UNIVERSITY OF CAMBRIDGE

Model-Independent Precision Constraints Using Dimension-6 Operators

Tevong You

Based on

-*The Universal One-Loop Effective Action*,
Aleksandra Drozd, **John Ellis, Jeremie Quevillon** and **TY**,
[arXiv:1512.03003]

-*Sensitivities of Prospective Future e+e− Colliders to Decoupled New Physics*,
John Ellis and **TY**
[arXiv:1510.04561]

-*Comparing EFT and Exact One-Loop Analyses of Non-Degenerate Stops*,
Aleksandra Drozd, **John Ellis, Jeremie Quevillon** and **TY**
JHEP 06 (2015) 028 [arXiv:1504.02409]

-*The Effective Standard Model after LHC Run I*,
John Ellis, **Veronica Sanz** and **TY**
JHEP 29 (2015) 007 [arXiv:1410.7703]

Outline

- Why SM EFT?
- Dimension-6 operators
- Present constraints
- Future constraints

Why SM EFT?

- 1930s-1970s: Beta decay, muon decay etc. -> Fermi theory

$$\mathcal{L}_{FERMI} = -\frac{G_F}{\sqrt{2}}(\bar{\Psi}\mathcal{O}_i\Psi)(\bar{\Psi}\mathcal{O}_i\Psi)$$

$$\mathcal{O}_{scalar} = 1$$
$$\mathcal{O}_{vector} = \gamma_\mu$$
$$\mathcal{O}_{A-V} = \gamma_5\gamma_\mu$$
$$\mathcal{O}_{tensor} = \sigma_{\mu\nu}$$
$$\mathcal{O}_{pseudoscalar} = \gamma_5$$

- Experimental data -> V-A structure

$$\mathcal{L}_{V-A} = -\frac{G_F}{\sqrt{2}}\bar{\Psi}\gamma_\mu(1-\gamma_5)\Psi\bar{\Psi}\gamma^\mu(1-\gamma_5)\Psi$$

- Pions -> Chiral perturbation theory (non-linear effective Lagrangian)

Why SM EFT?

- 1980s-2012: Discovery of weak bosons -> Non-linear effective Lagrangian for spontaneously-broken global symmetry (breaking mechanism unknown!)
- **Global** symmetry-breaking pattern gives low-energy effective theory regardless of UV mechanism responsible for it

$$SU(2) \times SU(2) \to SU(2)_V \qquad (\rho \equiv M_W/M_Z \cos\theta_w \sim 1)$$

$$\mathcal{L} = \frac{v^2}{4} \mathrm{Tr} D_\mu \Sigma^\dagger D^\mu \Sigma - m_i \bar{\psi}_L^i \Sigma \psi_R^i + \mathrm{h.c.}$$

$$\Sigma = \exp\left(i \frac{\sigma^a \pi^a}{v} \right)$$

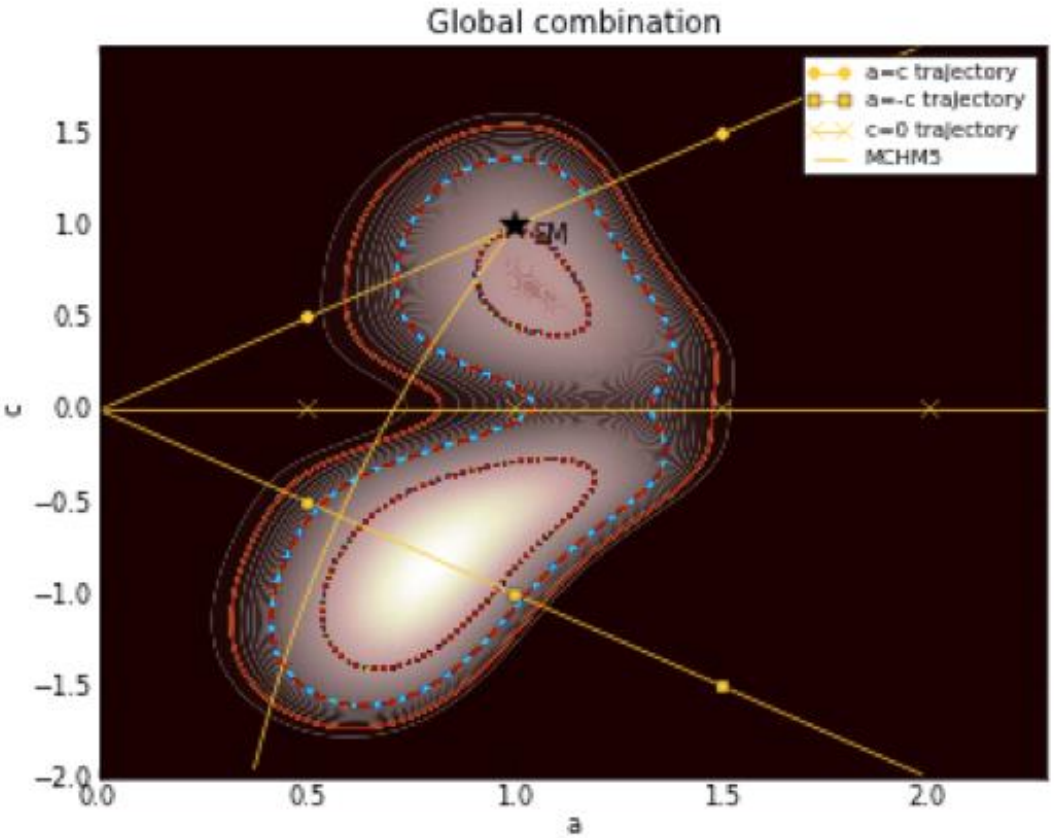
Why SM EFT?

- 2012: Discovery of a scalar -> Non-linear electroweak Lagrangian with general couplings to singlet scalar

$$\mathcal{L} = \frac{v^2}{4}\mathrm{Tr} D_\mu \Sigma^\dagger D^\mu \Sigma \left(1 + 2a\frac{h}{v} + b\frac{h^2}{v^2} + \ldots\right) - m_i \bar{\psi}_L^i \Sigma \left(1 + c\frac{h}{v} + \ldots\right) \psi_R^i + \mathrm{h.c.}$$
$$+ \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2} m_h^2 h^2 + d_3 \frac{1}{6}\left(\frac{3m_h^2}{v}\right) h^3 + d_4 \frac{1}{24}\left(\frac{3m_h^2}{v^2}\right) h^4 + \ldots \quad ,$$

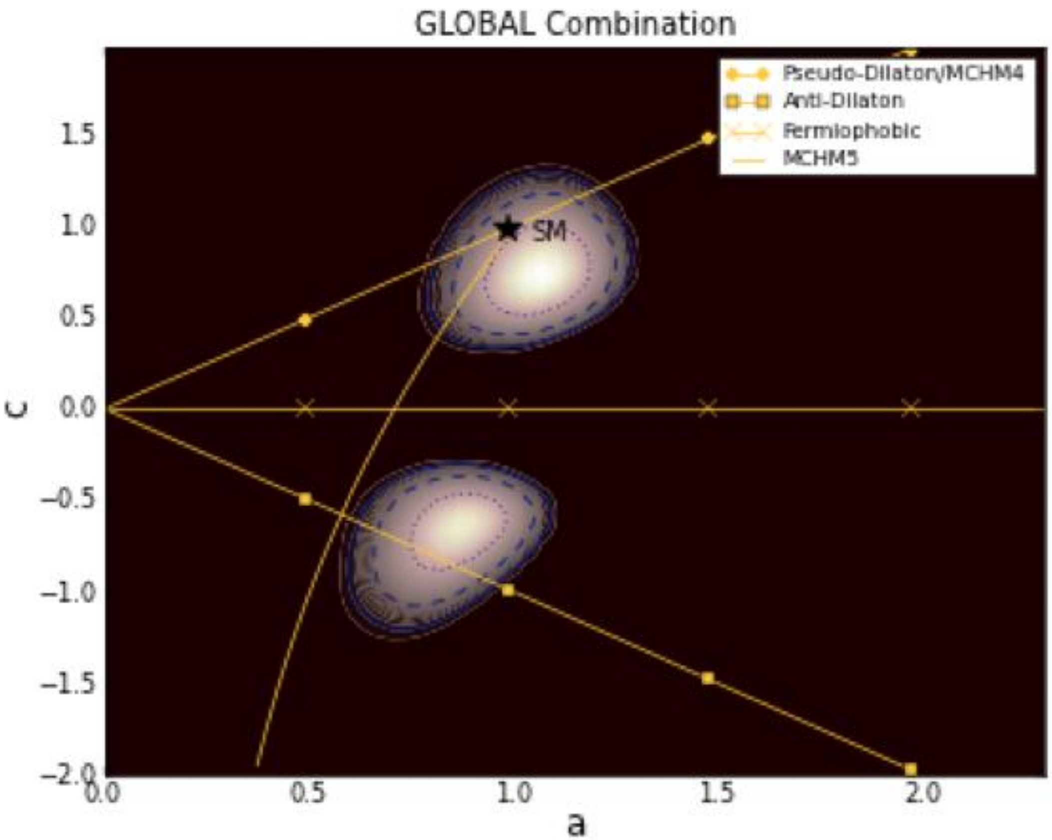
$$\Sigma = \exp\left(i\frac{\sigma^a \pi^a}{v}\right)$$

Why SM EFT?



March 2012 pre-discovery
J. Ellis and T.Y. [arXiv:1204.0464]

Why SM EFT?

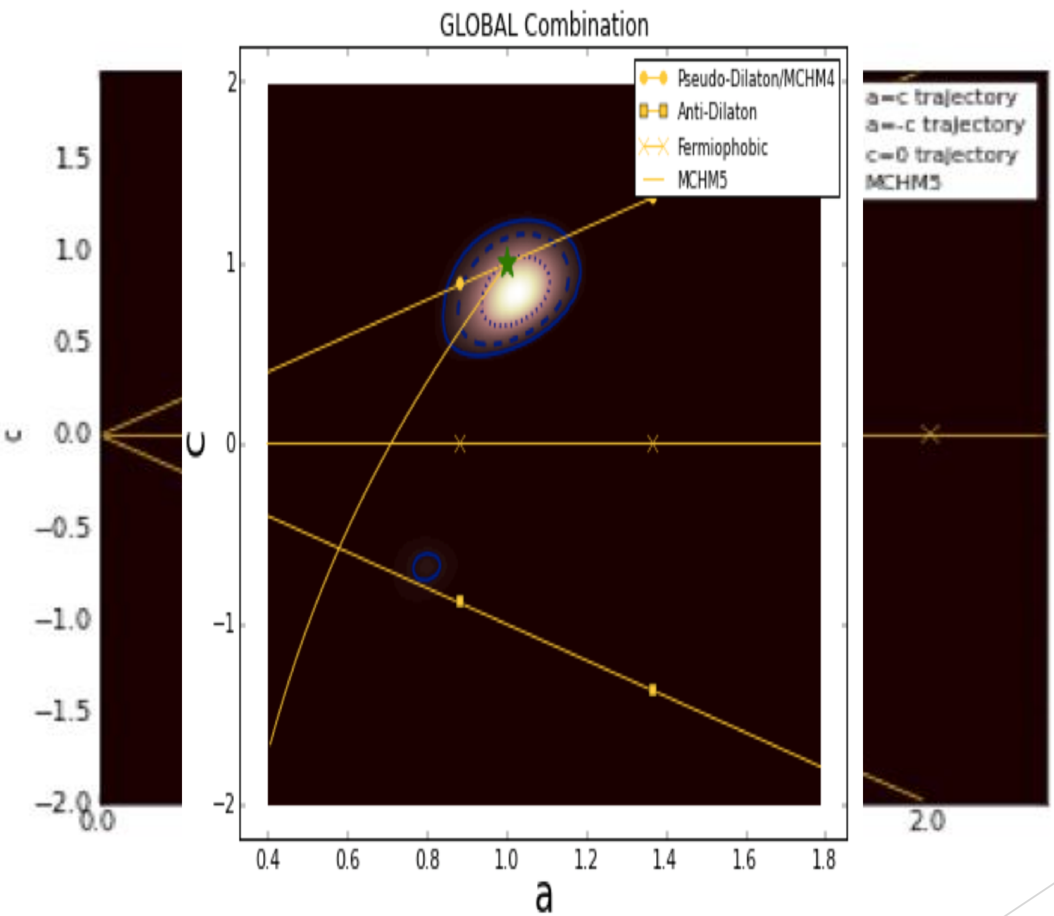


Moriond 2013

J. Ellis and T.Y. [arXiv:1303.1879]

Why SM EFT?

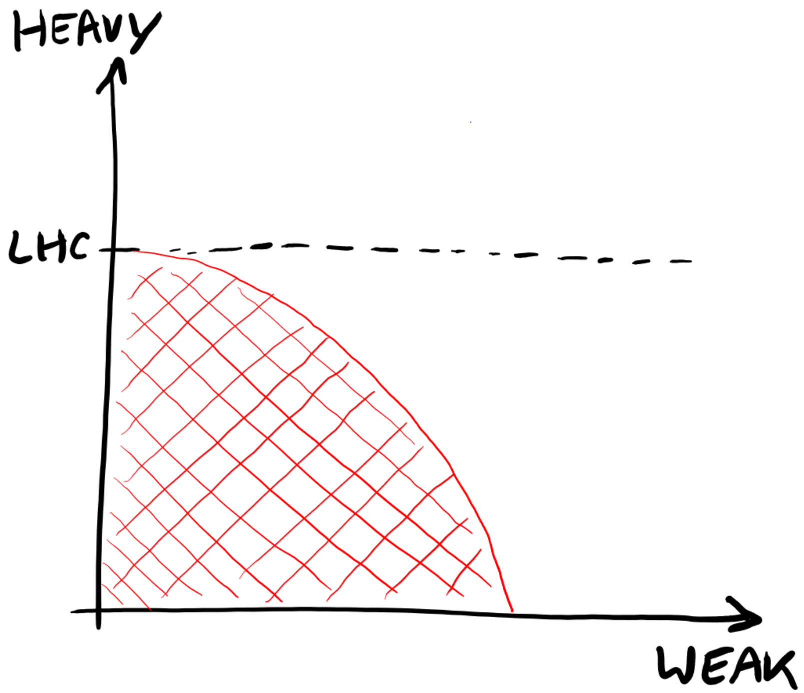
- To a first approximation Higgs is SM-like



July 2012 post-discovery
J. Ellis and T.Y. [arXiv:1207.1693]

Why SM EFT?

- Assume separation of electroweak scale and BSM physics



- SM EFT a systematic approach to decoupled new physics
- Job is now to classify phenomenology, from bottom-up and top-down
- Complements any information we might potentially get from direct discoveries of BSM resonances

Dimension-6 Operators

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
Q_L	**3**	**2**	$\frac{1}{6}$
q_R^u	**3**	**1**	$\frac{2}{3}$
q_R^d	**3**	**1**	$-\frac{1}{3}$
L_L	**1**	**2**	$-\frac{1}{2}$
l_R	**1**	**1**	-1
ϕ	**1**	**2**	$\frac{1}{2}$

$$\mathcal{L}_{SM} = \mathcal{L}_m + \mathcal{L}_g + \mathcal{L}_h + \mathcal{L}_y \quad + \quad \mathcal{L}_{\rm SM}^{\text{dim-6}} = \sum_i \frac{c_i}{\Lambda^2}\mathcal{O}_i$$

$$\begin{aligned}
\mathcal{L}_m &= \bar{Q}_L i\gamma^\mu D_\mu^L Q_L + \bar{q}_R i\gamma^\mu D_\mu^R q_R + \bar{L}_L i\gamma^\mu D_\mu^L L_L + \bar{l}_R i\gamma^\mu D_\mu^R l_R \\
\mathcal{L}_G &= -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W_{\mu\nu}^a W^{a\mu\nu} \\
\mathcal{L}_H &= (D_\mu^L\phi)^\dagger(D^{L\mu}\phi) - V(\phi) \\
\mathcal{L}_Y &= y_d\bar{Q}_L\phi q_R^d + y_u\bar{Q}_L\phi^c q_R^u + y_L\bar{L}_L\phi l_R + \text{h.c.} \quad ,
\end{aligned}$$

- First classified systematically by Buchmuller and Wyler (Nucl. Phys. B 268 (1986) 621)
- 59 dim-6 CP-even operators in a **non-redundant** basis, assuming MFV (Gradkowski et al [arXiv:1008.4884 [hep-ph]])

$$\begin{aligned}
\mathcal{O}_H &= \tfrac{1}{2}(\partial^\mu|H|^2)^2 \\
\mathcal{O}_T &= \tfrac{1}{2}\left(H^\dagger \overleftrightarrow{D}_\mu H\right)^2 \\
\mathcal{O}_6 &= \lambda|H|^6 \\
\mathcal{O}_W &= \tfrac{ig}{2}\left(H^\dagger\sigma^a \overleftrightarrow{D}^\mu H\right) D^\nu W_{\mu\nu}^a \\
\mathcal{O}_B &= \tfrac{ig'}{2}\left(H^\dagger \overleftrightarrow{D}^\mu H\right) \partial^\nu B_{\mu\nu}
\end{aligned}$$

$$\begin{aligned}
\mathcal{O}_{BB} &= g'^2|H|^2 B_{\mu\nu}B^{\mu\nu} \\
\mathcal{O}_{GG} &= g_s^2|H|^2 G_{\mu\nu}^A G^{A\mu\nu} \\
\mathcal{O}_{HW} &= ig(D^\mu H)^\dagger\sigma^a(D^\nu H)W_{\mu\nu}^a \\
\mathcal{O}_{HB} &= ig'(D^\mu H)^\dagger(D^\nu H)B_{\mu\nu} \\
\mathcal{O}_{3W} &= \tfrac{1}{3!}g\epsilon_{abc}W_\mu^{a\,\nu}W_{\nu\rho}^b W^{c\,\rho\mu}
\end{aligned}$$

$\mathcal{O}_{y_u} = y_u\|H\|^2\bar{Q}_L\widetilde{H}u_R + \text{h.c.}$	$\mathcal{O}_{y_d} = y_d\|H\|^2\bar{Q}_L H d_R + \text{h.c.}$	$\mathcal{O}_{y_e} = y_e\|H\|^2\bar{L}_L H e_R + \text{h.c.}$
$\mathcal{O}_R^u = (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{u}_R\gamma^\mu u_R)$ $\mathcal{O}_L^q = (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{Q}_L\gamma^\mu Q_L)$ $\mathcal{O}_L^{(3)\,q} = (iH^\dagger\sigma^a \overleftrightarrow{D}_\mu H)(\bar{Q}_L\sigma^a\gamma^\mu Q_L)$	$\mathcal{O}_R^d = (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{d}_R\gamma^\mu d_R)$	$\mathcal{O}_R^e = (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{e}_R\gamma^\mu e_R)$
$\mathcal{O}_{LL}^{(3)\,ql} = (\bar{Q}_L\sigma^a\gamma_\mu Q_L)(\bar{L}_L\sigma^a\gamma^\mu L_L)$		$\mathcal{O}_{LL}^{(3)\,l} = (\bar{L}_L\sigma^a\gamma^\mu L_L)(\bar{L}_L\sigma^a\gamma_\mu L_L)$

Basis adopted from Pomarol and Riva 1308.1426

(SILH basis Giudice et al. hep-ph/0703164)

Modifications of EWPO from dim-6 Operators

- (Pseudo-)Observables

$$\Gamma_Z = \Gamma_{had} + 3\Gamma_\ell + 3\Gamma_\nu \qquad R_\ell = \frac{\Gamma_{had}}{\Gamma_\ell} \qquad \sigma_{had} = 12\pi\frac{\Gamma_e \Gamma_{had}}{\hat{m}_Z^2 \Gamma_Z^2} \qquad A^f_{FB} = \frac{3}{4}A_e A_f \qquad M_W = c_W M_Z$$

$$R_q = \frac{\Gamma_q}{\Gamma_{had}}$$

- Depends on

$$\Gamma_f = \frac{\sqrt{2}G_F M_Z^2 \hat{M}_Z}{6\pi}\left[(g_L^f)^2 + (g_R^f)^2\right] \qquad A_f = \frac{(g_L^f)^2 - (g_R^f)^2}{(g_L^f)^2 + (g_R^f)^2}$$

$$g^f = T_f^3 - Q_f s_W^2 \qquad s_W^2 = \frac{1}{2} - \frac{1}{2}\sqrt{1 - \frac{4\pi\alpha}{\sqrt{2}G_F m_Z^2}}$$

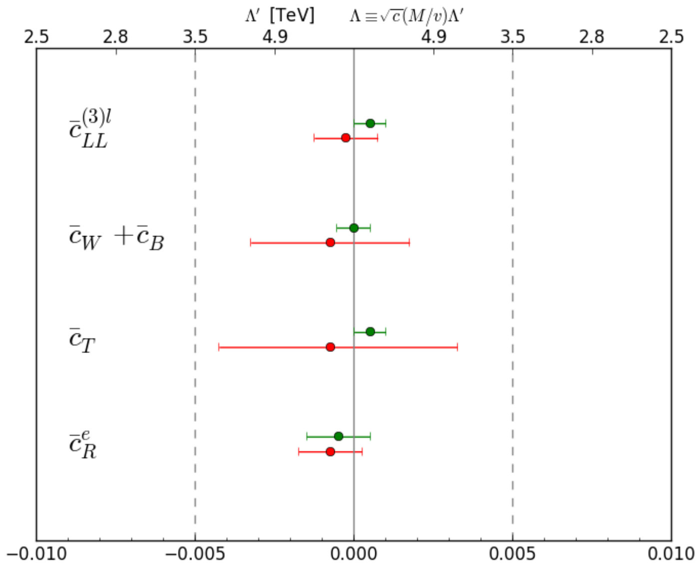
- Dim-6 operators can modify observables directly through Zff couplings contributions or indirectly through redefinitions of input observables

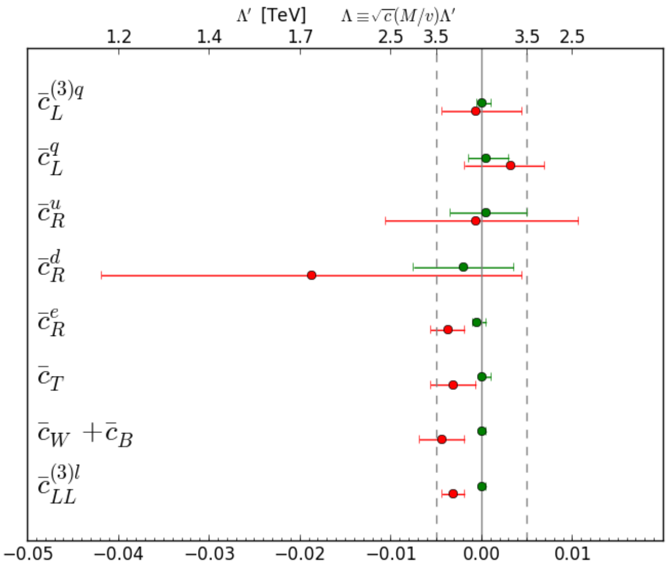
$$m_Z^2 = (m_Z^2)^0(1 + \Pi_{ZZ}) \qquad G_F = G_F^0(1 - \Pi_{WW}^0) \qquad \alpha(m_Z) = \alpha^0(m_Z)(1 + \Pi'_{\gamma\gamma})$$

SM EFT Present Constraints

- Marginalized constraints on a complete non-redundant basis of dim-6 operators affecting EWPTs

Ellis, Sanz and T.Y. 1410.7703

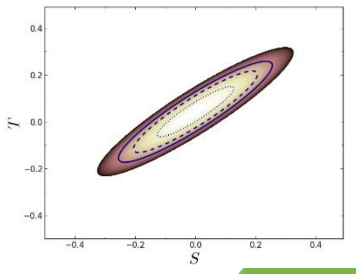




- S,T parameter corresponds to $(c_W + c_B)$, c_T subset

$$S = \frac{4\sin^2\theta_W}{\alpha(m_Z)}(\bar{c}_W + \bar{c}_B) \approx 119(\bar{c}_W + \bar{c}_B)$$

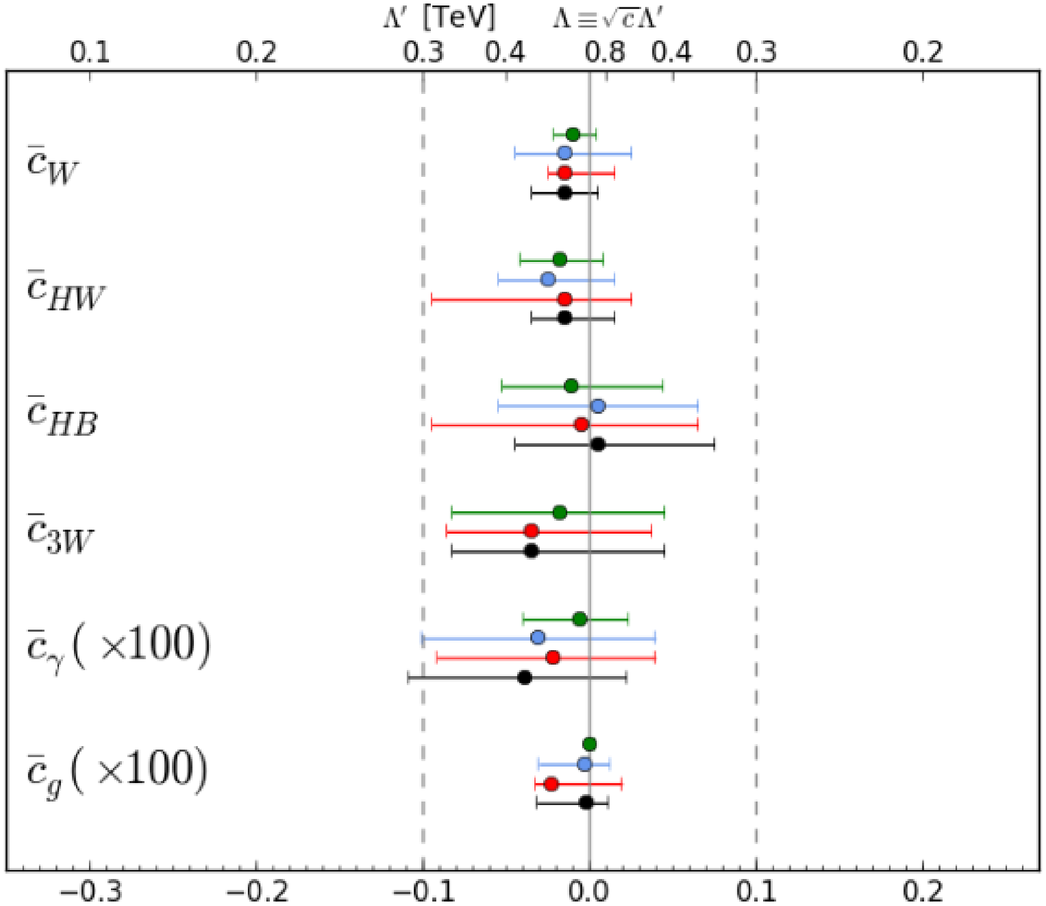
$$T = \frac{1}{\alpha(m_Z)}\bar{c}_T \approx 129\bar{c}_T\,.$$



SM EFT Present Constraints

- Constraints from LHC triple-gauge coupling measurements and Higgs physics

x 10 TeV



Ellis, Sanz and T.Y. 1410.7703

One-Loop Effective Action

- Calculate Wilson coefficients of dimension-6 operators using general expression

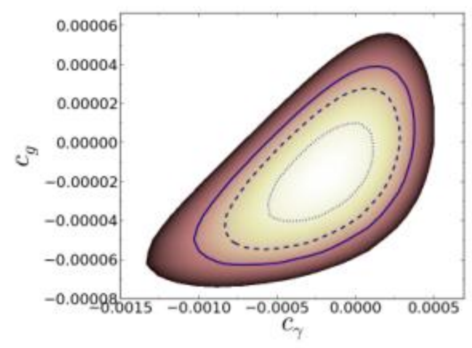
$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + (\Phi^\dagger F(x) + \text{h.c.}) + \Phi^\dagger(P^2 - M^2 - U(x))\Phi + \mathcal{O}(\Phi^3)\,,$$

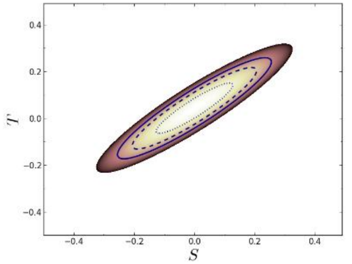
$$\begin{aligned}
\mathcal{L}^{\rm eff}_{\text{1-loop}}[\phi] \supset -ic_s \Big\{ & f_1^i + f_2^i U_{ii} + f_3^i G'^2_{\mu\nu,ij} + f_4^{ij} U_{ij}^2 \\
& + f_5^{ij}(P_\mu G'_{\mu\nu,ij})^2 + f_6^{ij}(G'_{\mu\nu,ij})(G'_{\nu\sigma,jk})(G'_{\sigma\mu,ki}) + f_7^{ij}[P_\mu, U_{ij}]^2 + f_8^{ijk}(U_{ij}U_{jk}U_{ki}) \\
& + f_9^{ij}(U_{ij}G'_{\mu\nu,jk}G'_{\mu\nu,ki}) \\
& + f_{10}^{ijkl}(U_{ij}U_{jk}U_{kl}U_{li}) + f_{11}^{ijk}U_{ij}[P_\mu, U_{jk}][P_\mu, U_{ki}] \\
& + f_{12,a}^{ij}\,[P_\mu,[P_\nu,U_{ij}]]\,[P_\mu,[P_\nu,U_{ji}]] + f_{12,b}^{ij}\,[P_\mu,[P_\nu,U_{ij}]]\,[P_\nu,[P_\mu,U_{ji}]] \\
& + f_{12,c}^{ij}\,[P_\mu,[P_\mu,U_{ij}]]\,[P_\nu,[P_\nu,U_{ji}]] \\
& + f_{13}^{ijk}U_{ij}U_{jk}G'_{\mu\nu,kl}G'_{\mu\nu,li} + f_{14}^{ijk}\,[P_\mu,U_{ij}]\,[P_\nu,U_{jk}]\,G'_{\nu\mu,ki} \\
& + \left(f_{15a}^{ijk}U_{i,j}[P_\mu,U_{j,k}] - f_{15b}^{ijk}[P_\mu,U_{i,j}]U_{j,k}\right)[P_\nu, G'_{\nu\mu,ki}] \\
& + f_{16}^{ijklm}(U_{ij}U_{jk}U_{kl}U_{lm}U_{mi}) + f_{17}^{ijkl}U_{ij}U_{jk}[P_\mu,U_{kl}][P_\mu,U_{li}] + f_{18}^{ijkl}U_{ij}[P_\mu,U_{jk}]U_{kl}[P_\mu,U_{li}] \\
& + f_{19}^{ijklmn}(U_{ij}U_{jk}U_{kl}U_{lm}U_{mn}U_{ni}) \Big\}\,.
\end{aligned}$$

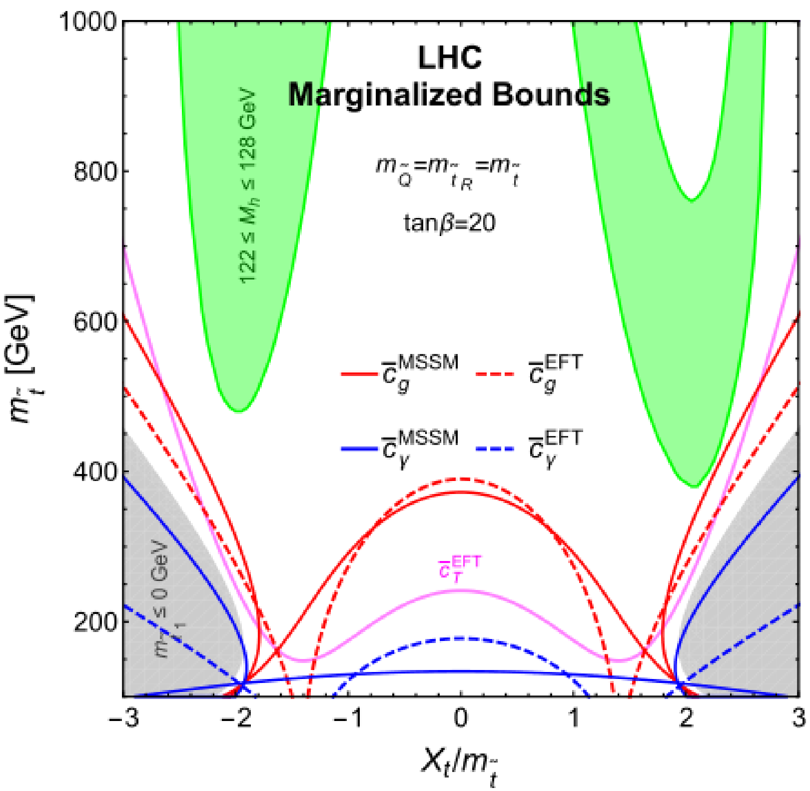
-A. Drozd, J. Ellis, J. Quevillon and TY [arXiv:1512.03003]

Translating EFT Constraints to MSSM Stops

Coeff.	Experimental constraints		95 % CL limit	deg. $m_{\tilde{t}_1}$, $X_t = 0$
$\bar{c}_g$	LHC	marginalized	$[-4.5, 2.2] \times 10^{-5}$	~ 410 GeV
		individual	$[-3.0, 2.5] \times 10^{-5}$	~ 390 GeV
$\bar{c}_\gamma$	LHC	marginalized	$[-6.5, 2.7] \times 10^{-4}$	~ 215 GeV
		individual	$[-4.0, 2.3] \times 10^{-4}$	~ 230 GeV
$\bar{c}_T$	LEP	marginalized	$[-10, 10] \times 10^{-4}$	~ 290 GeV
		individual	$[-5, 5] \times 10^{-4}$	~ 380 GeV
$\bar{c}_W + \bar{c}_B$	LEP	marginalized	$[-7, 7] \times 10^{-4}$	~ 185 GeV
		individual	$[-5, 5] \times 10^{-4}$	~ 195 GeV



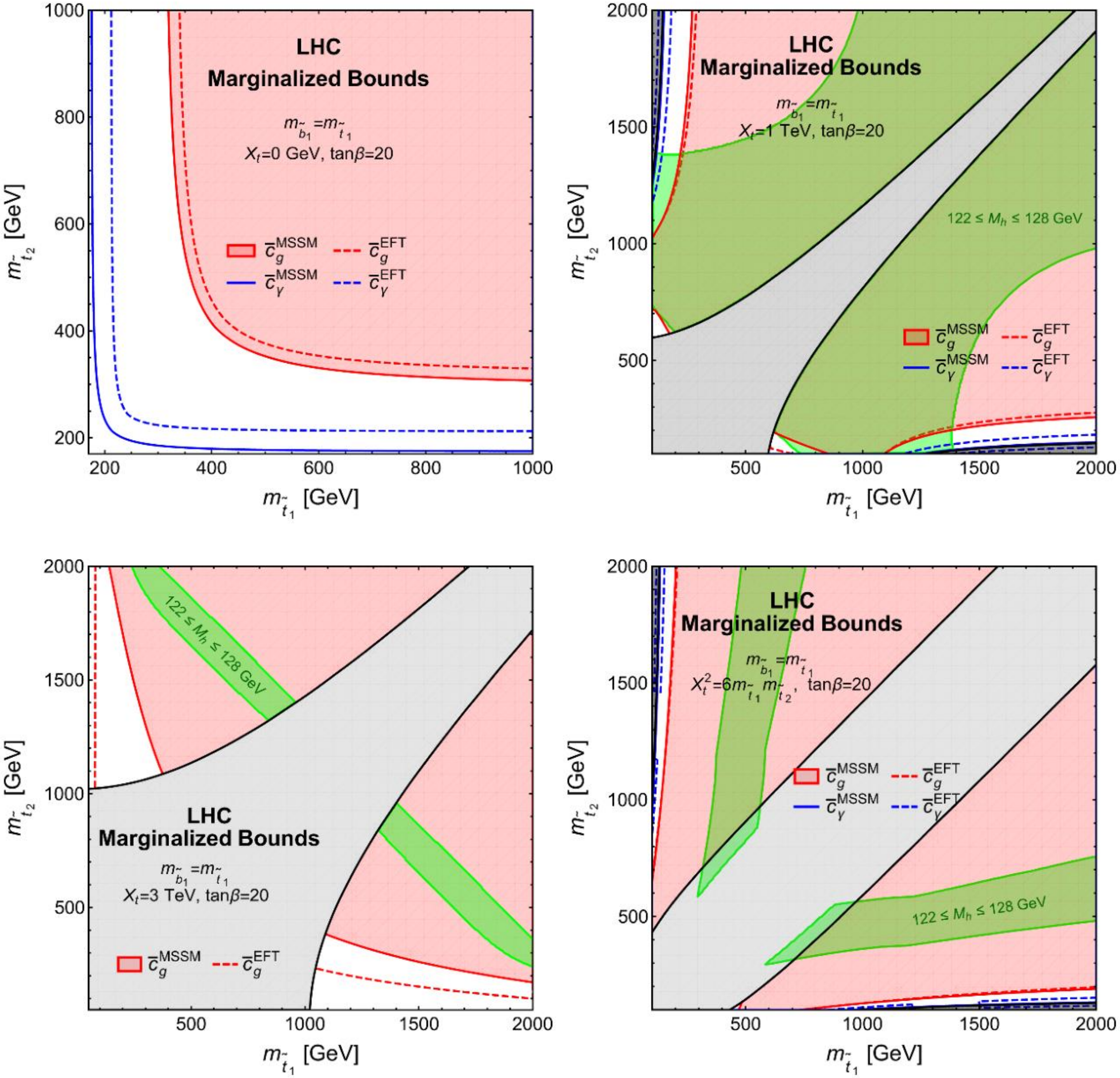




Drozd, Ellis, Quevillon and T.Y.
1504.02409

Translating EFT Constraints to MSSM Stops

- Non-degenerate stops



FCC-ee EWPT Constraints

best-of ee-FCC/TLEP #2: Precision EW measts

Asset: -- high luminosity (10^{12} Z decays + 10^8 Wpairs + 10^6 top pairs)
-- exquiste energy calibration up and above WW threshold

target precisions

Quantity	Present precision	Measured from	Statistical uncertainty	Systematic uncertainty
m_Z (keV)	91187500 ± 2100	Z Line shape scan	5 (6) keV	< 100 keV
Γ_Z (keV)	2495200 ± 2300	Z Line shape scan	8 (10) keV	< 100 keV
R_ℓ	20.767 ± 0.025	Z Peak	0.00010 (12)	< 0.001
N_ν	2.984 ± 0.008	Z Peak	0.00008 (10)	< 0.004
N_ν	2.92 ± 0.05	$Z\gamma$, 161 GeV	0.0010 (12)	< 0.001
R_b	0.21629 ± 0.00066	Z Peak	0.000003 (4)	< 0.000060
A_{LR}	0.1514 ± 0.0022	Z peak, polarized	0.000015 (18)	< 0.000015
m_W (MeV)	80385 ± 15	WW threshold scan	0.3 (0.4)MeV	< 0.5 MeV
m_{top} (MeV)	173200 ± 900	$t\bar{t}$ threshold scan	10 (12) MeV	< 10 MeV

Also -- $\Delta\sin^2\theta_W \approx 10^{-6}$ from Z peak AFBs
-- $\Delta\alpha_S = 0.0001$ from W and Z hadronic widths
-- orders of magnitude on FCNCs and rare decays etc. etc.

Design study to establish possibility of achieving corresponding precision theoretical calculations.

FCC-ee EWPT Constraints

Parametric and theoretical uncertainties

- We assume that theoretical uncertainties will be reduced by calculating three-loop contributions of $O(\alpha^2\alpha_s)$ and $O(\alpha^3)$.

	TLEP direct	Parametric uncertainty α_s	$\Delta\alpha^{(5)}_{\rm had}$	M_Z	m_t	m_h	Total	Theoretical uncertainty current	future
δM_W [MeV]	± 0.64	± 0.36	± 0.91	± 0.13	± 0.10	± 0.14	± 1.00	± 4	± 1
$\delta\Gamma_Z$ [MeV]	± 0.1	± 0.3	± 0.0	± 0.0	± 0.0	± 0.0	± 0.3	± 0.5	± 0.1
$\delta\mathcal{A}_\ell$ [10^{-5}]	± 2.1	± 1.6	± 13.7	± 0.6	± 0.4	± 0.9	± 13.9	± 37.0	± 11.8

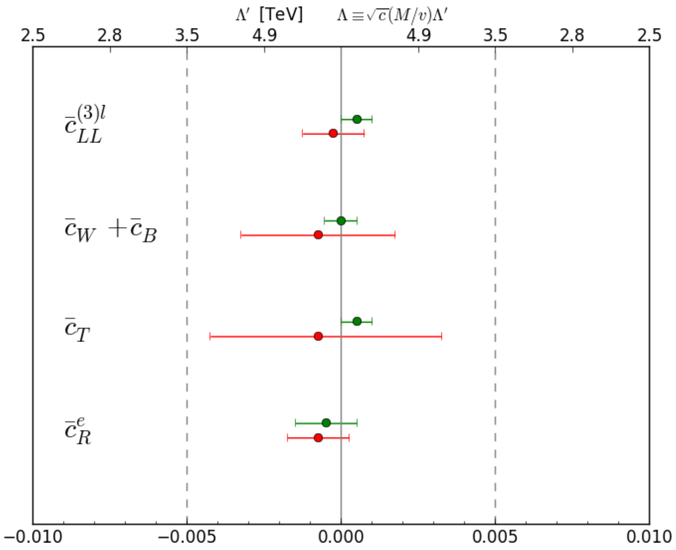
$$\delta \sin^2\theta^{\rm lept}_{\rm eff} = 4.7\times 10^{-5} \;\rightarrow\; 1.5\times 10^{-5}$$

- Parametric uncertainties are dominated by $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$.

- Theoretical calculations at three-loop level are necessary to reach the TLEP precision.

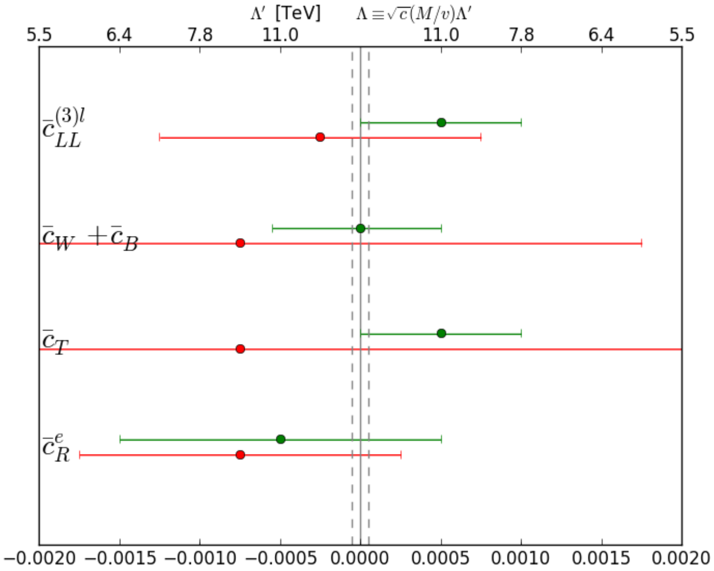
Satoshi Mishima (Univ. of Rome)

FCC-ee EWPT Constraints



LEP

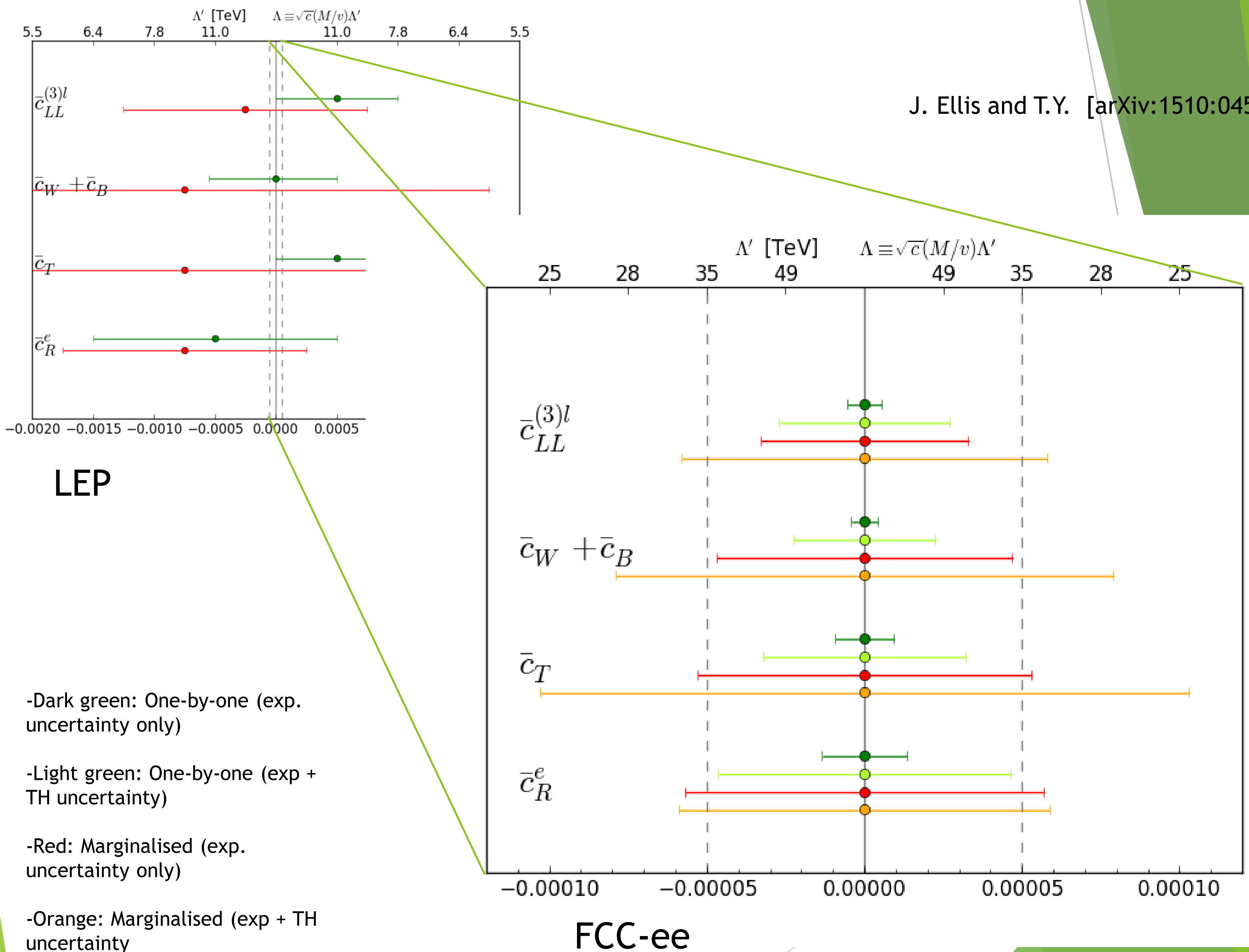
FCC-ee EWPT Constraints



LEP

FCC-ee EWPT Constraints

J. Ellis and T.Y. [arXiv:1510:04561]



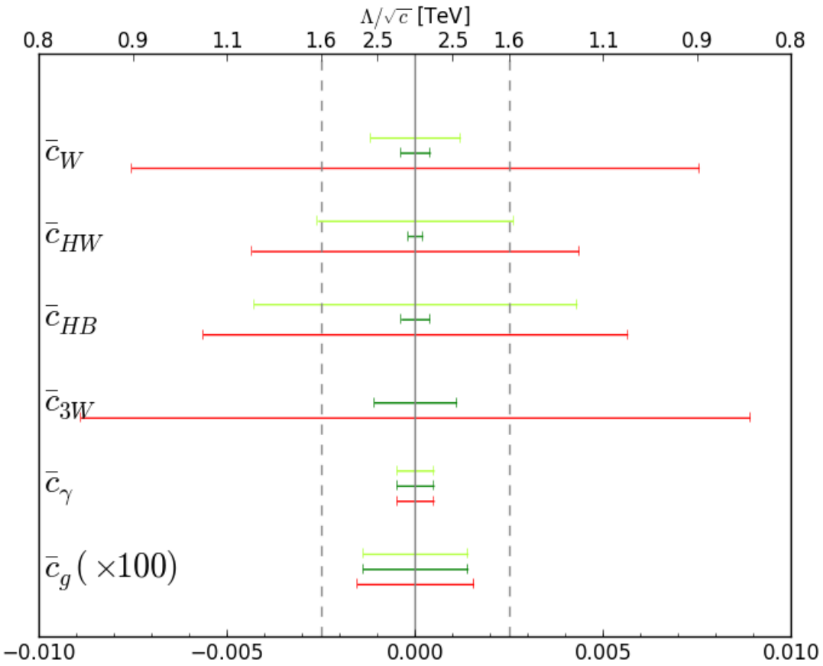
-Dark green: One-by-one (exp. uncertainty only)

-Light green: One-by-one (exp + TH uncertainty)

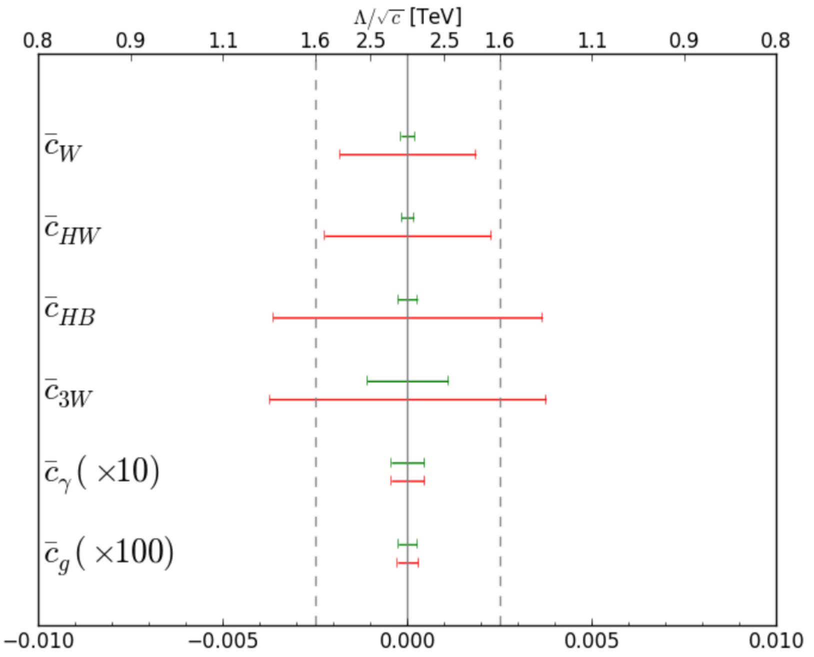
-Red: Marginalised (exp. uncertainty only)

-Orange: Marginalised (exp + TH uncertainty

Future Higgs Constraints



ILC

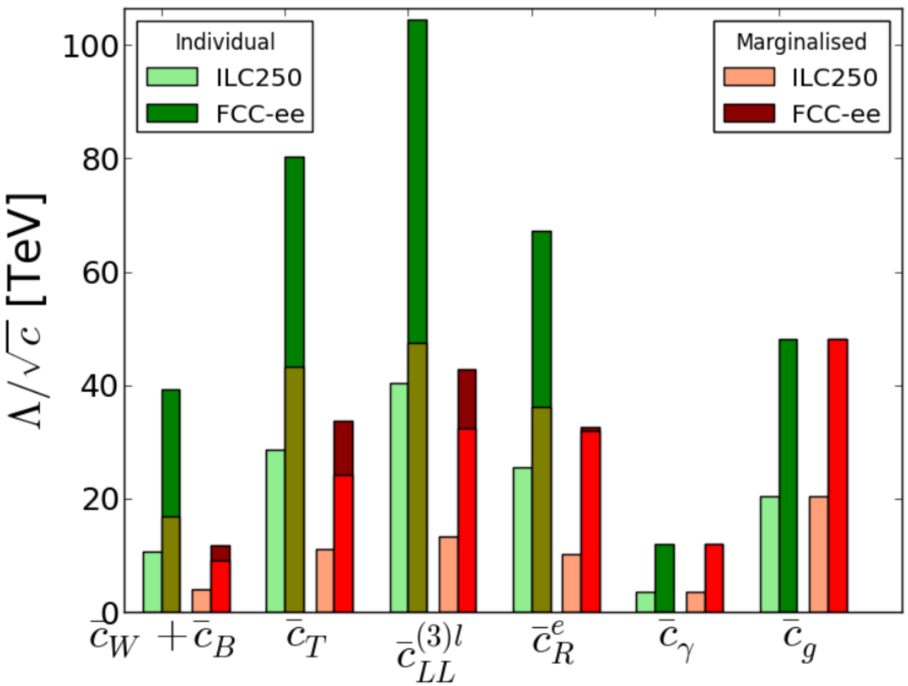


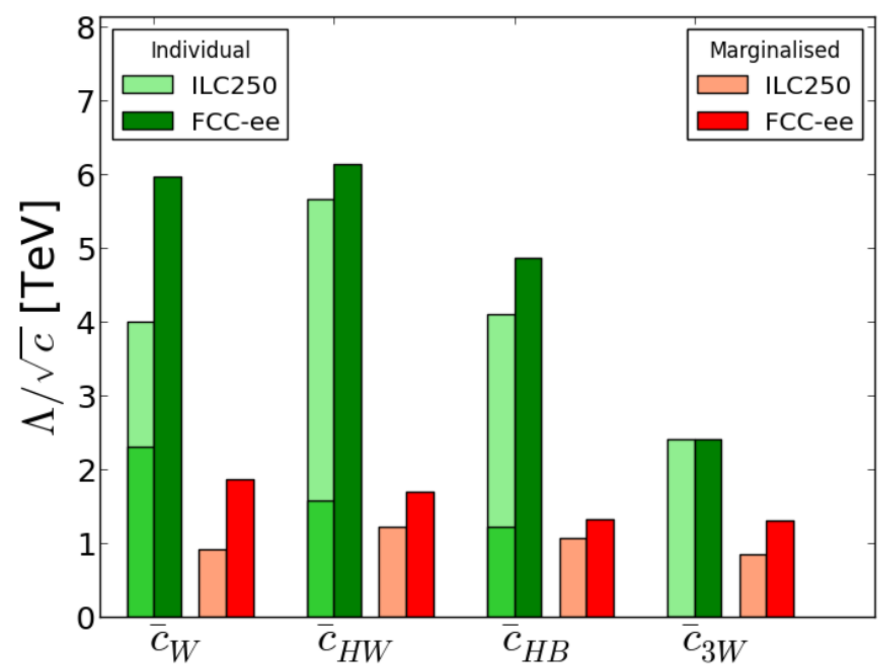
FCC-ee

(light green: without TGC)

J. Ellis and T.Y. [arXiv:1510:04561]

Future e+e- Constraints





J. Ellis and T.Y. [arXiv:1510:04561]

Future Constraints to MSSM Stops

Coeff.	Experimental constraints		95 % CL limit	deg. $m_{\tilde{t}_1}$ $X_t = 0$	deg. $m_{\tilde{t}_1}$ $X_t = m_{\tilde{t}}/2$
$\bar{c}_g$	$\mathrm{ILC}^{1150\mathrm{fb}^{-1}}_{250\mathrm{GeV}}$	marginalized	$[-7.7, 7.7] \times 10^{-6}$	~ 675 GeV	~ 520 GeV
		individual	$[-7.5, 7.5] \times 10^{-6}$	~ 680 GeV	~ 545 GeV
	FCC-ee	marginalized	$[-3.0, 3.0] \times 10^{-6}$	~ 1065 GeV	~ 920 GeV
		individual	$[-3.0, 3.0] \times 10^{-6}$	~ 1065 GeV	~ 915 GeV
$\bar{c}_\gamma$	$\mathrm{ILC}^{1150\mathrm{fb}^{-1}}_{250\mathrm{GeV}}$	marginalized	$[-3.4, 3.4] \times 10^{-4}$	~ 200 GeV	~ 40 GeV
		individual	$[-3.3, 3.3] \times 10^{-4}$	~ 200 GeV	~ 35 GeV
	FCC-ee	marginalized	$[-6.4, 6.4] \times 10^{-5}$	~ 385 GeV	~ 250 GeV
		individual	$[-6.3, 6.3] \times 10^{-5}$	~ 390 GeV	~ 260 GeV
$\bar{c}_T$	$\mathrm{ILC}^{1150\mathrm{fb}^{-1}}_{250\mathrm{GeV}}$	marginalized	$[-3, 3] \times 10^{-4}$	~ 480 GeV	~ 285 GeV
		individual	$[-7, 7] \times 10^{-5}$	~ 930 GeV	~ 780 GeV
	FCC-ee	marginalized	$[-3, 3] \times 10^{-5}$	~ 1410 GeV	~ 1285 GeV
		individual	$[-0.9, 0.9] \times 10^{-5}$	~ 2555 GeV	~ 2460 GeV
$\bar{c}_W + \bar{c}_B$	$\mathrm{ILC}^{1150\mathrm{fb}^{-1}}_{250\mathrm{GeV}}$	marginalized	$[-2, 2] \times 10^{-4}$	~ 230 GeV	~ 170 GeV
		individual	$[-6, 6] \times 10^{-5}$	~ 340 GeV	~ 470 GeV
	FCC-ee	marginalized	$[-2, 2] \times 10^{-5}$	~ 545 GeV	~ 960 GeV
		individual	$[-0.8, 0.8] \times 10^{-5}$	~ 830 GeV	~ 1590 GeV

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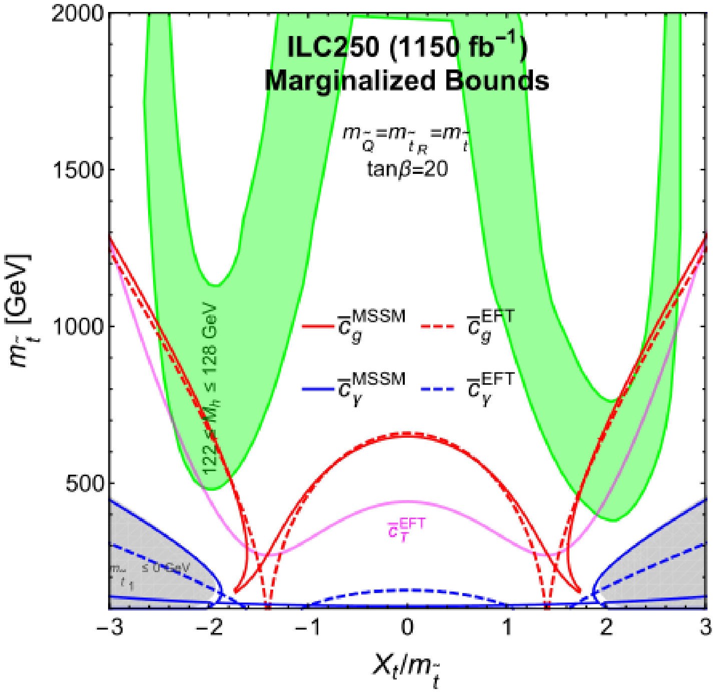
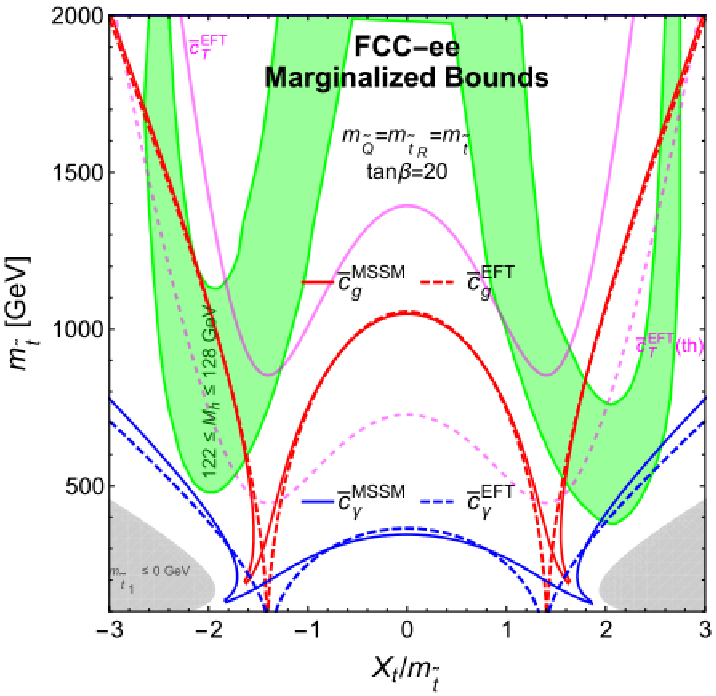
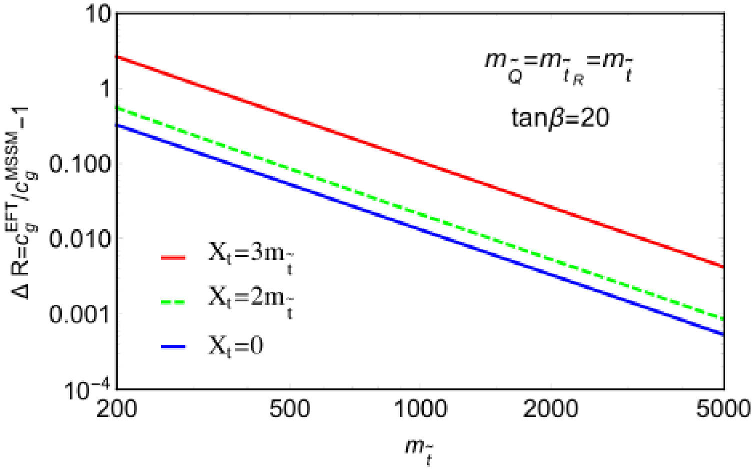
1504.02409f

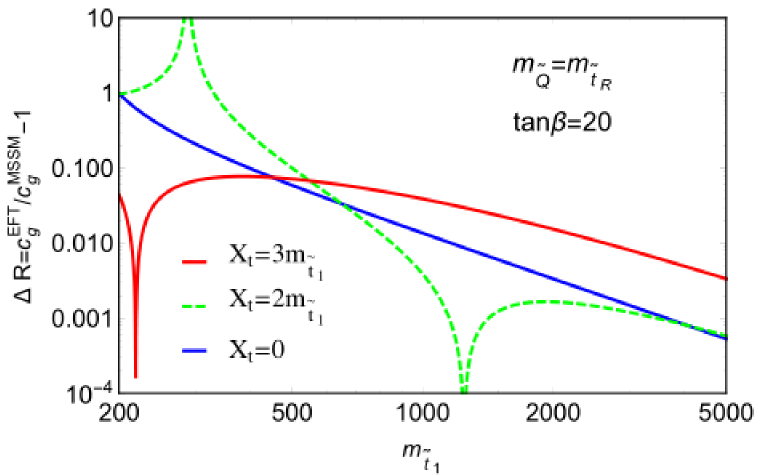


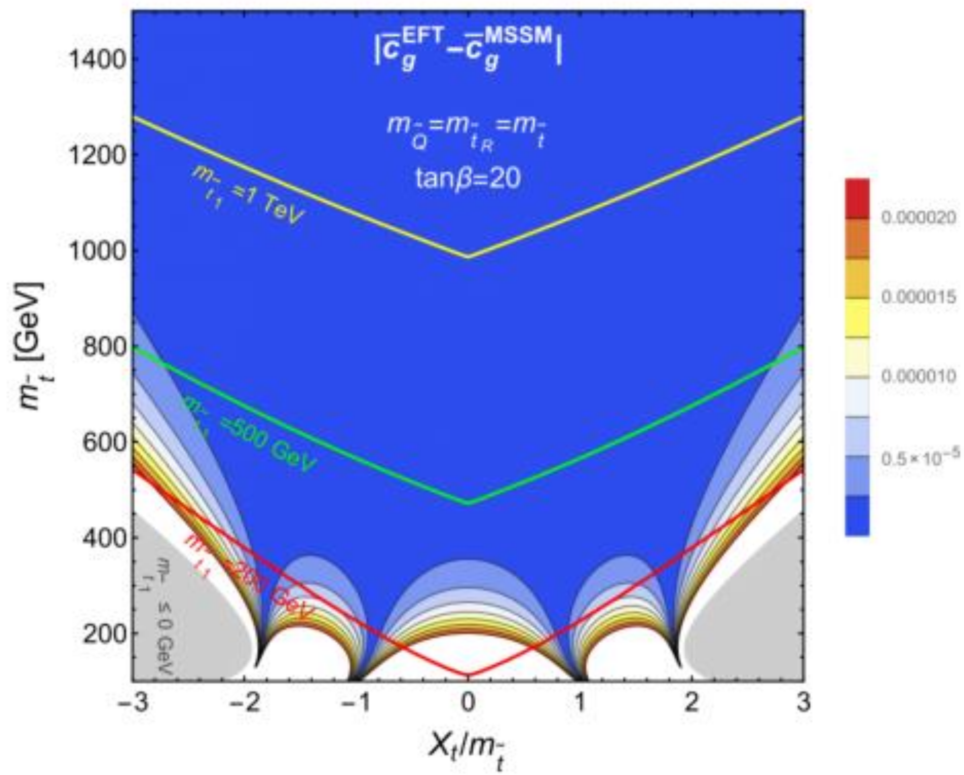


EFT Validity for Stops

- Operators > dim-6 become important when EFT cut-off/stop mass is too low
- Compare EFT dim-6 vs full MSSM amplitude

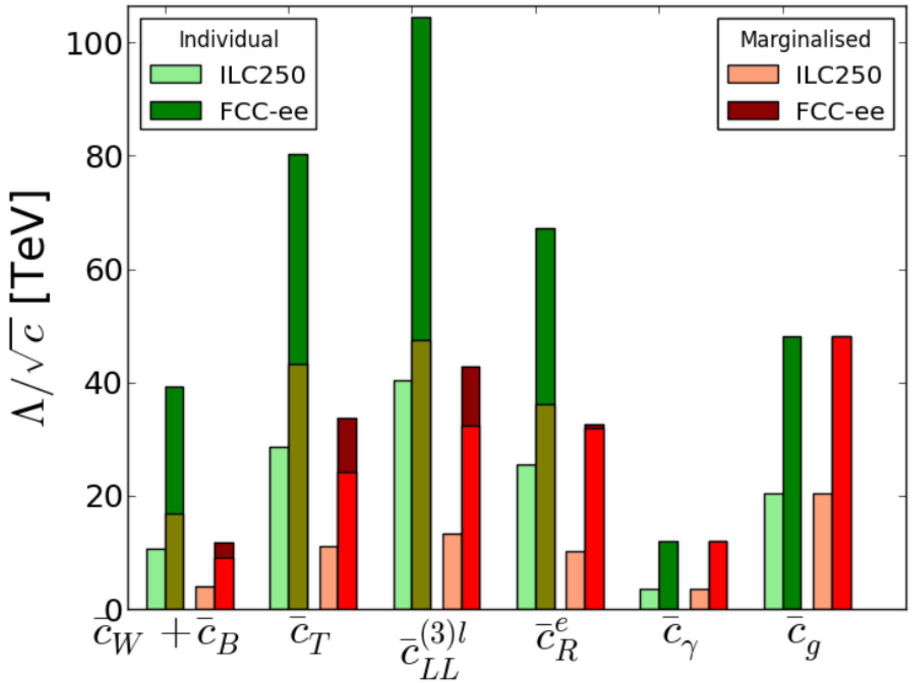




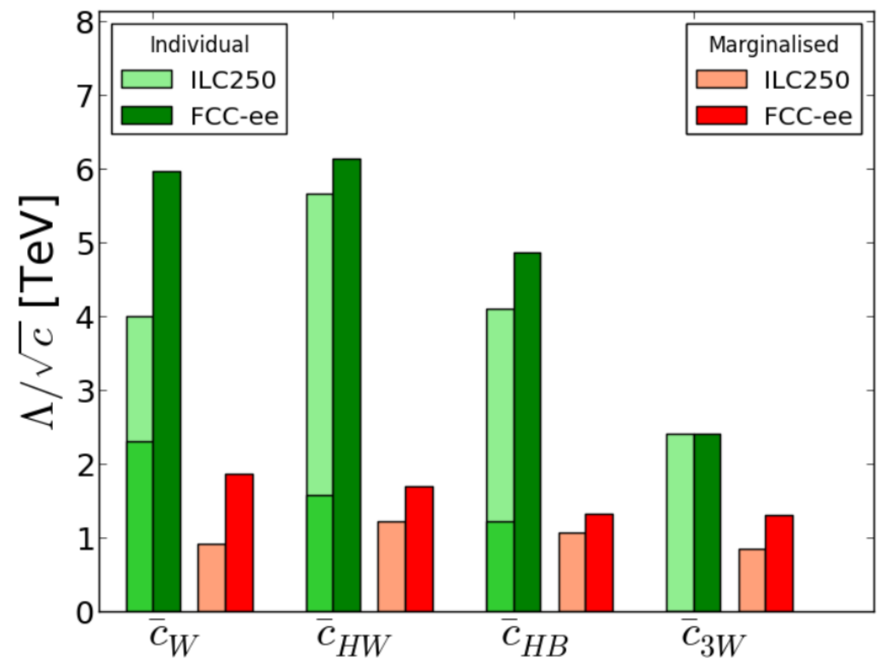


Conclusion

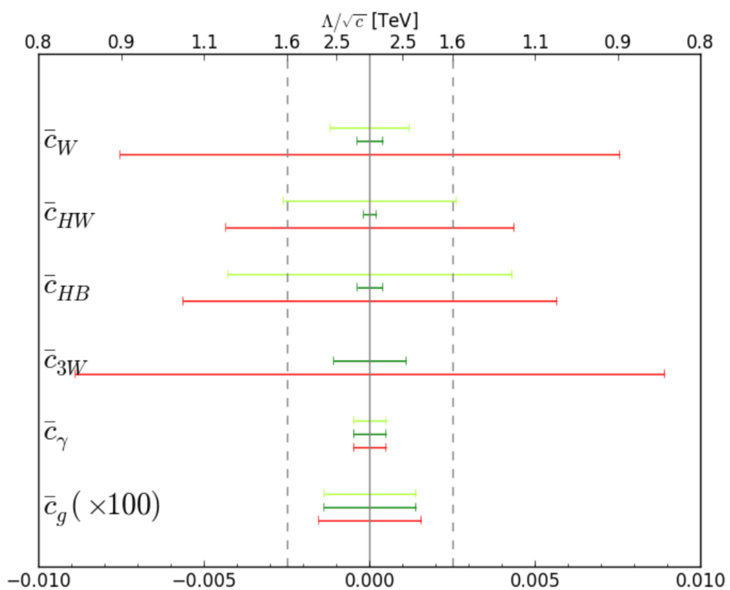
- SM EFT is the Fermi theory of the 21st century
- Wilson coefficients are our windows to heavy new physics
- Future precision may probe even loop-induced operators at the TeV scale
- Universal one-loop effective action for dim-6 operators facilitates systematic comparison of experiment and theory
- Any discovery of direct resonance will rely on precision indirect measurements to complete the picture



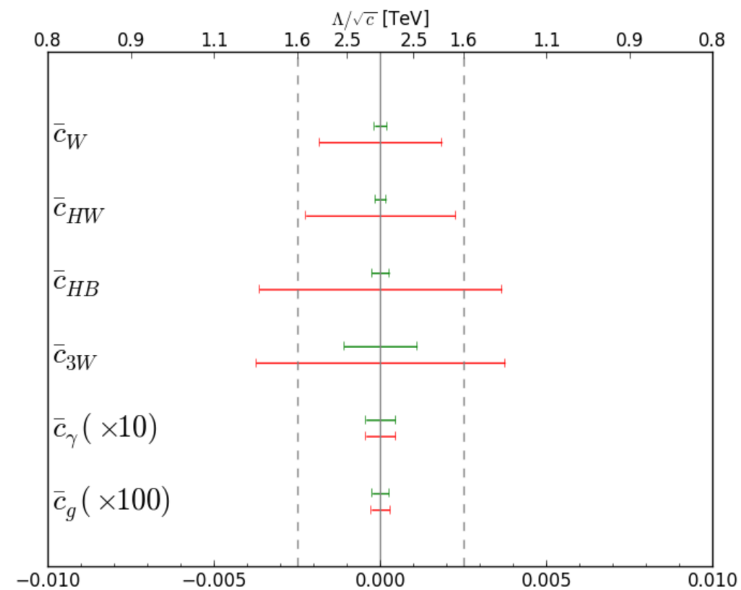
Individual
ILC250
FCC-ee
Marginalised
ILC250
FCC-ee
$\Lambda/\sqrt{c}$ [TeV]
0
20
40
60
80
100
$\bar{c}_W + \bar{c}_B$
$\bar{c}_T$
$\bar{c}_{LL}^{(3)l}$
$\bar{c}_R^e$
$\bar{c}_\gamma$
$\bar{c}_g$



Individual
ILC250
FCC-ee
Marginalised
ILC250
FCC-ee
$\Lambda/\sqrt{c}$ [TeV]
0
1
2
3
4
5
6
7
8
$\bar{c}_W$
$\bar{c}_{HW}$
$\bar{c}_{HB}$
$\bar{c}_{3W}$



$\Lambda/\sqrt{c}$ [TeV]
0.8 0.9 1.1 1.6 2.5 2.5 1.6 1.1 0.9 0.8
$\bar{c}_W$
$\bar{c}_{HW}$
$\bar{c}_{HB}$
$\bar{c}_{3W}$
$\bar{c}_\gamma$
$\bar{c}_g\,(\times 100)$
−0.010 −0.005 0.000 0.005 0.010



$\Lambda/\sqrt{c}$ [TeV]
0.8 0.9 1.1 1.6 2.5 2.5 1.6 1.1 0.9 0.8
$\bar{c}_W$
$\bar{c}_{HW}$
$\bar{c}_{HB}$
$\bar{c}_{3W}$
$\bar{c}_\gamma\,(\times 10)$
$\bar{c}_g\,(\times 100)$
−0.010 −0.005 0.000 0.005 0.010

Tevong You

One-Loop Effective Action

$$\mathrm{I}[\mathrm{q}^{2\alpha}]^{nm\cdots p}_{i\,j\,\cdots l} = \int \frac{d^4\mathrm{q}}{(2\pi)^4}\int d\xi\, \mathrm{q}^{2\alpha}\, (\Delta_{\xi,i})^n\, (\Delta_{\xi,j})^m \cdots (\Delta_{\xi,l})^p$$

$$\Delta_{\xi,i} = 1/(q^2 - \xi m_i^2)$$

- Universal coefficients

$$\begin{aligned}
f_9^{ij} &= \frac{1}{2}\Big(\big(\mathrm{I}^{31}_{ij} - \mathrm{I}[\mathrm{q}^2]^{41}_{ij}\big)\, m_i^2 + \big(-\mathrm{I}[\mathrm{q}^2]^{14}_{ij} - \mathrm{I}[\mathrm{q}^2]^{23}_{ij} - \mathrm{I}[\mathrm{q}^2]^{32}_{ij} + \mathrm{I}^{13}_{ij} + \mathrm{I}^{22}_{ij}\big)\, m_j^2\Big)\,, \\
f_{10}^{ijkl} &= \mathrm{I}^{2111}_{ijkl} m_i^2\,, \\
f_{11}^{ijk} &= \big(\mathrm{I}^{212}_{ijk} - \mathrm{I}[\mathrm{q}^2]^{213}_{ijk}\big)\, m_i^2 + \big(\mathrm{I}^{122}_{ijk} - \mathrm{I}[\mathrm{q}^2]^{123}_{ijk}\big)\, m_j^2 + \big(-\mathrm{I}[\mathrm{q}^2]^{132}_{ijk} - \mathrm{I}[\mathrm{q}^2]^{222}_{ijk} - \mathrm{I}[\mathrm{q}^2]^{312}_{ijk} + \mathrm{I}^{122}_{ijk} + \mathrm{I}^{212}_{ijk}\big)\, m_k^2\,, \\
f_{12,a}^{ij} &= \frac{1}{3}\big(2\mathrm{I}[\mathrm{q}^4]^{25}_{ij} - 3\mathrm{I}[\mathrm{q}^2]^{24}_{ij} + \mathrm{I}[\mathrm{q}^2]^{33}_{ij} + \mathrm{I}[\mathrm{q}^2]^{42}_{ij} + \mathrm{I}^{23}_{ij} - \mathrm{I}^{32}_{ij}\big)\, m_i^2\,, \\
f_{12,b}^{ij} &= f_{12,a}^{ij}\,, \\
f_{12,c}^{ij} &= \frac{1}{3}\big(-3\mathrm{I}[\mathrm{q}^2]^{24}_{ij} + \mathrm{I}^{23}_{ij} + 2\mathrm{I}[\mathrm{q}^4]^{25}_{ij} - 2\mathrm{I}[\mathrm{q}^2]^{33}_{ij} - 2\mathrm{I}[\mathrm{q}^2]^{42}_{ij} + 2\mathrm{I}^{32}_{ij}\big)\, m_i^2\,, \\
f_{13}^{ijk} &= \frac{1}{2}\Big(\big(\mathrm{I}^{311}_{ijk} - \mathrm{I}[\mathrm{q}^2]^{411}_{ijk}\big)\, m_i^2 + \big(-\mathrm{I}[\mathrm{q}^2]^{141}_{ijk} - \mathrm{I}[\mathrm{q}^2]^{231}_{ijk} - \mathrm{I}[\mathrm{q}^2]^{321}_{ijk} + \mathrm{I}^{131}_{ijk} + \mathrm{I}^{221}_{ijk}\big)\, m_j^2 \\
&\qquad + \big(-\mathrm{I}[\mathrm{q}^2]^{114}_{ijk} - \mathrm{I}[\mathrm{q}^2]^{123}_{ijk} - \mathrm{I}[\mathrm{q}^2]^{132}_{ijk} - \mathrm{I}[\mathrm{q}^2]^{213}_{ijk} - \mathrm{I}[\mathrm{q}^2]^{222}_{ijk} - \mathrm{I}[\mathrm{q}^2]^{312}_{ijk} + \mathrm{I}^{113}_{ijk} + \mathrm{I}^{122}_{ijk} + \mathrm{I}^{212}_{ijk}\big)\, m_k^2\Big)\,, \\
f_{14}^{ijk} &= \big(\mathrm{I}[\mathrm{q}^2]^{132}_{ijk} + 2\mathrm{I}[\mathrm{q}^2]^{141}_{ijk} + \mathrm{I}[\mathrm{q}^2]^{231}_{ijk} - 2\mathrm{I}^{131}_{ijk}\big)\, m_j^2\,, \\
f_{15a}^{ijk} &= \frac{1}{3}\big(-\mathrm{I}[\mathrm{q}^2]^{312}_{ijk} - 2\mathrm{I}[\mathrm{q}^2]^{411}_{ijk} + 2\mathrm{I}^{311}_{ijk}\big)\, m_i^2 \\
&\quad + \frac{1}{3}\big(-\mathrm{I}[\mathrm{q}^2]^{132}_{ijk} - 2\mathrm{I}[\mathrm{q}^2]^{141}_{ijk} - \mathrm{I}[\mathrm{q}^2]^{222}_{ijk} + 2\mathrm{I}^{131}_{ijk} + 2\mathrm{I}^{221}_{ijk} - 2\mathrm{I}[\mathrm{q}^2]^{231}_{ijk} - 2\mathrm{I}[\mathrm{q}^2]^{321}_{ijk}\big)\, m_j^2 \\
&\quad + \frac{1}{3}\big(2\mathrm{I}[\mathrm{q}^2]^{114}_{ijk} + \mathrm{I}[\mathrm{q}^2]^{123}_{ijk} + \mathrm{I}[\mathrm{q}^2]^{213}_{ijk} - 2\mathrm{I}^{113}_{ijk}\big)\, m_k^2\,, \\
f_{15b}^{ijk} &= \frac{1}{3}\big(\mathrm{I}[\mathrm{q}^2]^{312}_{ijk} + \mathrm{I}[\mathrm{q}^2]^{321}_{ijk} + 2\mathrm{I}[\mathrm{q}^2]^{411}_{ijk} - 2\mathrm{I}^{311}_{ijk}\big)\, m_i^2 \\
&\quad + \frac{1}{3}\big(-2\mathrm{I}[\mathrm{q}^2]^{141}_{ijk} - \mathrm{I}[\mathrm{q}^2]^{231}_{ijk} + 2\mathrm{I}^{131}_{ijk}\big)\, m_j^2 \\
&\quad + \frac{1}{3}\big(-2\mathrm{I}[\mathrm{q}^2]^{114}_{ijk} - 2\mathrm{I}[\mathrm{q}^2]^{123}_{ijk} - 2\mathrm{I}[\mathrm{q}^2]^{132}_{ijk} - \mathrm{I}[\mathrm{q}^2]^{213}_{ijk} - \mathrm{I}[\mathrm{q}^2]^{222}_{ijk} + 2\mathrm{I}^{113}_{ijk} + 2\mathrm{I}^{122}_{ijk}\big)\, m_k^2\,, \\
f_{16}^{ijklm} &= \mathrm{I}^{21111}_{ijklm} m_i^2\,, \\
f_{17}^{ijkl} &= \big(-\mathrm{I}[\mathrm{q}^2]^{2113}_{ijkl} + \mathrm{I}^{2112}_{ijkl}\big)\, m_i^2 + \big(\mathrm{I}^{1212}_{ijkl} - \mathrm{I}[\mathrm{q}^2]^{1213}_{ijkl}\big)\, m_j^2 + \big(\mathrm{I}^{1122}_{ijkl} - \mathrm{I}[\mathrm{q}^2]^{1123}_{ijkl}\big)\, m_k^2 \\
&\quad + \big(-\mathrm{I}[\mathrm{q}^2]^{1132}_{ijkl} - \mathrm{I}[\mathrm{q}^2]^{1222}_{ijkl} - \mathrm{I}[\mathrm{q}^2]^{1312}_{ijkl} - \mathrm{I}[\mathrm{q}^2]^{2122}_{ijkl} - \mathrm{I}[\mathrm{q}^2]^{2212}_{ijkl} - \mathrm{I}[\mathrm{q}^2]^{3112}_{ijkl} \\
&\quad + \mathrm{I}^{1122}_{ijkl} + \mathrm{I}^{1212}_{ijkl} + \mathrm{I}^{2112}_{ijkl}\big) m_l^2\,, \\
f_{18}^{ijkl} &= \big(-\mathrm{I}[\mathrm{q}^2]^{2113}_{ijkl} - \mathrm{I}[\mathrm{q}^2]^{2122}_{ijkl} - \mathrm{I}[\mathrm{q}^2]^{2131}_{ijkl} + \mathrm{I}^{2112}_{ijkl} + \mathrm{I}^{2121}_{ijkl}\big)\, m_i^2 \\
&\quad + \big(-\mathrm{I}[\mathrm{q}^2]^{1312}_{ijkl} - \mathrm{I}[\mathrm{q}^2]^{2212}_{ijkl} - \mathrm{I}[\mathrm{q}^2]^{3112}_{ijkl} + \mathrm{I}^{1212}_{ijkl} + \mathrm{I}^{2112}_{ijkl}\big)\, m_l^2\,, \\
f_{19}^{ijklmn} &= \mathrm{I}^{211111}_{ijklmn} m_i^2\,.
\end{aligned} \tag{2.5}$$

etc.

One-Loop Effective Action

$$\mathrm{I}[\mathrm{q}^{2\alpha}]^{nm\cdots p}_{i\,j\,\cdots l} = \int \frac{d^4\mathrm{q}}{(2\pi)^4} \int d\xi\, \mathrm{q}^{2\alpha} \left(\Delta_{\xi,i}\right)^n \left(\Delta_{\xi,j}\right)^m \cdots \left(\Delta_{\xi,l}\right)^p$$

$$\Delta_{\xi,i} = 1/(q^2 - \xi m_i^2)$$

- Universal coefficients

$$\tilde{f}_5^{ij} = \frac{6m_i^4 m_j^2 - 3m_i^2 m_j^4 - 6m_i^2 m_j^4 \ln\left(\frac{m_i^2}{m_j^2}\right) - m_i^6 - 2m_j^6}{54\Delta m_{ij}^8} + \frac{1}{270}\left(\frac{7}{m_i^2} - \frac{9}{m_j^2}\right),$$

$$\tilde{f}_6^{ij} = \frac{7}{180m_i^2} - \frac{1}{20m_j^2},$$

$$\tilde{f}_7^{ij} = \frac{-2m_i^4 m_j^2 + \frac{5}{2}m_i^2 m_j^4 + 2m_i^4 m_j^2 \ln\left(\frac{m_i^2}{m_j^2}\right) + m_i^2 m_j^4 \ln\left(\frac{m_i^2}{m_j^2}\right) - \frac{m_i^6}{2}}{\Delta m_{ij}^8},$$

$$\tilde{f}_8^{ijk} = \frac{m_i^2 m_j^2 m_k^2 + m_i^2 m_j^2 m_k^2 \ln\left(m_i^2\right) - m_i^4 m_j^2 - m_i^4 m_k^2 + m_i^6 - m_i^6 \ln\left(m_i^2\right)}{\Delta m_{ij}^4 \Delta m_{ik}^4} + \frac{m_i^2 m_j^2 \ln\left(m_j^2\right)}{\Delta m_{ij}^4 \Delta m_{jk}^2} - \frac{m_i^2 m_k^2 \ln\left(m_k^2\right)}{\Delta m_{ik}^4 \Delta m_{jk}^2},$$

$$\tilde{f}_9^{ij} = \frac{m_i^4 m_j^2 + m_i^2 m_j^4 + 2m_i^4 m_j^2 \ln\left(\frac{m_i^2}{m_j^2}\right) - 2m_i^2 m_j^4 \ln\left(\frac{m_i^2}{m_j^2}\right) - m_i^6 - m_j^6}{4\Delta m_{ij}^8},$$

$$\tilde{f}_{10}^{ijkl} = \left\{\frac{-m_i^2 m_j^2 + m_i^2 m_j^2 \ln\left(m_j^2\right) + m_i^4 - m_i^4 \ln\left(m_i^2\right)}{2\Delta m_{ij}^4 \Delta m_{jk}^2 \Delta m_{jl}^2} + (j \leftrightarrow k) - (j \leftrightarrow l)\right\} + \frac{m_i^2 \ln\left(m_i^2\right)}{2\Delta m_{ij}^2 \Delta m_{ik}^2 \Delta m_{il}^2},$$

$$\begin{aligned}\tilde{f}_{11}^{ijk} = \frac{1}{2\Delta m_{ij}^2 \Delta m_{ik}^6 \Delta m_{jk}^6}\Bigg(& 3m_i^2 m_j^4 m_k^4 - 3m_i^4 m_j^2 m_k^4 + 6m_i^2 m_j^2 m_k^6 \ln\left(\frac{m_i^2}{m_j^2}\right) - 6m_i^2 m_j^4 m_k^4 \ln\left(\frac{m_i^2}{m_k^2}\right) \\ &+6m_i^4 m_j^2 m_k^4 \ln\left(\frac{m_j^2}{m_k^2}\right) + 2m_i^2 m_j^6 m_k^2 \ln\left(\frac{m_i^2}{m_k^2}\right) - 2m_i^6 m_j^2 m_k^2 \ln\left(\frac{m_j^2}{m_k^2}\right) - m_i^4 m_j^6 + m_i^6 m_j^4 \\ &-3m_i^2 m_k^8 + 4m_i^4 m_k^6 - m_i^6 m_k^4 - 2m_i^2 m_k^8 \ln\left(\frac{m_i^2}{m_k^2}\right) + 3m_j^2 m_k^8 - 4m_j^4 m_k^6 + m_j^6 m_k^4 + 2m_j^2 m_k^8 \ln\left(\frac{m_j^2}{m_k^2}\right)\Bigg),\end{aligned}$$

$$\tilde{f}_{12,a}^{ij} = \frac{28m_i^6 m_j^2 - 28m_i^2 m_j^6 - 12m_i^6 m_j^2 \ln\left(\frac{m_i^2}{m_j^2}\right) - 36m_i^4 m_j^4 \ln\left(\frac{m_i^2}{m_j^2}\right) - 12m_i^2 m_j^6 \ln\left(\frac{m_i^2}{m_j^2}\right) + m_i^8 - m_j^8}{36\Delta m_{ij}^{12}},$$

$$\tilde{f}_{12,b}^{ij} = \tilde{f}_{12,a}^{ij},$$

etc.

One-Loop Effective Action

$$\mathrm{I}[\mathrm{q}^{2\alpha}]^{nm\cdots p}_{i\,j\,\cdots l} = \int \frac{d^4\mathrm{q}}{(2\pi)^4} \int d\xi\, \mathrm{q}^{2\alpha} \left(\Delta_{\xi,i}\right)^n \left(\Delta_{\xi,j}\right)^m \cdots \left(\Delta_{\xi,l}\right)^p$$

$$\Delta_{\xi,i} = 1/(q^2 - \xi m_i^2)$$

- Universal coefficients

$$\tilde{f}_5^{ij} = \frac{6m_i^4m_j^2 - 3m_i^2m_j^4 - 6m_i^2m_j^4 \ln\left(\frac{m_i^2}{m_j^2}\right) - m_i^6 - 2m_j^6}{54\Delta m_{ij}^8} + \frac{1}{270}\left(\frac{7}{m_i^2} - \frac{9}{m_j^2}\right),$$

$$\tilde{f}_6^{ij} = \frac{7}{180m_i^2} - \frac{1}{20m_j^2},$$

$$\tilde{f}_7^{ij} = \frac{-2m_i^4m_j^2 + \frac{5}{2}m_i^2m_j^4 + 2m_i^4m_j^2 \ln\left(\frac{m_i^2}{m_j^2}\right) + m_i^2m_j^4 \ln\left(\frac{m_i^2}{m_j^2}\right) - \frac{m_i^6}{2}}{\Delta m_{ij}^8},$$

$$\tilde{f}_8^{ijk} = \frac{m_i^2m_j^2m_k^2 + m_i^2m_j^2m_k^2 \ln\left(m_i^2\right) - m_i^4m_j^2 - m_i^4m_k^2 + m_i^6 - m_i^6 \ln\left(m_i^2\right)}{\Delta m_{ij}^4 \Delta m_{ik}^4} + \frac{m_i^2m_j^2 \ln\left(m_j^2\right)}{\Delta m_{ij}^4 \Delta m_{jk}^2} - \frac{m_i^2m_k^2 \ln\left(m_k^2\right)}{\Delta m_{ik}^4 \Delta m_{jk}^2},$$

$$\tilde{f}_9^{ij} = \frac{m_i^4m_j^2 + m_i^2m_j^4 + 2m_i^4m_j^2 \ln\left(\frac{m_i^2}{m_j^2}\right) - 2m_i^2m_j^4 \ln\left(\frac{m_i^2}{m_j^2}\right) - m_i^6 - m_j^6}{4\Delta m_{ij}^8},$$

$$\tilde{f}_{10}^{ijkl} = \left\{\frac{-m_i^2m_j^2 + m_i^2m_j^2 \ln\left(m_j^2\right) + m_i^4 - m_i^4 \ln\left(m_i^2\right)}{2\Delta m_{ij}^4 \Delta m_{jk}^2 \Delta m_{jl}^2} + (j \leftrightarrow k) - (j \leftrightarrow l)\right\} + \frac{m_i^2 \ln\left(m_i^2\right)}{2\Delta m_{ij}^2 \Delta m_{ik}^2 \Delta m_{il}^2},$$

$$\begin{aligned}\tilde{f}_{11}^{ijk} = \frac{1}{2\Delta m_{ij}^2 \Delta m_{ik}^6 \Delta m_{jk}^6}\Bigg(& 3m_i^2m_j^4m_k^4 - 3m_i^4m_j^2m_k^4 + 6m_i^2m_j^2m_k^6 \ln\left(\frac{m_i^2}{m_j^2}\right) - 6m_i^2m_j^4m_k^4 \ln\left(\frac{m_i^2}{m_k^2}\right) \\ &+ 6m_i^4m_j^2m_k^4 \ln\left(\frac{m_j^2}{m_k^2}\right) + 2m_i^2m_j^6m_k^2 \ln\left(\frac{m_i^2}{m_k^2}\right) - 2m_i^6m_j^2m_k^2 \ln\left(\frac{m_j^2}{m_k^2}\right) - m_i^4m_j^6 + m_i^6m_j^4 \\ &- 3m_i^2m_k^8 + 4m_i^4m_k^6 - m_i^6m_k^4 - 2m_i^2m_k^8 \ln\left(\frac{m_i^2}{m_k^2}\right) + 3m_j^2m_k^8 - 4m_j^4m_k^6 + m_j^6m_k^4 + 2m_j^2m_k^8 \ln\left(\frac{m_j^2}{m_k^2}\right)\Bigg),\end{aligned}$$

$$\tilde{f}_{12,a}^{ij} = \frac{28m_i^6m_j^2 - 28m_i^2m_j^6 - 12m_i^6m_j^2 \ln\left(\frac{m_i^2}{m_j^2}\right) - 36m_i^4m_j^4 \ln\left(\frac{m_i^2}{m_j^2}\right) - 12m_i^2m_j^6 \ln\left(\frac{m_i^2}{m_j^2}\right) + m_i^8 - m_j^8}{36\Delta m_{ij}^{12}},$$

$$\tilde{f}_{12,b}^{ij} = \tilde{f}_{12,a}^{ij},$$

etc.

Degenerate mass limit:

$$f_5 = -\frac{i}{(4\pi)^2 60m^2}, \quad f_{11} = \frac{i}{(4\pi)^2 12m^4}, \quad f_{15a} = \frac{i}{(4\pi)^2 60m^4},$$

$$f_6 = -\frac{i}{(4\pi)^2 90m^2}, \quad f_{12,a} = 0, \quad f_{15b} = \frac{i}{(4\pi)^2 60m^4},$$

$$f_7 = -\frac{i}{(4\pi)^2 12m^2}, \quad f_{12,b} = 0, \quad f_{16} = -\frac{i}{(4\pi)^2 60m^6},$$

$$f_8 = -\frac{i}{(4\pi)^2 6m^2}, \quad f_{12,c} = \frac{i}{(4\pi)^2 120m^4}, \quad f_{17} = -\frac{i}{(4\pi)^2 20m^6},$$

$$f_9 = -\frac{i}{(4\pi)^2 12m^2}, \quad f_{13} = \frac{i}{(4\pi)^2 24m^4}, \quad f_{18} = -\frac{i}{(4\pi)^2 30m^6},$$

$$f_{10} = \frac{i}{(4\pi)^2 24m^4}, \quad f_{14} = \frac{-i}{(4\pi)^2 60m^4}, \quad f_{19} = \frac{i}{(4\pi)^2 120m^8}.$$

One-Loop Effective Action

- e.g. MSSM stop

$$G'_{\mu\nu} = \begin{pmatrix} W'^a_{\mu\nu}\tau^a + Y_{\tilde{Q}} B'_{\mu\nu}\mathbb{1} & 0 \\ 0 & -Y_{\tilde{t}_R} B'_{\mu\nu} \end{pmatrix}$$

$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + \cancel{(\Phi^\dagger F(x) + \text{h.c.})} + \Phi^\dagger (P^2 - M^2 - U(x))\Phi + \mathcal{O}(\Phi^3)\,,$$

(R-parity)

$$M^2 = \begin{pmatrix} m^2_{\tilde{Q}} & 0 \\ 0 & m^2_{\tilde{t}_R} \end{pmatrix} \qquad \Phi = (\tilde{Q}\,, \tilde{t}^*_R).$$

$$U = \begin{pmatrix} (h_t^2 + \frac{1}{2} g_2^2 c_\beta^2)\tilde{H}\tilde{H}^\dagger + \frac{1}{2} g_2^2 s_\beta^2 H H^\dagger - \frac{1}{2}(g_1^2 Y_{\tilde{Q}} c_{2\beta} + \frac{1}{2} g_2^2)|H|^2 & h_t X_t \tilde{H} \\ h_t X_t \tilde{H}^\dagger & (h_t^2 - \frac{1}{2} g_1^2 Y_{\tilde{t}_R} c_{2\beta})|H|^2 \end{pmatrix}$$

$$\begin{aligned}
{}^{\rm eff}_{\text{1-loop}}[\phi] \supset -ic_s \Big\{ & f_1^i + f_2^i U_{ii} + f_3^i G'^2_{\mu\nu,ij} + f_4^{ij} U^2_{ij} \\
& + f_5^{ij}(P_\mu G'_{\mu\nu,ij})^2 + f_6^{ij}(G'_{\mu\nu,ij})(G'_{\nu\sigma,jk})(G'_{\sigma\mu,ki}) + f_7^{ij}[P_\mu, U_{ij}]^2 + f_8^{ijk}(U_{ij}U_{jk}U_{ki}) \\
& + f_9^{ij}(U_{ij}G'_{\mu\nu,jk}G'_{\mu\nu,ki}) \\
& + f_{10}^{ijkl}(U_{ij}U_{jk}U_{kl}U_{li}) + f_{11}^{ijk}U_{ij}[P_\mu, U_{jk}][P_\mu, U_{ki}] \\
& + f_{12,a}^{ij}\,[P_\mu,[P_\nu,U_{ij}]]\,[P_\mu,[P_\nu,U_{ji}]] + f_{12,b}^{ij}\,[P_\mu,[P_\nu,U_{ij}]]\,[P_\nu,[P_\mu,U_{ji}]] \\
& + f_{12,c}^{ij}\,[P_\mu,[P_\mu,U_{ij}]]\,[P_\nu,[P_\nu,U_{ji}]] \\
& + f_{13}^{ijk}U_{ij}U_{jk}G'_{\mu\nu,kl}G'_{\mu\nu,li} + f_{14}^{ijk}\,[P_\mu,U_{ij}]\,[P_\nu,U_{jk}]\,G'_{\nu\mu,ki} \\
& + \left(f_{15a}^{ijk}U_{i,j}[P_\mu,U_{j,k}] - f_{15b}^{ijk}[P_\mu,U_{i,j}]U_{j,k}\right)[P_\nu, G'_{\nu\mu,ki}] \\
& + f_{16}^{ijklm}(U_{ij}U_{jk}U_{kl}U_{lm}U_{mi}) + f_{17}^{ijkl}U_{ij}U_{jk}[P_\mu,U_{kl}][P_\mu,U_{li}] + f_{18}^{ijkl}U_{ij}[P_\mu,U_{jk}]U_{kl}[P_\mu,U_l \\
& + f_{19}^{ijklmn}(U_{ij}U_{jk}U_{kl}U_{lm}U_{mn}U_{ni}) \Big\}\,.
\end{aligned}$$

One-Loop Effective Action

- e.g. MSSM stop

$$G'_{\mu\nu} = \begin{pmatrix} W'^a_{\mu\nu}\tau^a + Y_{\tilde{Q}} B'_{\mu\nu}\mathbb{1} & 0 \\ 0 & -Y_{\tilde{t}_R} B'_{\mu\nu} \end{pmatrix}$$

$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + (\Phi^\dagger F(x) + \text{h.c.}) + \Phi^\dagger(P^2 - M^2 - U(x))\Phi + \mathcal{O}(\Phi^3)\,,$$

(R-parity)

$$M^2 = \begin{pmatrix} m^2_{\tilde{Q}} & 0 \\ 0 & m^2_{\tilde{t}_R} \end{pmatrix} \qquad \Phi = (\tilde{Q}\,, \tilde{t}^*_R).$$

$$U = \begin{pmatrix} (h_t^2 + \frac{1}{2}g_2^2 c_\beta^2)\tilde{H}\tilde{H}^\dagger + \frac{1}{2}g_2^2 s_\beta^2 H H^\dagger - \frac{1}{2}(g_1^2 Y_{\tilde{Q}} c_{2\beta} + \frac{1}{2}g_2^2)|H|^2 & h_t X_t \tilde{H} \\ h_t X_t \tilde{H}^\dagger & (h_t^2 - \frac{1}{2}g_1^2 Y_{\tilde{t}_R} c_{2\beta})|H|^2 \end{pmatrix}$$

$$\begin{aligned} {}^{\rm eff}_{\text{1-loop}}[\phi] \supset -ic_s\Big\{ & f_1^i + f_2^i U_{ii} + f_3^i G'^2_{\mu\nu,ij} + f_4^{ij} U^2_{ij} \\ & + f_5^{ij}(P_\mu G'_{\mu\nu,ij})^2 + f_6^{ij}(G'_{\mu\nu,ij})(G'_{\nu\sigma,jk})(G'_{\sigma\mu,ki}) + f_7^{ij}[P_\mu, U_{ij} \\ & + f_9^{ij}(U_{ij}G'_{\mu\nu,jk}G'_{\mu\nu,ki}) \\ & + f_{10}^{ijkl}(U_{ij}U_{jk}U_{kl}U_{li}) + f_{11}^{ijk}U_{ij}[P_\mu, U_{jk}][P_\mu, U_{ki}] \\ & + f^{ij}_{12,a}[P_\mu,[P_\nu,U_{ij}]][P_\mu,[P_\nu,U_{ji}]] + f^{ij}_{12,b}[P_\mu,[P_\nu,U_{ij}]][P_\nu, \\ & + f^{ij}_{12,c}[P_\mu,[P_\mu,U_{ij}]][P_\nu,[P_\nu,U_{ji}]] \\ & + f_{13}^{ijk}U_{ij}U_{jk}G'_{\mu\nu,kl}G'_{\mu\nu,li} + f_{14}^{ijk}[P_\mu,U_{ij}][P_\nu,U_{jk}]G'_{\nu\mu,ki} \\ & + \left(f^{ijk}_{15a}U_{i,j}[P_\mu,U_{j,k}] - f^{ijk}_{15b}[P_\mu,U_{i,j}]U_{j,k}\right)[P_\nu, G'_{\nu\mu,ki}] \\ & + f_{16}^{ijklm}(U_{ij}U_{jk}U_{kl}U_{lm}U_{mi}) + f_{17}^{ijkl}U_{ij}U_{jk}[P_\mu,U_{kl}][P_\mu,U_{li}] + \\ & + f_{19}^{ijklmn}(U_{ij}U_{jk}U_{kl}U_{lm}U_{mn}U_{ni})\Big\}. \end{aligned}$$

$$c_{WB} = -\frac{g_2^2 c_{2\beta} + 2h_t^2}{48 m^2_{\tilde{Q}}} + \tilde{X}_t^2\left(\frac{33 m^4_{\tilde{Q}} m^2_{\tilde{t}_R} - 3 m^2_{\tilde{Q}} m^4_{\tilde{t}_R} + 5 m^6_{\tilde{Q}} + m^6_{\tilde{t}_R}}{24 m^2_{\tilde{Q}}\left(m^2_{\tilde{Q}} - m^2_{\tilde{t}_R}\right)^4} - \frac{m^2_{\tilde{Q}} m^2_{\tilde{t}_R}\left(2m^2_{\tilde{Q}} + m^2_{\tilde{t}_R}\right)\ln\left(\frac{m^2_{\tilde{Q}}}{m^2_{\tilde{t}_R}}\right)}{2\left(m^2_{\tilde{Q}} - m^2_{\tilde{t}_R}\right)^5}\right),$$

$$c_W = \tilde{X}_t^2\left(\frac{-8 m^2_{\tilde{Q}} m^2_{\tilde{t}_R} + m^4_{\tilde{Q}} - 17 m^4_{\tilde{t}_R}}{12\left(m^2_{\tilde{Q}} - m^2_{\tilde{t}_R}\right)^4} + \frac{\left(3 m^2_{\tilde{Q}} m^4_{\tilde{t}_R} + m^6_{\tilde{t}_R}\right)\ln\left(\frac{m^2_{\tilde{Q}}}{m^2_{\tilde{t}_R}}\right)}{2\left(m^2_{\tilde{Q}} - m^2_{\tilde{t}_R}\right)^5}\right),$$

$$c_B = \tilde{X}_t^2\left(\frac{-8 m^2_{\tilde{Q}} m^2_{\tilde{t}_R} - 23 m^4_{\tilde{Q}} + 7 m^4_{\tilde{t}_R}}{12\left(m^2_{\tilde{Q}} - m^2_{\tilde{t}_R}\right)^4} - \frac{\left(-12 m^4_{\tilde{Q}} m^2_{\tilde{t}_R} + 3 m^2_{\tilde{Q}} m^4_{\tilde{t}_R} - 4 m^6_{\tilde{Q}} + m^6_{\tilde{t}_R}\right)\ln\left(\frac{m^2_{\tilde{Q}}}{m^2_{\tilde{t}_R}}\right)}{6\left(m^2_{\tilde{Q}} - m^2_{\tilde{t}_R}\right)^5}\right),$$

$$c_D = \tilde{X}_t^2\left(\frac{10 m^2_{\tilde{Q}} m^2_{\tilde{t}_R} + m^4_{\tilde{Q}} + m^4_{\tilde{t}_R}}{2\left(m^2_{\tilde{Q}} - m^2_{\tilde{t}_R}\right)^4} - \frac{3 m^2_{\tilde{Q}} m^2_{\tilde{t}_R}\left(m^2_{\tilde{Q}} + m^2_{\tilde{t}_R}\right)\ln\left(\frac{m^2_{\tilde{Q}}}{m^2_{\tilde{t}_R}}\right)}{\left(m^2_{\tilde{Q}} - m^2_{\tilde{t}_R}\right)^5}\right),$$

etc.

One-Loop Effective Action

- e.g. MSSM stop

$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + (\Phi^\dagger F(x) + \text{h.c.}) + \Phi^\dagger(P^2 - M$$

(R-parity)

$$M^2 = \begin{pmatrix} m^2_{\tilde{Q}} & 0 \\ 0 & m^2_{\tilde{t}_R} \end{pmatrix}$$

$$U = \Big((h_t^2 + \tfrac{1}{2}g_2^2 c_\beta^2)\tilde{H}\tilde{H}^\dagger \quad \ldots \; s_\beta^2 H H^\dagger - \tfrac{1}{2}(g_1 \ldots \quad h_t X_t \tilde{H}^\dagger$$

$$\begin{aligned}
{}^{\rm eff}_{\text{1-loop}}[\phi] \supset -ic_s\Big\{ & f_1^i + f_2^i U_{ii} + f_3^i G'^2_{\mu\nu,ij} + f_4^{ij} U_{ij}^2 \\
& + f_5^{ij}(P_\mu G'_{\mu\nu,ij})^2 + f_6^{ij}(G'_{\mu\nu,ij})(G'_{\nu\sigma,jk})(G'_{\sigma\mu,ki}) + f_7^{ij}[P_\mu, U_{ij} \\
& + f_9^{ij}(U_{ij}G'_{\mu\nu,jk}G'_{\mu\nu,ki}) \\
& + f_{10}^{ijkl}(U_{ij}U_{jk}U_{kl}U_{li}) + f_{11}^{ijk}U_{ij}[P_\mu, U_{jk}][P_\mu, U_{ki}] \\
& + f_{12,a}^{ij}[P_\mu,[P_\nu,U_{ij}]]\,[P_\mu,[P_\nu,U_{ji}]] + f_{12,b}^{ij}[P_\mu,[P_\nu,U_{ij}]]\,[P_\nu, \\
& + f_{12,c}^{ij}[P_\mu,[P_\mu,U_{ij}]]\,[P_\nu,[P_\nu,U_{ji}]] \\
& + f_{13}^{ijk}U_{ij}U_{jk}G'_{\mu\nu,kl}G'_{\mu\nu,li} + f_{14}^{ijk}[P_\mu,U_{ij}]\,[P_\nu,U_{jk}]\,G'_{\nu\mu,ki} \\
& + \left(f_{15a}^{ijk}U_{i,j}[P_\mu,U_{j,k}] - f_{15b}^{ijk}[P_\mu,U_{i,j}]U_{j,k}\right)[P_\nu, G'_{\nu\mu,ki}] \\
& + f_{16}^{ijklm}(U_{ij}U_{jk}U_{kl}U_{lm}U_{mi}) + f_{17}^{ijkl}U_{ij}U_{jk}[P_\mu,U_{kl}][P_\mu,U_{li}] + \\
& + f_{19}^{ijklmn}(U_{ij}U_{jk}U_{kl}U_{lm}U_{mn}U_{ni})\Big\}.
\end{aligned}$$

	X_t^0	X_t^2	X_t^4	X_t^6
c_6	f_8	f_{10}	f_{16}	f_{19}
c_H	f_7	f_{11}	$f_{17},\ f_{18}$	-
c_T	f_7	f_{11}	$f_{17},\ f_{18}$	-
c_R	f_7	f_{11}	f_{17}	-
c_{GG}	f_9	f_{13}	-	-
c_{WW}	f_9	$f_{13},\ f_{14}$	-	-
c_{BB}	f_9	$f_{13},\ f_{14}$	-	-
c_{WB}	f_9	$f_{13},\ f_{14}$	-	-
c_W	-	$f_{15a},\ f_{15b}$	-	-
c_B	-	$f_{15a},\ f_{15b}$	-	-
c_D	-	f_{12c}	-	-

$$c_{WB} = -\frac{g_2^2 c_{2\beta} + 2h_t^2}{48 m^2_{\tilde{Q}}} + \bar{X}_t^2\left(\frac{33 m^4_{\tilde{Q}} m^2_{\tilde{t}_R} - 3m^2_{\tilde{Q}} m^4_{\tilde{t}_R} + 5m^6_{\tilde{Q}} + m^6_{\tilde{t}_R}}{24 m^2_{\tilde{Q}}\left(m^2_{\tilde{Q}} - m^2_{\tilde{t}_R}\right)^4} - \frac{m^2_{\tilde{Q}} m^2_{\tilde{t}_R}\left(2m^2_{\tilde{Q}} + m^2_{\tilde{t}_R}\right)\ln\left(\frac{m^2_{\tilde{Q}}}{m^2_{\tilde{t}_R}}\right)}{2\left(m^2_{\tilde{Q}} - m^2_{\tilde{t}_R}\right)^5}\right),$$

$$c_W = \bar{X}_t^2\left(\frac{-8m^2_{\tilde{Q}} m^2_{\tilde{t}_R} + m^4_{\tilde{Q}} - 17 m^4_{\tilde{t}_R}}{12\left(m^2_{\tilde{Q}} - m^2_{\tilde{t}_R}\right)^4} + \frac{\left(3m^2_{\tilde{Q}} m^4_{\tilde{t}_R} + m^6_{\tilde{t}_R}\right)\ln\left(\frac{m^2_{\tilde{Q}}}{m^2_{\tilde{t}_R}}\right)}{2\left(m^2_{\tilde{Q}} - m^2_{\tilde{t}_R}\right)^5}\right),$$

$$c_B = \bar{X}_t^2\left(\frac{-8m^2_{\tilde{Q}} m^2_{\tilde{t}_R} - 23 m^4_{\tilde{Q}} + 7 m^4_{\tilde{t}_R}}{12\left(m^2_{\tilde{Q}} - m^2_{\tilde{t}_R}\right)^4} - \frac{\left(-12 m^4_{\tilde{Q}} m^2_{\tilde{t}_R} + 3m^2_{\tilde{Q}} m^4_{\tilde{t}_R} - 4m^6_{\tilde{Q}} + m^6_{\tilde{t}_R}\right)\ln\left(\frac{m^2_{\tilde{Q}}}{m^2_{\tilde{t}_R}}\right)}{6\left(m^2_{\tilde{Q}} - m^2_{\tilde{t}_R}\right)^5}\right),$$

$$c_D = \bar{X}_t^2\left(\frac{10 m^2_{\tilde{Q}} m^2_{\tilde{t}_R} + m^4_{\tilde{Q}} + m^4_{\tilde{t}_R}}{2\left(m^2_{\tilde{Q}} - m^2_{\tilde{t}_R}\right)^4} - \frac{3m^2_{\tilde{Q}} m^2_{\tilde{t}_R}\left(m^2_{\tilde{Q}} + m^2_{\tilde{t}_R}\right)\ln\left(\frac{m^2_{\tilde{Q}}}{m^2_{\tilde{t}_R}}\right)}{\left(m^2_{\tilde{Q}} - m^2_{\tilde{t}_R}\right)^5}\right),$$

etc.