A higher-spin theory of the magneto-roton

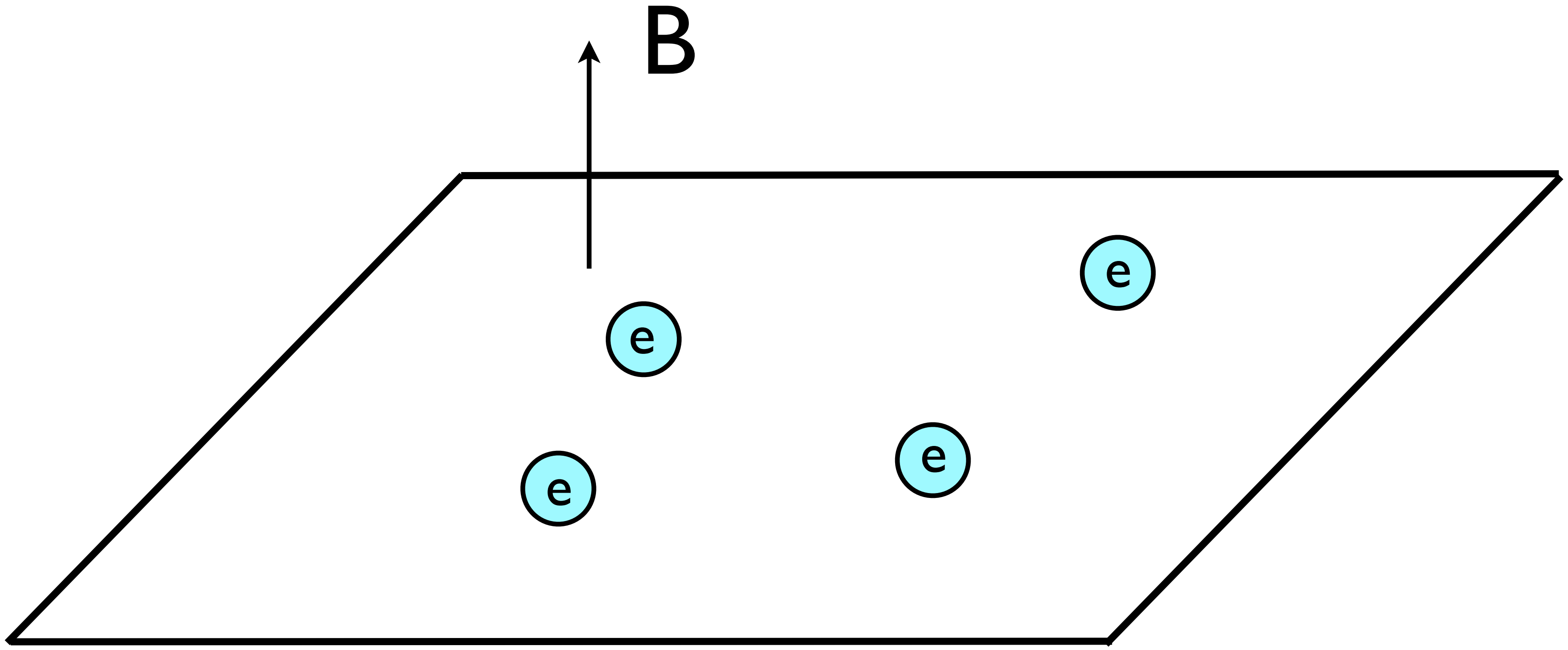
Dam Thanh Son (University of Chicago)
CERN June 21, 2016

Plan

- Fractional quantum Hall physics (modern view)
- Bosonization of the Fermi surface
- Magneto-rotons

Ref:
Siavash Golkar, Dung Nguyen, DTS, Matt Roberts 1602.08499

Microscopic problem



$$H = \sum_a \frac{(\mathbf{p}_a + e\mathbf{A}_a)^2}{2m} + \sum_{\langle a,b \rangle} \frac{e^2}{|\mathbf{x}_a - \mathbf{x}_b|}$$

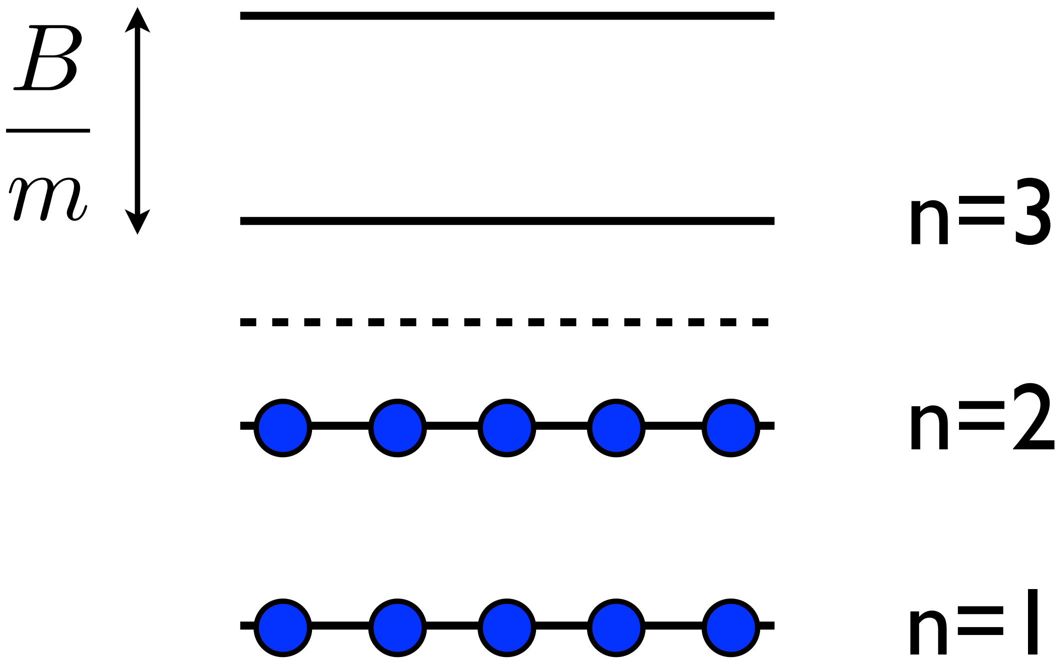
Find the ground state and low-energy excitations

2 types of QH effects

- Quantum Hall state = gapped state
- Two type of quantum Hall effects
  - integer
  - fractional

Integer quantum Hall state

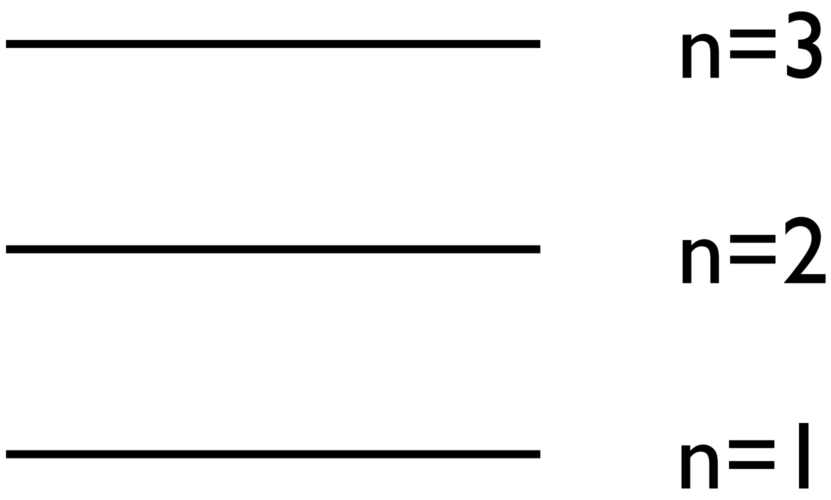
- electrons filling n Landau levels



When $\rho = n\frac{B}{2\pi}$

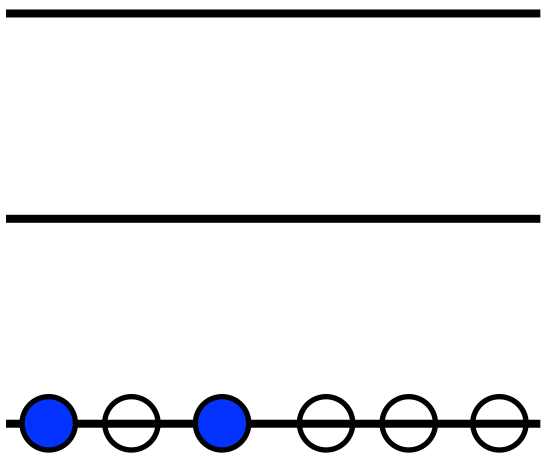
energy gap: $\frac{B}{m}$

Fractional QHE



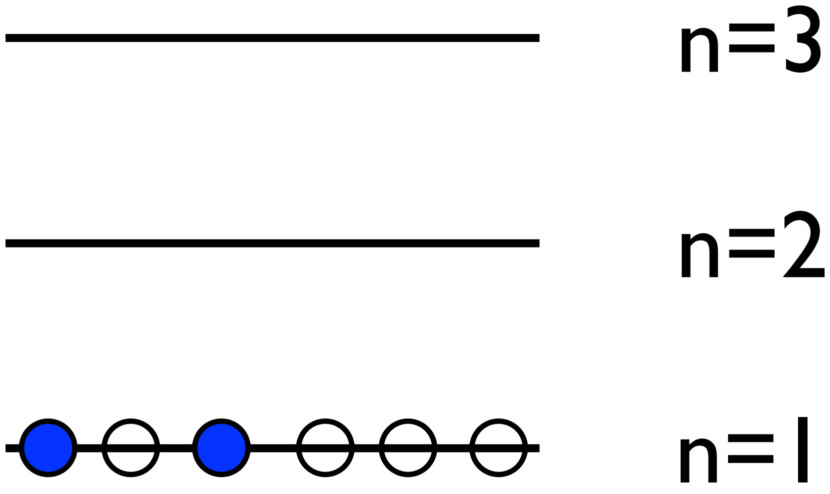
Filling fraction $\nu = \dfrac{\rho}{B/2\pi}$

Fractional QHE



Filling fraction $\nu = \frac{\rho}{B/2\pi}$

Fractional QHE



n=3

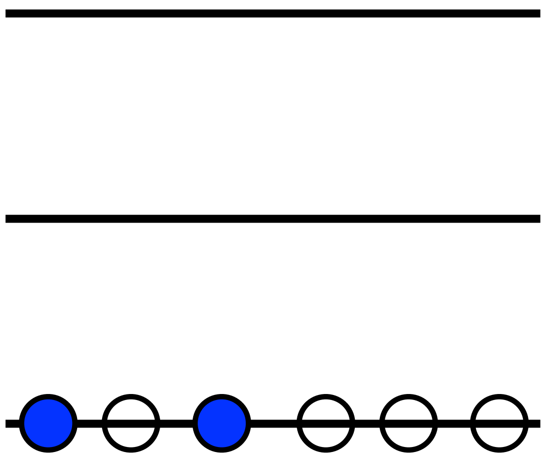
n=2

n=1

Filling fraction $\nu = \frac{\rho}{B/2\pi}$

$$\nu = \frac{1}{3}$$

Fractional QHE



n=3

n=2

n=1

Filling fraction $\nu = \frac{\rho}{B/2\pi}$

$$\nu = \frac{1}{3}$$

Large ground-state degeneracy without interactions

Experiments: energy gap for certain rational filling fractions, most prominently ν=N/(2N+1) and (N+1)/(2N+1)

Modern theory of FQHE

- Traditionally, FQHE is treated in condensed matter theory with flux attachment, composite fermion Jain; Lopez, Fradkin, Halperin Lee Read
- The modern version of the theory relies on a peculiar field-theoretical duality
- Refs: DTS arXiv:1502.03446
  - Metlitski, Vishwanath; Senthil, Wang...
- field theory "derivations": Karch, Tong 1606.01893; Seiberg, Senthil, Wang, Witten 1606.01989

Particle-vortex duality

- Duality between two (2+1)d field theories

background

$$\mathcal{L} = i\bar{\Psi}\gamma^\mu(\partial_\mu - iA_\mu)\Psi$$

$$\mathcal{L} = i\bar{\psi}\gamma^\mu(\partial_\mu - ia_\mu)\psi - \frac{1}{2}\frac{1}{2\pi}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu a_\lambda$$

dynamical

Ψ, ψ: two-component fermions

Fermionic version of a well-known bosonic duality: complex scalar = Abelian Higgs model

More precise statement

Seiberg, Senthil, Wang, Witten

$$\mathcal{L}[\Psi, A] + \frac{1}{2}\frac{1}{4\pi}AdA \Leftrightarrow$$

$$\mathcal{L}[\psi, a] - \frac{1}{2}\frac{1}{4\pi}ada + \frac{1}{2\pi}adb - \frac{2}{4\pi}bdb - \frac{1}{2\pi}Adb$$

Naively integrating b in the second action

$$\mathcal{L}[\psi, a] - \frac{1}{2}\frac{1}{2\pi}Ada + \frac{1}{2}\frac{1}{4\pi}AdA$$

Particle-vortex duality

original fermion	composite fermion
magnetic field	density
density	magnetic field

$$\mathcal{L} = i\bar{\psi}\gamma^{\mu}(\partial_{\mu} - ia_{\mu})\psi - \frac{1}{2}\frac{1}{2\pi}\epsilon^{\mu\nu\lambda}A_{\mu}\partial_{\nu}a_{\lambda}$$

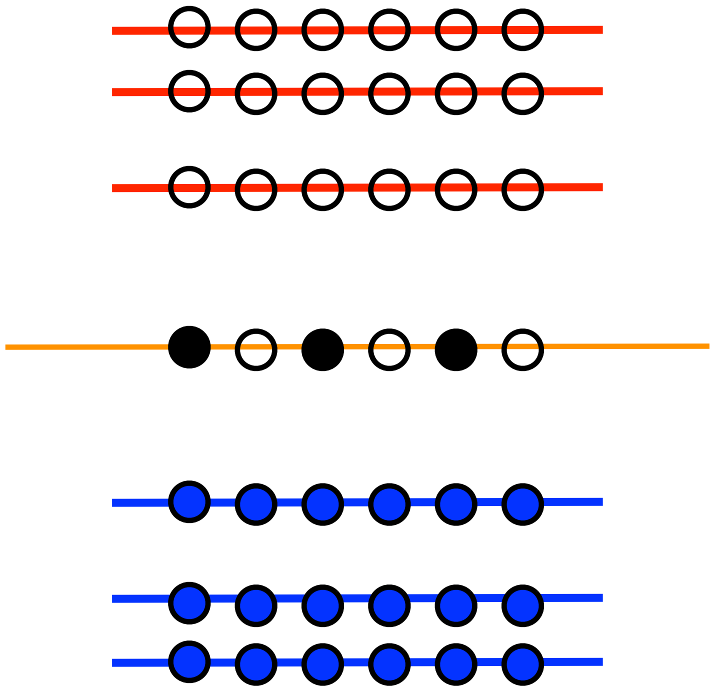
$$j^{\mu} = \frac{\delta S}{\delta A_{\mu}} = \frac{1}{2\pi}\epsilon^{\mu\nu\lambda}\partial_{\nu}a_{\lambda}$$

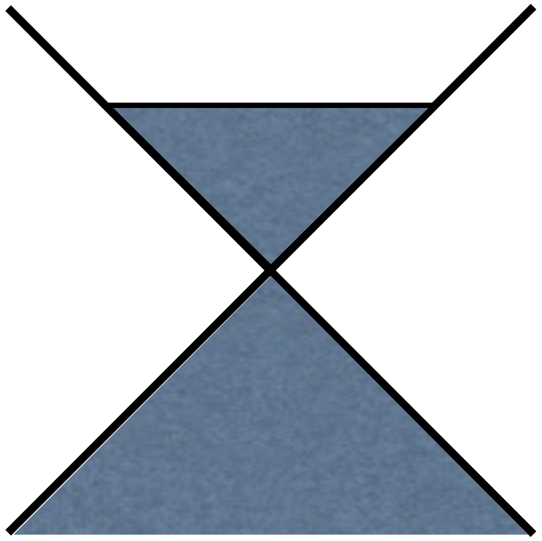
$$\frac{\delta S}{\delta a_0} = 0 \longrightarrow \langle \psi\bar{\gamma}^{0}\psi \rangle = \frac{B}{4\pi}$$

Consequences of duality for FQHE

electron Ψ

composite
fermion ψ





$\mu = 0, \quad B \neq 0$

$\rho \neq 0 \qquad b = 0$

half-filled Landau level

Fermi liquid of CFs

At and away from half-filling

- Well-known fact in QH physics: half-filled Landau level behaves as Fermi liquid
- Duality maps a state with filling fraction of the zeroth energy Landau level nu to composite fermion with filling fraction

$$\nu_{\mathrm{CF}} = -\frac{1}{4(\nu - \frac{1}{2})}$$

In particular

$$\nu = \frac{N}{2N+1} \to \nu_{\mathrm{CF}} = N + \frac{1}{2}$$

Relativistic IQHE

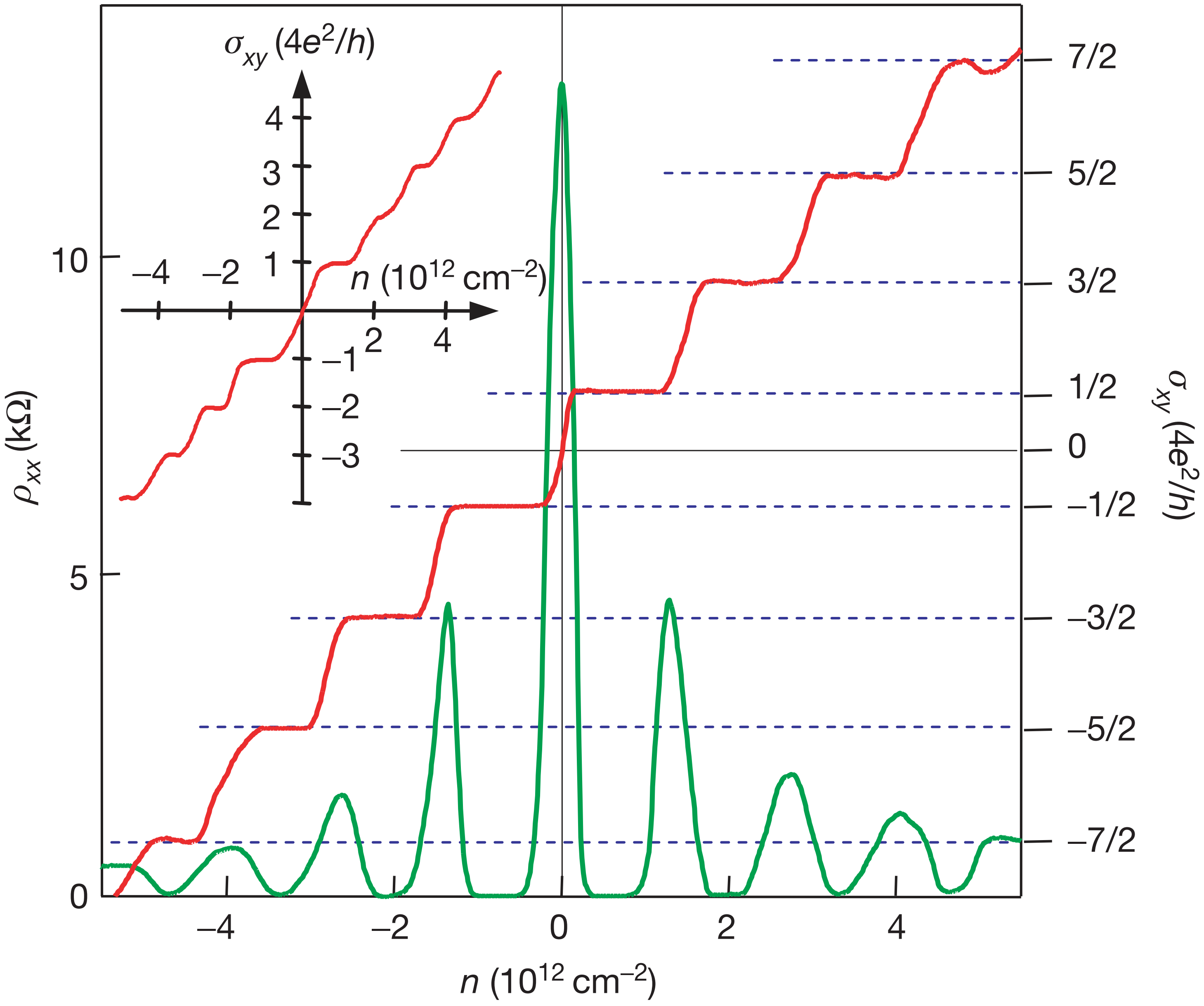
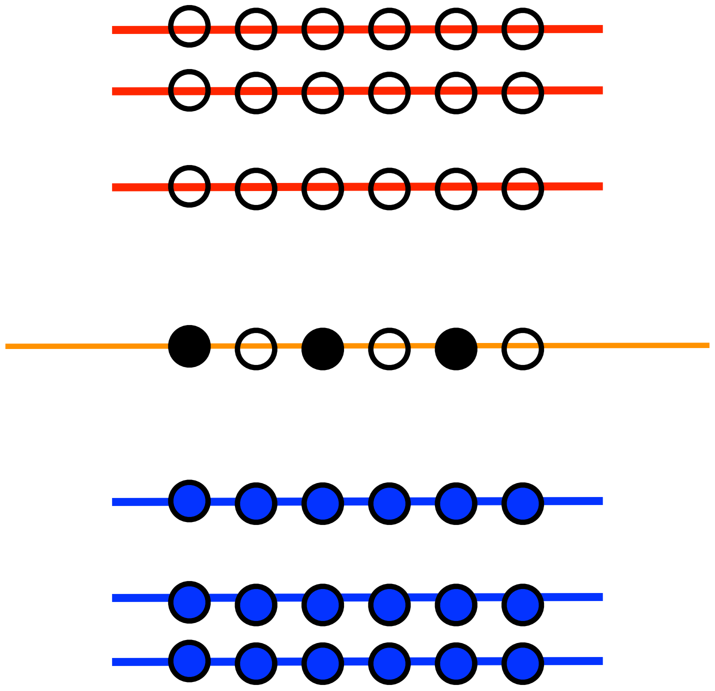


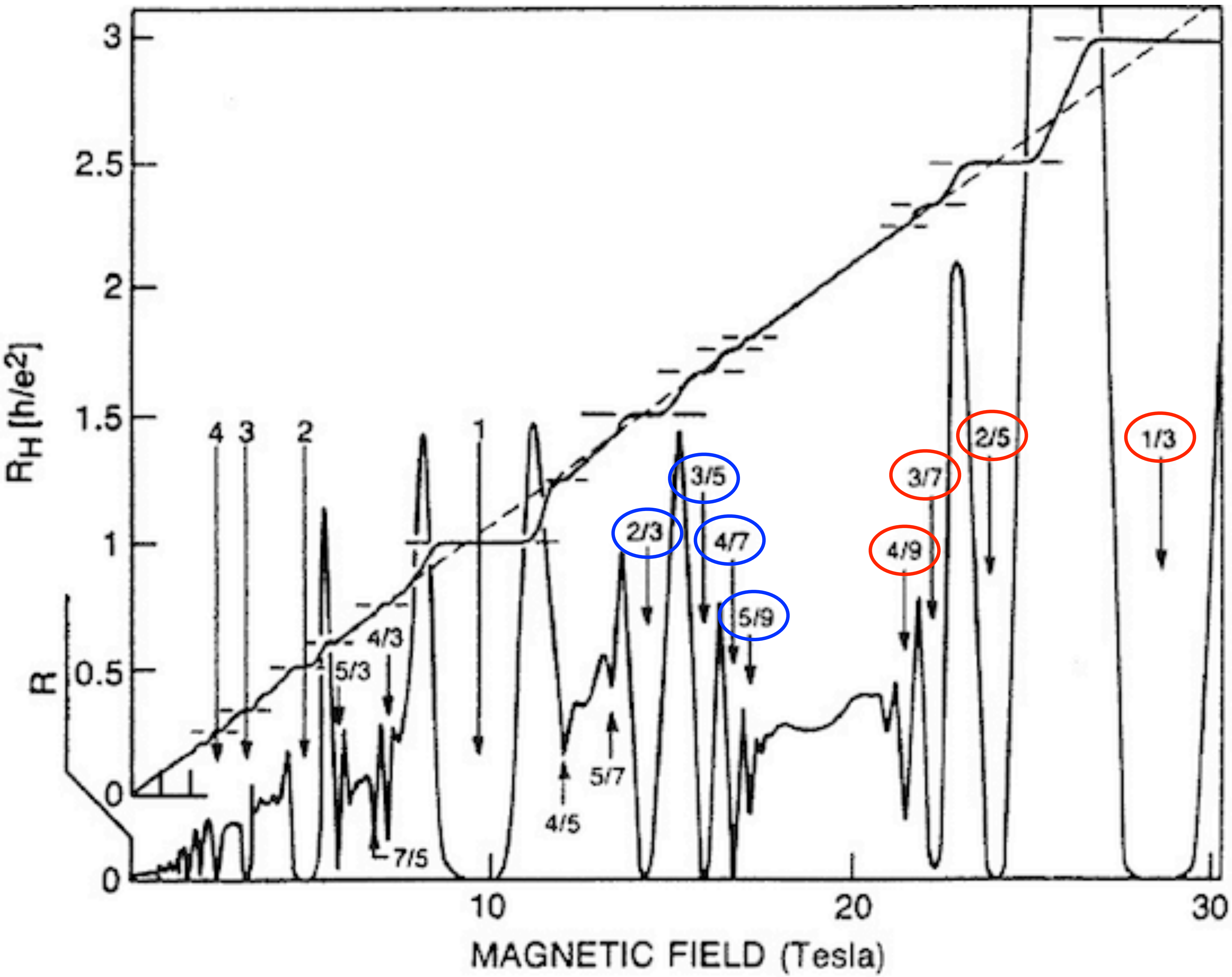
Figure 4 | QHE for massless Dirac fermions. Hall conductivity σ_{xy} and longitudinal resistivity ρ_{xx} of graphene as a function of their concentration at $B = 14$ T and $T = 4$ K. $\sigma_{xy} \equiv (4e^2/h)\nu$ is calculated from the measured

$$\nu = N + \frac{1}{2}$$



Novoselov et al 2005

Jain’s sequences of plateaux



$$\nu = \frac{n+1}{2n+1} \qquad \nu = \frac{n}{2n+1}$$

Summary of part 1

- Fractional quantum Hall states with nu=N/(2N+1) corresponds to integer quantum Hall states of the CFs at nu=N+1/2

$$\frac{b}{p_F^2} = \frac{1}{2N+1}$$

Magneto-roton

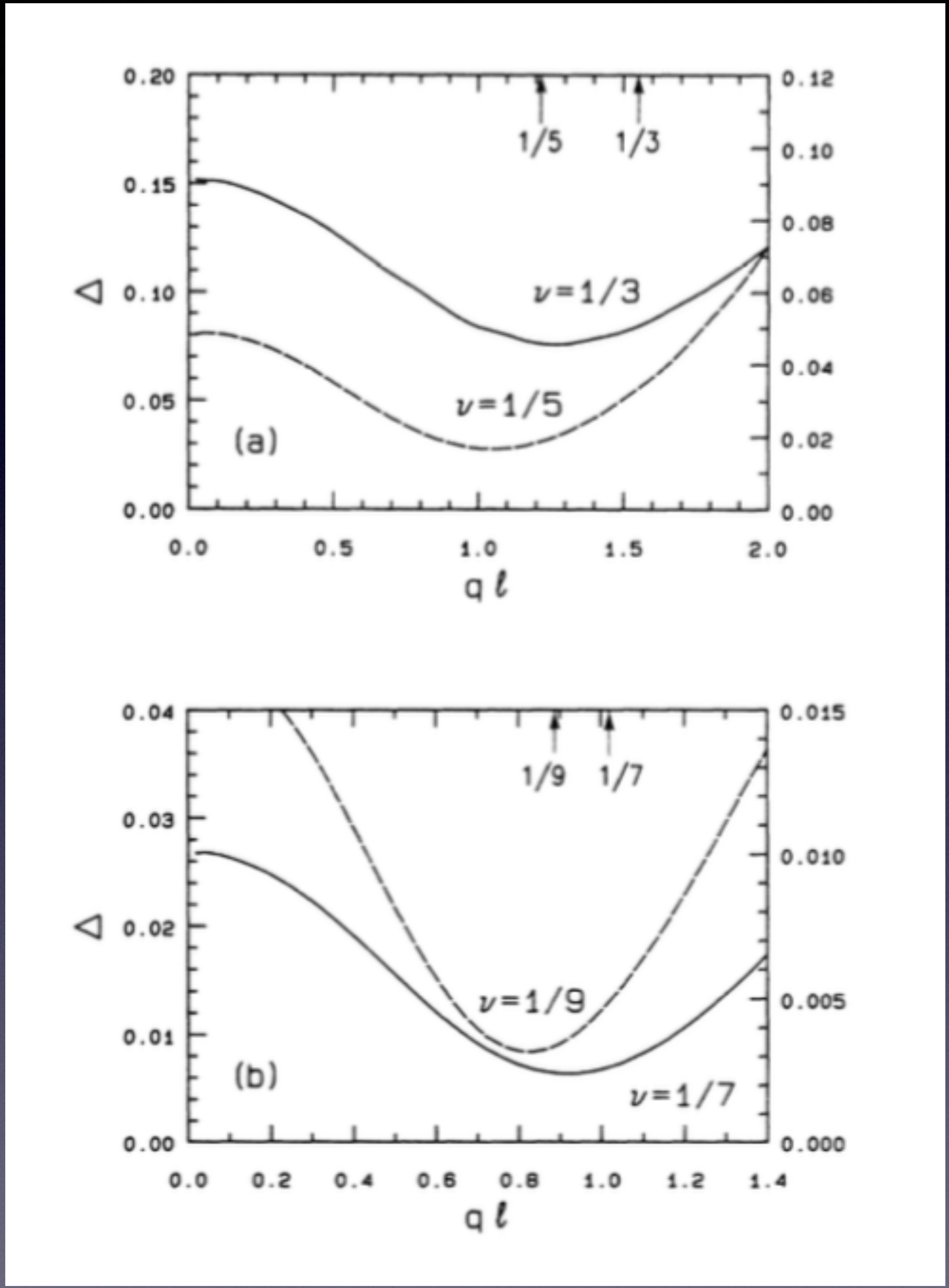
- Magneto-roton: minimum at nonzero momentum in dispersion curved of a neutral excitation
- First predicted by Girvin, MacDonald and Platzman (GMP 1984)
  - in analogy with Feynman's theory of the roton in superfluid helium
- Observed experimentally ~ 1990s
- But experiments seem to show a richer picture than in the original theory

- Laughlin 1/n state: GMP ansatz for density wave

$$\psi_{\mathbf{k}}(r_i) = \rho_{\mathbf{k}}\psi_{\text{Laughlin}}(r_i),$$

$$\rho_{\mathbf{k}} = \sum_j \exp(-i\mathbf{k}\cdot\mathbf{r}_j)$$

- Dispersion minimum at $q\ell_B \approx 1$



S. M. Girvin, A. H. MacDonald, and P. M. Platzman, Phys. Rev. B **33**, 2481 (1986).

$\nu = 1/3$

- Laughlin 1/n state: GMP ansatz for density wave

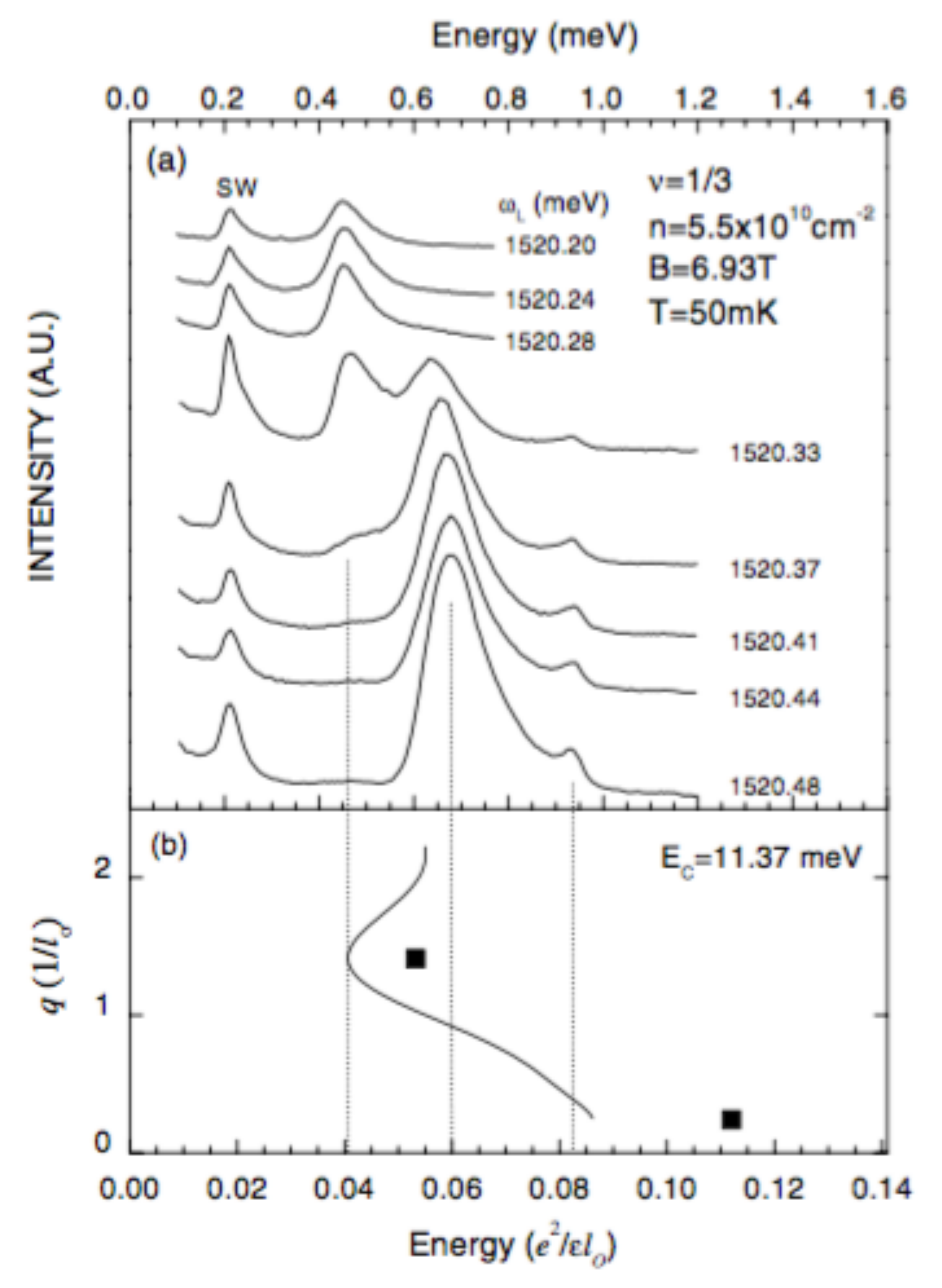
$$\psi_{\mathbf{k}}(r_i) = \rho_{\mathbf{k}} \psi_{\mathrm{Laughlin}}(r_i),$$

$$\rho_{\mathbf{k}} = \sum_j \exp(-i\mathbf{k} \cdot \mathbf{r}_j)$$

- Dispersion minimum at $q\ell_B \approx 1$

- Observed experimentally for 1/3

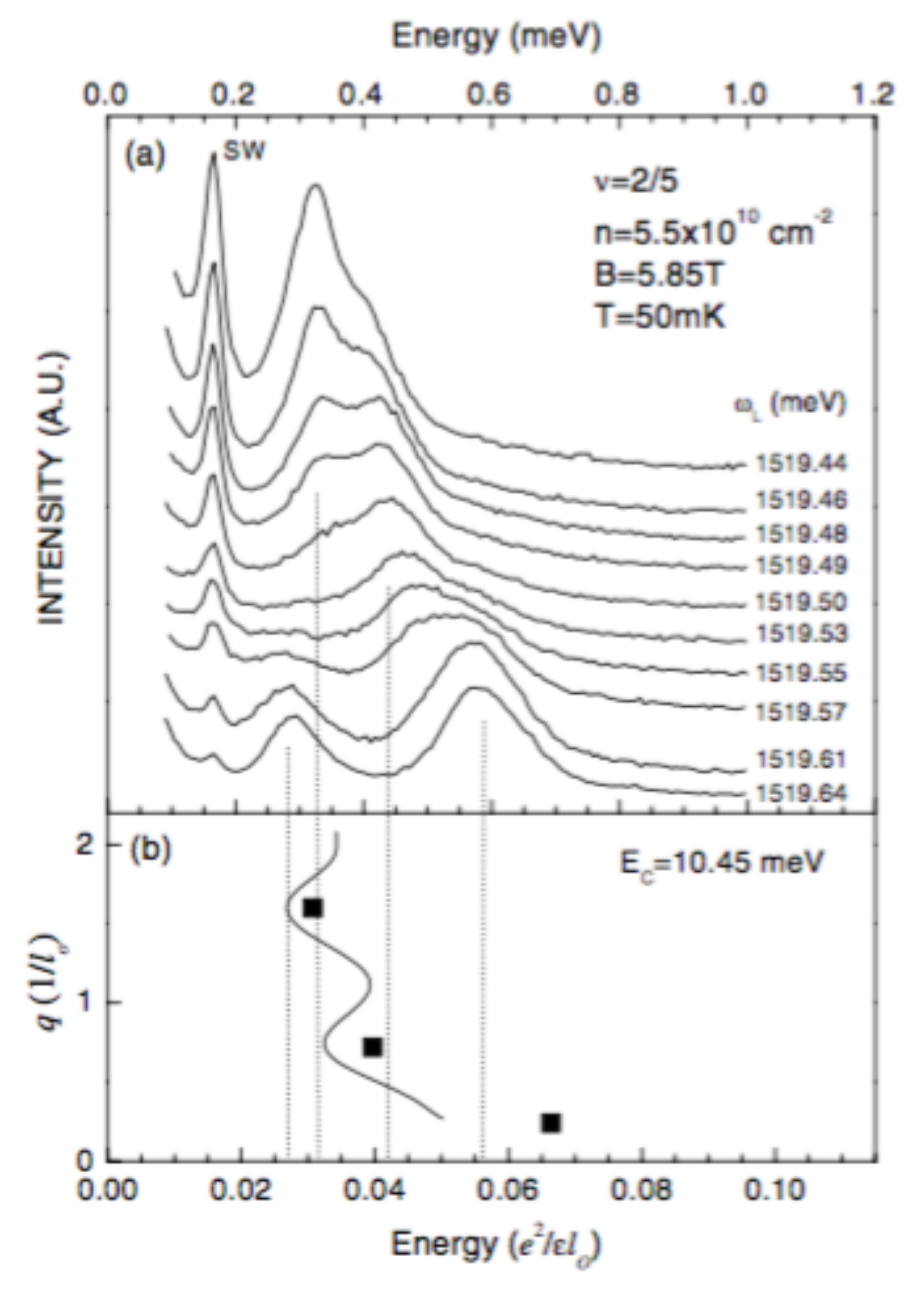
A. Pinczuk, B. S. Dennis, L. N. Pfeiffer, and K. West, Phys. Rev. Lett. 70, 3983 (1993)



M. Kang, A. Pinczuk, B. S. Dennis, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. 86, 2637 (2000).

$\nu = 2/5$

- Higher Jain states have density waves with more than one minimum
- Can we use composite fermion description of fractional states to understand the magneto-roton?

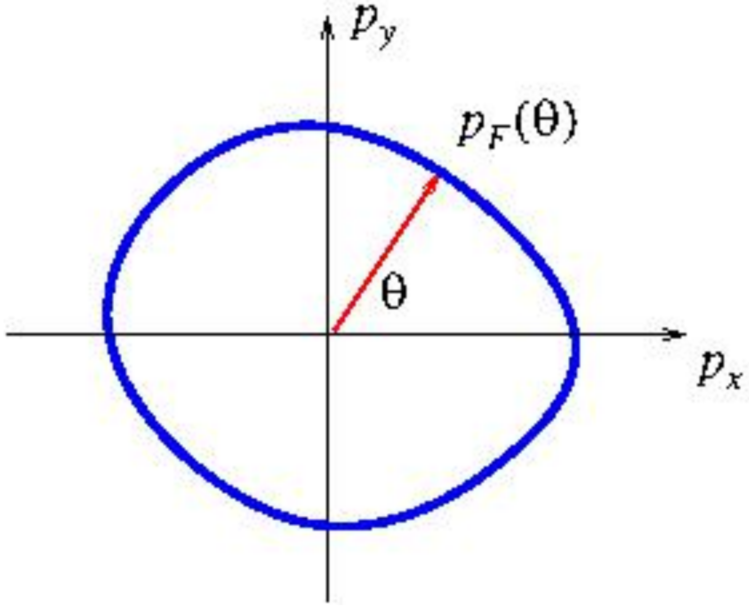


M. Kang, A. Pinczuk, B. S. Dennis, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. 86, 2637 (2000).

The theoretical problem

- As we have see above, the state with ν=N/(2N+1) corresponds to CFs in a magnetic field $b=p_F^2/(2N+1)$
- Problem: to determine the spectrum of excitations of a Fermi liquid, coupled with a U(1) gauge field, in a small magnetic field
- Proposal: near half-filling (N>>1) low-energy modes are fluctuations of the shape of the Fermi surface

Bosonic excitations



- Low-energy, long-wavelength excitations: fluctuations of the shape of the Fermi surface

$$p_F(t, \mathbf{x}, \theta) = p_F^0 + \sum_{n=-\infty}^{\infty} u_n(t, \mathbf{x})\, e^{-in\theta}.$$

- Infinite number of bosonic fields, one per spin

Coupling to gauge field

- The composite fermion is coupled to the dynamical gauge field a_μ
- This will has an effect of freezing out fluctuations of the charge and current fluctuations
  - $a_0=0$, $a_1=a_{-1}=0$

Semiclassical description

- In small magnetic field, fermion executes large orbits → semiclassical description

$$\hat{F} = \int \frac{d\mathbf{x}\, d\mathbf{p}}{(2\pi)^2}\, F(\mathbf{x}, \mathbf{p}) n_{\mathbf{p}}(\mathbf{x})$$

n_p = 1 inside Fermi surface and 0 outside: F=F[u(x,θ)]

Postulate commutation relation

$$[\hat{F},\, \hat{G}] = \int \frac{d\mathbf{x}\, d\mathbf{p}}{(2\pi)^2}\, \{F,\, G\}(\mathbf{x}, \mathbf{p}) n_{\mathbf{p}}(\mathbf{x})$$

$$\{F,\, G\} = \frac{\partial F}{\partial p_i}\frac{\partial G}{\partial x_i} - \frac{\partial G}{\partial x_i}\frac{\partial F}{\partial p_i} - b\epsilon^{ij}\frac{\partial F}{\partial p_i}\frac{\partial G}{\partial p_j},$$

Commutator of shape deformations

- One can derive the commutation relation between u's:

(Haldane, 1992)

$$[u_m(\mathbf{q}),\, u_n(\mathbf{q}')] = \frac{\pi}{p_F}\left[\frac{\overset{\nabla \times \mathbf{a}\ \swarrow}{2bm}}{p_F}\delta_{m+n,0} + \delta_{m+n,1}q_+ + \delta_{m+n,-1}q_-\right](2\pi)^2\delta(\mathbf{q}+\mathbf{q}')$$

$$q_\pm = q_x \pm iq_y$$

Landau's Fermi liquid theory with its predictions at T=0 (zero sound etc) is recovered from the quadratic Hamiltonian

$$H = \frac{v_F p_F}{4\pi}\int d\mathbf{x} \sum_{n=-\infty}^{\infty}(1+F_n)u_n(\mathbf{x})u_{-n}(\mathbf{x}),$$

Semiclassical gauged Fermi surface

- In the case of the composite fermions in FQHE, modes with n=0 and n=±1 has to be excluded from the algebra
- The algebra can then be written as

$$[u_m(\mathbf{q}),\, u_{-n}(\mathbf{q}')] = C_{mn}\delta(\mathbf{q}+\mathbf{q}')$$

$$C_{mn} \sim \begin{pmatrix} 2 & z & 0 & 0 & \dots \\ z & 3 & z & 0 & \dots \\ 0 & z & 4 & z & \dots \\ 0 & 0 & z & 5 & \dots \\ \dots & \dots & \dots & \dots & \dots \end{pmatrix}, \quad z = \frac{2N+1}{2} q\ell_B$$

$$\ell_B = \frac{1}{\sqrt{B}}$$

Near zero momentum

$$[u_m,\, u_n] \sim m\delta_{m+n,0}$$

- At q=0: (u_{-n}, u_n) form pairs of creations/annihilation operators (b≠0)
- For the quadratic Hamiltonian of the Landau's Fermi liquid theory

$$\omega_n^{(0)} = n(1+F_n)\omega_c, \quad \omega_c = \frac{b}{m_*}\,.$$

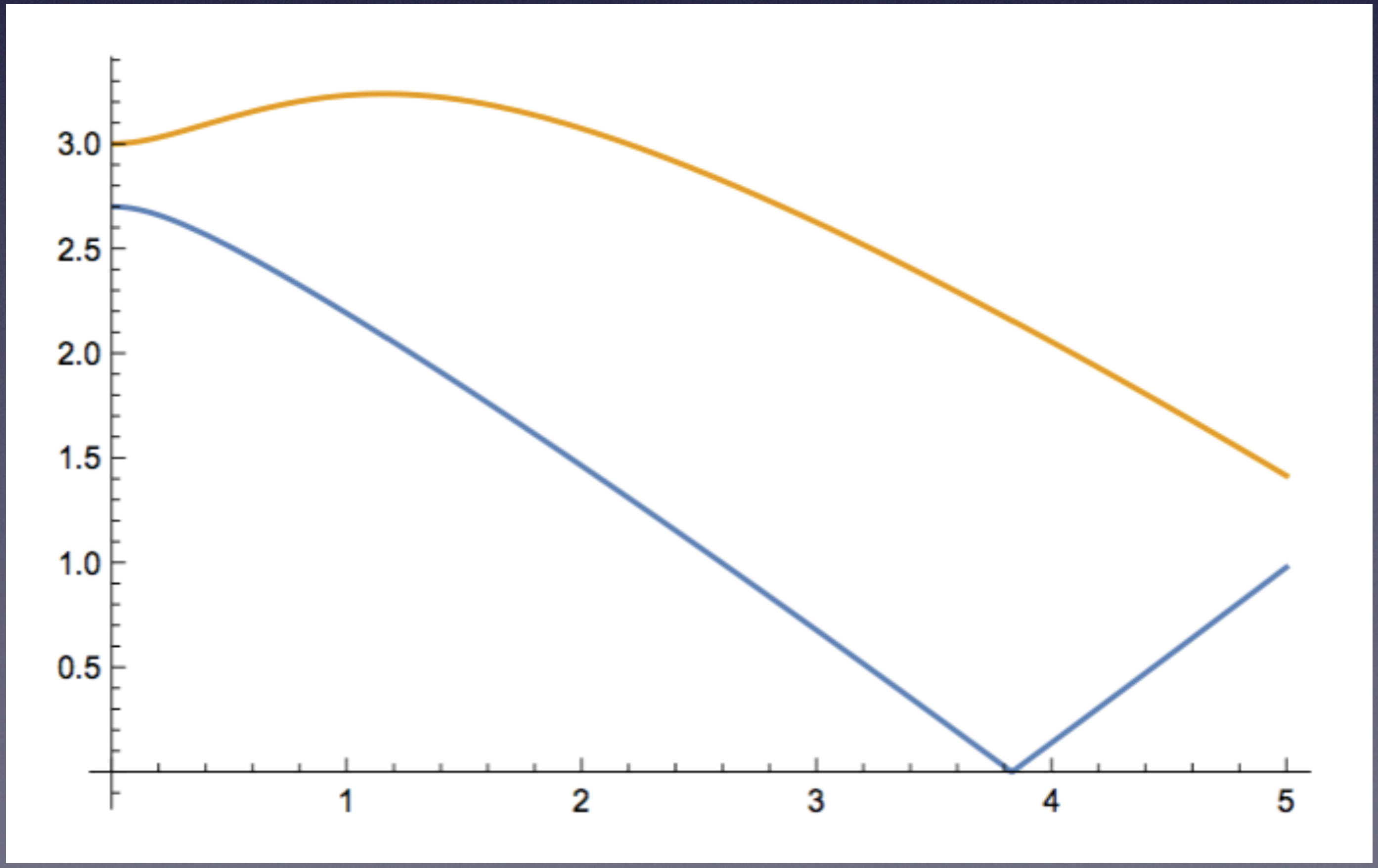
- Finite momentum: solve equation of motion

$$[\omega - n(1+F_n)\omega_c]u_n = \frac{v_F q}{2}[(1+F_{n-1})u_{n-1} + (1+F_{n+1})u_{n+1}]$$

- b.c. $u_{-1,0,1} = 0$

- Multiple branches

- Minima at ω=0



- Where are zero frequency modes?

$$\begin{aligned} n(1+F_n)\omega_c u_n + \frac{v_F q}{2}[(1+F_{n-1})u_{n-1} \\ +(1+F_{n+1})u_{n+1}] = 0 \end{aligned}$$

- Solution: $u_n = \frac{(-1)^n}{1+F_n} J_n\left(\frac{p_F q}{b}\right)$

- Recall our boundary conditions $u_{\pm 1} = 0$:

$$J_1(p_F q/b) = 0$$

- Translates to $q\ell_B = z_i \frac{b}{B} = \frac{z_i}{2N+1}$
  - where z_i are zeroes of $J_1(z)$
- Independent of Landau parameters!

$$\nu = \frac{N}{2N+1}, \ \frac{N+1}{2N+1}$$

$$[u_m(\mathbf{q}),\, u_{-n}(\mathbf{q}')] = C_{mn}\delta(\mathbf{q}+\mathbf{q}')$$

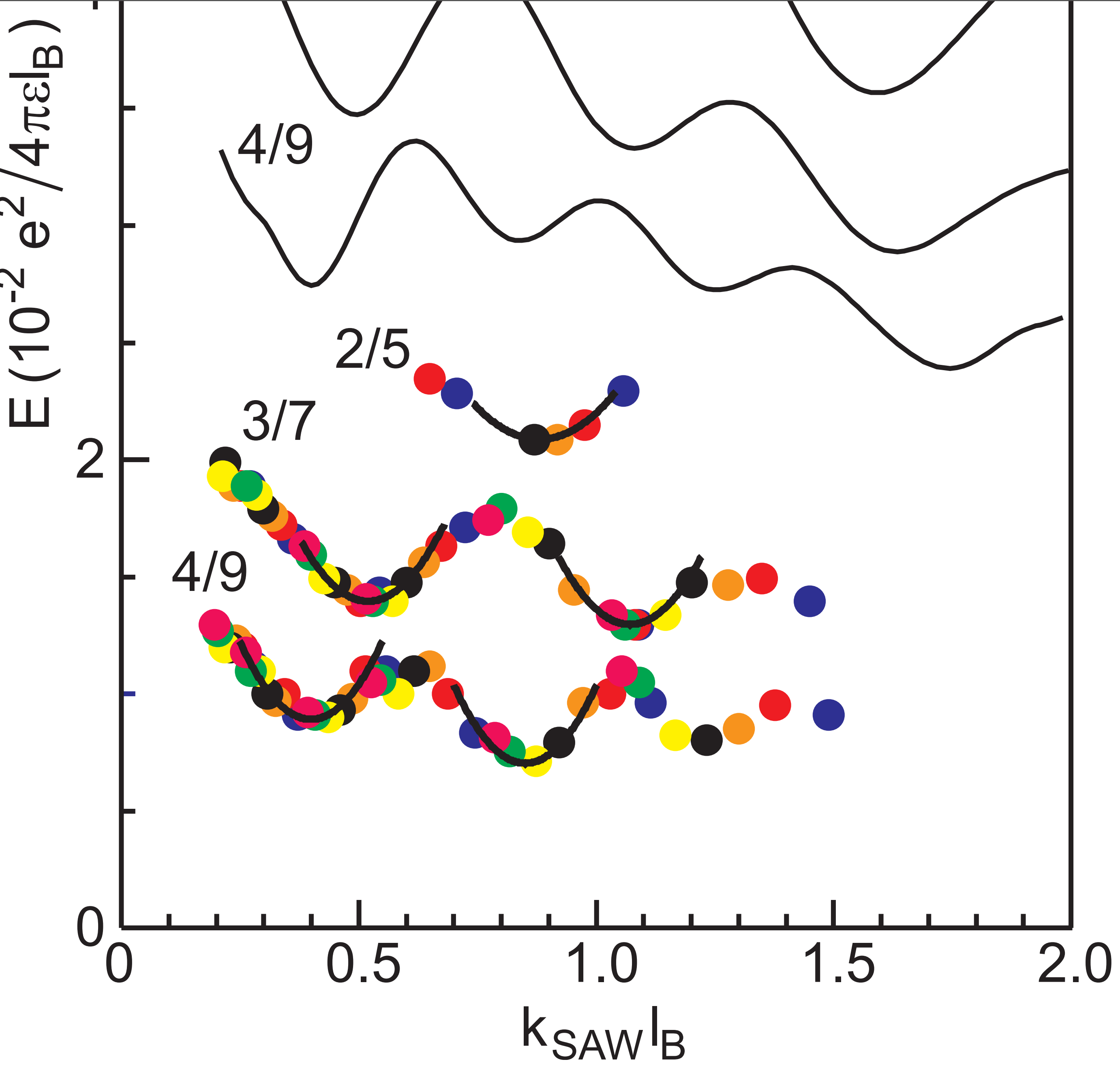
$$C_{mn} \sim \begin{pmatrix} 2 & z & 0 & 0 & \dots \\ z & 3 & z & 0 & \dots \\ 0 & z & 4 & z & \dots \\ 0 & 0 & z & 5 & \dots \\ \dots & \dots & \dots & \dots & \dots \end{pmatrix}, \qquad z = \frac{2N+1}{2} q\ell_B$$

- Commutator matrix is degenerate at $q\ell_B = z_i/(2N+1)$ where $J_1(z_i)=0$ $\quad \nu = N/(2N+1)$
- For Hamiltonian quadratic in u's: zero eigenvalues at these momentum. In real life, energy is not zero but reaches minima: magneto-rotons

$$\nu = \frac{n}{2n+1}$$

$$q\ell_B = \frac{z_i}{2n+1}$$

$$J_1(z_i) = 0$$

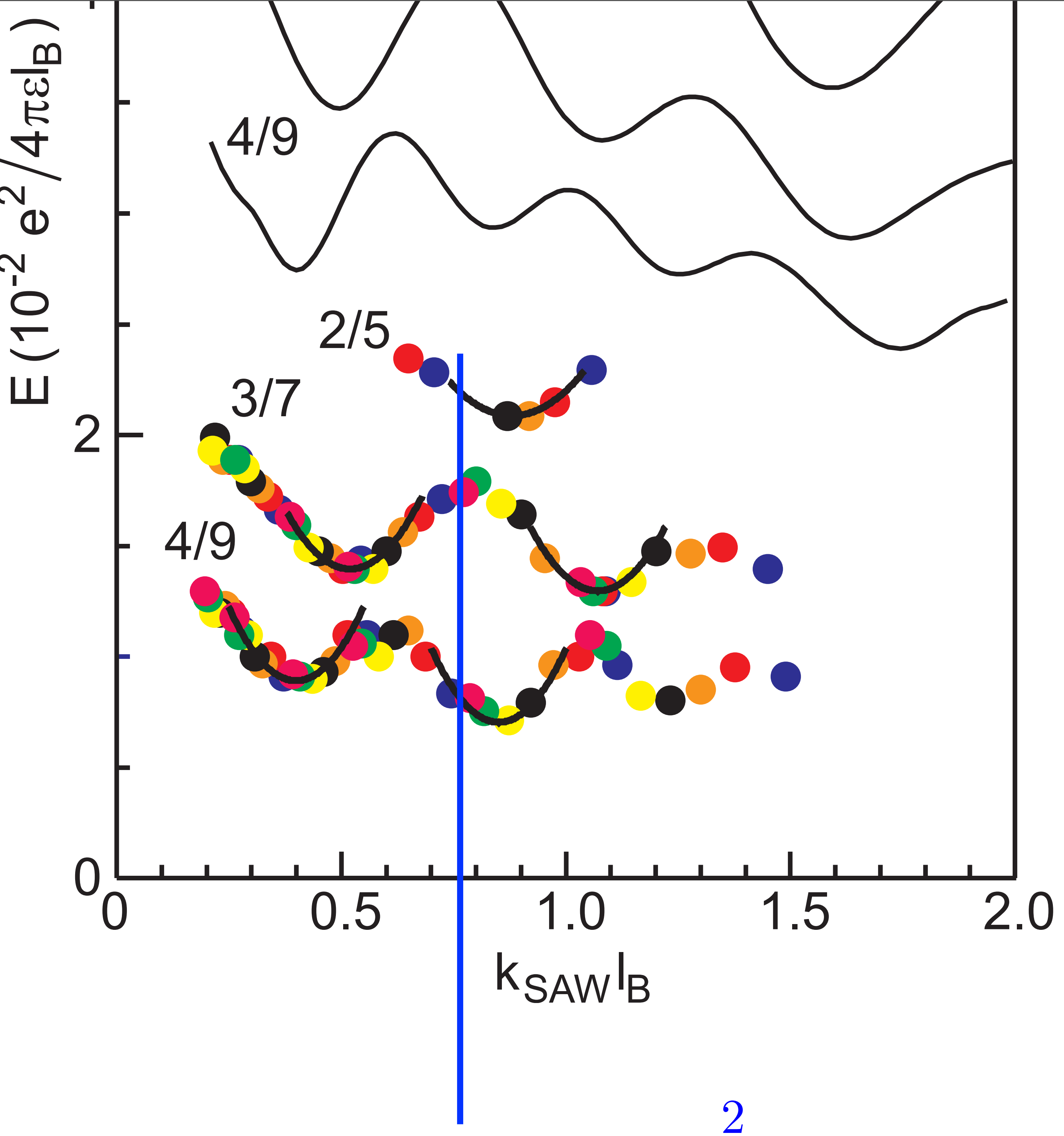


Kukushkin et al, Science 324 (2009)

$$\nu = \frac{n}{2n+1}$$

$$q\ell_B = \frac{z_i}{2n+1}$$

$$J_1(z_i) = 0$$



0.77

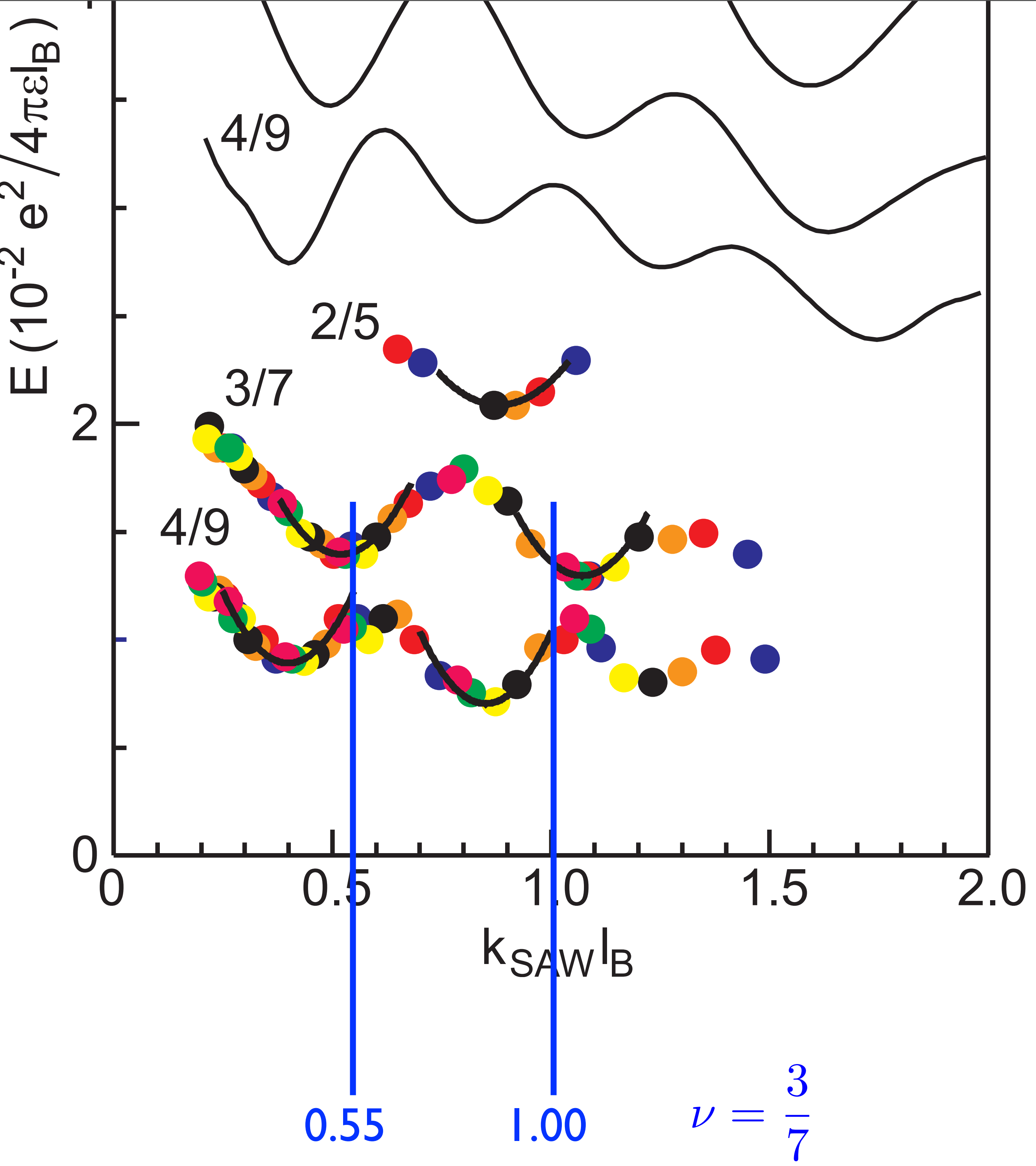
$$\nu = \frac{2}{5}$$

Kukushkin et al, Science 324 (2009)

$$\nu = \frac{n}{2n+1}$$

$$q\ell_B = \frac{z_i}{2n+1}$$

$$J_1(z_i) = 0$$

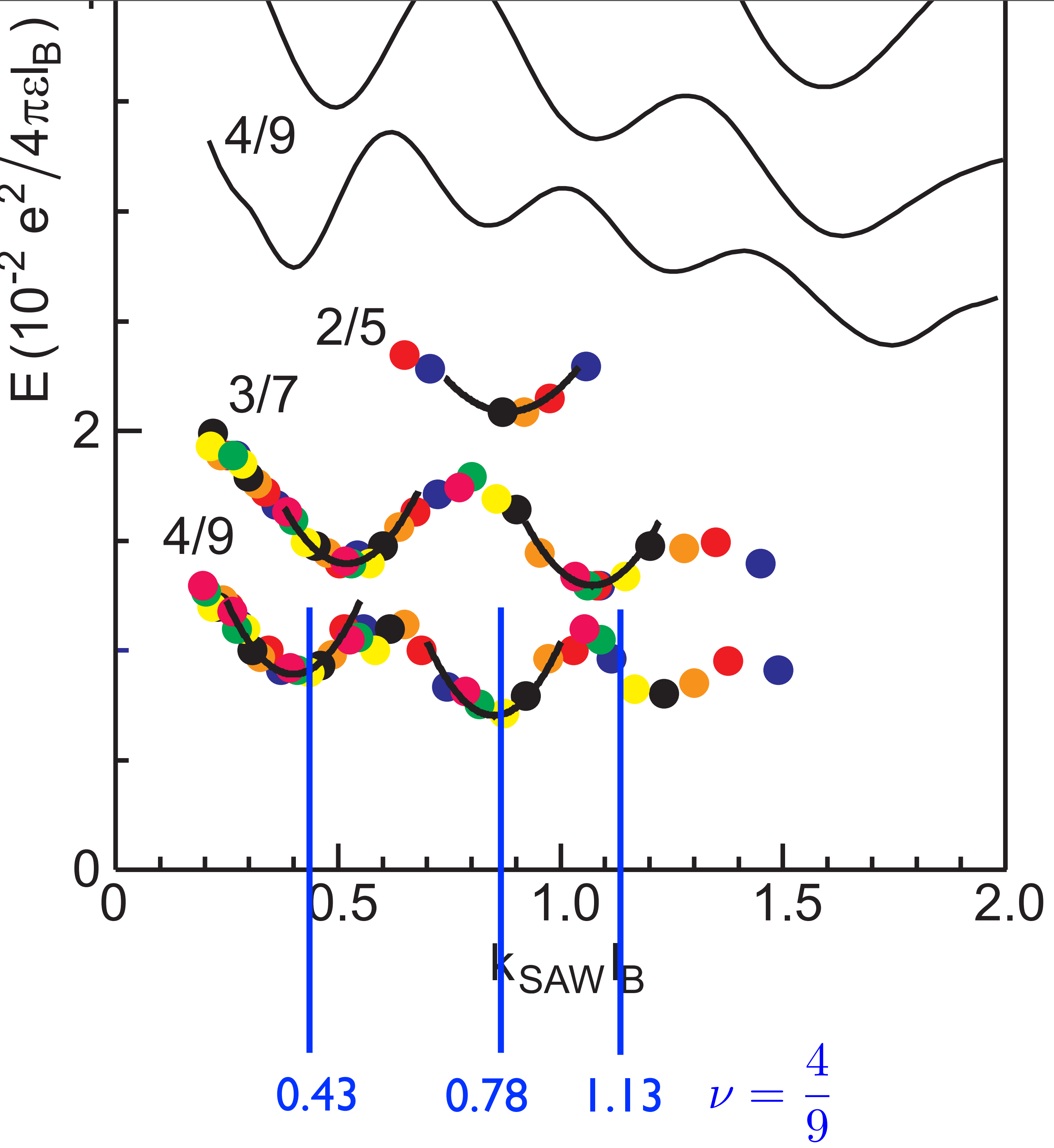


Kukushkin et al, Science 324 (2009)

$$\nu = \frac{n}{2n+1}$$

$$q\ell_B = \frac{z_i}{2n+1}$$

$$J_1(z_i) = 0$$

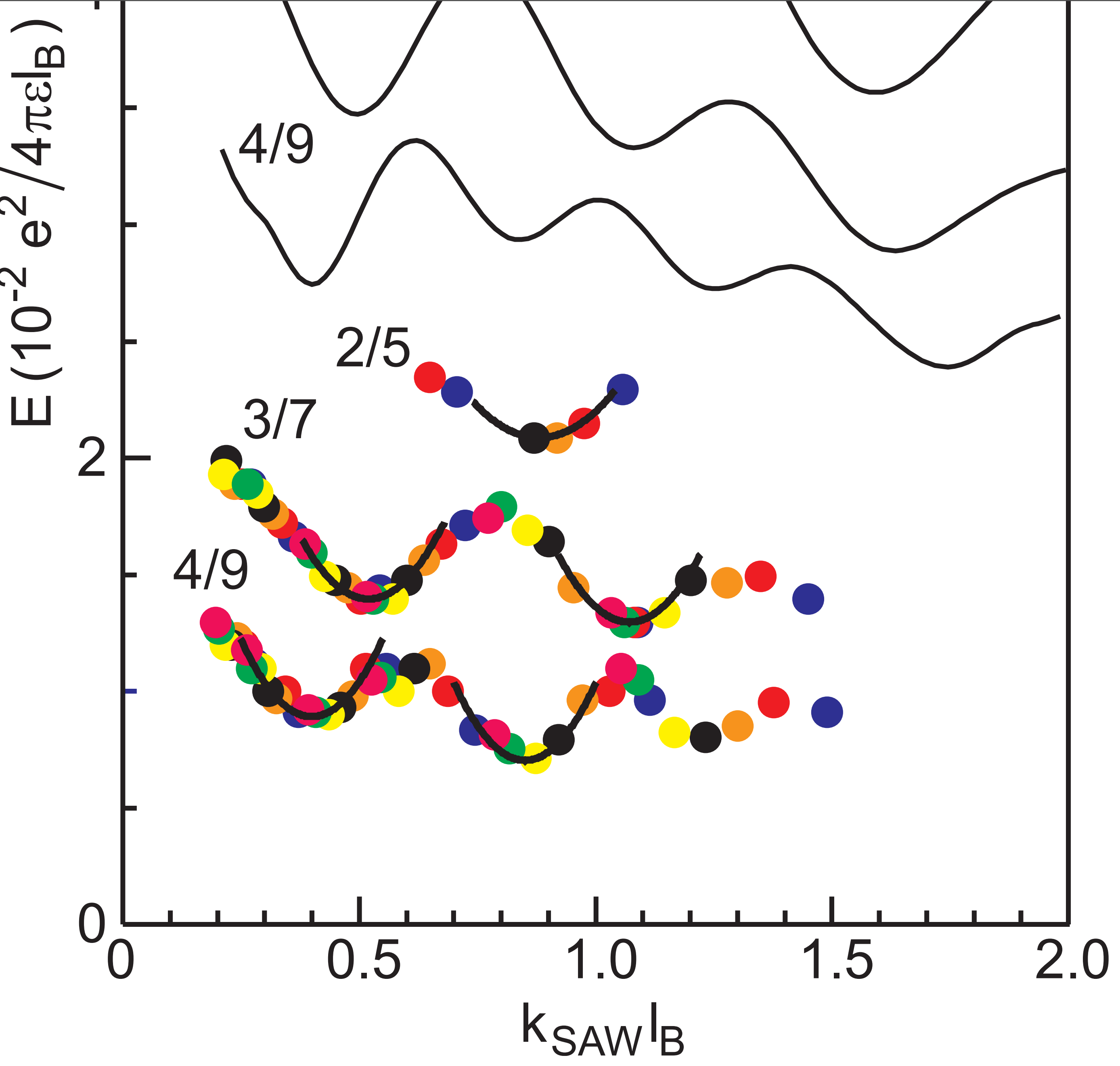


Kukushkin et al, Science 324 (2009)

$$\nu = \frac{n}{2n+1}$$

$$q\ell_B = \frac{z_i}{2n+1}$$

$$J_1(z_i) = 0$$



Kukushkin et al, Science 324 (2009)

Conclusion

- Low energy neutral excitations of FQH states with $\nu=N/(2N+1)$, $N>>1$ are fluctuations of the shape of the Fermi surface
- Magneto-roton minima at values dictated by kinemetics (commutation relations), independent of Hamiltonian
- Though the theory is undoubtedly imperfect, very good fit with experimental data
- Bessel functions: holographic interpretation?