Causal Dynamical Triangulations: The emergence of spacetime

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What is quantum gravity?

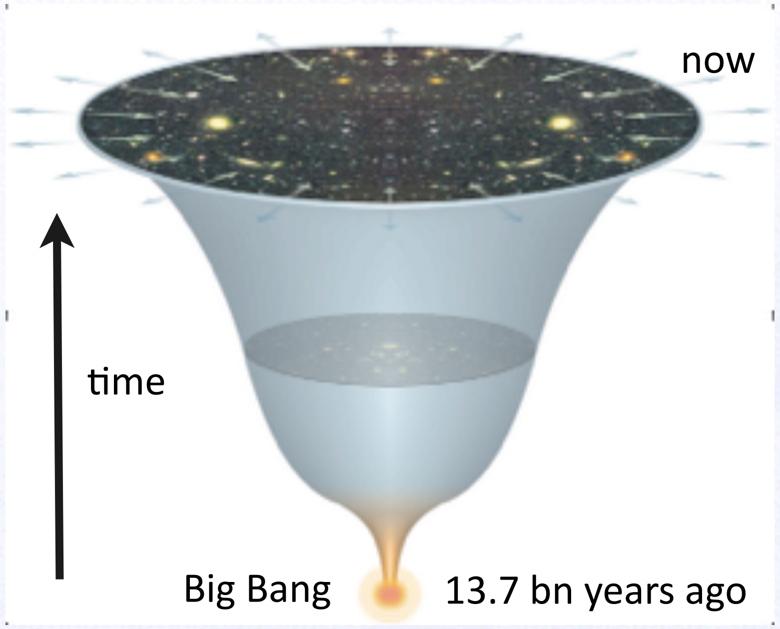
- *Quantum gravity* is the putative fundamental quantum theory underlying the classical field theory of General Relativity.
- It is *the* missing piece in our theoretical understanding of the four fundamental interactions.
- We do not know whether quantum gravity can/must be understood as part of a grand unifying dynamical principle.
- Applying the logic of Einstein's General Relativity, quantum gravity should also describe the dynamics of *spacetime* on all scales.
- The length scale at which quantum properties of the gravitational field must be taken into account is the *Planck length*

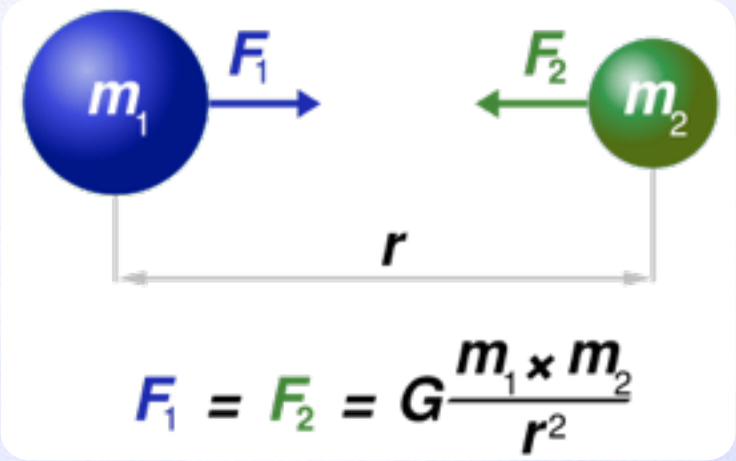
$$\ell_{\mathrm{Pl}} = \sqrt{\frac{G_{\mathrm{N}}\hbar}{c^3}} \approx 1.6 \times 10^{-35} m$$

(yes, this is *really* small; gravity is special!)

Questions that quantum gravity should answer

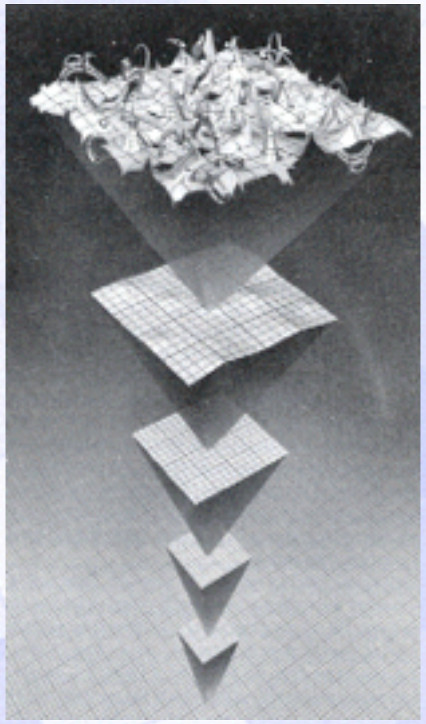
- What was the quantum behaviour of the very early universe?
- Are space and time fundamental or merely emergent on macroscopic scales?





- Can we *derive* gravitational attraction from first-principles quantum dynamics @ ℓ_{Pl}?

- What is the quantum microstructure of spacetime? Can we use it to *explain* the observed large-scale de Sitter nature of our universe? Can we make de Sitter space "emerge"?



zooming in on the Planck scale

Quantum gravity: where do we stand?

• perturbative quantum gravity "does not work" (non-renormalizable)

• we have several nonperturbative candidate theories, working from different premises (some are more promising than others …)

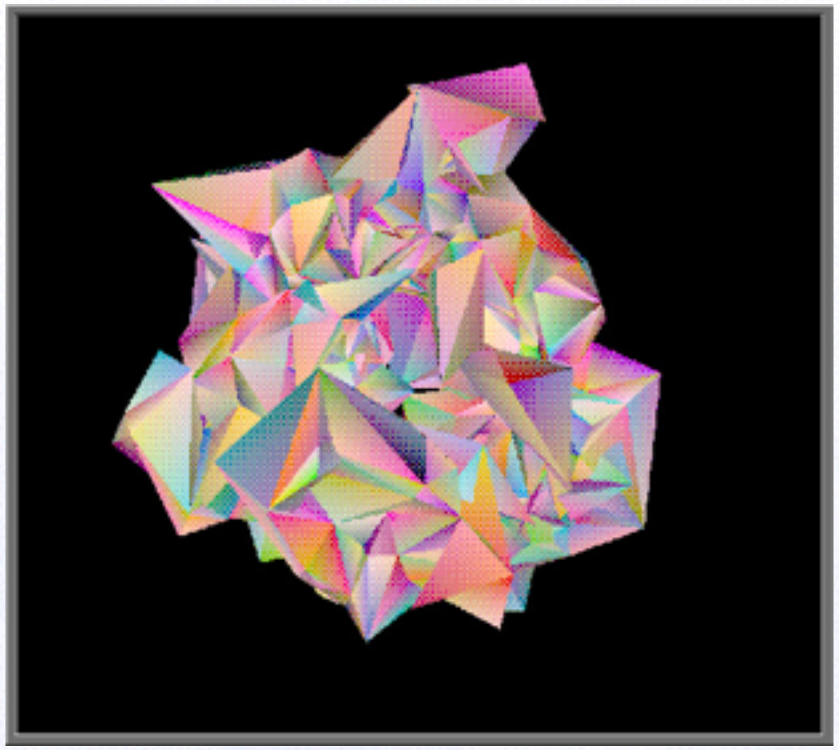
• they are too incomplete and/or have too many free parameters to make any solid predictions; comparing them is also (still) difficult

• there is little if any quantum gravity phenomenology to speak of

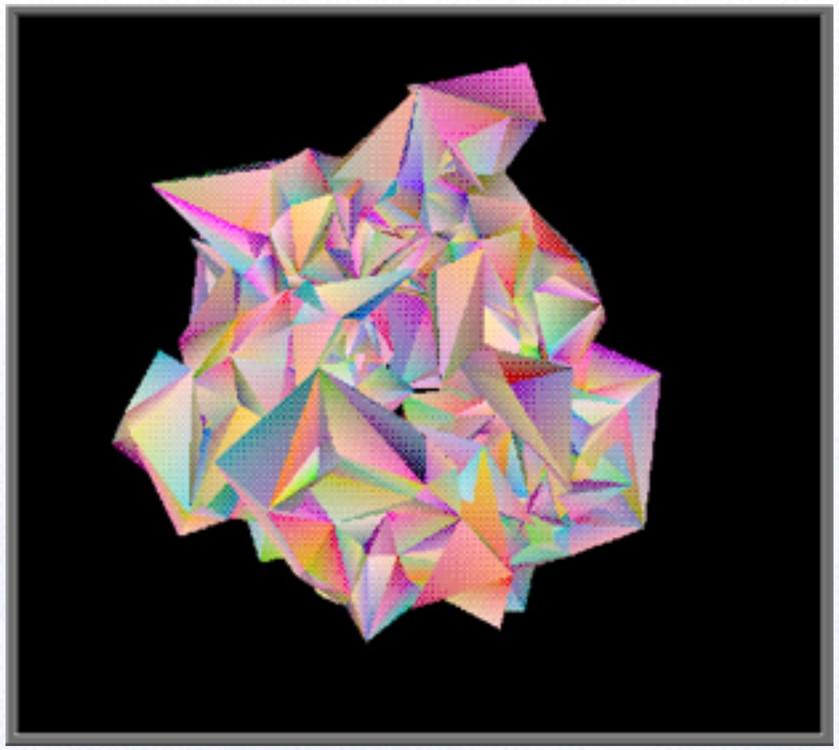
• in the absence of experimental verification, and with $l_{\mathrm{Pl}} = l_{\mathrm{LHC}} \times 10^{-16}$, it is difficult to nail down what constitutes true "progress"

Today I will present some evidence that there has been considerable progress in a specific nonperturbative formulation of quantum gravity, based on Causal Dynamical Triangulations, and that it is a very concrete realization of the "emergence of spacetime".

Quantum gravity:



What is the correct approach?



Quantum gravity:

~~What is the correct approach?~~

What is the most fruitful approach?

Less may be more

Formulations with “exotic” ingredients lead to an *embarassment of riches* (many free parameters, no predictive power). - Isn’t there something simpler (using “good old QFT”) that has not been tried?

YES: in nonperturbative(*) quantum gravity it has been fruitful to be “radically conservative”, in the sense of

(i) being minimalist in terms of ingredients/prior assumptions, with little background structure and few free parameters,
(ii) using standard *quantum field-theoretic* methods and
(iii) using nonperturbative computational tools for quantitative evaluation.

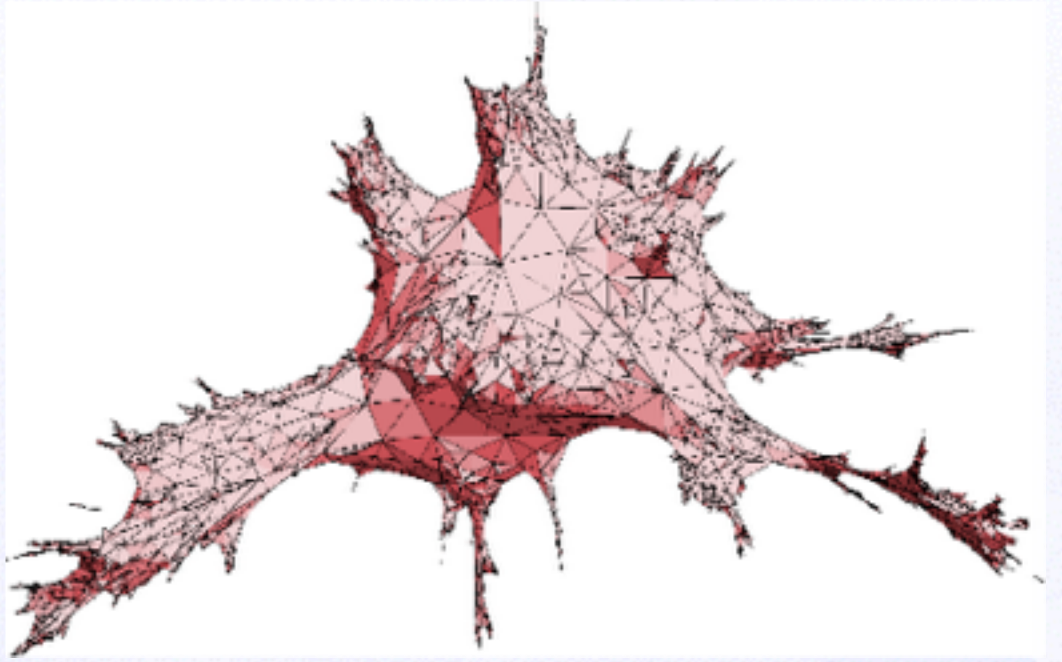
CDT quantum gravity embodies these ideas - it is “as simple as possible, but not simpler” and leads to nontrivial results.

(*)nonperturbative = allowing for large quantum fluctuations, not just linear perturbations around a fixed, classical background metric

The Story of (Causal) Dynamical Triangulations

This approach to quantum gravity (1998) grew out of a confluence of ideas:

- the primacy of *pure geometry* in the sense of Einstein's rods and clocks (measuring distances, not metrics $g_{\mu\nu}$);
- using *powerful numerical methods* to describe such geometry far away from a flat-space, perturbative regime;



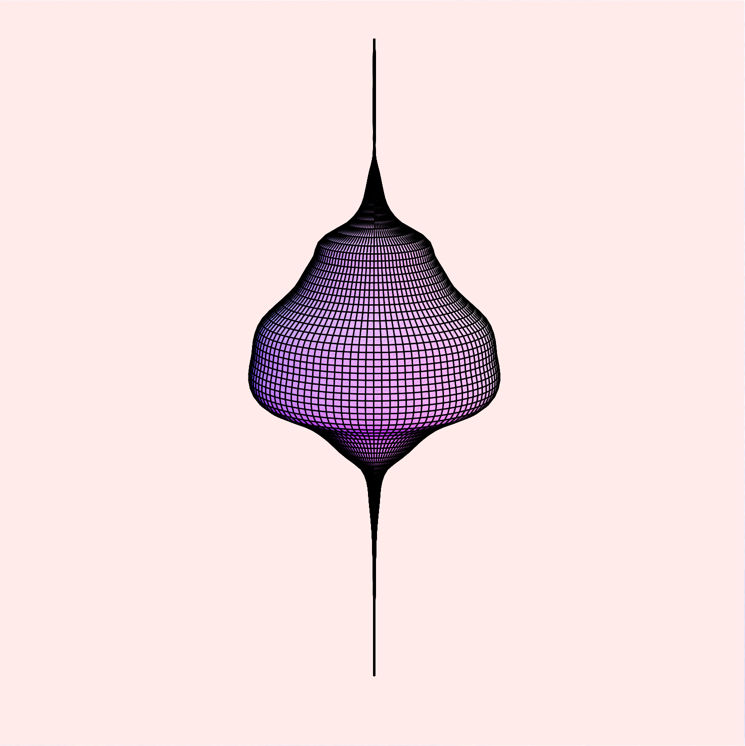
A typical path integral history (glued from triangles in 2d quantum gravity)

- subsequently, the realization that the imposition of a local *causal structure* on path integral histories appears to be necessary to obtain a good classical limit in four dimensions (DT → CDT)

(J. Ambjørn, A. Görlich, J. Jurkiewicz & RL, "Nonperturbative Quantum Gravity", Physics Report 519 (2012) 127 [arXiv: 1203.3591])

The Emergence of Classical Spacetime from Causal Dynamical Triangulations (CDT)

CDT is currently the only candidate quantum theory of gravity which can generate *dynamically* a spacetime with semiclassical properties from pure quantum excitations, without using a background metric.



Other key results:

- crucial role of causal structure
- scale-dependent dimensionality (2 → 4)
- nontrivial phase structure
- second-order phase transitions!
- applicability of renormalization group methods

Quantum Gravity from CDT*

is a *nonperturbative* implementation of the gravitational path integral,

Newton's constant

$$Z(G_N,\Lambda) = \int_{\substack{\text{spacetimes}\\ g\in\mathcal{G}}} \mathcal{D}g\; \mathrm{e}^{iS^{\mathrm{EH}}_{\mathrm{G_N},\Lambda}[g]}$$

cosmological constant

Einstein-Hilbert action

much in the spirit of lattice quantum field theory, but based on *dynamical* triangular lattices, reflecting the dynamical nature of spacetime geometry:

$$Z(G_N,\Lambda) := \lim_{\substack{a\to 0\\ N\to\infty}} \sum_{\substack{\text{inequiv.}\\ \text{triangul.s}\\ T\in\mathcal{G}_{a,N}}} \frac{1}{C(T)}\, \mathrm{e}^{iS^{\mathrm{Regge}}_{\mathrm{G_N},\Lambda}[T]}$$

UV cutoff

building blocks

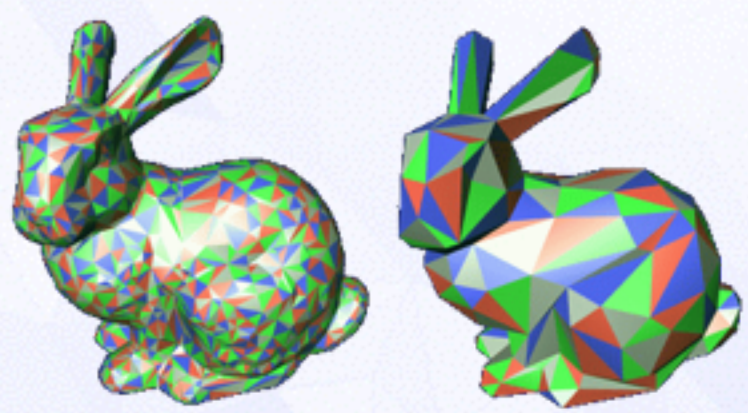
|Aut(T)|

This describes "pure gravity"; inclusion of matter fields is straightforward.

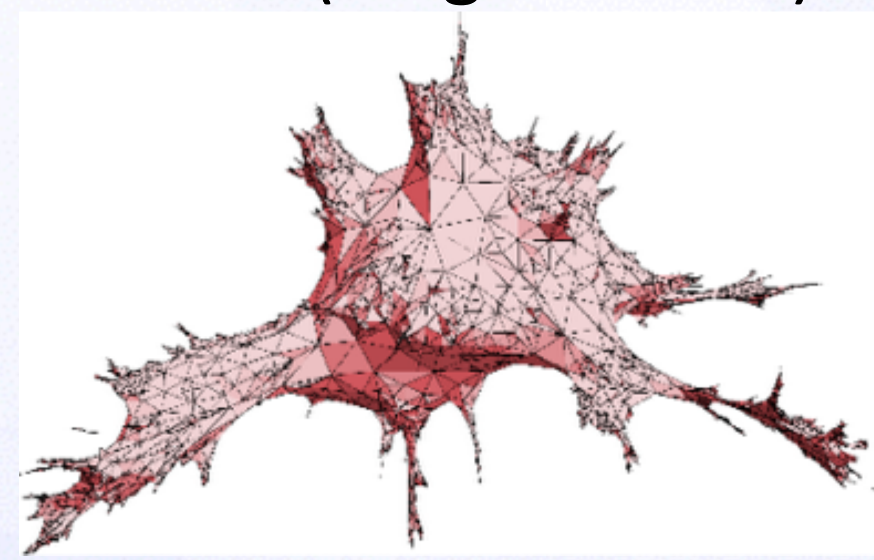
* some recent contributors: J. Ambjørn, D. Benedetti, T. Budd, J. Cooperman, D. Coumbe, B. Durhuus, J. Gizbert-Studnicki, L. Glaser, A. Görlich, J. Henson, A. Ipsen, T. Jonsson, S. Jordan, J. Jurkiewicz, N.Klitgaard, A. Kreienbuehl, J. Laiho, B. Ruijl, Y. Sato, Y. Watabiki, J. Wheater …

Key ingredients of the CDT approach:

- representing curved spacetimes by piecewise flat triangulations makes the path integral well defined at an intermediate (“regularized") stage

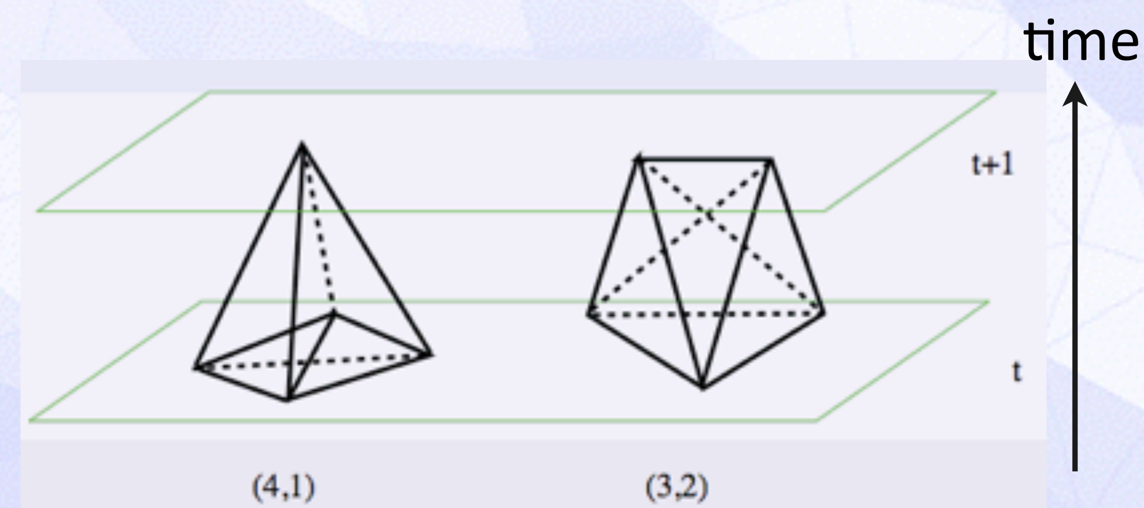


approximating a given *classical* curved surface through triangulation



Quantum Theory: approximating the space of *all* curved geometries by a space of triangulations

- crucial to obtain a semiclassical limit: spacetimes must have causal structure
- crucial in d = 4: nonperturbative comput. tools (Monte Carlo simulations) to extract quantitative results



simplicial 4d building blocks of CDT

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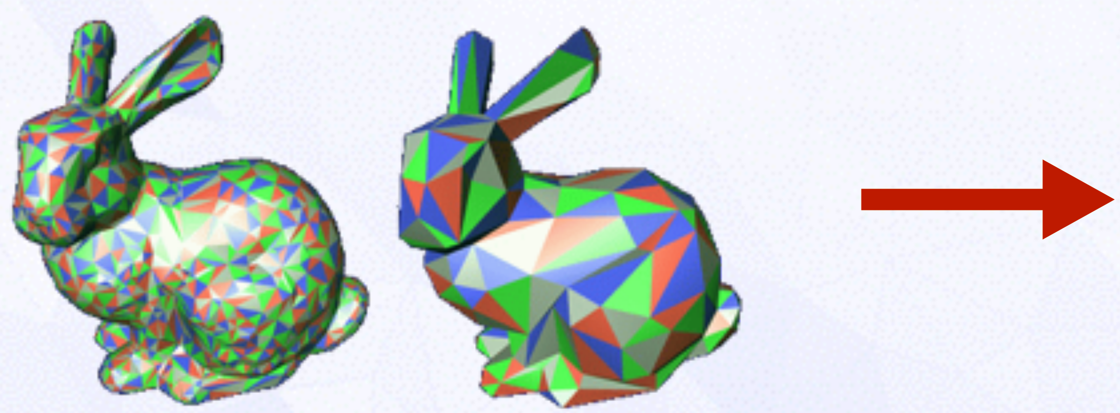

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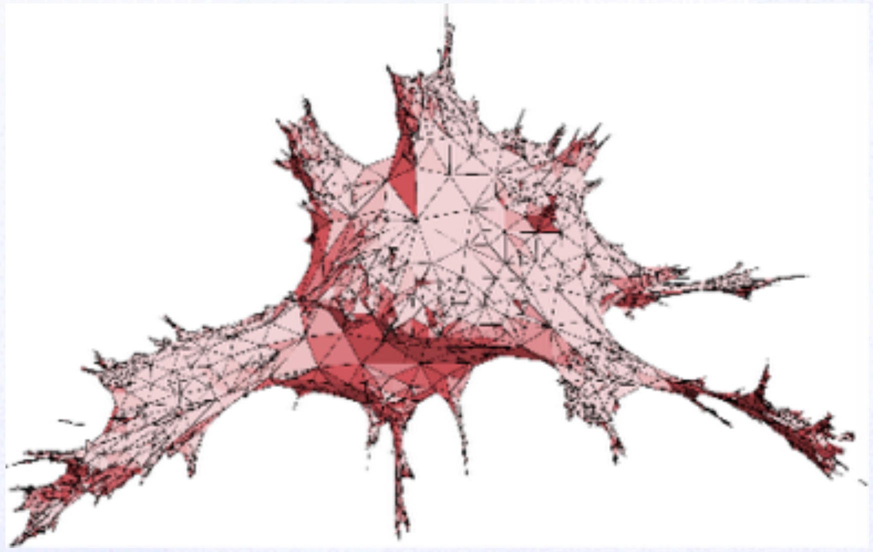

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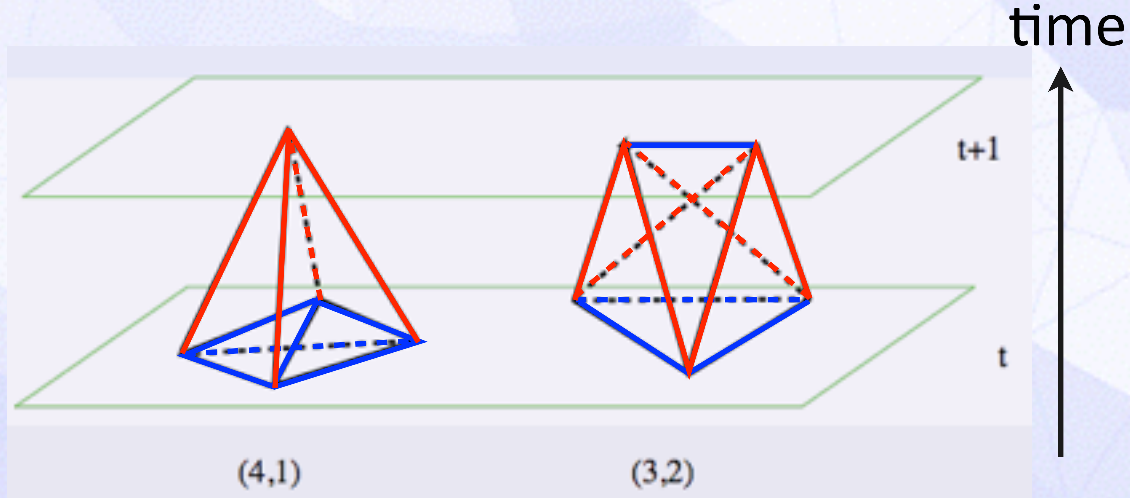


simplicial 4d building blocks of CDT

spacelike edge, squared length a^2

timelike edge, squared length $-\alpha a^2$, $\alpha > 0$

What makes CDT Quantum Gravity unique?

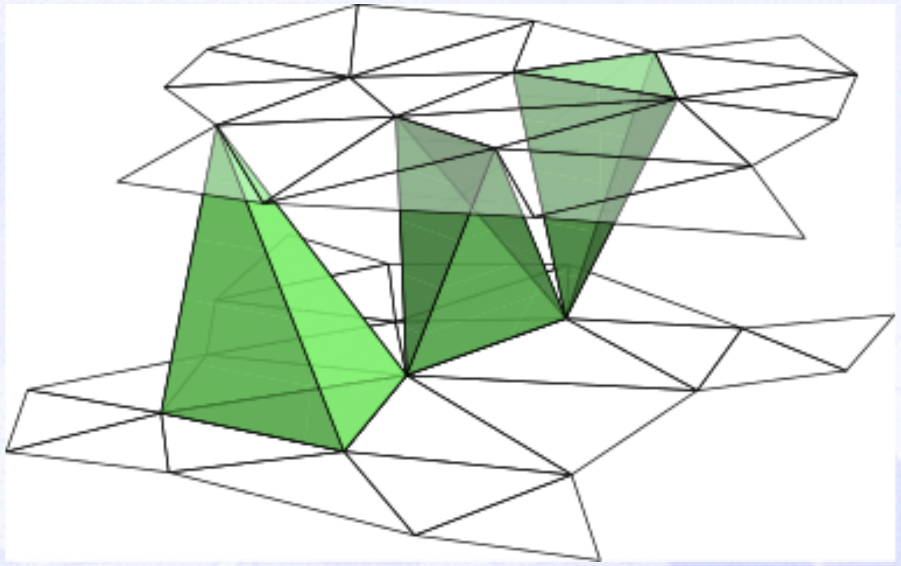
Imagine you wanted to do a nonperturbative path integral (PI) …

- usual problem: cannot evaluate complex PI and there is no Wick rotation - do Euclidean QG instead, i.e. $\int Dg \exp(-S^{eu})$?
  - ☑ CDT has a well-defined analytic continuation; "Wick-rotated" Lorentzian PI is *not* equivalent to the Euclidean PI
- usual problem: there are redundancies because of diffeomorphism or other gauge symmetries, leading to unwanted divergences
  - ☑ CDT has no residual gauge symmetries, works with geometries
- frequent problem: PI highly divergent, no unique renormalization
  - ☑ number of configurations in CDT exponentially bounded
- frequent problem: cannot do any computations, cannot evaluate PI
  - ☑ CDT amenable to MC simulations; quantitative results, falsifiable!
- usual problem: why should PI lead to a unitary theory?
  - ☑ CDT reflection-positive w.r.t. discrete "proper time", hence unitary!

What have we achieved?

The combination of these highly desirable features in CDT has been exploited to good effect in research that is ongoing.

In the remaining time, I will describe several insights and highlights from this research program, to illustrate that CDT Quantum Gravity has also achieved some ***unique results*****.**



a piece of causal triangulation (spacetime dimension d = 3)

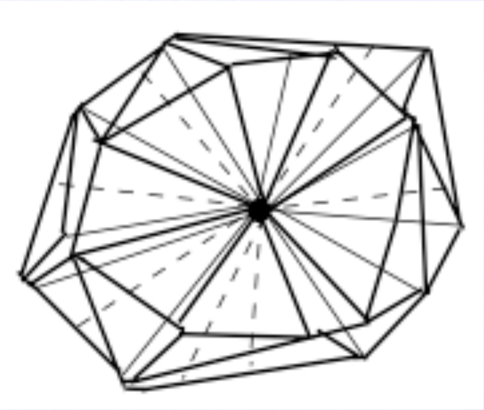
Nonperturbative “geometry” behaves strangely

Isn’t it obvious that by gluing together four-dimensional building blocks, one will obtain a (quantum) spacetime of dimension 4?

No. Generically it does not happen when quantum fluctuations are large.

This was only gradually understood, using computer “experiments”. In DT models prior to CDT, one of two things happened to ‘quantum geometry’:

it polymerized (small G_N^{bare}), $d_H = 2$

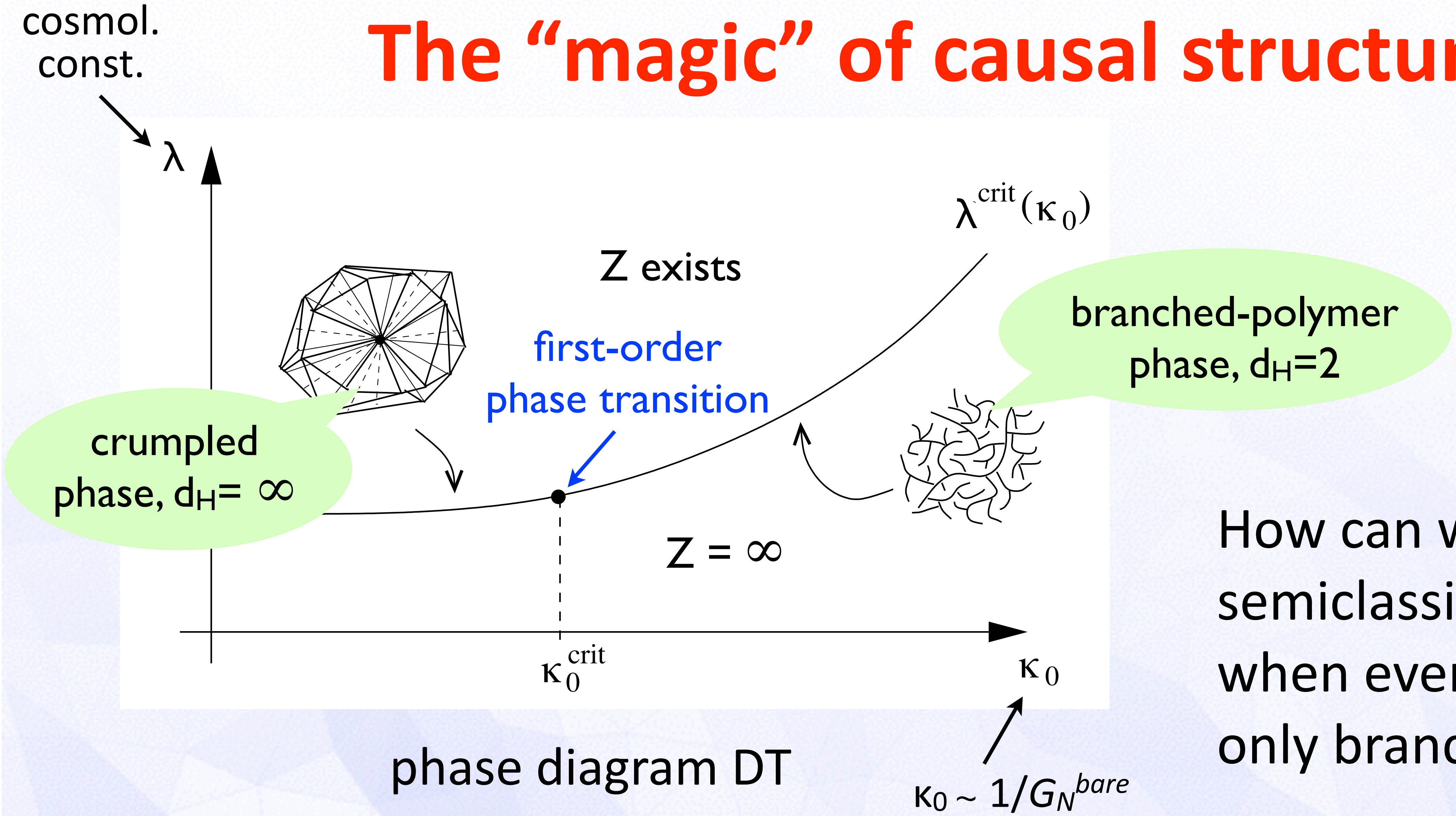


it crumpled (large G_N^{bare}), $d_H = \infty$

This degenerate behaviour is generic for (Euclidean) DT in dimension d > 2. [For d = 2, one obtains Liouville quantum gravity.]

<u>*Causal* DT was invented to cure this problem and appears to do so!</u>

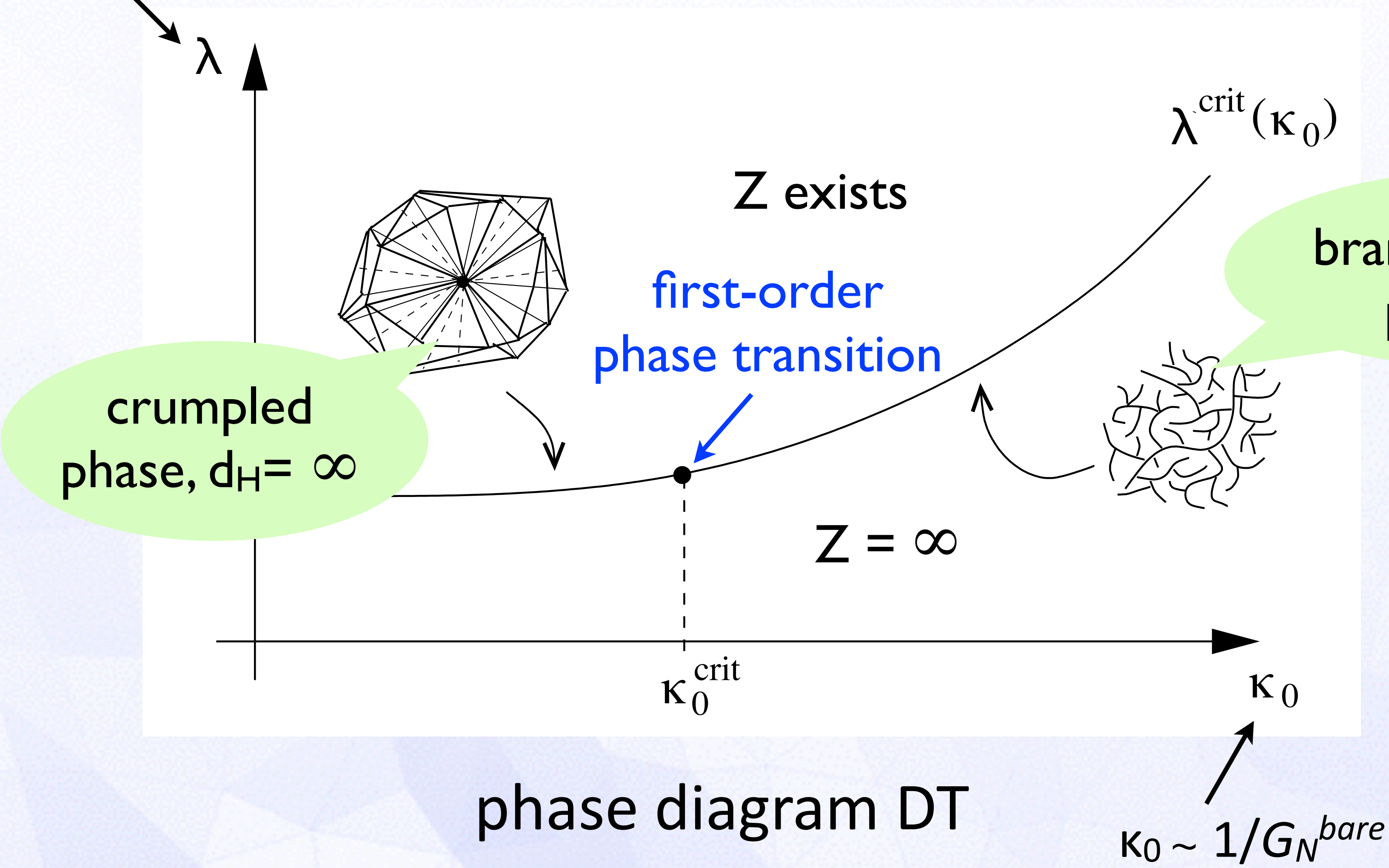
The “magic” of causal structure



phase diagram DT

How can we observe semiclassical behaviour, when everyone else finds only branched polymers?

The “magic” of causal structure

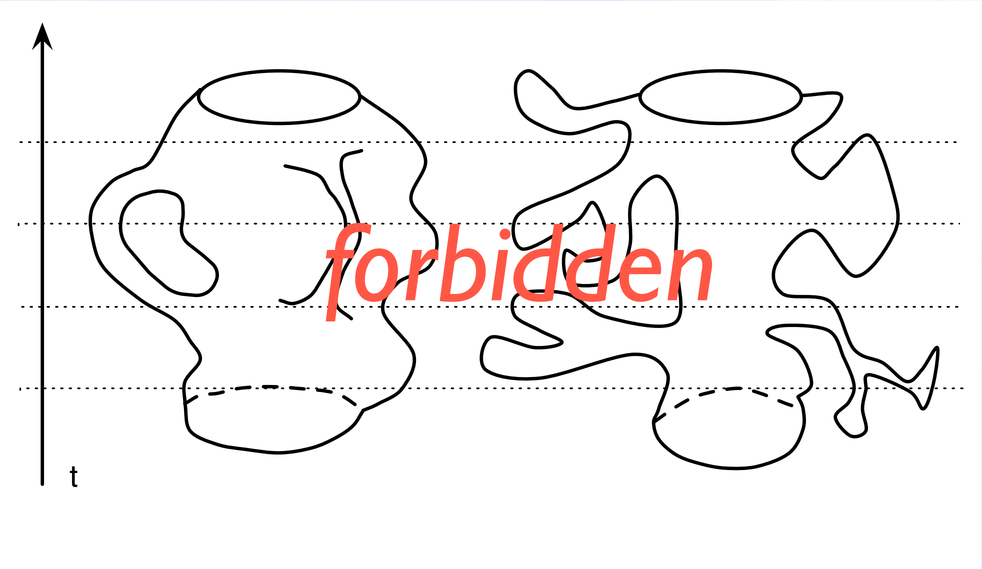


phase diagram DT

How can we observe semiclassical behaviour, when everyone else finds only branched polymers?

<u>It’s the causal structure, stupid!</u>

Building blocks have a Lorentzian (= light cone) structure, and gluing rules ensure a *well-behaved causal structure* overall. ‘Baby universes’ are forbidden, and spatial topology does not change.



N.B.: singular “trouser points”

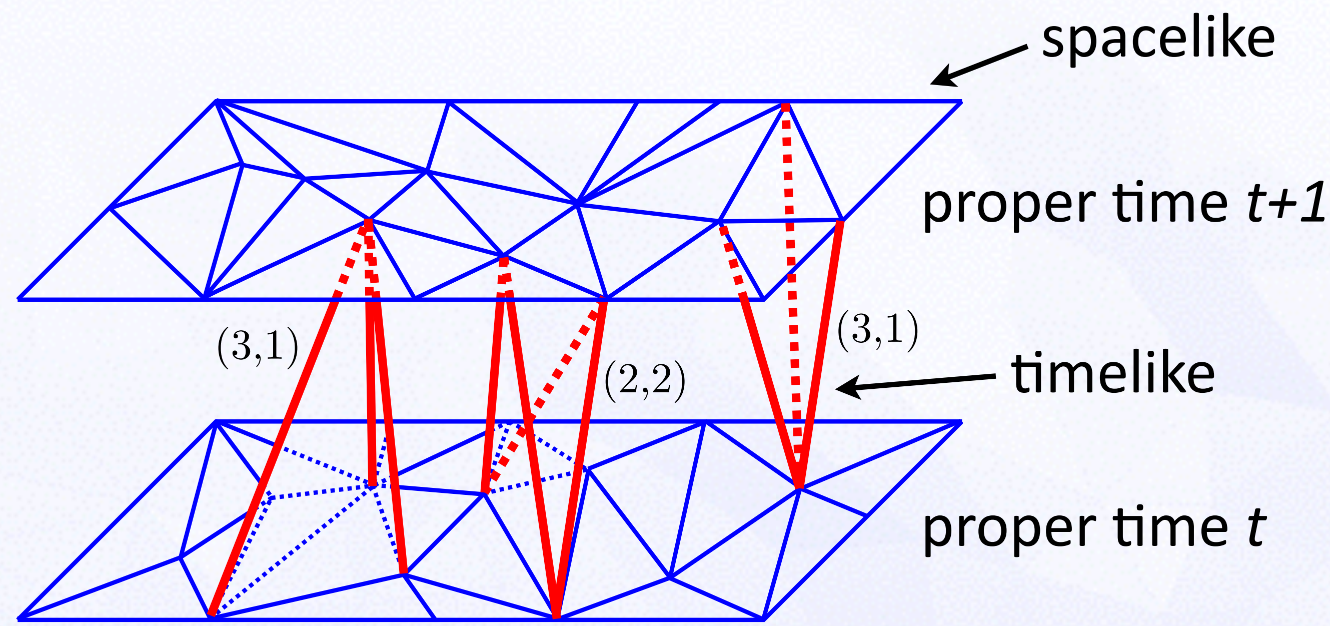
Causal structure vs. proper time foliation

Which is responsible for the "good" behaviour of CDT?

Does the preferred time/ foliation affect the results?

Not a gauge choice (no coords).

Continuum interpretation of "t" only on large scales.

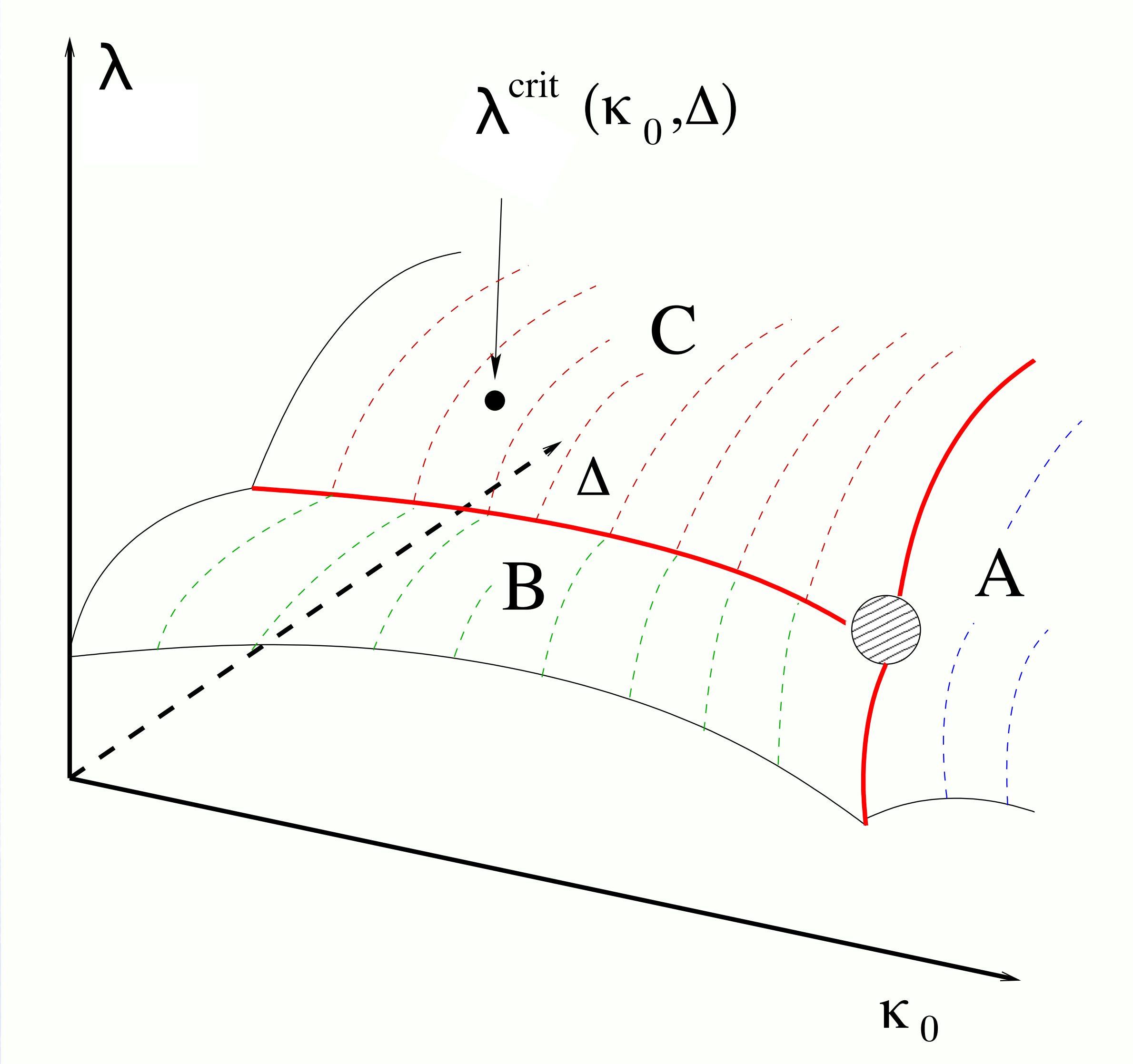


"sliced" structure of 2+1 CDT

We have introduced a new version of CDT quantum gravity, "*Locally Causal Dynamical Triangulations (LCDT)*", where the causal structure and the preferred time are dissociated (in fact, there is no preferred time), and have repeated the standard analysis of the phase structure and the volume profiles for 2+1 CDT quantum gravity. Key CDT findings appear unaltered!

⇒ The causal structure is responsible; the time foliation is very convenient, but not strictly necessary. (S. Jordan and R.L., Phys. Lett. B 724 (2013) 155; Phys. Rev. D 88 (2013) 044055)

Phase diagram of CDT quantum gravity in 4D



The CDT gravitational action is *simple*:

$$S_{\mathrm{eu}}^{\mathrm{Regge}} = -\kappa_0 N_2 + N_4(c\kappa_0 + \lambda) + \\ +\Delta(2N_4^{(4,1)} + N_4^{(3,2)})$$

λ ~ cosmological constant

κ_0 ~ $1/G_N$ inverse Newton's constant

Δ ~ relative time/space scaling

c ~ numerical constant, >0

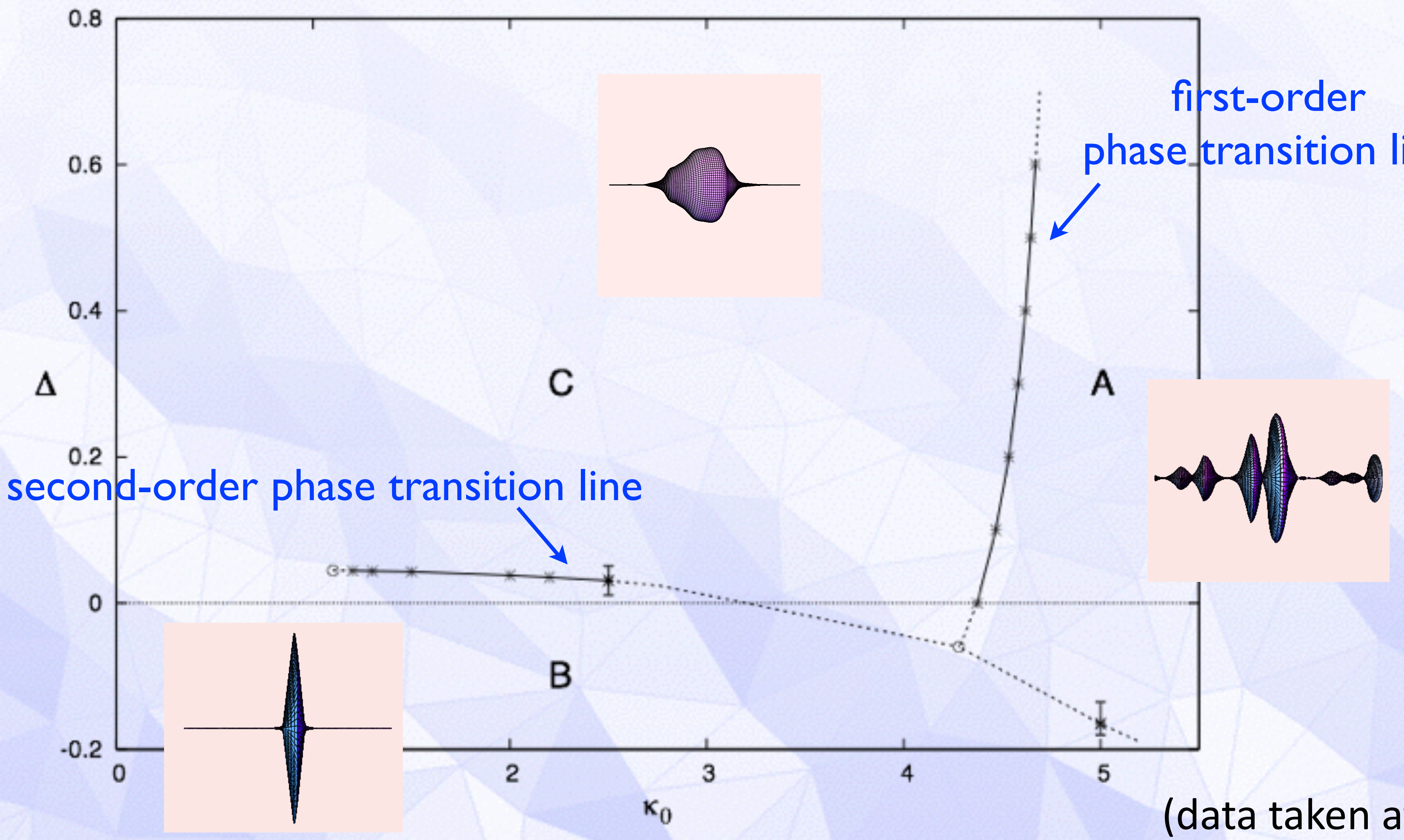
N_i ~ # of triangular building blocks of dimension i

The partition function is defined for $\lambda > \lambda^{crit}(\kappa_0,\Delta)$;
approaching the critical surface from above = taking infinite-volume limit.
red lines ~ phase transitions

(J. Ambjørn, J. Jurkiewicz, RL, PRD 72 (2005) 064014;
J. Ambjørn, A. Görlich, S. Jordan, J. Jurkiewicz, RL, PLB 690 (2010) 413)

Phase diagram of CDT quantum gravity I

Unlike DT, CDT in d = 4 exhibits a phase of extended geometry with Hausdorff dimension 4. On the hypersurface $\lambda = \lambda^{crit}$, the dynamically generated quantum universe has an overall shape that depends on the phase. Only "phase C" has a large-scale limit compatible with GR.



(data taken at N_4=80.000)

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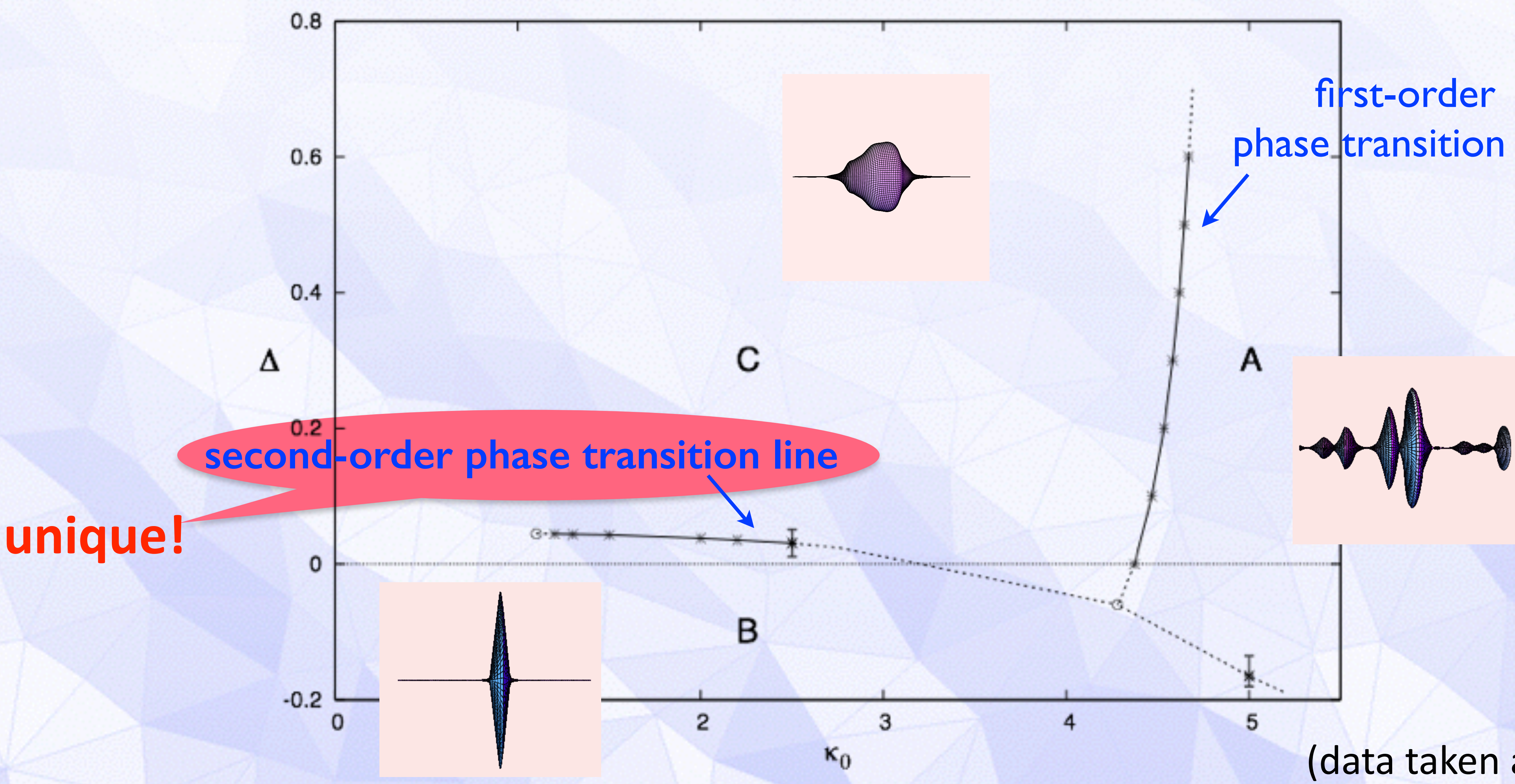
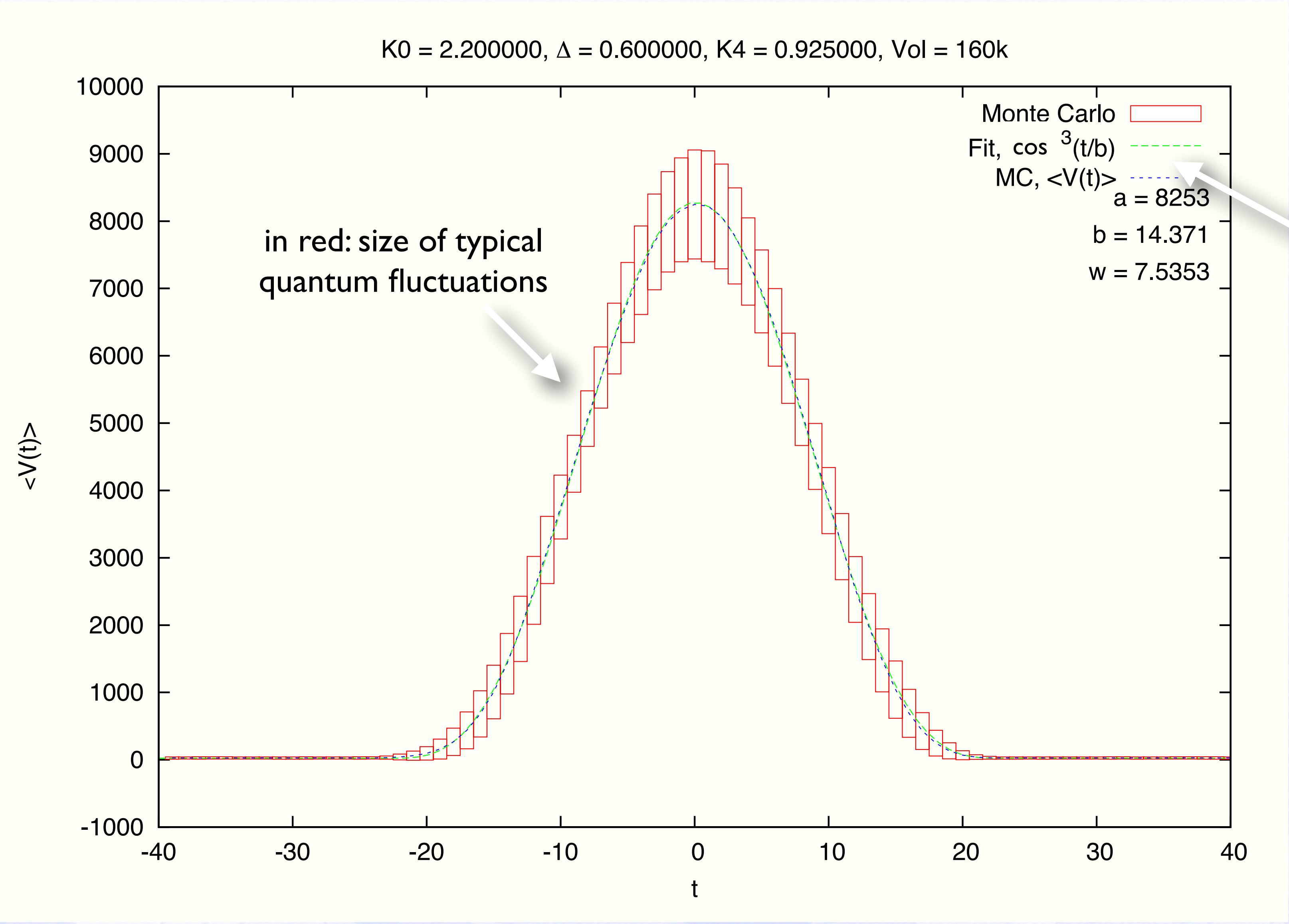


(data taken at N_4=80.000)

(J. Ambjørn, S. Jordan, J. Jurkiewicz, R.L., PRL 107 (2011) 211303; PRD 85 (2012) 124044)

"CDT Classic": universal de Sitter-like volume profile



fit to $\cos^3(t/b)$

The average volume profile $<V_3(i)>$ of the universe, as function of Euclidean proper time *t*, matches to great accuracy a corresponding GR minisuperspace calculation. (N.B.: also compatible with HL gravity).

The classical line element of Euclidean de Sitter space, derived by *assuming homogeneity and isotropy a priori*, as function of Euclidean proper time *t=iτ*, is

$$ds^2 = dt^2 + a(t)^2 d\Omega_{(3)}^2 = dt^2 + c^2 \cos^2\left(\frac{t}{c}\right) d\Omega_{(3)}^2$$

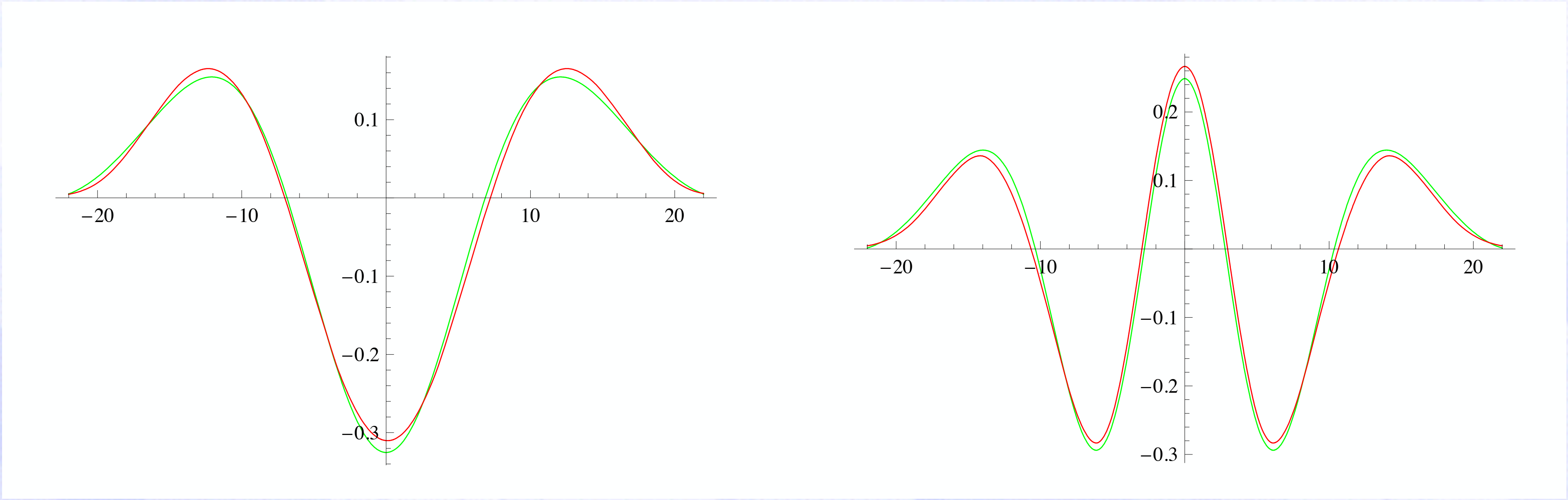
scale factor

volume el. S^3

In addition, expanding the minisuperspace action around the de Sitter solution,

$$S_{\rm eu}(V_3) = S(V_3^{\rm dS}) + \kappa \int dt\ \delta V_3(t) \hat{H} \delta V_3(t)$$

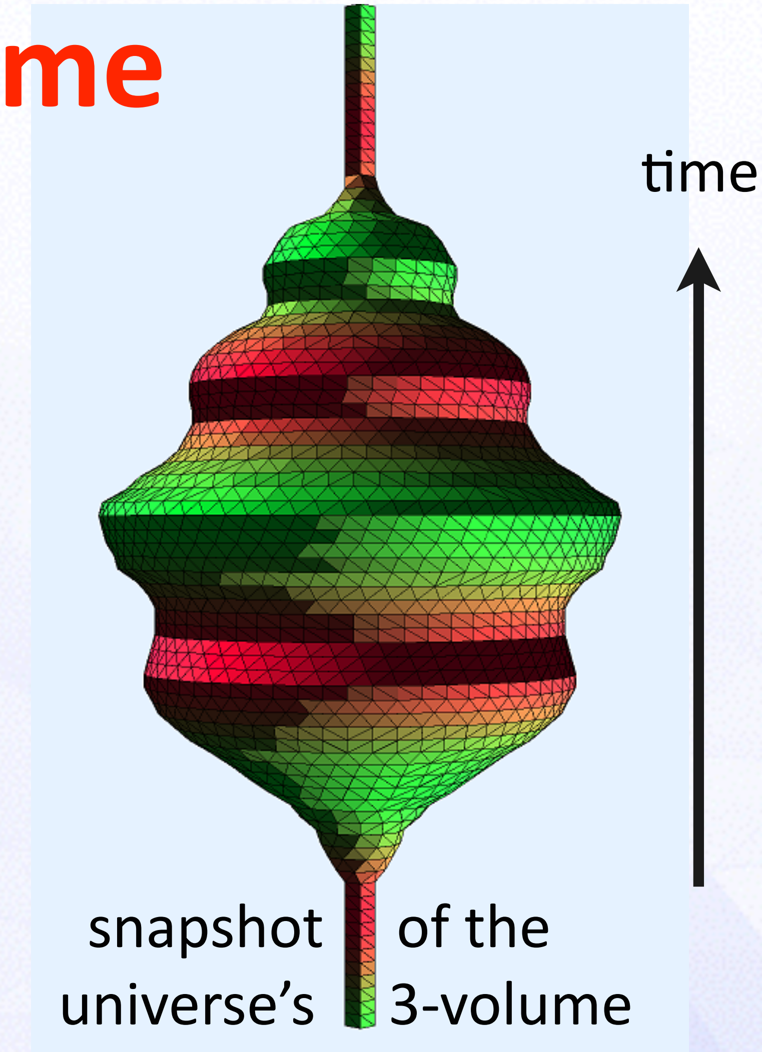
the eigenmodes of $\hat{H}$ match well with those extracted from the simulations:



(N.B.: no further fitting necessary)

(J. Ambjørn, A. Görlich, J. Jurkiewicz, RL, PRL 100 (2008) 091304, PRD 78 (2008) 063544, NPB 849 (2011) 144 (with J. Gizbert-Studnicki, T. Trzesniewski))

Dynamical emergence of spacetime (out of “quantum foam”)



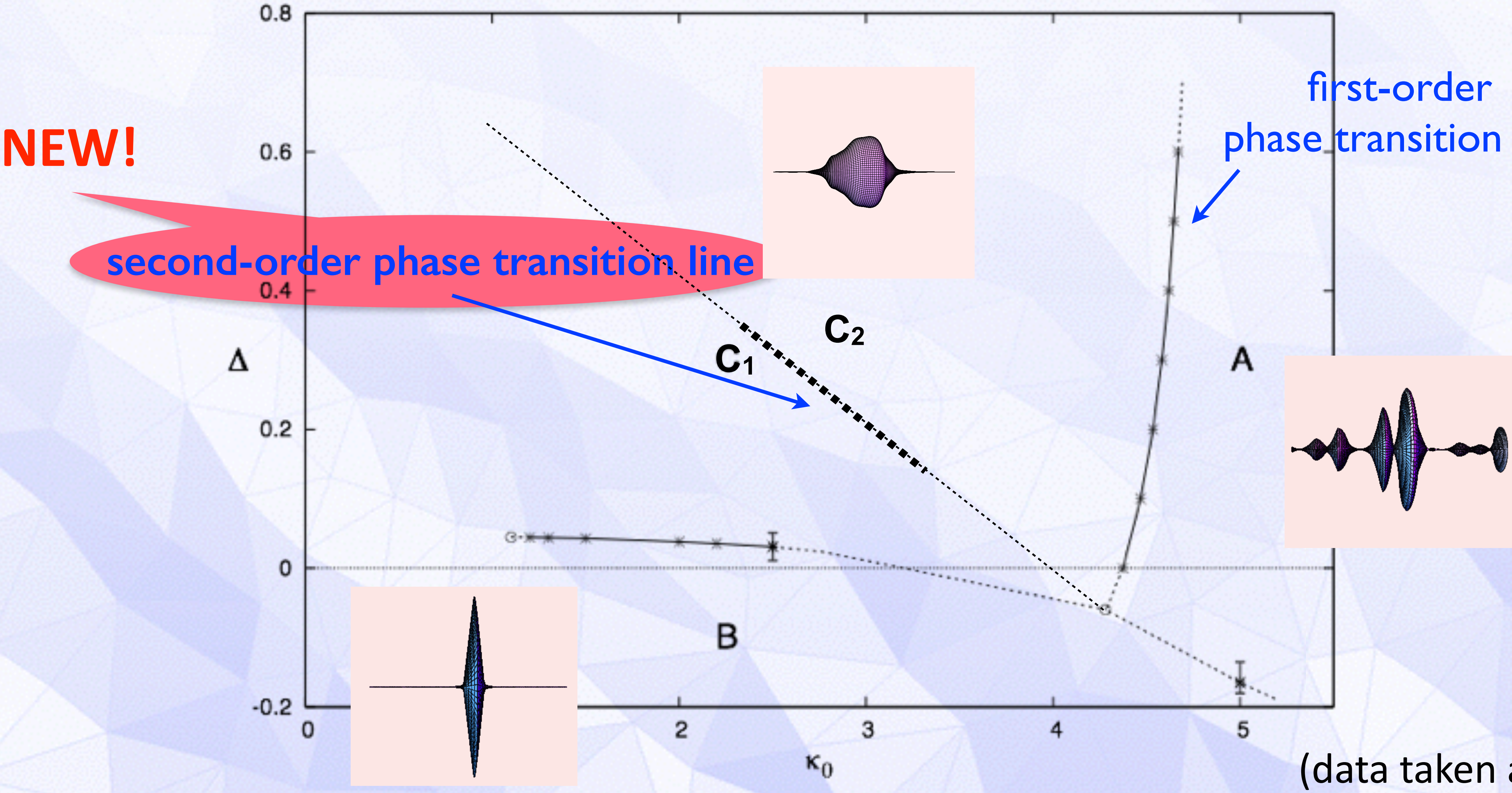
We conclude that for suitable bare couplings, CDT quantum gravity dynamically produces a “quantum spacetime”, that is, a ground state (“vacuum”), whose macroscopic scaling properties are ***four-dimensional*** and whose macroscopic shape is that of a well known cosmology, ***de Sitter space***. In background-independent gravity, this is unprecedented.

It is brought about by a ***nonperturbative*** mechanism, with “energy” (the bare action) and “entropy” (the measure, i.e. number of microscopic spacetime configurations) contributing in equal measure.

The region in coupling constant space where we see interesting physics is far away from the perturbative regime and quantum fluctuations are large. N.B.: universe is tiny and local geometry is highly nonclassical!

Phase diagram of CDT quantum gravity II

Recent simulations of the system using a small time extension of just two time steps have revealed that there is yet another second-order transition line, dividing phase C into C_1 ("bifurcation phase") and C_2, and related to a breaking of homogeneity and isotropy of geometry.



(data taken at N_4=80.000)

How do the new phases C_1 and C_2 differ?

The new transition was found when considering the transfer matrix between two adjacent spatial slices at times t and t+a, the path integral

$$\langle T^{(3)}(t+a)|M|T^{(3)}(t)\rangle = \sum_{T:T^{(3)}(t)\to T^{(3)}(t+a)} \frac{1}{C(T)} \mathrm{e}^{iS^{\mathrm{Regge}}[T]}$$

We use a reduced version which only keeps track of the 3-volume $V_3(t)$. In phase C_2 we find to good precision (after Wick rotation):

$$\langle n|M_{C_2}|m\rangle = \mathrm{e}^{-\frac{1}{\Gamma}\left[\frac{(n-m)^2}{(n+m)}+\mu(n+m)^{1/3}-\lambda(n+m)\right]}$$

where *m*, *n* denote the 3-volumes at times *t* and *t+a*. Compare this to the minisuperspace action à la Hartle/Hawking generating dS space:

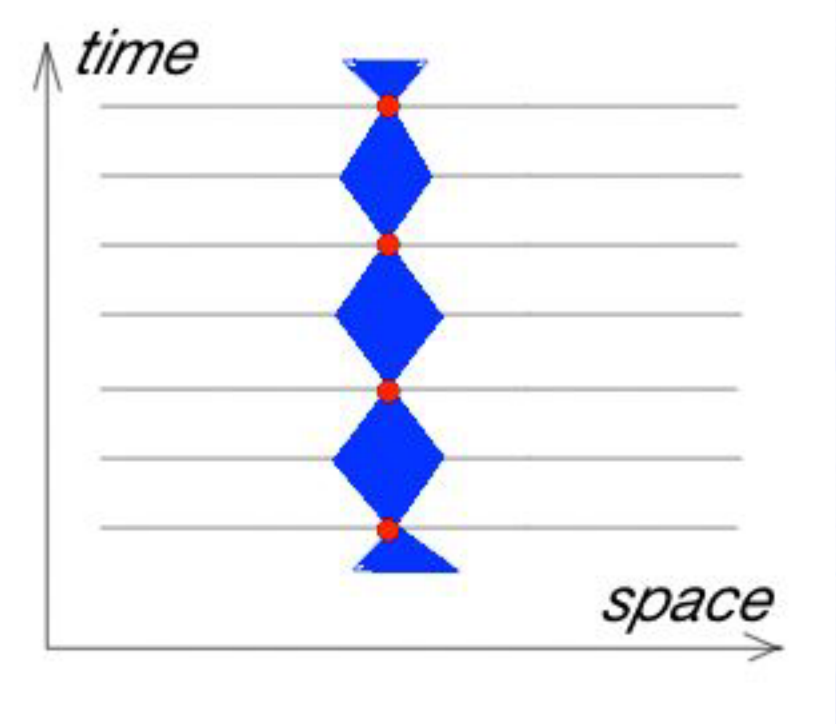
$$S_{\mathrm{mini}} = \frac{1}{24\pi G_N}\int dt\sqrt{g_{tt}}\left(\frac{g^{tt}\dot{V}_3^2(t)}{V_3(t)} + k_2 V_3^{1/3}(t) - \lambda V_3(t)\right)$$

Instead, in the "bifurcation phase" C_1 and in phase B we find a double-peak structure as function of the difference of the neighbouring 3-volumes:

$$\langle n|M_{C_1}|m\rangle = (\mathrm{e}^{-\frac{1}{\Gamma}\frac{(n-m-c[n+m])^2}{(n+m)}} + \mathrm{e}^{-\frac{1}{\Gamma}\frac{(n-m+c[n+m])^2}{(n+m)}})\,\mathrm{e}^{-\frac{1}{\Gamma}[\mu(n+m)^{1/3}-\lambda(n+m)]}$$

This is related to the appearance of a vertex of very high order on alternating spatial slices, collapsing into a single such vertex as we cross into phase B.

Having identified a suitable order parameter, our tentative conclusion is that the C_1-C_2 transition is second order, and a new prime candidate for looking for continuum gravity! Physically, it is related to a breaking of homogeneity and isotropy of spatial and spacetime geometry.



(J. Ambjørn, J. Gizbert-Studnicki, A. Görlich, J. Jurkiewicz, JHEP 1406 (2014) 034; D.Coumbe, J. Gizbert-Studnicki, J. Jurkiewicz, arXiv:1510.086; J. Ambjørn, J. Gizbert-Studnicki, A. Görlich, J. Jurkiewicz, N.Klitgaard, R.L., to appear)

Making contact with continuum physics

Note that CDT quantum gravity does not postulate any fundamental discreteness at the Planck scale, nor have we found so far any evidence for it dynamically.

All results I talked about are obtained in a scaling limit of infinitely many building blocks (using finite-size scaling); only then do they stand a chance of being universal.

Analogous to how one proceeds in QCD on the lattice, CDT uses a dynamical lattice regularization to try to construct a theory of nonperturbative quantum gravity.

This raises …

… some important questions

- Is there a continuum limit where physical observables become independent of the UV cut-off and of regularisation “artefacts”?
- Does QG exist as a nontrivial QFT when the UV regulator is removed?
- Do standard lattice renormalization methods apply?
- Can we confirm the presence of an ultraviolet fixed point as predicted in the asymptotic safety scenario?
- What *is* the UV theory/completion?

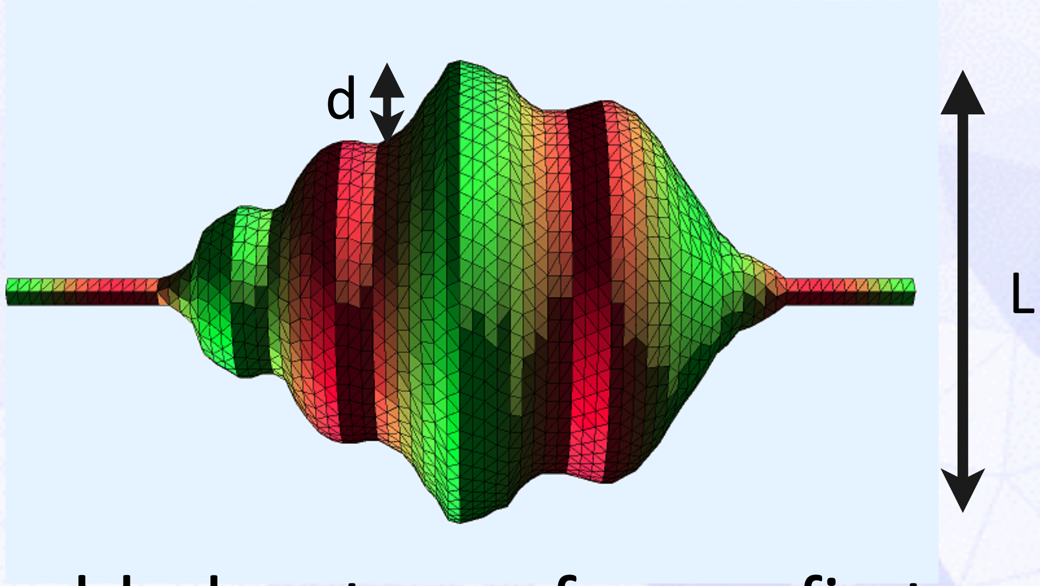
These are relevant physical questions, but highly nontrivial in nonperturbative quantum gravity, where there is no a priori background metric or measuring grid, and “geometry” and “length” are generated *dynamically*. A “naïve” correlator G(x,y) and associated correlation length are not well defined.

Standard renormalization *can* be applied!

Having located lines of second-order transition points, we want to investigate the scaling behaviour of the theory in their vicinity.

We are interested in renormalization group (RG) flows probing ever shorter distances. Since there is no correlation length immediately available, we let the linear lattice size $N_4^{1/4} \to \infty$ while keeping physics constant.

Idea: use the length scales associated with the dynamically generated de Sitter universe in CDT to define physical "yardsticks".



Under simplifying assumptions this has enabled us to perform a first explicit study of such RG flows near the B-C_1 transition. (J. Ambjørn, A. Görlich, J. Jurkiewicz, A. Kreienbühl, RL, CQG 31 (2014) 1650)

An analogous study needs to be done near the C_1-C_2 phase transition.

Summary and conclusions

CDT quantum gravity enjoys a number of nice features that enable it to reach where other nonperturbative approaches don't, and obtain some highly nontrivial results. Today I focused on

- the emergence of macroscopic 4D geometry
- its dependence on the presence of a microscopic causal structure (no known instance where causal structure itself emerges)
- the structure of the phase diagram with lines of second-order phase transitions, which provide natural candidates for taking a scaling limit

Good old quantum field theory, without exotic ingredients and adapted to the case of dynamical geometry, may provide the answer to quantum gravity after all.

Work is in progress on identifying and measuring more observables, to complete the theory further and eventually predict observable effects.

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The End