delta m_t \sim 100 \text{ MeV}$$

$$\delta \Gamma_t \sim 30 \text{ MeV}$$

$$\delta \alpha_s \sim 0.001$$

$$\delta y_t \sim 30\%$$

[Martinez,Miquel'02;

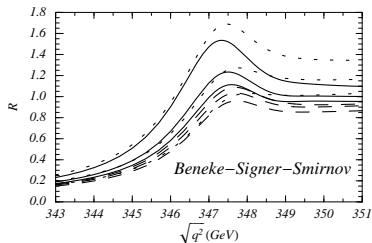
Seidel,Simon,Tesar,Poss'13;

Horiguchi,Ishikawa,Suehara,

Fujii,Sumino,Kiyo,Yamamoto'14]

# $e^+e^- \rightarrow t\bar{t}$ at NNLO: 2000

[Hoang,Beneke,Melnikov,Nagano,Ota,Penin,Pivovarov,Signer,Smirnov,Sumino,Teubner,Yakovlev,Yelkhovsky'00]



- NNLO corrections large
- stabilization of **peak position** (“threshold mass”)
- no stability in **normalization** of peak

$\Gamma_t$  large  $\Leftrightarrow$  perturbation theory must work!

# $\sigma_{\text{tot}}(e^+ e^- \rightarrow t\bar{t})$ in pNRQCD



$$\sigma(s) = \sigma_0 \text{Im} \left[ \Pi(q^2 = s + i\epsilon) + Z \text{ boson contr.} \right]$$

$$\Pi = \frac{N_c}{2m_t^2} c_V \left[ c_V - \frac{E}{m_t} \left( c_V + \frac{d_V}{3} \right) \right] G(E) + \dots$$

⇒ needed:

1. matching coefficients  $c_V$  1 loop: [Källen, Sarby'55]

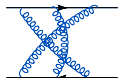
2 loops: [Czarnecki,Melnikov'97; Beneke,Signer,Smirnov'97]; 3 loops: [Marquard,Piclum,Seidel,Steinhauser'06'08'14]

and  $d_V$  1 loop: [Luke,Savage'98]

2.  $G(E)$  N<sup>3</sup>LO: [Beneke,Kiyo,Penin'07; Beneke,Kiyo'08; Beneke,Kiyo,Schuller'13]

- $N^3\text{LO}$  correction to static potential ( $a_3$ )

[Smirnov,Smirnov,Steinhauser'08; Smirnov,Smirnov,Steinhauser'09; Anzai,Kiyo,Sumino'09]



- ultra-soft corrections to  $G(E)$  [Beneke,Kiyo'08]

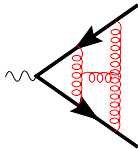
- Coulomb contribution to third order to  $G(E)$  [Beneke,Kiyo,Schuller'05]

- Single and double non-Coulomb insertion for  $G(E)$



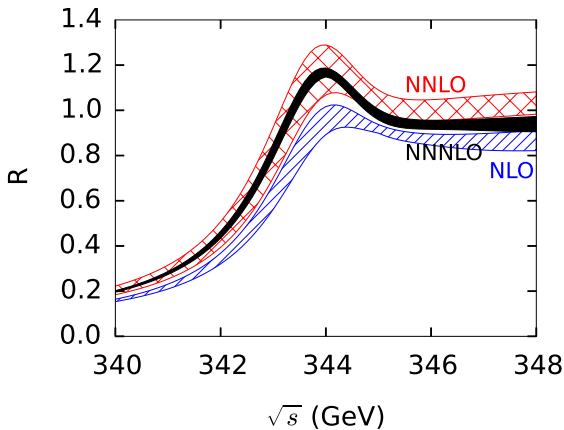
[Beneke,Kiyo,Schuller]

- QCD-NRQCD matching for vector current to 3 loops ( $c_V^{(3)}$ )



[Marquard,Piclum,Seidel,Steinhauser'06'08'14]

$$\sigma(e^+e^- \rightarrow t\bar{t})$$



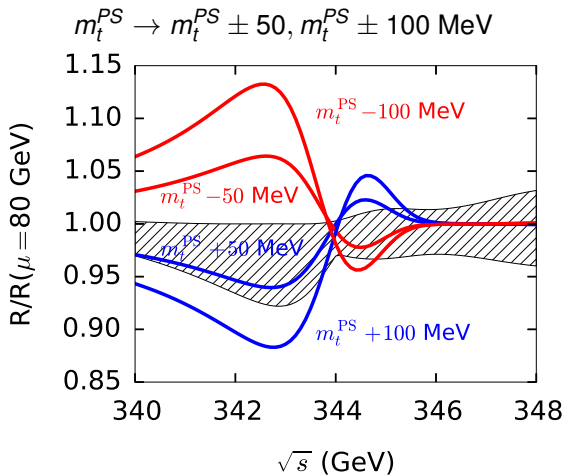
$$50 \text{ GeV} \leq \mu \leq 350 \text{ GeV}$$

$$m_t^{\text{PS}} = 171.3 \text{ GeV}$$

$$R = \frac{\sigma(e^+e^- \rightarrow t\bar{t})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

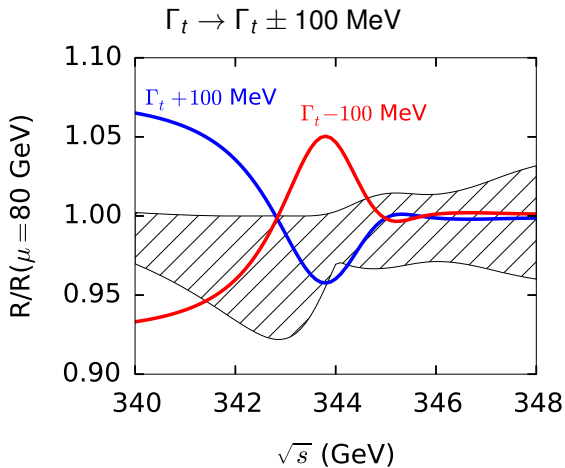
[Beneke,Kiyo,Marquard,Penin,Piclum,Steinhauser'15]

# Parameter variation



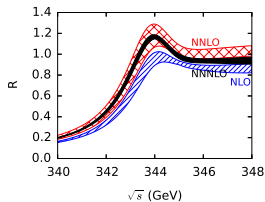
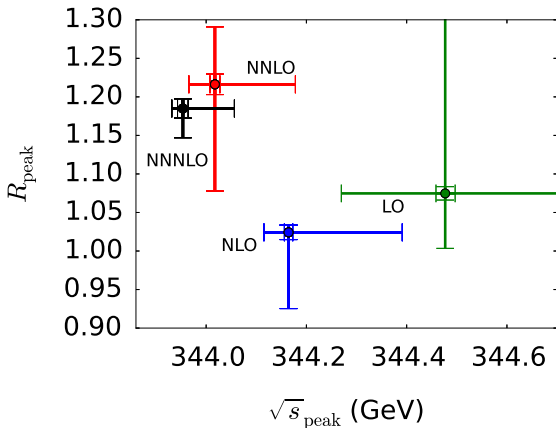
$$50 \text{ GeV} \leq \mu \leq 350 \text{ GeV} \leftrightarrow \pm 3\%$$

# Parameter variation



[Beneke,Kiyo,Marquard,Penin,Piclum,Steinhauser'15]

# $R_{\text{peak}}$ vs $\sqrt{s_{\text{peak}}}$

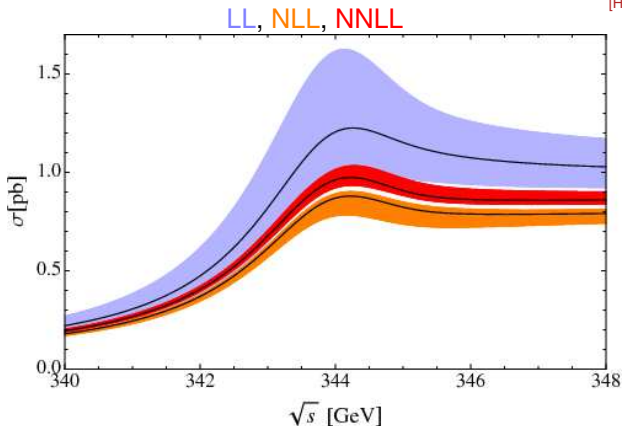


$N^3\text{LO}: \delta(\sqrt{s_{\text{peak}}}) \approx \pm 60 \text{ MeV}$

[Beneke, Kiyo, Marquard, Penin, Piclum, Steinhauser'15]

# $e^+e^- \rightarrow t\bar{t}$ at NNLL: resum $(\alpha_s \ln v)^n$

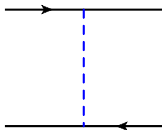
[Hoang,Stahlhofen'14]



- NNLL not complete
- no overlap of bands from  $\mu$  variation
- $\delta\sigma/\sigma = \pm 5\%$  (peak)
- see also: [Hoang,Manohar,Stewart,Teubner'02; Pineda,Signer'07]

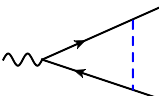
- $v^2 \sim \alpha_s^2 \sim y_t^2 \sim \alpha$

- Higgs: potential



$$\delta V_H = -\frac{y_t^2}{2m_H^2}$$

[Eiras,Steinhauser'06; Beneke,Maier,Piclum,Rauh'15]

production current   $\Leftrightarrow c_v, \mathcal{O}(y_t^2)$  and  $\mathcal{O}(\alpha_s y_t^2)$

[Grzadkowski,Kühn,Krawczyk,Stuart'86; Guth,Kühn'91; Hoang,ReiBer'06; Eiras,Steinhauser'06]

- Non-resonant production:  $e^+ e^- \rightarrow W^+ W^- b \bar{b}$

NLO [Hoang,ReiBer'06; Beneke,Jantzen,Ruiz-Femenia'10; Penin,Piclum'12]

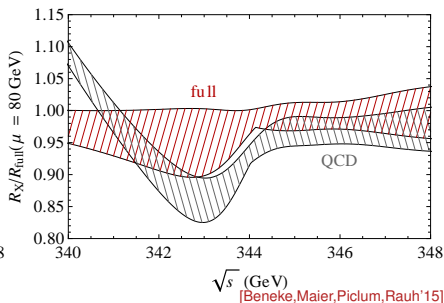
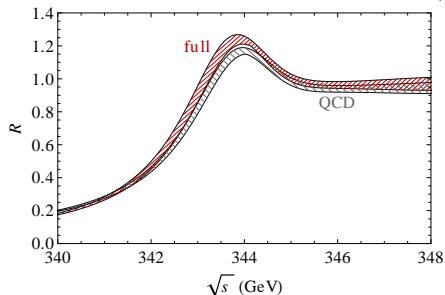
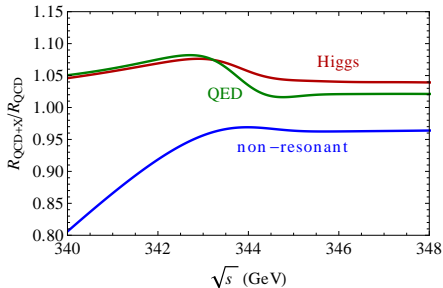
NNLO not complete [Penin,Piclum'12; Jantzen,Ruiz-Femenia'13; Ruiz-Femenia'14]

- $\delta V_{\text{QED}} = -\frac{4\pi\alpha Q_f^2}{|\vec{q}|^2}$

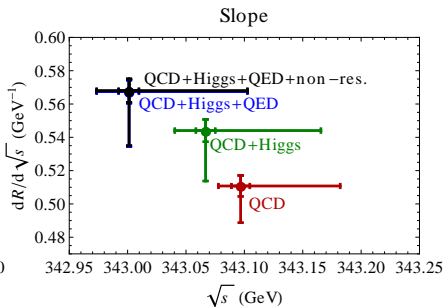
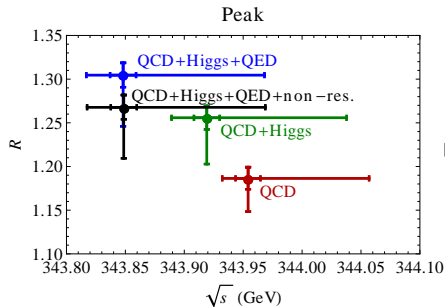
- $P$  wave production ( $\gamma^\mu \gamma^5$  coupling of  $Z$  boson)

[Penin,Pivovarov'99; Beneke,Piclum,Rauh'13]

# Beyond-QCD effects – numerical results

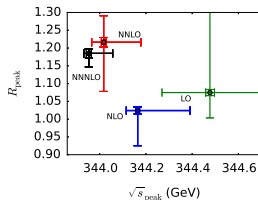


# Beyond-QCD effects – numerical results

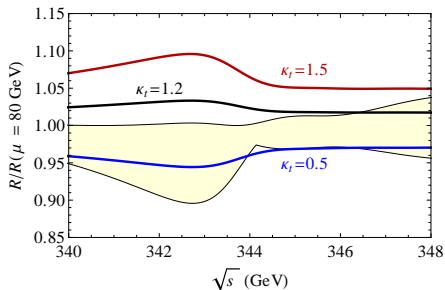


Higgs:  $-35$  MeV; QED:  $-71$  MeV

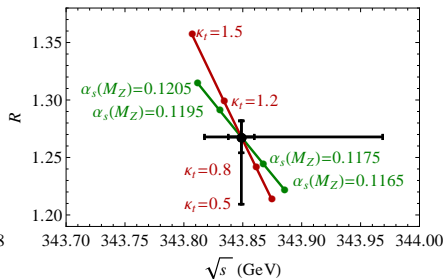
$\Leftrightarrow \delta m_t$ :  $-53$  MeV



# Beyond-QCD effects – numerical results



sensitivity:  $\mathcal{O}(20\%)$



[Beneke, Maier, Piclum, Rauh'15]



II.  $m^{\overline{\text{MS}}} - m^{\text{OS}}$  to 4 loops

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} G_{\mu\nu}^2 + \sum_q \bar{\psi}_q (\not{D} - m_q) \psi_q$$

- OS mass
- $\overline{\text{MS}}$  mass
- PS mass
- 1S mass
- RS mass
- kinetic mass
- ...

[Beneke'98]

[Hoang,Smith,Stelzer,Willenbrock'99]

[Pineda'01]

[Bigi,Shifman,Uraltsev,Vainshtein'97]

Wanted:  $m^{\text{thr}} - m^{\overline{\text{MS}}}$  relation

Needed:  $m^{\text{thr}} - m^{\text{OS}}$  and  $m^{\text{OS}} - m^{\overline{\text{MS}}}$  relations

# Example: PS mass

1. defined via relation to pole mass

[Beneke'98]

$$m^{\text{PS}}(\mu_f) = m^{\text{OS}} - \delta m(\mu_f)$$

$$\delta m(\mu_f) = -\frac{1}{2} \int_{|\vec{q}| < \mu_f} \frac{d^3 q}{(2\pi)^3} V(\vec{q})$$

$V(\vec{q})$ : static potential  
known to 3 loops

[Smirnov, Smirnov, Steinhäuser'09;  
Anzai, Kiyo, Sumino'09]

$$= \mu_f \frac{C_F \alpha_s}{\pi} \left\{ 1 + \frac{\alpha_s}{4\pi} a_1 + \left(\frac{\alpha_s}{4\pi}\right)^2 a_2 + \left(\frac{\alpha_s}{4\pi}\right)^3 a_3 \right\}$$

$\Leftrightarrow m^{\text{PS}} - m^{\text{OS}}$  relation known to N<sup>3</sup>LO (top:  $\mu_f = 80$  GeV)

2. use  $m^{\text{OS}} - m^{\overline{\text{MS}}}$  relation

$$m^{\text{OS}} = m^{\overline{\text{MS}}} \left( 1 + \frac{\alpha_s}{\pi} c_m^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 c_m^{(2)} + \left(\frac{\alpha_s}{\pi}\right)^3 c_m^{(3)} + \left(\frac{\alpha_s}{\pi}\right)^4 c_m^{(4)} + \dots \right)$$

needed to 4 loops

# $Z_m^{\text{OS}}$ : known results

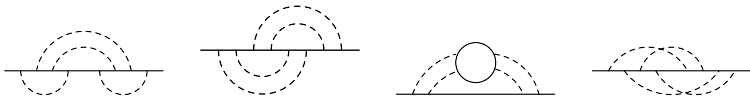
-  [Tarrach'81]
-  [Gray,Broadhurst,Grafe,Schilcher'90]
-  [Chetyrkin,Steinhauser'99; Melnikov, v. Ritbergen'00; Marquard,Mihaila,Piclum,Steinhauser'07]
- $n_f^2$   [Lee,Marquard,Smirnov,Smirnov,Steinhauser'13]
- full  [Marquard,Smirnov,Smirnov,Steinhauser'15]

- $Z_m^{\overline{\text{MS}}}$  known to 4 loops [Chetyrkin'97; Larin,van Ritbergen,Vermaseren'97]  
(5 loops: [Baikov,Chetyrkin,Kühn'14])
- electroweak corrections

[Hempfling,Kniehl'94; Jegerlehner,Kalmykov'03; Faisst,Kühn,Veretin'04; Martin'05; Eiras,Steinhauser'05]

# Some technical details

- generation of amplitudes: QGRAF [Nogueira'91]  
manipulation/transformation to FORM: q2e/exp  
[Harlander,Seidelsticker,Steinhauser'97;Seidelsticker'97]
- map to  $\sim 100$  families (topologies)



- reduce to MIs: FIRE5 [Smirnov'14] and crusher [Marquard,Seidel]
- minimize MIs over all topologies: tsort [Pak'11]  
 $\Leftrightarrow$  386 4-loop OS MIs
- compute MIs with FIESTA [Smirnov'14'15]

**Note:**  $m^{\text{OS}} - m^{\overline{\text{MS}}}$  relation in terms of MIs known **analytically**

$\Leftrightarrow$  systematic improvement possible

# 4-loop coefficient

$$m^{\overline{\text{MS}}} = m^{\text{OS}} \left( 1 + \frac{\alpha_s}{\pi} z_m^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 z_m^{(2)} + \left(\frac{\alpha_s}{\pi}\right)^3 z_m^{(3)} + \left(\frac{\alpha_s}{\pi}\right)^4 z_m^{(4)} + \dots \right)$$

charm

$$z_m^{(4)} \Big|_{n_f=3} = -1744.8 \pm 21.5 - 703.48 l_{\text{OS}} - 122.97 l_{\text{OS}}^2 \\ - 14.234 l_{\text{OS}}^3 - 0.75043 l_{\text{OS}}^4$$

bottom

$$z_m^{(4)} \Big|_{n_f=4} = -1267.0 \pm 21.5 - 500.23 l_{\text{OS}} - 83.390 l_{\text{OS}}^2 \\ - 9.9563 l_{\text{OS}}^3 - 0.514033 l_{\text{OS}}^4$$

top

$$z_m^{(4)} \Big|_{n_f=5} = -859.96 \pm 21.5 - 328.94 l_{\text{OS}} - 50.856 l_{\text{OS}}^2 \\ - 6.4922 l_{\text{OS}}^3 - 0.33203 l_{\text{OS}}^4$$

$$l_{\text{OS}} = \ln(\mu^2/M^2)$$

- $\ln(\mu^2/M^2)$  known from RGE
- constant term: uncertainty  $< 3\%$

# $\overline{\text{MS}}$ —OS relation for top and bottom

$$\begin{aligned}m_t^{\text{OS}} &= m_t^{\overline{\text{MS}}} \left[ 1 + 0.4244 \alpha_s + 0.8345 \alpha_s^2 + 2.375 \alpha_s^3 + (8.49 \pm 0.25) \alpha_s^4 \right] \\ &= 163.643 + 7.557 + 1.617 + 0.501 + (0.195 \pm 0.005) \text{ GeV}\end{aligned}$$

$$\begin{aligned}m_t^{\text{OS}} &= m_t^{\overline{\text{MS}}} \left[ 1 + 0.4244 \alpha_s + 0.8345 \alpha_s^2 + 2.375 \alpha_s^3 + (8.49 \pm 0.25) \alpha_s^4 \right] \\ &= 163.643 + 7.557 + 1.617 + 0.501 + (0.195 \pm 0.005) \text{ GeV}\end{aligned}$$

$$\begin{aligned}m_b^{\text{OS}} &= m_b^{\overline{\text{MS}}} \left[ 1 + 0.4244 \alpha_s + 0.9401 \alpha_s^2 + 3.045 \alpha_s^3 + (12.57 \pm 0.38) \alpha_s^4 \right] \\ &= 4.163 + 0.401 + 0.201 + 0.148 + (0.138 \pm 0.004) \text{ GeV}\end{aligned}$$

- **top**: good/reasonable convergence
- **bottom**: no convergence

# $\overline{\text{MS}}$ — threshold top mass relation

input #loops	$m_t^{\overline{\text{MS}}}(m_t^{\overline{\text{MS}}})$		
	$m^{\text{PS}} =$	$m^{\text{1S}} =$	$m^{\text{RS}} =$
1	168.204	172.227	171.215
2	164.311	165.045	164.847
	163.713	163.861	163.853

1-2 GeV

# $\overline{\text{MS}}$ — threshold top mass relation

input #loops	$m_t^{\overline{\text{MS}}}(m_t^{\overline{\text{MS}}})$		
	$m^{\text{PS}} =$	$m^{\text{1S}} =$	$m^{\text{RS}} =$
	168.204	172.227	171.215
1	164.311	165.045	164.847
2	163.713	163.861	163.853
3	163.625	163.651	163.663

1-2 GeV  
 $\approx$  200 MeV

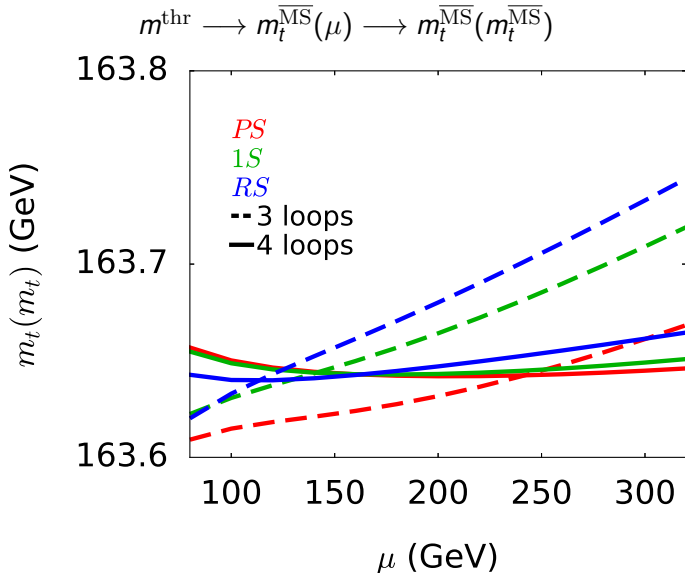
# $\overline{\text{MS}}$ — threshold top mass relation

input #loops	$m_t^{\overline{\text{MS}}}(m_t^{\overline{\text{MS}}})$			
	$m^{\text{PS}} =$	$m^{\text{1S}} =$	$m^{\text{RS}} =$	
	168.204	172.227	171.215	
1	164.311	165.045	164.847	
2	163.713	163.861	163.853	1-2 GeV
3	163.625	163.651	163.663	$\sim 200$ MeV
4	163.643	163.643	163.643	$\sim 20$ MeV

input #loops	$m_t^{\overline{\text{MS}}}(m_t^{\overline{\text{MS}}})$			
	$m^{\text{PS}} =$	$m^{\text{1S}} =$	$m^{\text{RS}} =$	
	168.204	172.227	171.215	
1	164.311	165.045	164.847	
2	163.713	163.861	163.853	1-2 GeV
3	163.625	163.651	163.663	$\lesssim 200$ MeV
4	163.643	163.643	163.643	$\lesssim 20$ MeV
4 ( $\times 1.03$ )	163.637	163.637	163.637	

- 3 loops:  $\lesssim 200$  MeV
  - 4 loops: {18, 8, 20} MeV
  - 3% uncertainty  $\hat{=} 6$  MeV
  - $\delta m_t^{\text{fcc}} \lesssim 50$  MeV
- }  $\Leftrightarrow$  {9, 7, 11} MeV uncertainty  
in the  $m_t^{\overline{\text{MS}}} - m_t^{\text{thr}}$  relation

# $\overline{\text{MS}}$ — threshold top mass relation



- $\sigma(e^+e^- \rightarrow t\bar{t})$  to N<sup>3</sup>LO QCD,  $\pm 3\%$  uncertainty
- beyond-QCD (Higgs, QED, non-resonant)
- 4-loop  $\overline{\text{MS}}$ -OS relation for heavy quarks
- Precise  $m^{\overline{\text{MS}}}-m^{\text{thr}}$  relations to N<sup>3</sup>LO
- $\delta m_t \lesssim 50$  MeV within reach