



# Overview: from (many) qubits to space-time

### Román Orús

Institut für Physik, Johannes Gutenberg-Universität, Mainz (Germany)

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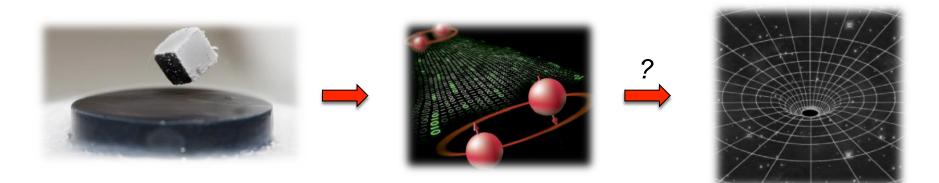
c.f. talks by Latorre, Molina-Vilaplana, Pastawski, Wen, Maldacena

### **Motivation**

Condensed Matter

#### Quantum Information

#### Space-time?



(Objects that look like) space-times seem to emerge from the *entanglement structure* of quantum many-body states

(and we were not thinking about gravity at all...)

This talk: overview of some ideas along these lines

## Outline



1) Review of TNs

2) PEPS and emergent Hamiltonians

3) Symmetric TNs and emergent spin networks

4) MERA and emergent AdS/CFT

5) Summary & open questions

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# Entanglement obeys area-law





#### key resource in quantum information

teleportation, quantum algorithms, quantum error correction, quantum cryptography...





2d system

#### key resource in quantum information

teleportation, quantum algorithms, quantum error correction, quantum cryptography...

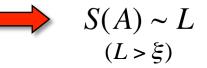
 $\rho_{A} = \mathrm{tr}_{E}(|\Psi\rangle\langle\Psi|)$ 

 $S(A) = -\mathrm{tr}(\rho_A \log \rho_A)$ 

Reduced density matrix of subsystem A

Entanglement entropy (von Neumann entropy)

For many ground states







2d system

#### key resource in quantum information

teleportation, quantum algorithms, quantum error correction, quantum cryptography...

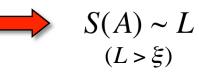
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Reduced density matrix of subsystem A

Entanglement entropy (von Neumann entropy)

For many ground states



#### In d dimensions

Generic state

 $S(A) \sim L^d$ (volume)

Ground states of (most) local Hamiltonians

 $S(A) \sim L^{d-1}$ (area)

Srednicki, Plenio, Eisert, Dreißig, Cramer, Wolf...

Locality of interactions 🔶 area-law

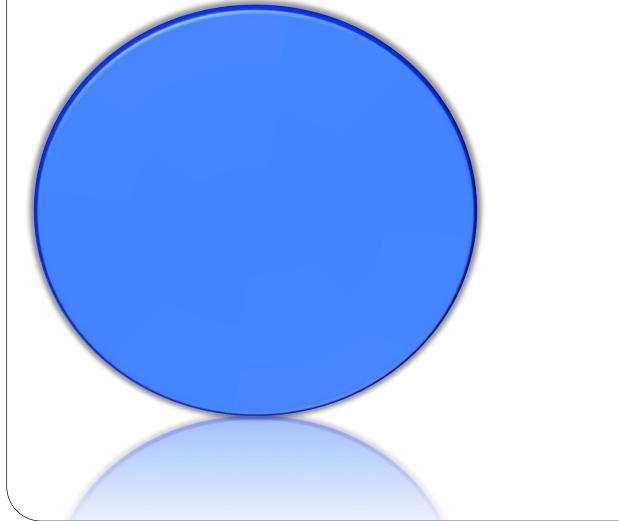


# Many-body Hilbert space is far too large

# Hilbert space is a convenient illusion

JG

Hilbert space of a N-body many-body system



# Hilbert space is a convenient illusion

JG

Hilbert space of a N-body many-body system

> , Set of area-law states Y. Ge, J. Eisert, arXiv:1411.2995

Set of TN states (low-energy eigenstates of local Hamiltonians)

Set of product states (mean field)

# Hilbert space is a convenient illusion

JG

Hilbert space of a N-body many-body system

Most states here are not even reachable by a time evolution with a local Hamiltonian in polynomial time

Poulin, Qarry, Somma, Verstraete, PRL 106 170501 (2011) "Exploration" time ~  $O(10^{10^{23}})$  sec.

Compare to... Age of the universe ~  $O(10^{17})$  sec.

Set of area-law states
 Y. Ge, J. Eisert, arXiv:1411.2995

Set of TN states (low-energy eigenstates of local Hamiltonians)

Set of product states (mean field)

We need a language to target the relevant corner of quantum states directly

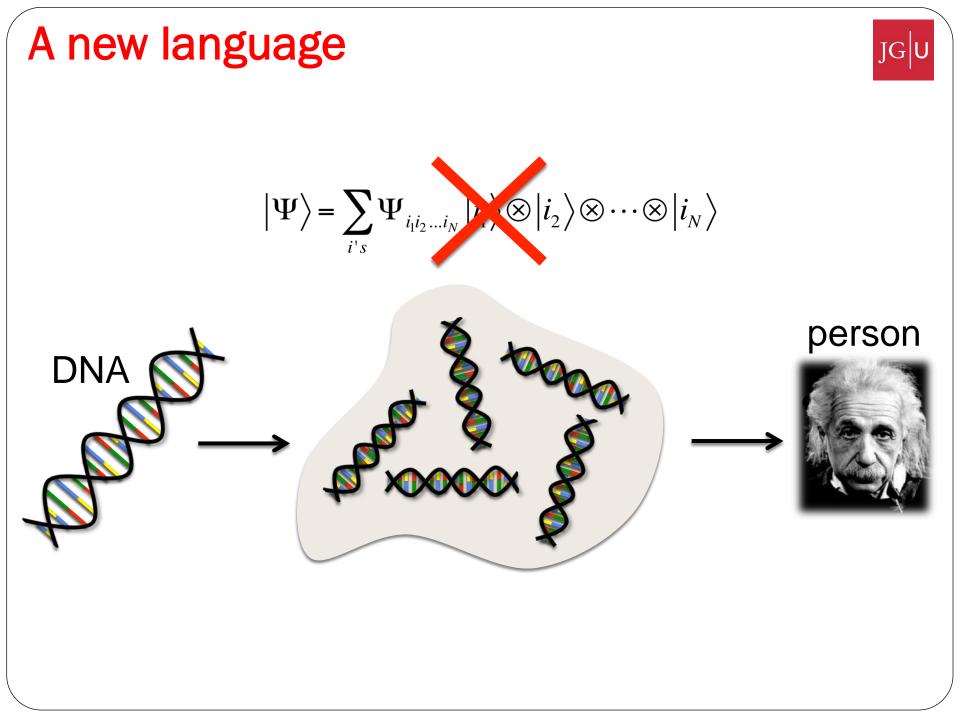


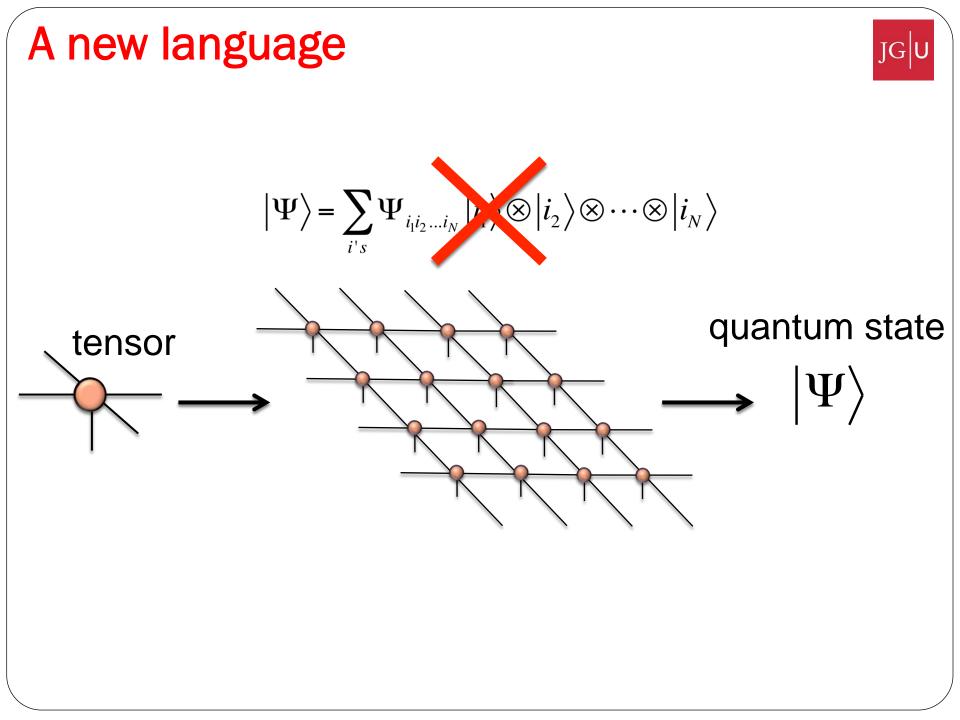
## **Tensor Networks**

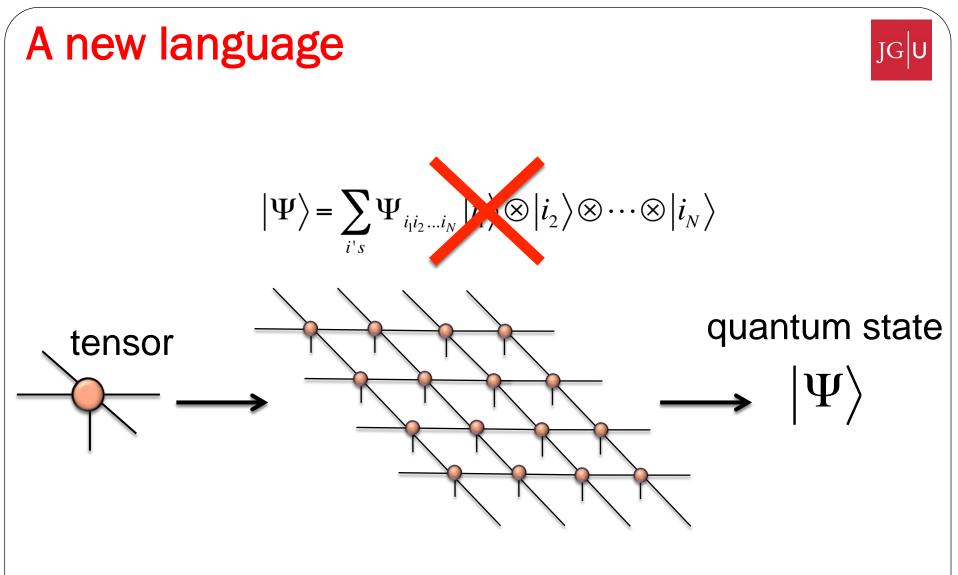
### A new language



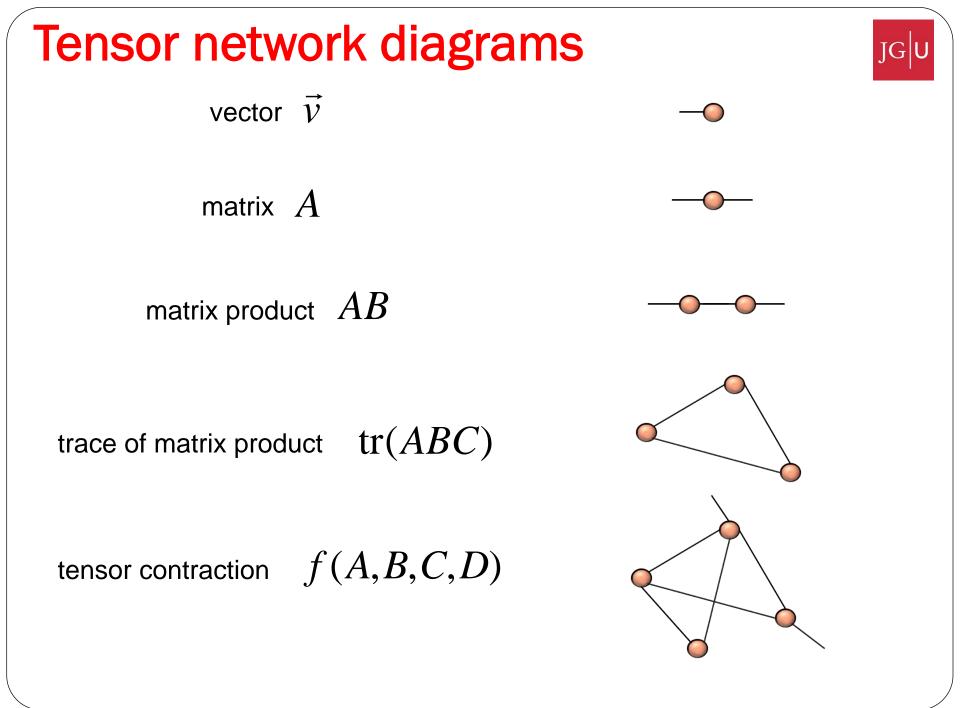
 $|\Psi\rangle = \sum_{i's} \Psi_{i_1 i_2 \dots i_N} \otimes |i_2\rangle \otimes \dots \otimes |i_N\rangle$ 





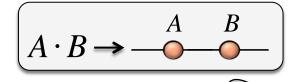


Tensors are local building blocks for the quantum state (like a DNA, or LEGO)



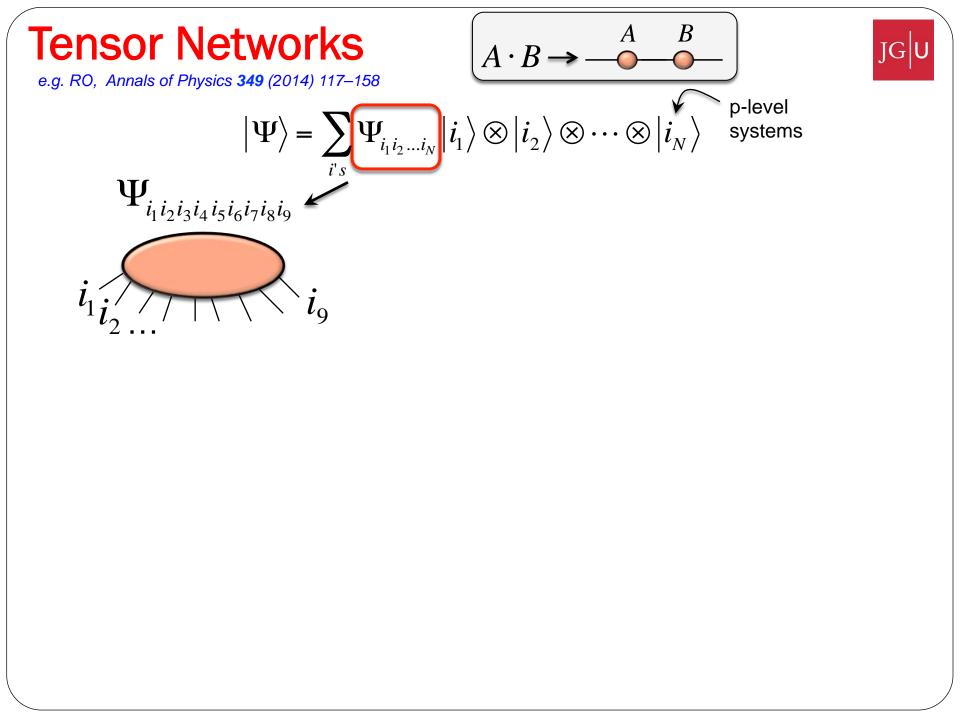
### **Tensor Networks**

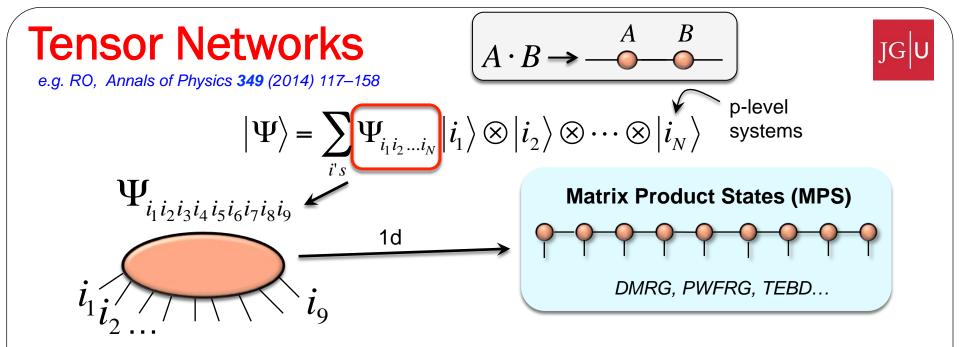


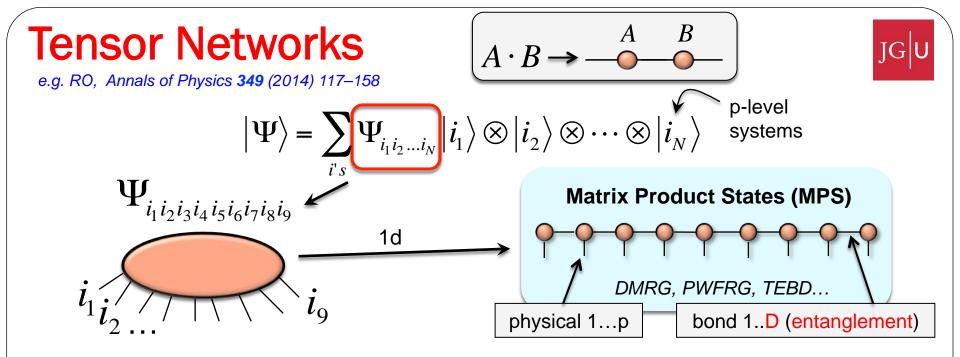


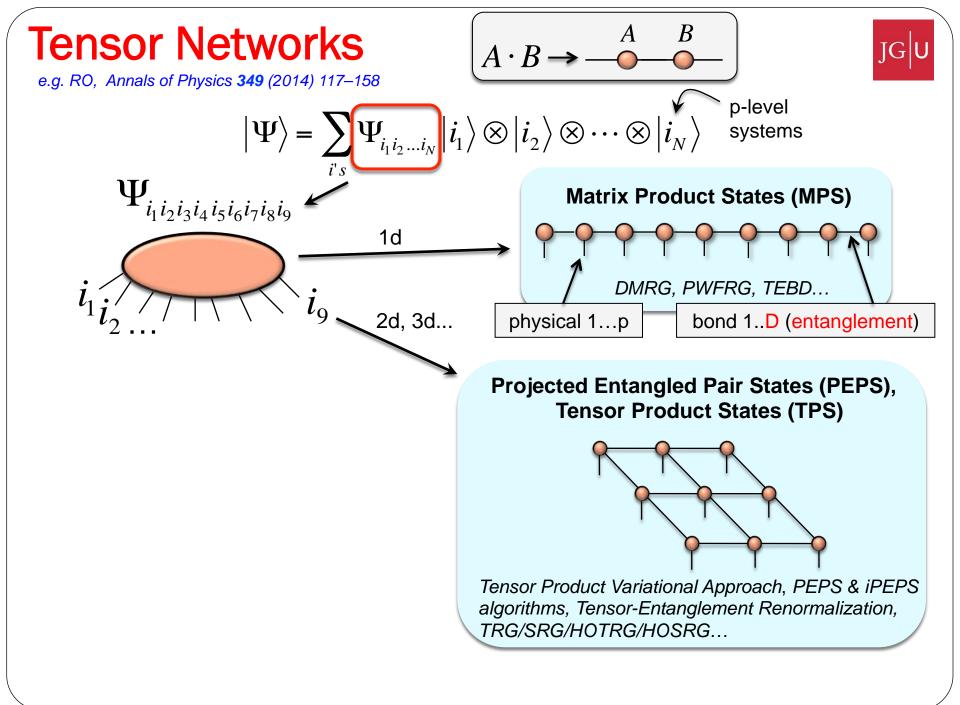


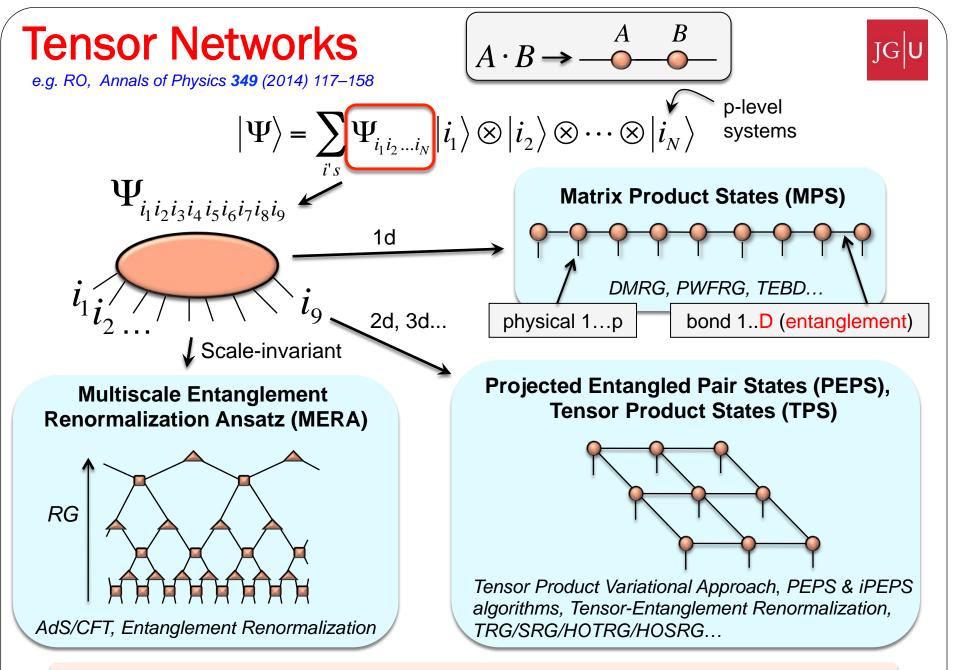
 $|\Psi\rangle = \sum_{i's} \Psi_{i_1 i_2 \dots i_N} |i_1\rangle \otimes |i_2\rangle \otimes \dots \otimes |i_N\rangle \quad \text{p-level systems}$ 









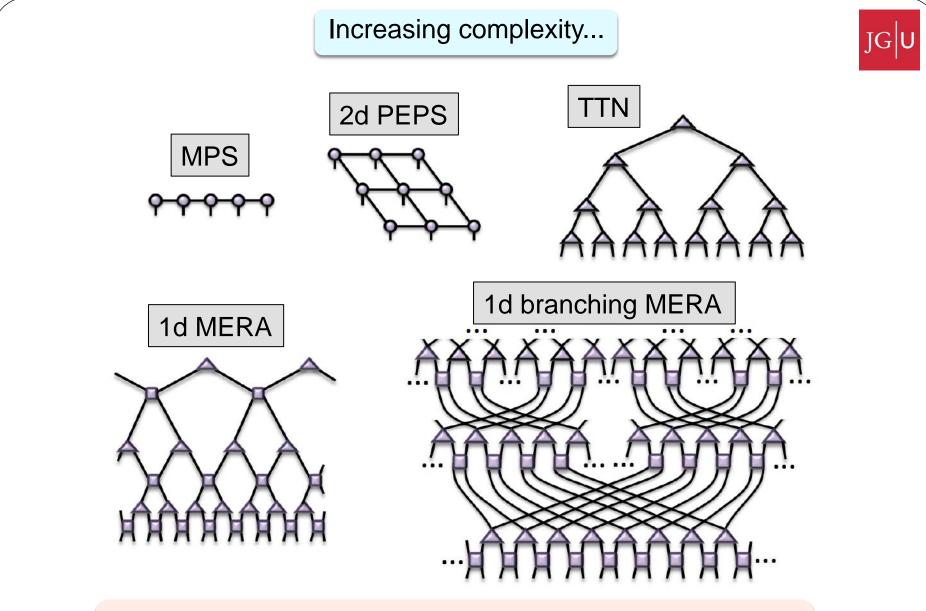


Efficient O(poly(N)), satisfy area-law, low-energy eigenstates of local Hamiltonians

# Comparison



	MPS in 1d ♀-♀-♀-♀	PEPS in 2d	MERA in 1d
Ent. entropy	S(L) = O(1)	S(L) = O(L)	$S(L) = O(\log L)$
Exact contraction	efficient	inefficient	efficient
Corr. length	finite	finite & infinite	finite & infinite
To/from	1d Ham.	2d Ham.	1d Ham.
Tensors	arbitrary	arbitrary	constrained



Exact in many cases Variational ansatz for numerical simulations (e.g. DMRG)

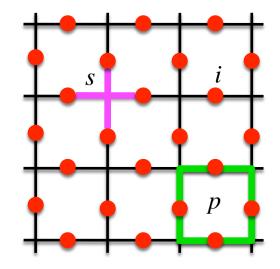
# Exact example 1: Kitaev's Toric Code



$$H = -J\sum_{s} A_{s} - J\sum_{p} B_{p}$$

$$A_s = \prod_{i \in s} \sigma_i^x$$
 star operator

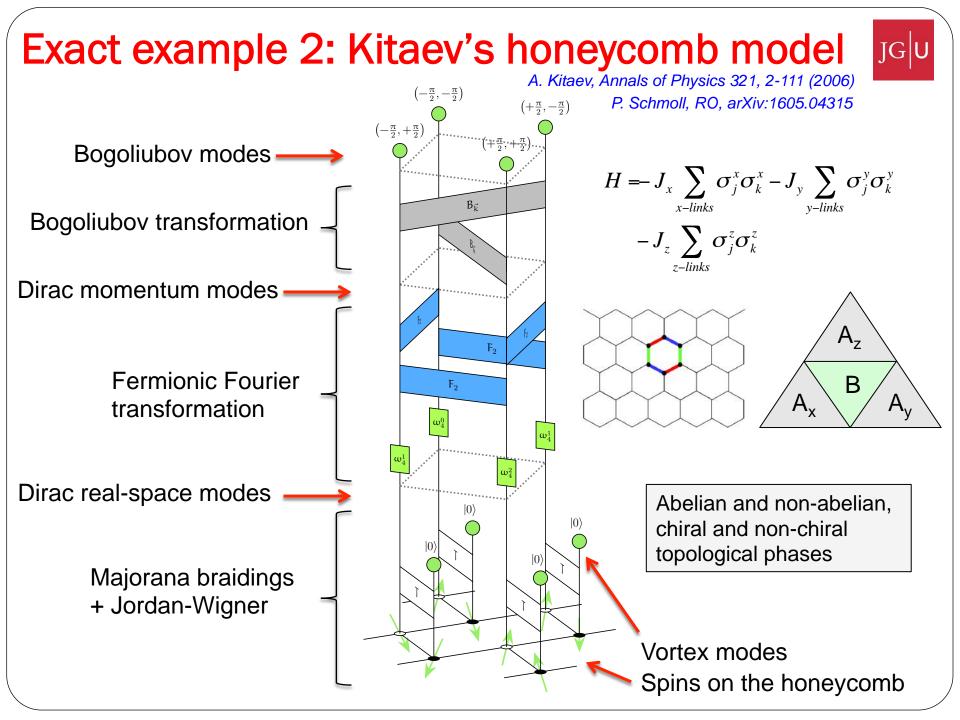
 $B_p = \prod_{i \in p} \sigma_i^z$  plaquette operator



#### Simplest known model with "topological order"

Ground state (and in fact all eigenstates) are PEPS with D=2

$$\frac{1}{1} + \frac{1}{1} = \frac{2}{1} + \frac{2}{2} = \frac{2}{2} + \frac{2}{1} + \frac{1}{1} = \frac{1}{2} + \frac{1}{2} = 1$$
  
And another tensor rotated 90°



## Outline



1) Review of TNs

2) PEPS and emergent Hamiltonians

3) Symmetric TNs and emergent spin networks

4) MERA and emergent AdS/CFT

5) Summary & open questions

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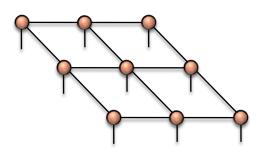
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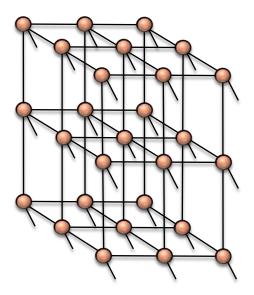
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## **Projected Entangled Pair States (PEPS)**



2d systems

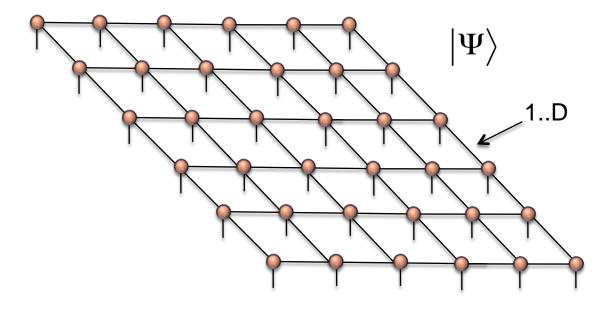


3d systems

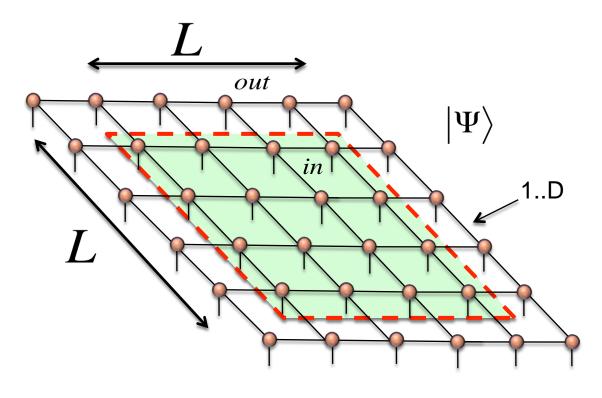


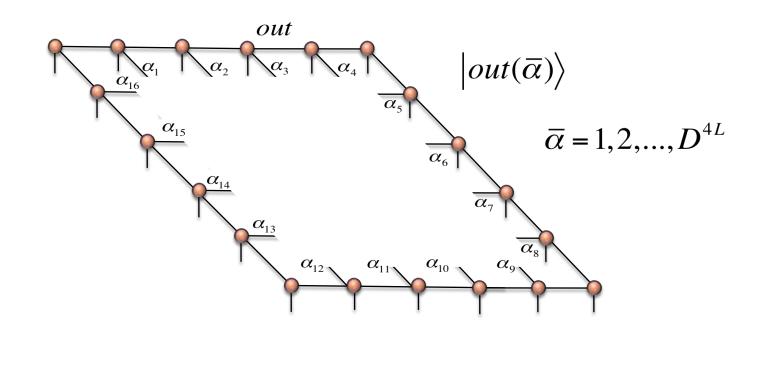
# PEPS obey 2d area-law

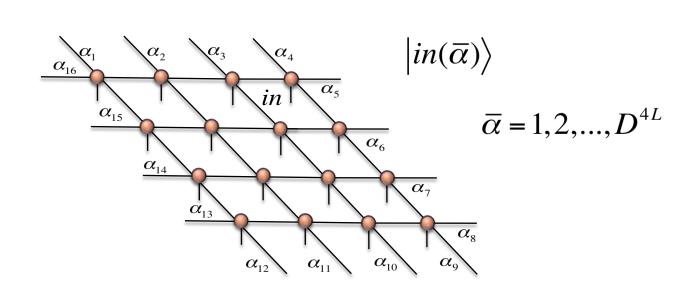


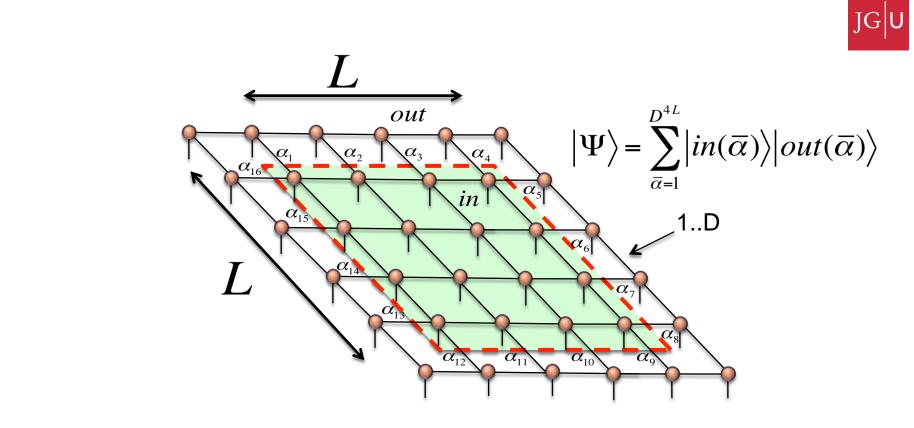




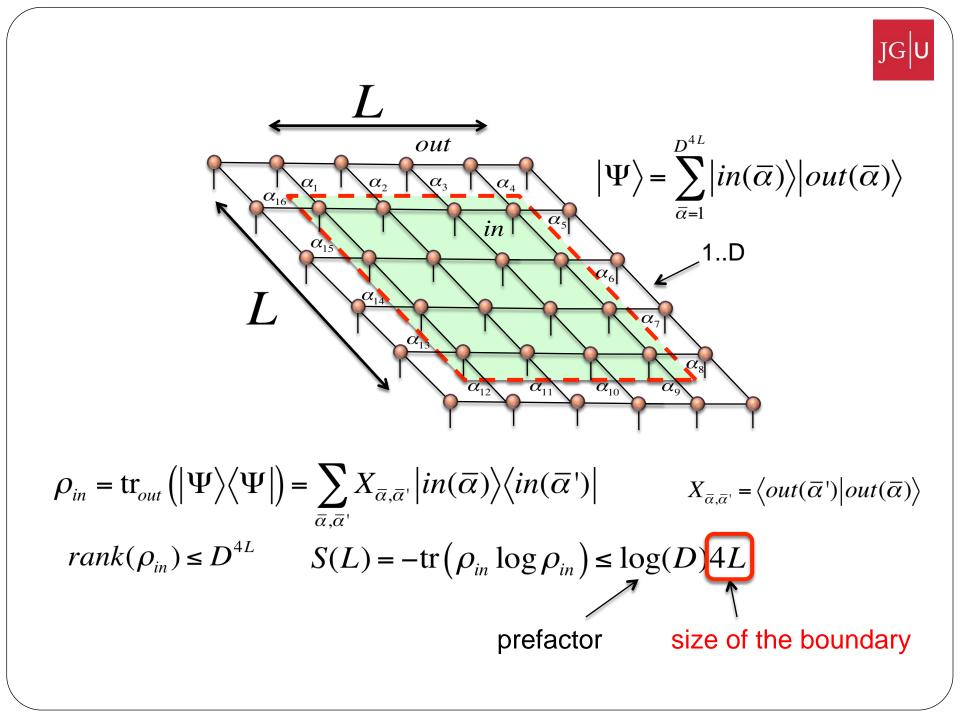








$$\begin{split} \rho_{in} &= \operatorname{tr}_{out} \left( \left| \Psi \right\rangle \left\langle \Psi \right| \right) = \sum_{\overline{\alpha}, \overline{\alpha}'} X_{\overline{\alpha}, \overline{\alpha}'} \left| in(\overline{\alpha}) \right\rangle \left\langle in(\overline{\alpha}') \right| \qquad X_{\overline{\alpha}, \overline{\alpha}'} = \left\langle out(\overline{\alpha}') \right| out(\overline{\alpha}) \right\rangle \\ rank(\rho_{in}) &\leq D^{4L} \qquad S(L) = -\operatorname{tr} \left( \rho_{in} \log \rho_{in} \right) \leq \log(D) 4L \end{split}$$

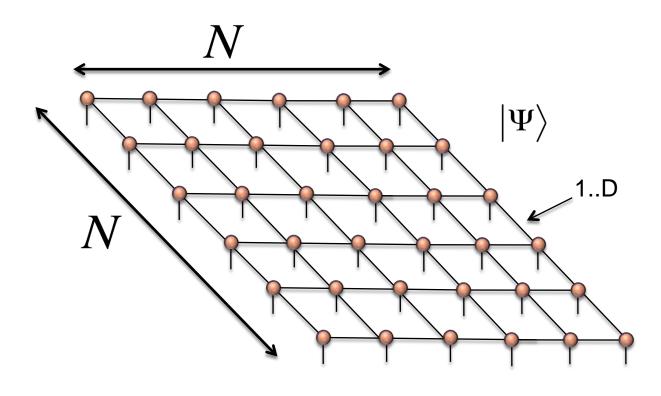




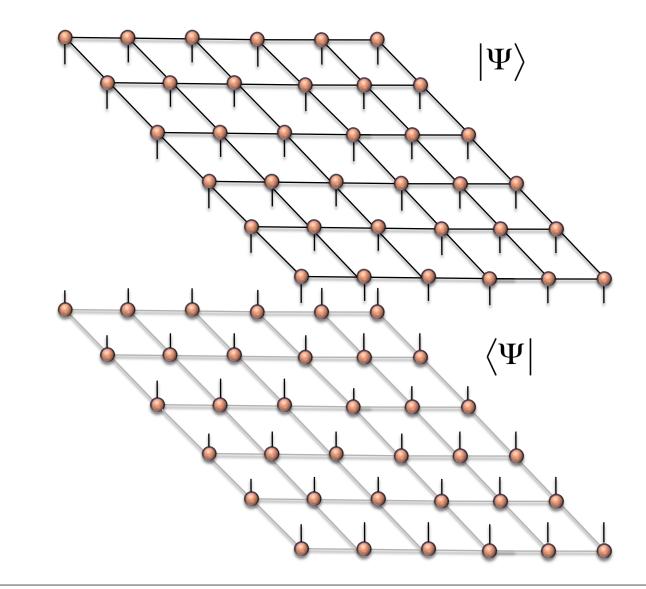
### **PEPS & Entanglement Hamiltonians**

e.g. I. Cirac et al, PRB 83, 245134 (2011), N. Schuch et al, PRL 111, 090501 (2013)

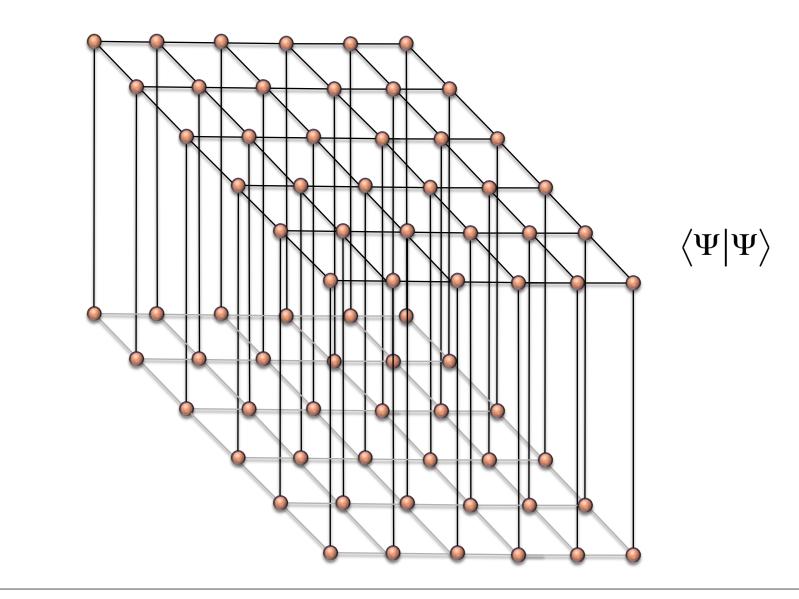


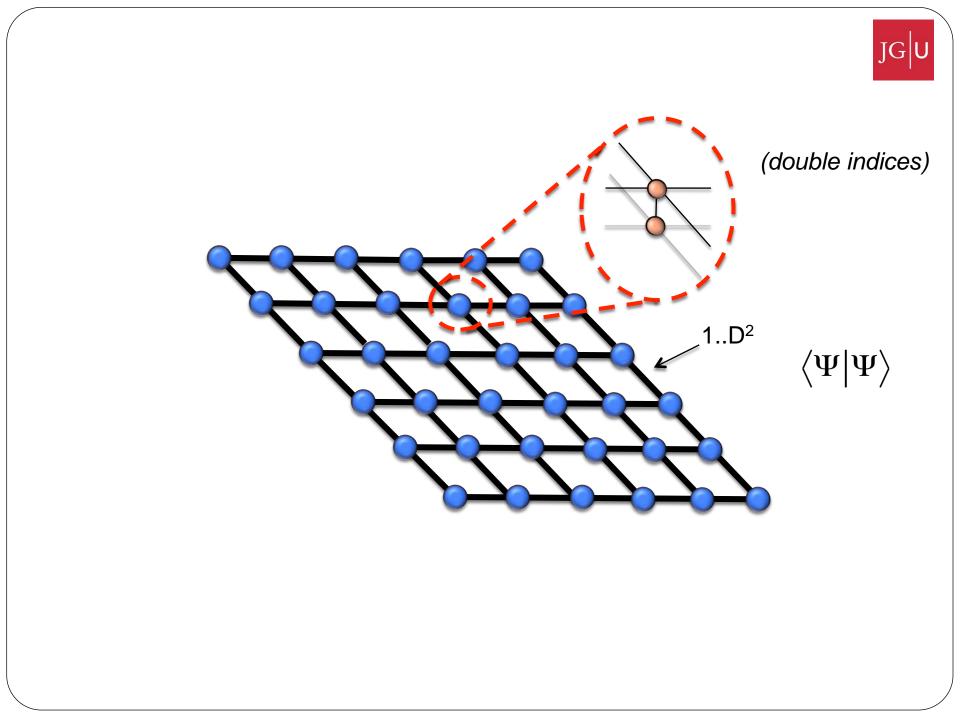




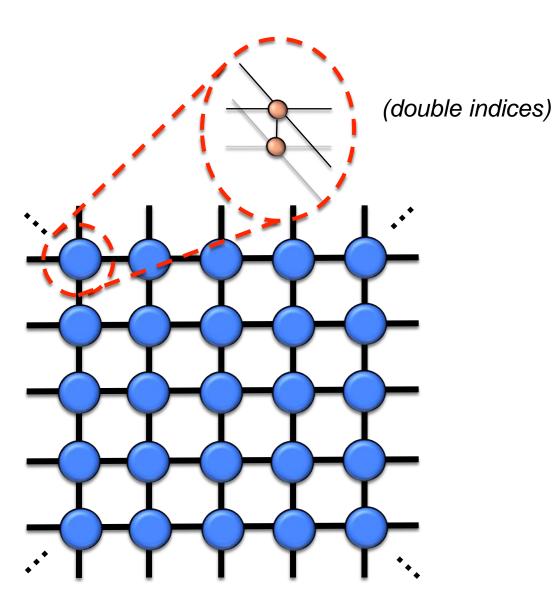




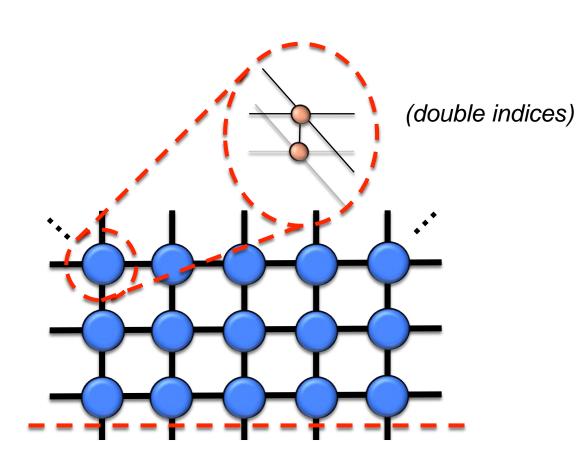




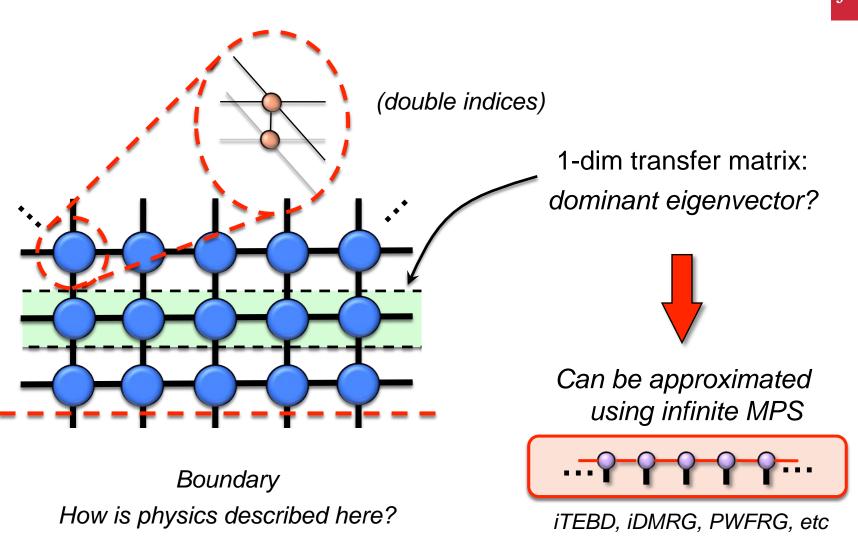








Boundary How is physics described here?

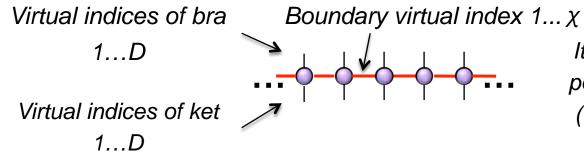




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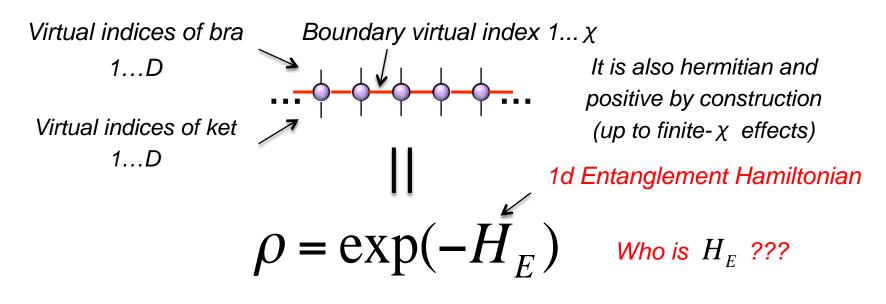
Remember it has double indices...



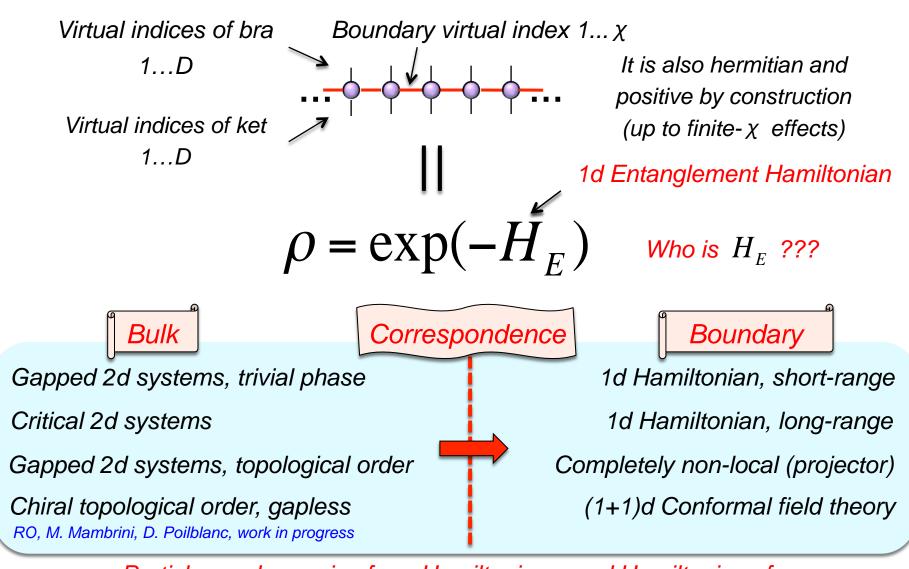


It is also hermitian and positive by construction (up to finite- $\chi$  effects)









Particles and energies from Hamiltonians, and Hamiltonians from networks of entanglement + bulk-boundary correspondence

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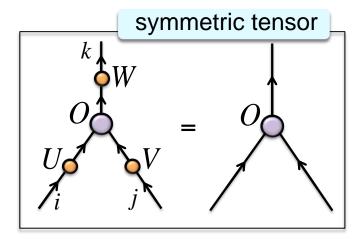
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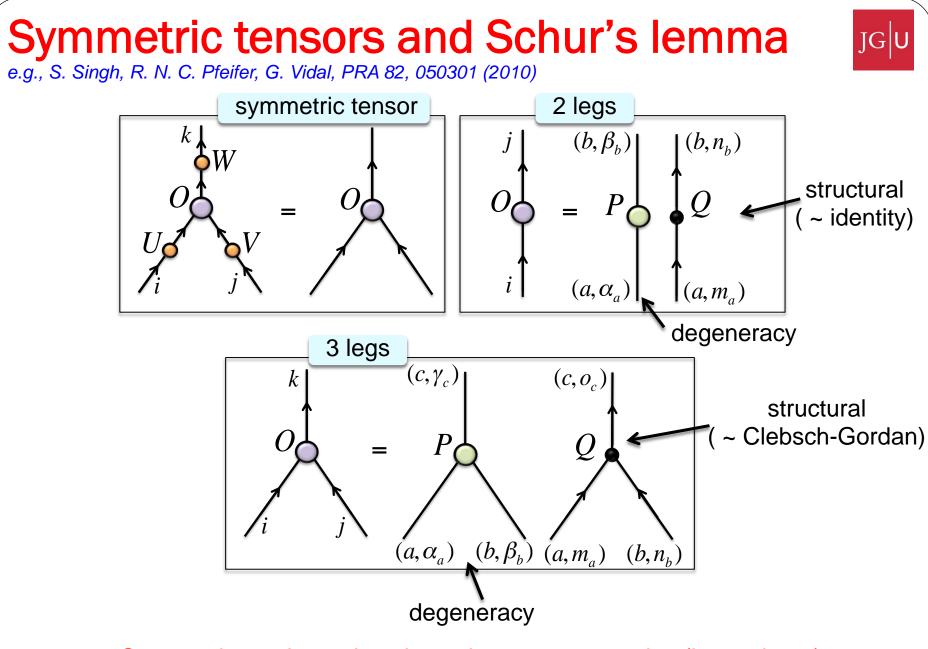
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## Symmetric tensors and Schur's lemma

JGU

e.g., S. Singh, R. N. C. Pfeifer, G. Vidal, PRA 82, 050301 (2010)

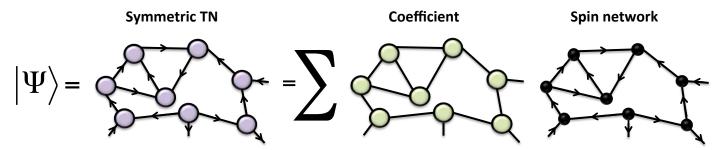




Structural part depends only on the group properties (intertwiners)

### **Emergent spin networks**

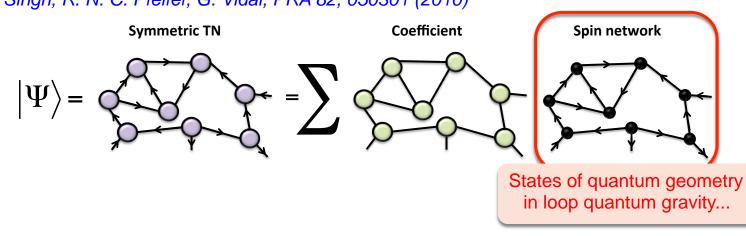
e.g., S. Singh, R. N. C. Pfeifer, G. Vidal, PRA 82, 050301 (2010)





#### Emergent spin networks e.g., S. Singh, R. N. C. Pfeifer, G. Vidal, PRA 82, 050301 (2010)





#### **Emergent spin networks** e.g., S. Singh, R. N. C. Pfeifer, G. Vidal, PRA 82, 050301 (2010) Coefficient Spin network Symmetric TN $|\Psi\rangle$ States of quantum geometry in loop quantum gravity... Global Local (gauge)

JG|L

Global and gauge symmetries are handled naturally

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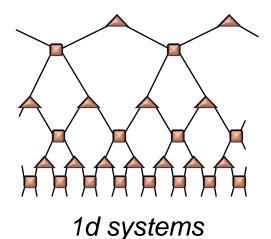
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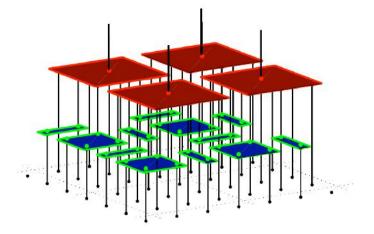
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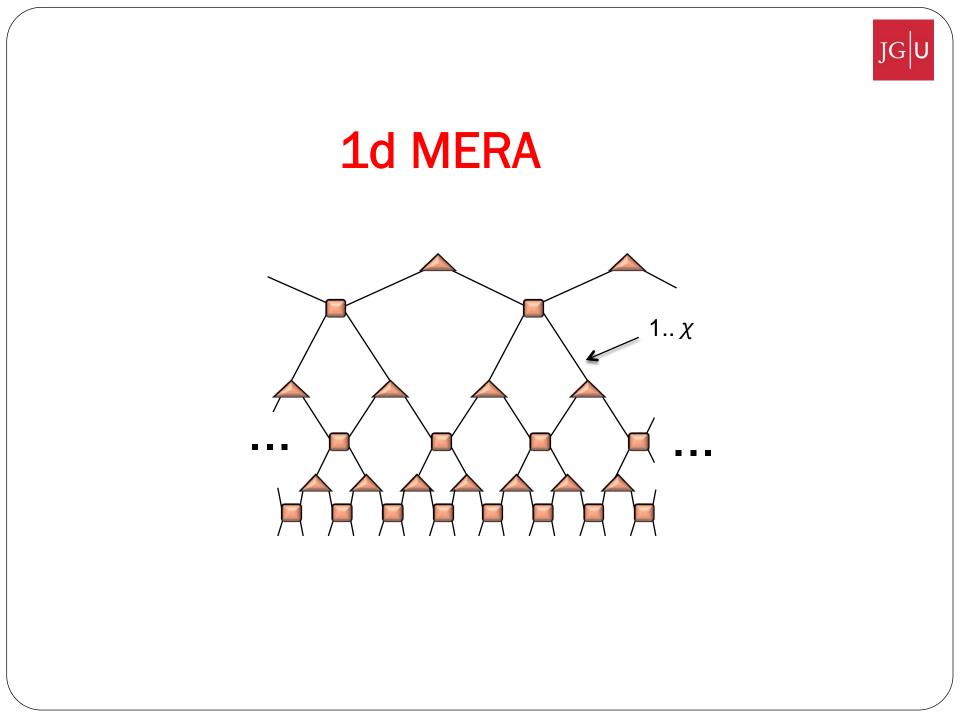


# Multiscale Entanglement Renormalization Ansatz (MERA)





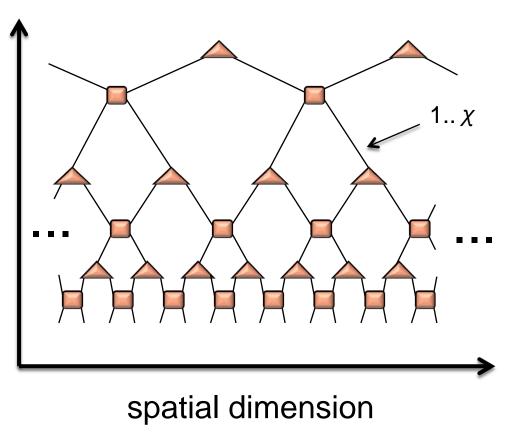
2d systems





# 1d MERA

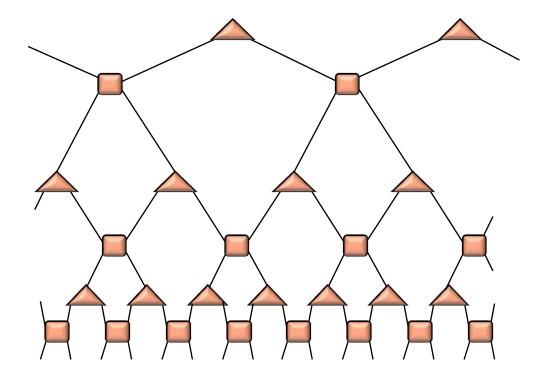
holographic dimension (RG)

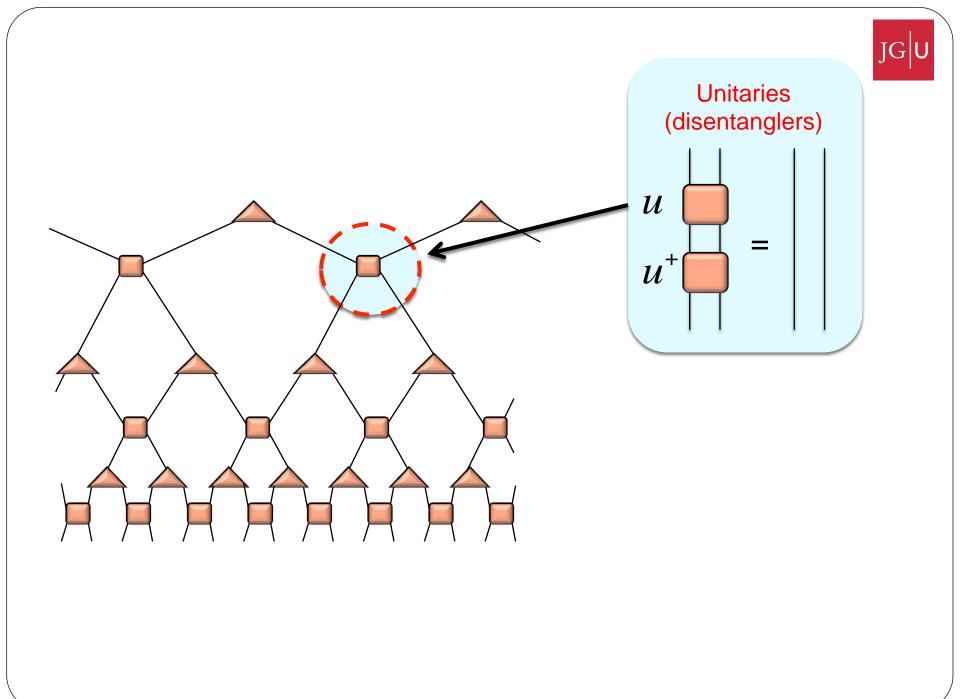


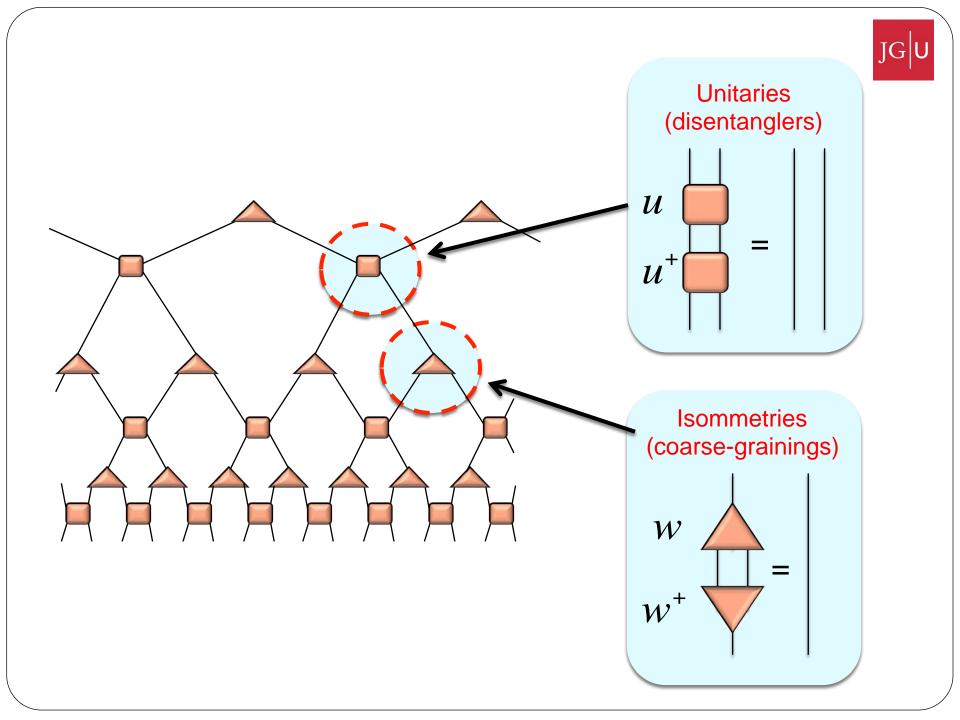


# **Tensors obey constraints**











#### **Reason:**

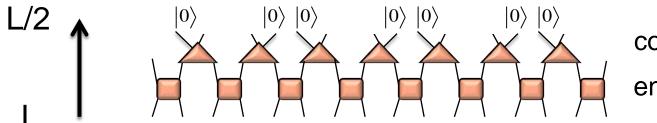
# entanglement is built locally at all length scales



# 

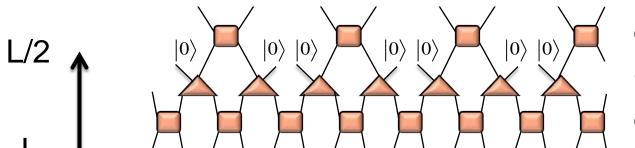
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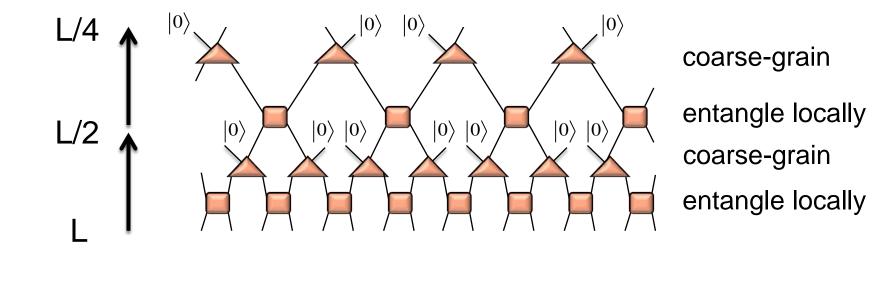
coarse-grain entangle locally

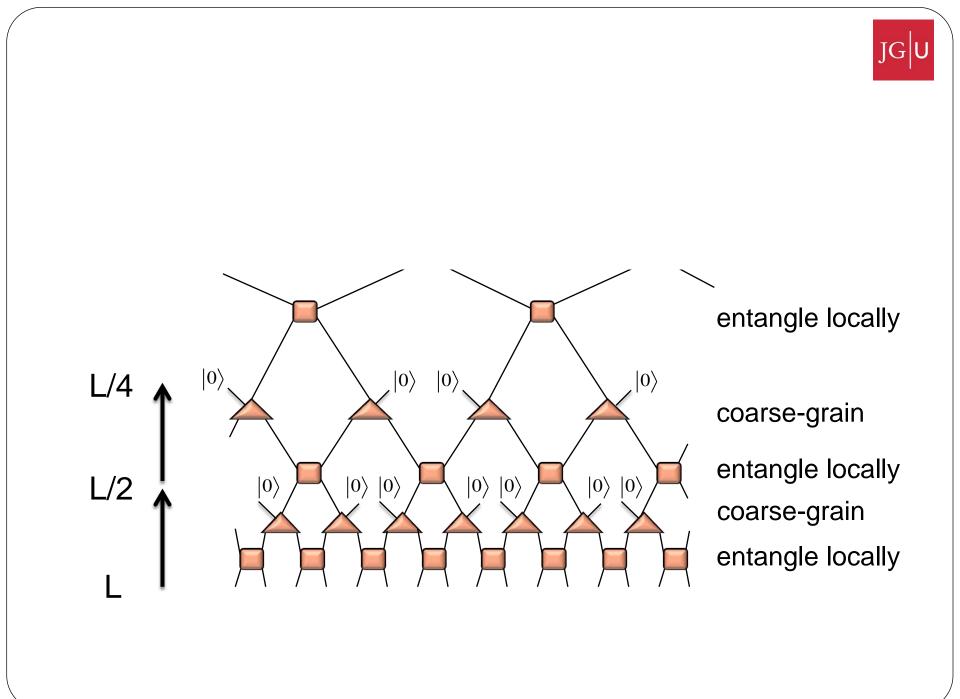


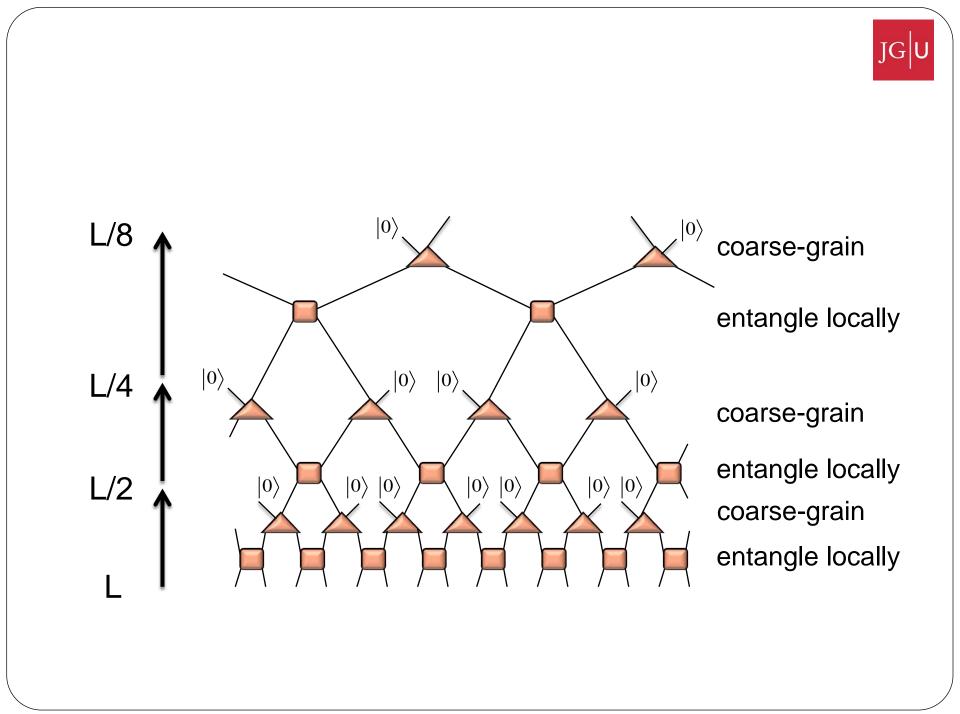


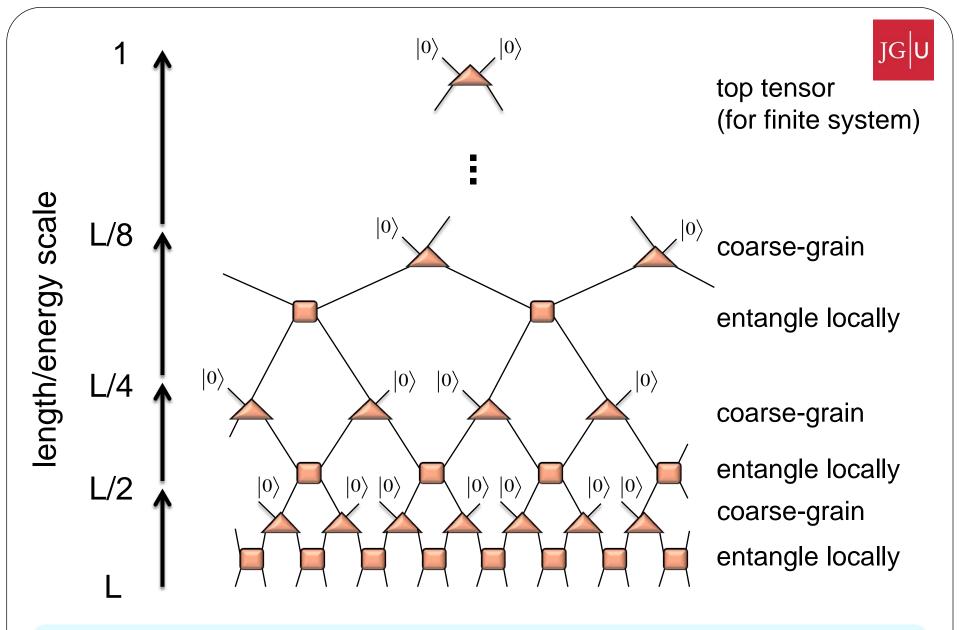
entangle locally coarse-grain entangle locally



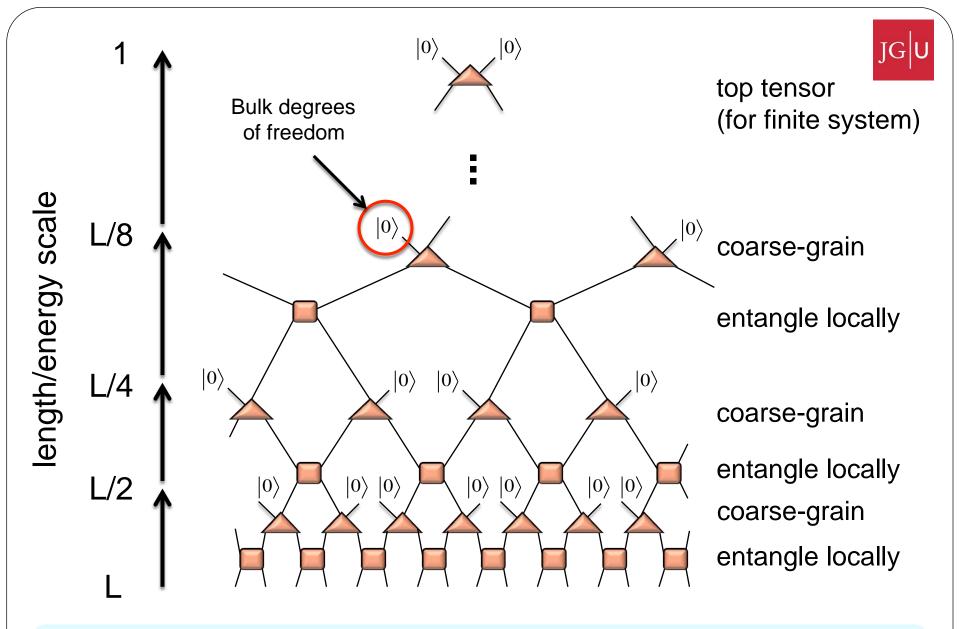








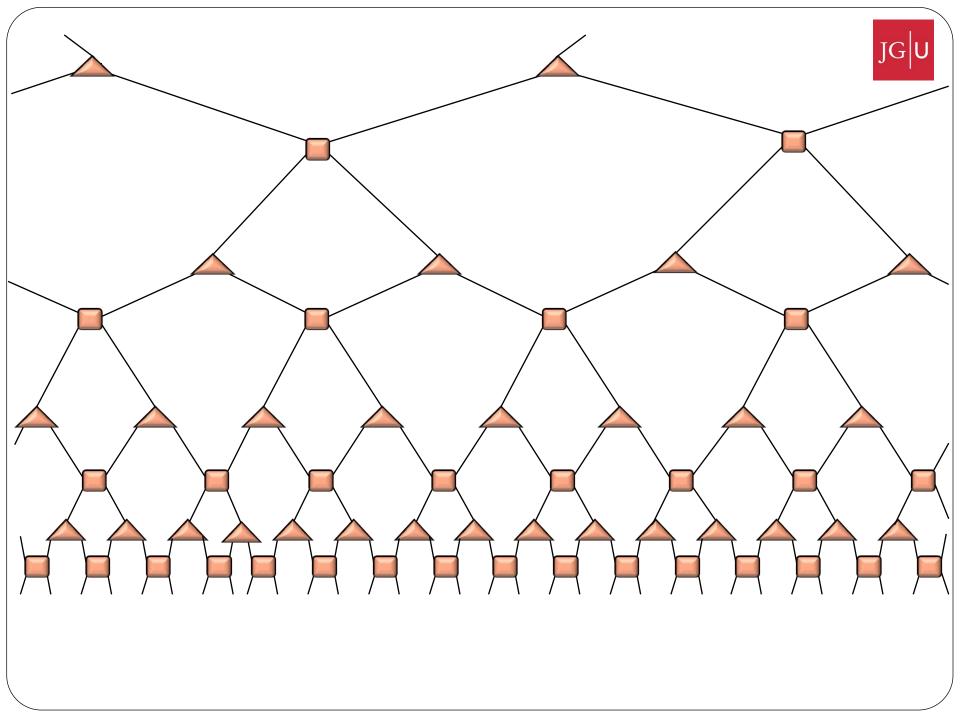
Extra dimension defines an RG flow: Entanglement Renormalization

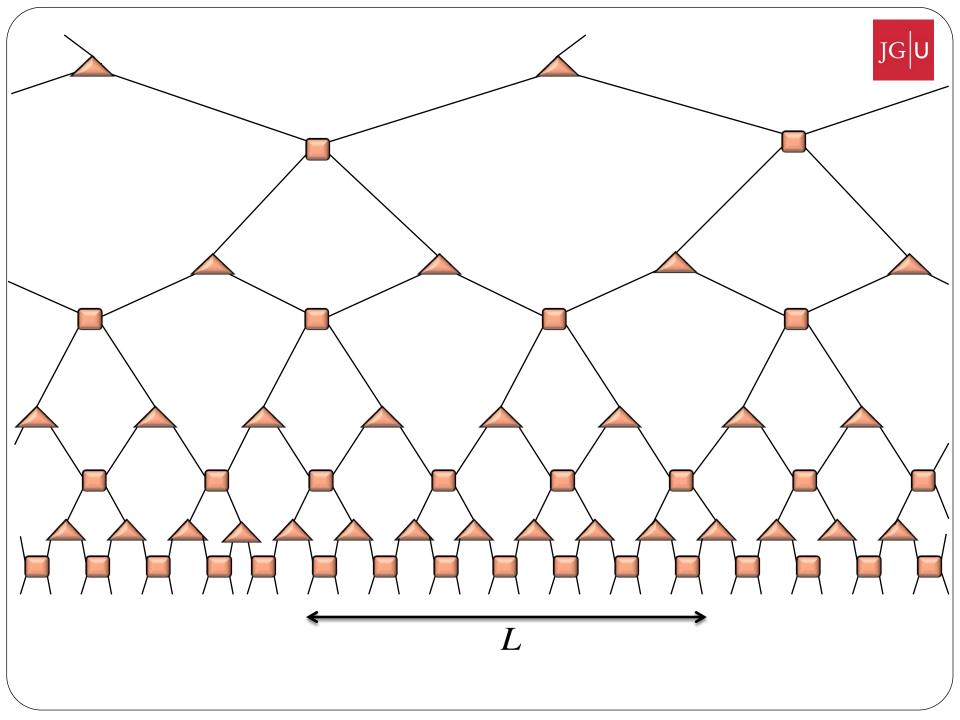


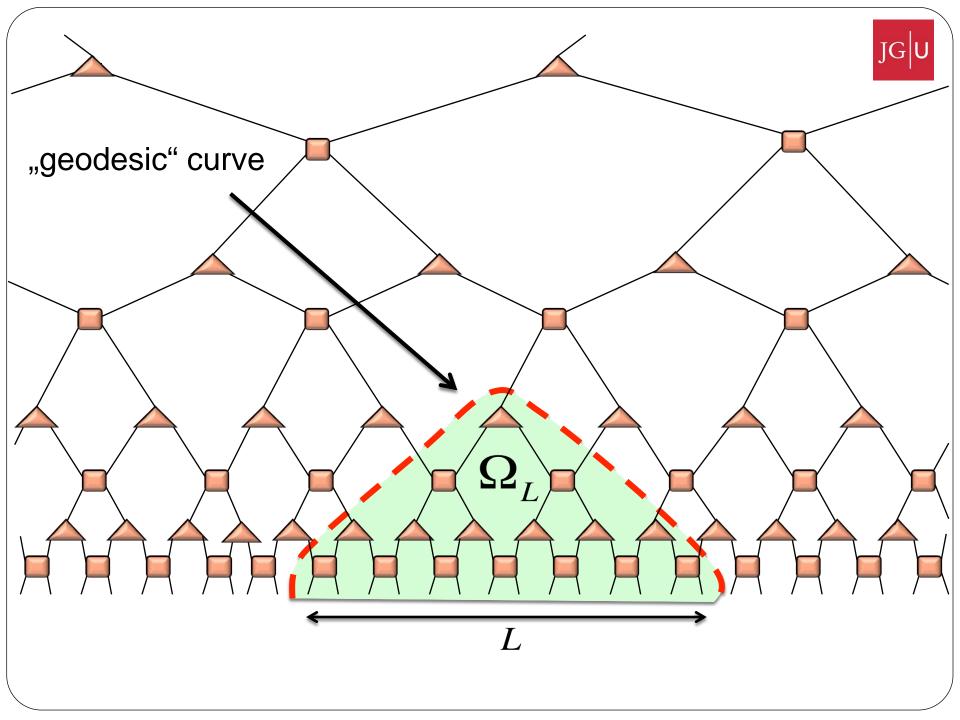
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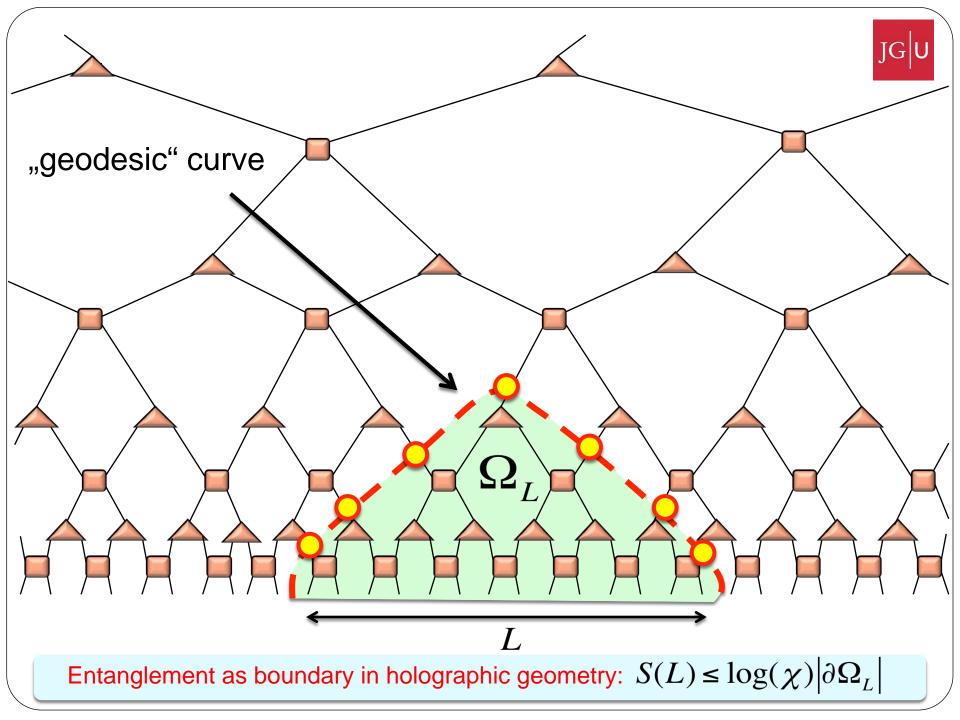


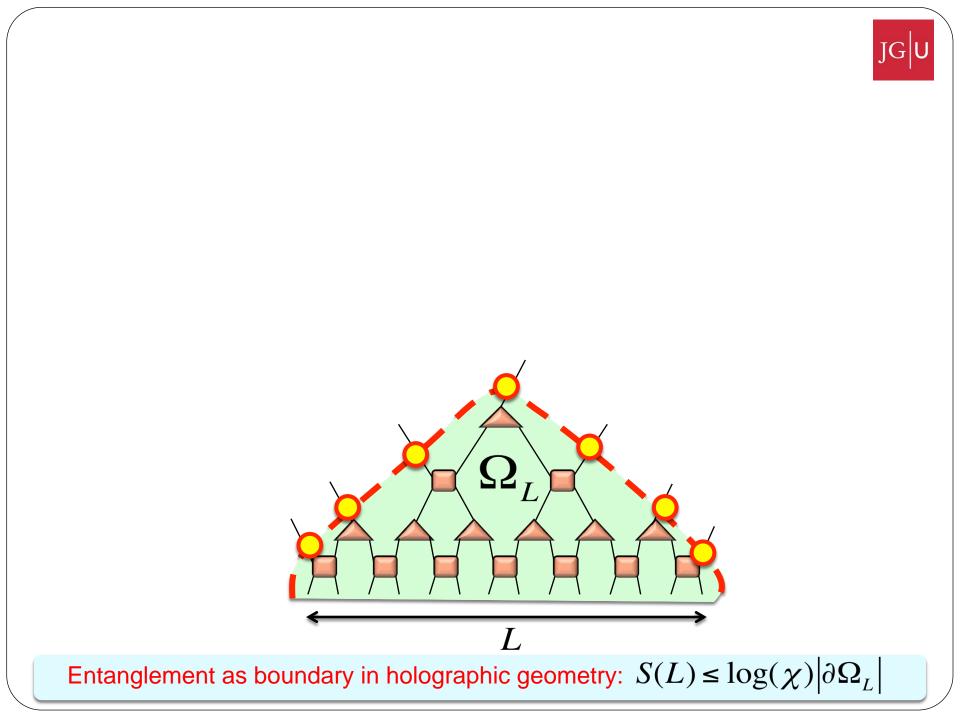
## Entropy of 1d MERA

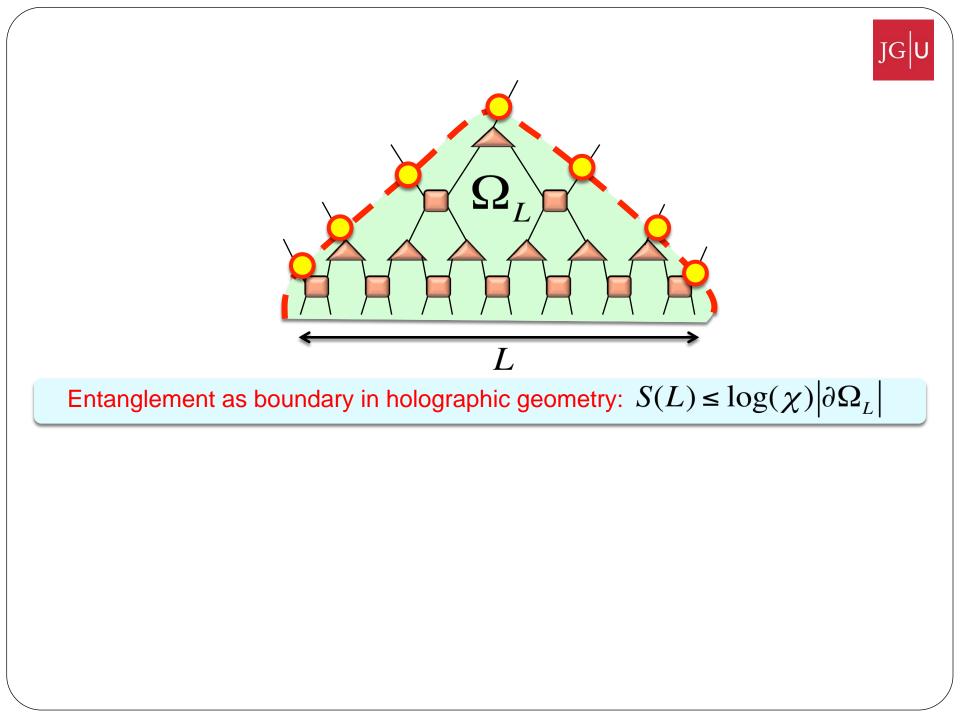


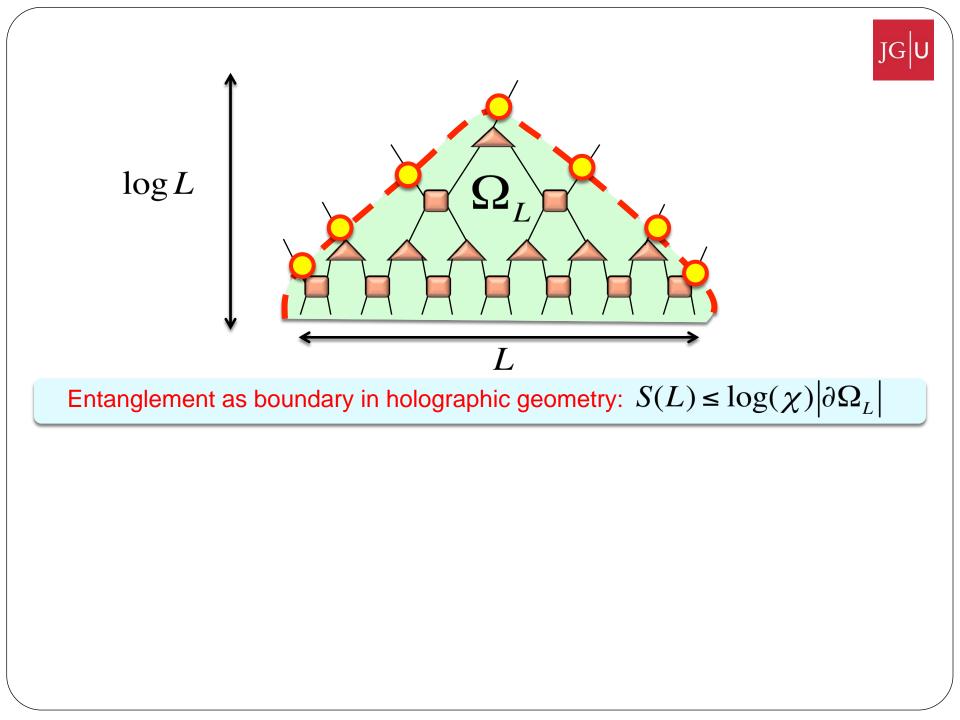


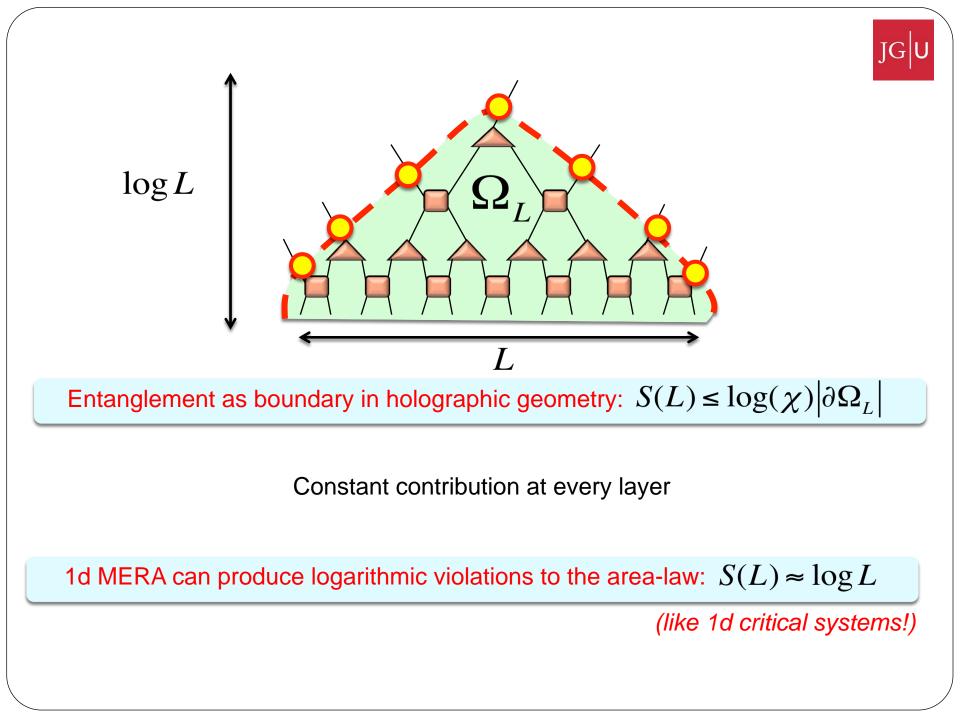














#### MERA & AdS/CFT

e.g. B. Swingle, PRD 86, 065007 (2012), G. Evenbly, G. Vidal, JSTAT 145:891-918 (2011)

## Emergent space-time MERA AdS/CFT $\downarrow I = 0$ I = -1 $I = -\infty$ $(= u_{III})$

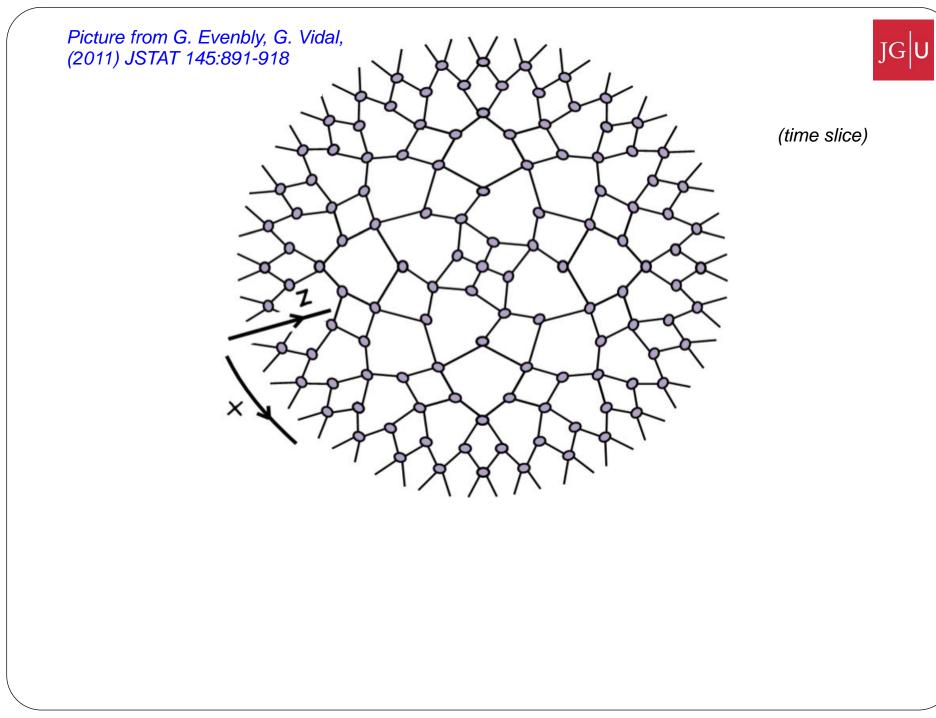
 $S_A \propto \text{Min}[\#\text{Bonds}(\gamma_A)]$ 

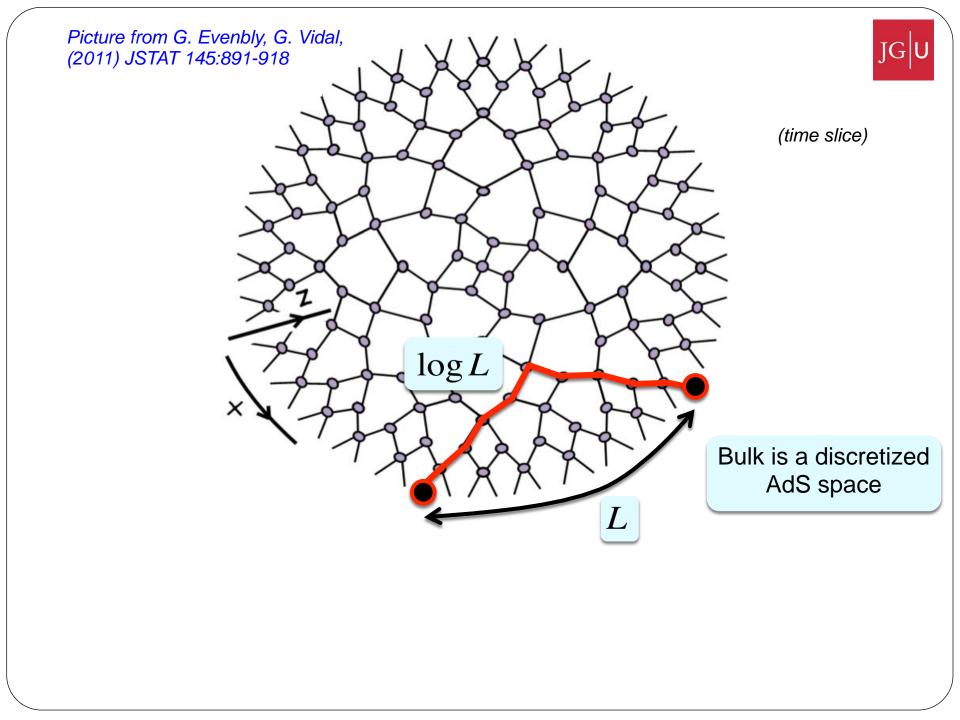
 $S_A \propto \text{Min}[\text{Area}]$ 

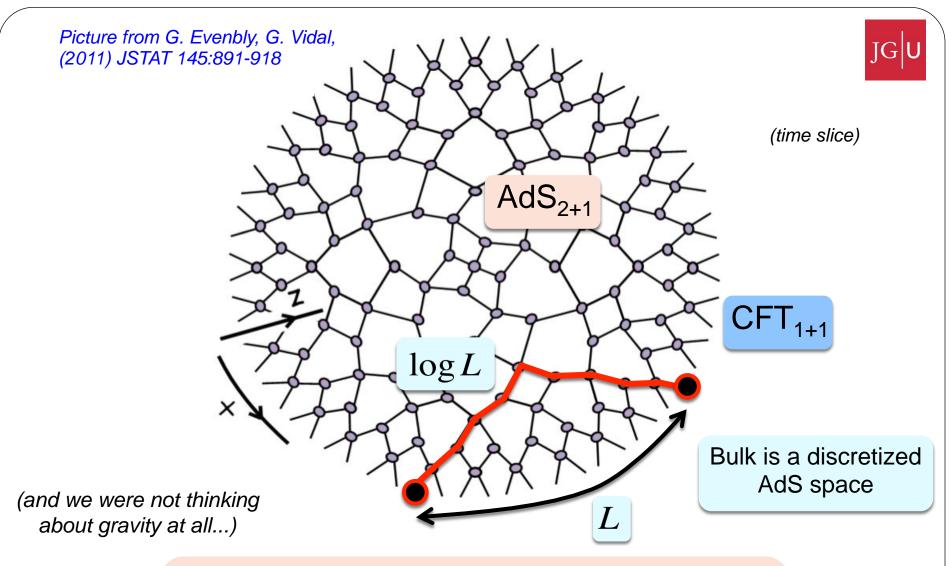
JG|l

Picture from M. Nozaki, S. Ryu, T. Takayanagi, JHEP10(2012)193

#### MERA entropy ~ Ryu-Takayanagi prescription





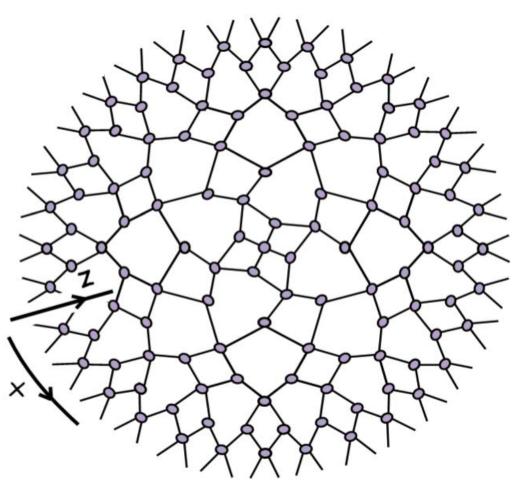


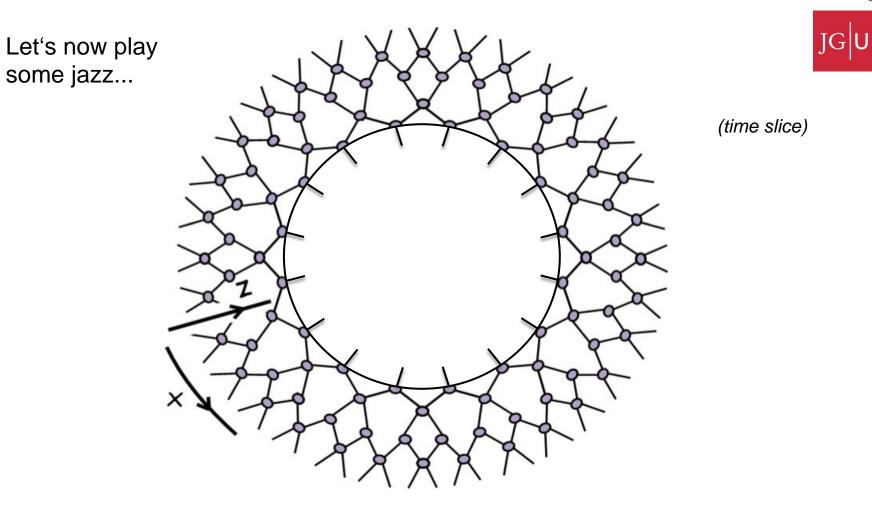
For a scale-invariant MERA, the tensors of a critical model with a CFT limit correspond to a "gravitational" description in a discretized AdS space: "lattice" realization of AdS/CFT correspondence

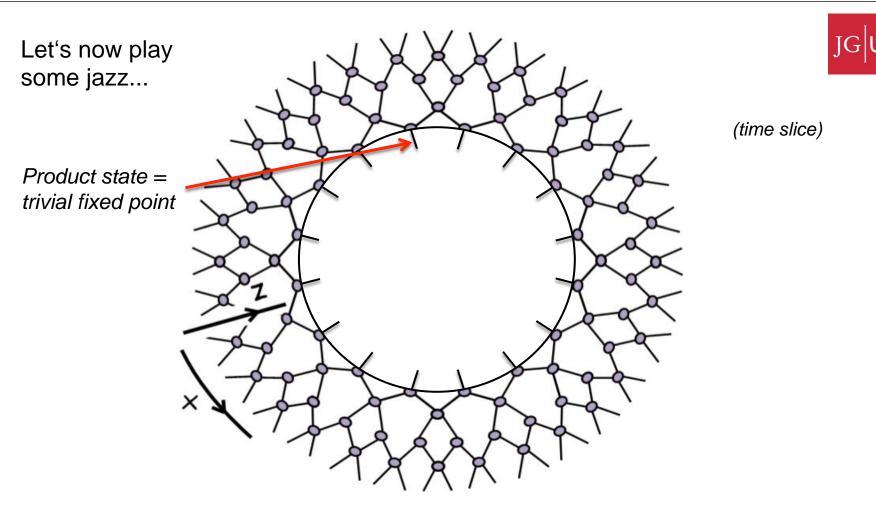


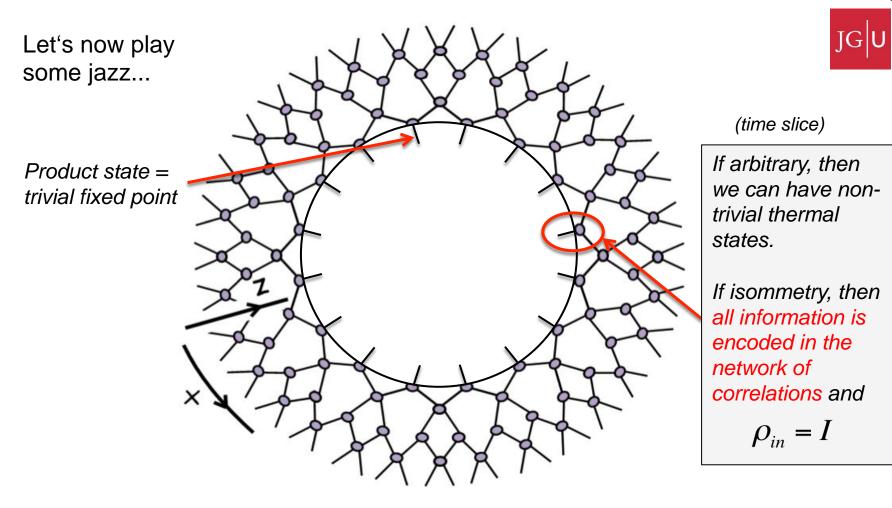
(time slice)

Let's now play some jazz...



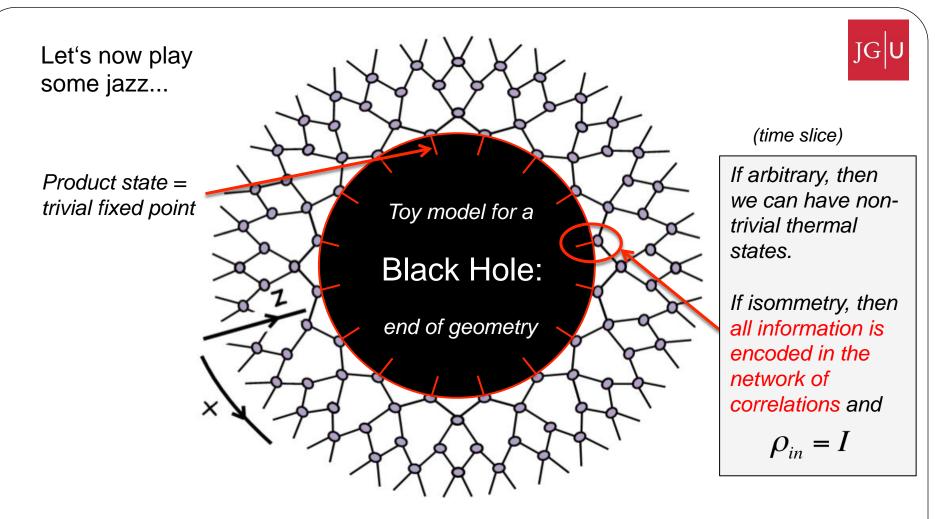






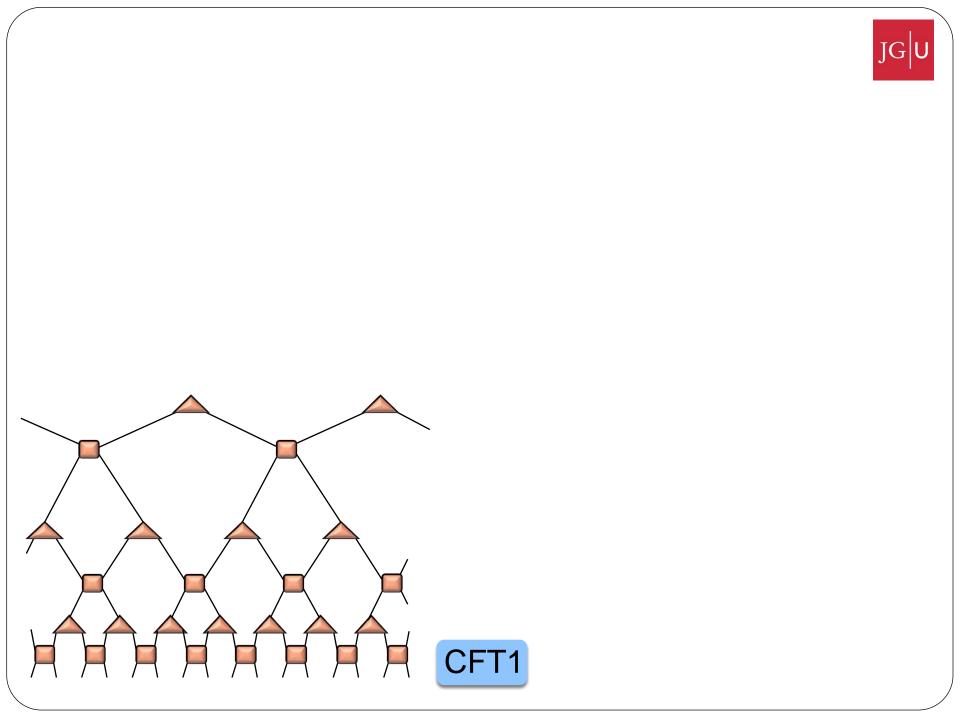
$$\rho_{in} = tr_{out} \left( |\Psi\rangle \langle \Psi| \right) \right]$$
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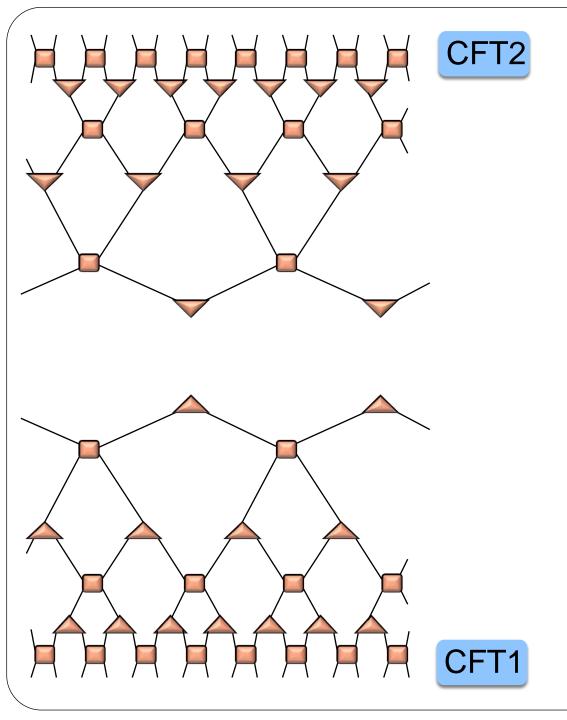
Same **thermal** spectrum (entanglement Hamiltonian) finite temperature, scale invariance broken



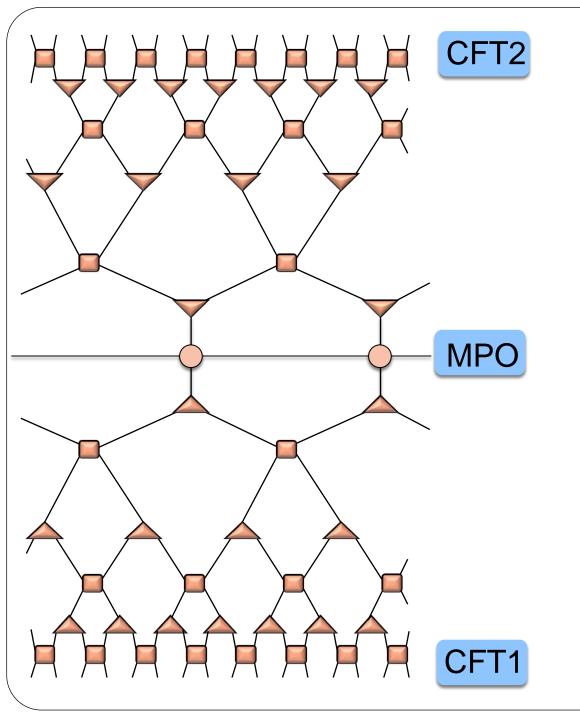
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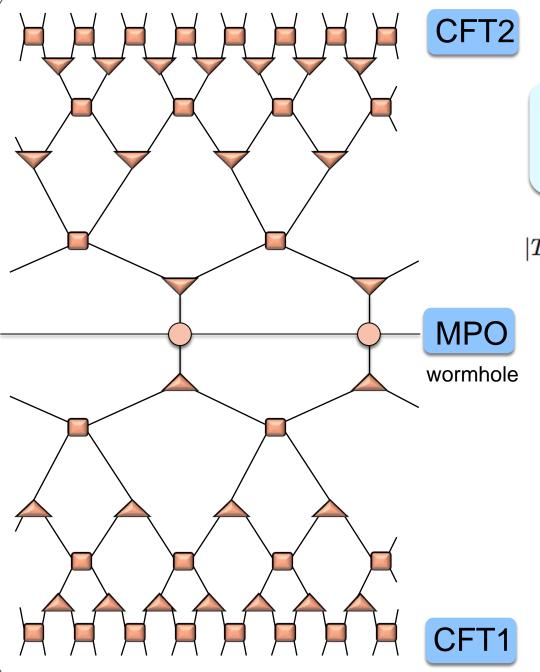




JG







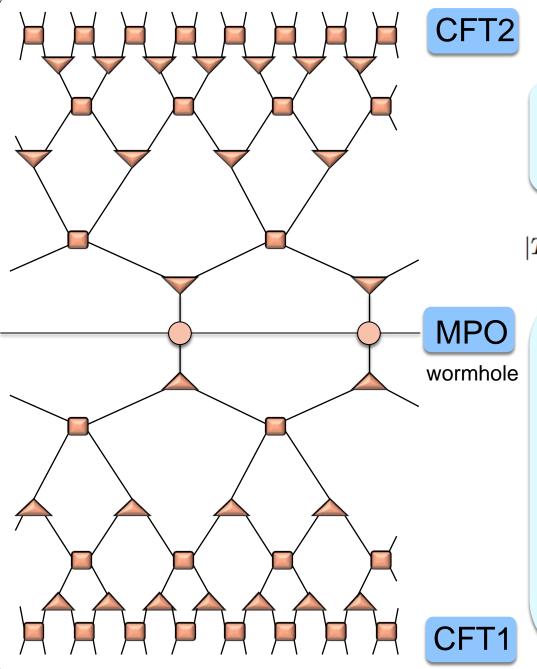
e.g., T. Hartman, J. Maldacena, JHEP05(2013)014

Thermofield double state

JGU

Eternal AdS black-hole

$$|TFD
angle = rac{1}{\sqrt{Z(eta)}}\sum_n e^{-eta E_n/2}|n
angle_1|n
angle_2$$



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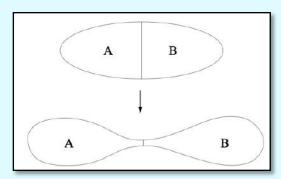
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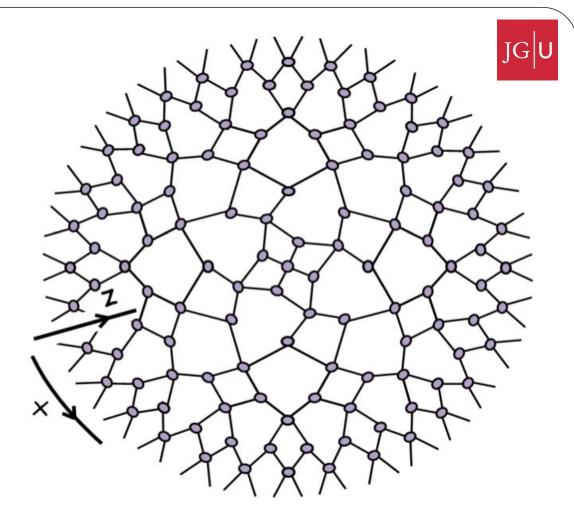
Entanglement connects upper and lower spacetimes



M. Van Raamsdonk, arXiv:0907.2939

ER=EPR, Maldacena & Susskind





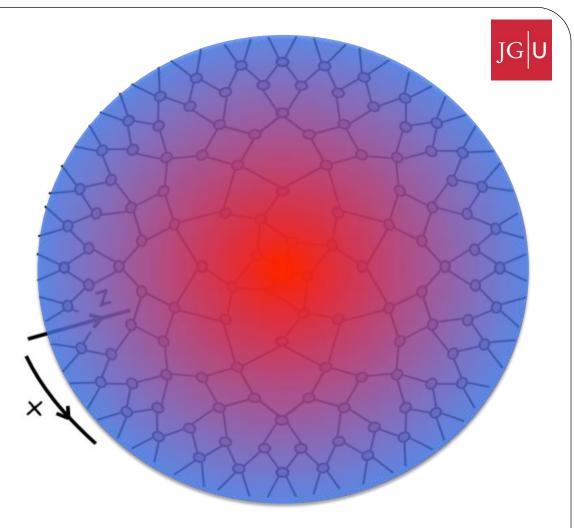
#### cMERA

(continuum)

$$\left|\psi\right\rangle = Pe^{-i\int_{u^{2}}^{u^{1}} \left(K(u)+L\right)du} \left|\Omega\right\rangle$$

J. Haegeman et al, Phys. Rev. Lett. 110, 100402 (2013)

- K(u) Disentangler generator
  - *L* Isommetry generator



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K(u) Disentangler generator

*L* Isommetry generator

$$g_{uu}(u)du^2 = \mathcal{N}^{-1}\left(1 - \left|\langle \Psi(u)|e^{iL\cdot du}|\Psi(u+du)
ight|^2
ight|^2$$

Measures the density of strength of disentanglers. Compatible with AdS metric

M. Nozaki, S. Ryu, T. Takayanagi, JHEP10(2012)193

curvature ~ change of entanglement at every length scale

## Outline



 $\checkmark$ 

#### 1) Review of TNs

2) PEPS and emergent Hamiltonians

3) Symmetric TNs and emergent spin networks

4) MERA and emergent AdS/CFT

5) Summary & open questions

## Outline



#### 1) Review of TNs

2) PEPS and emergent Hamiltonians



3) Symmetric TNs and emergent spin networks



4) MERA and emergent AdS/CFT

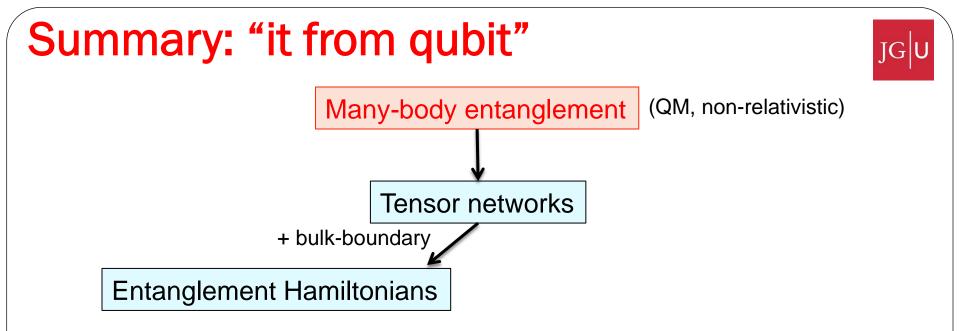
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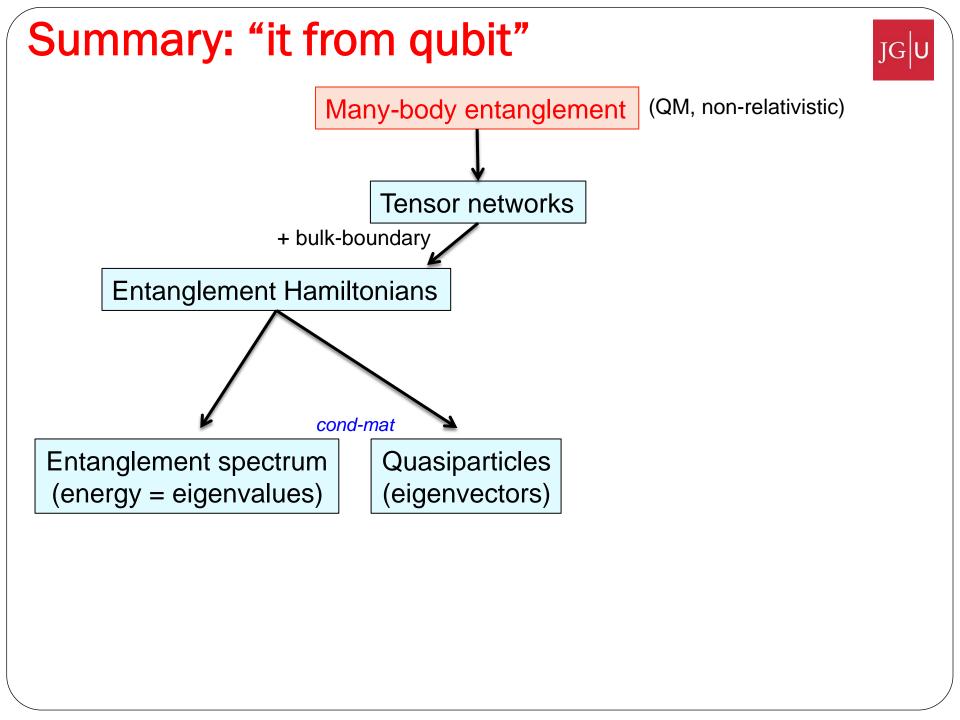
## Summary: "it from qubit"

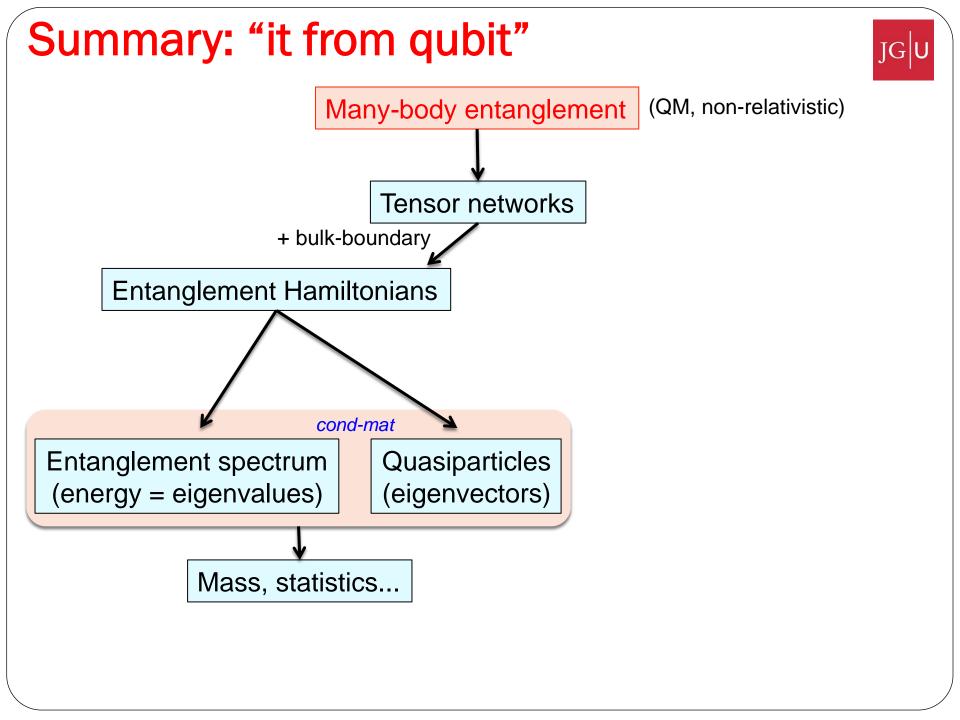


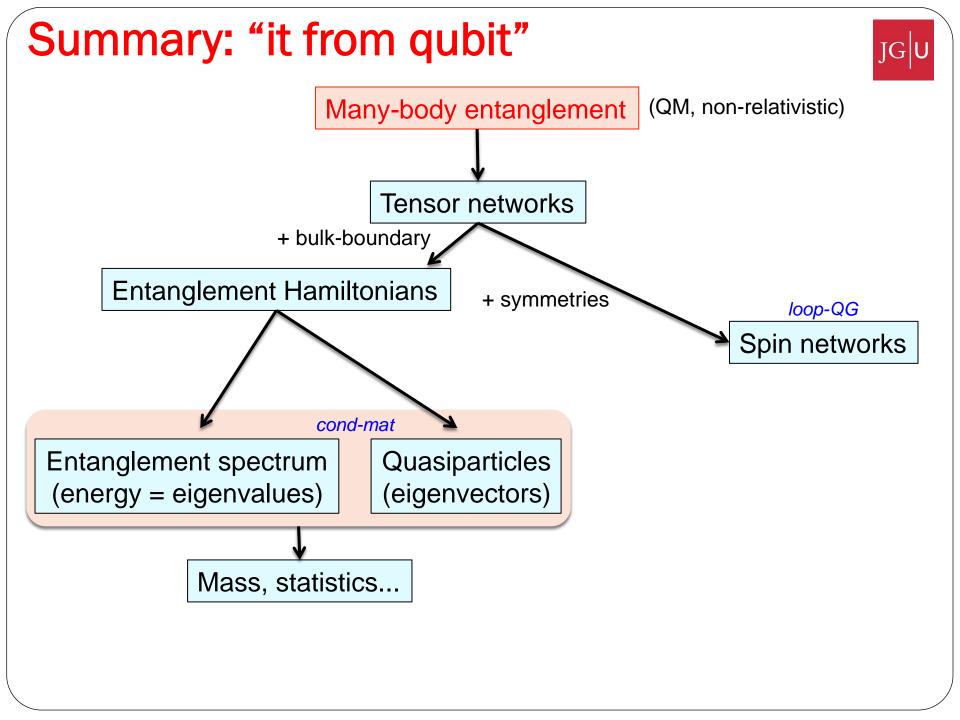
Many-body entanglement (QM, non-relativistic)

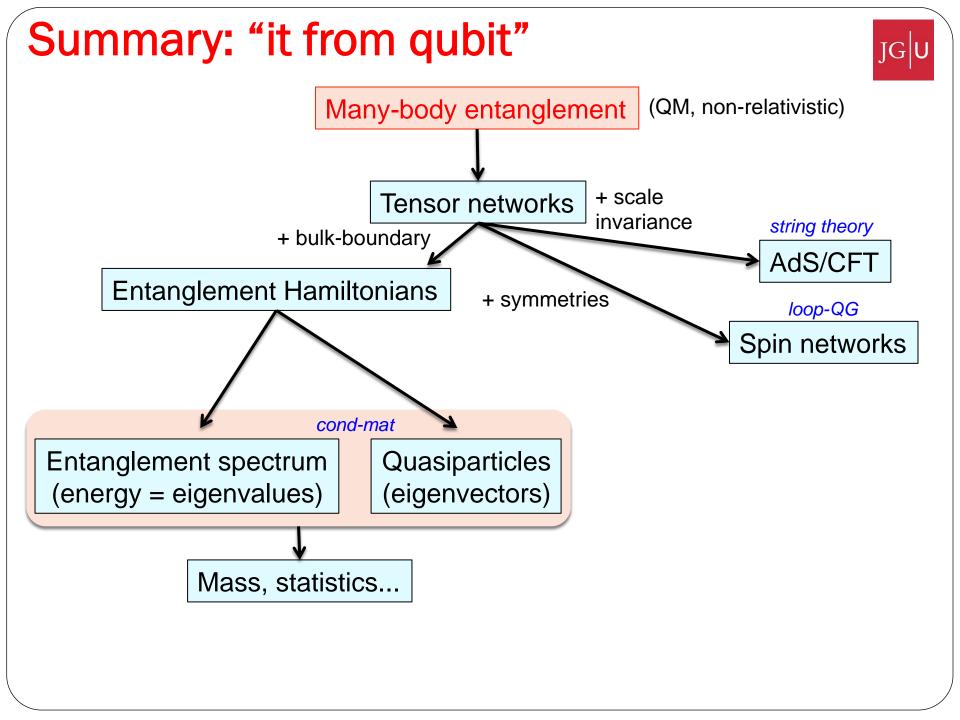
# Summary: "it from qubit" JGU (QM, non-relativistic) Many-body entanglement Tensor networks

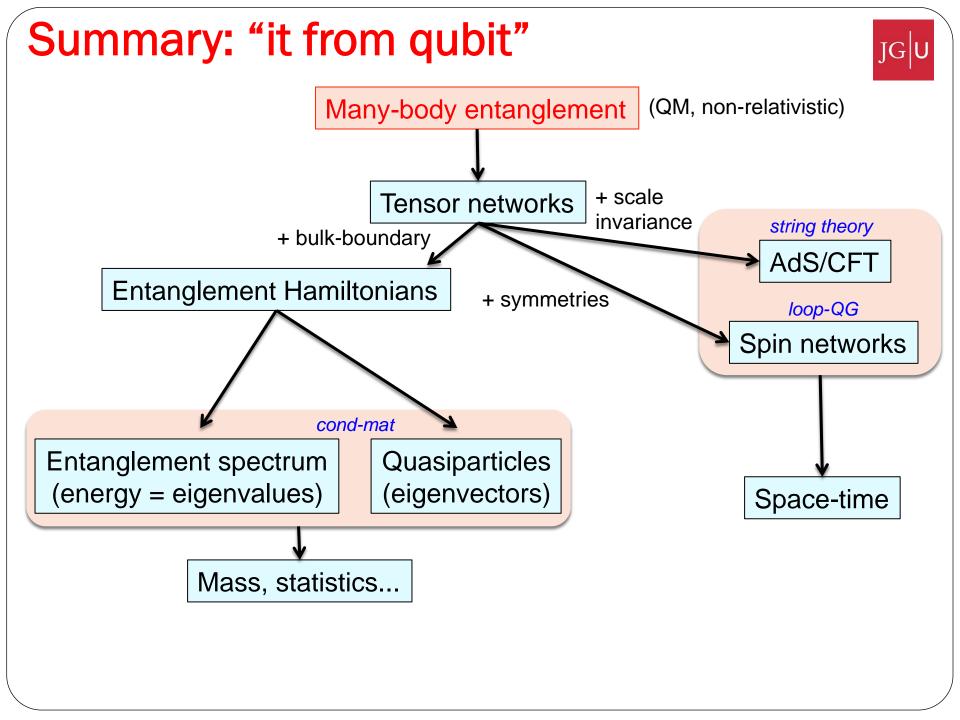


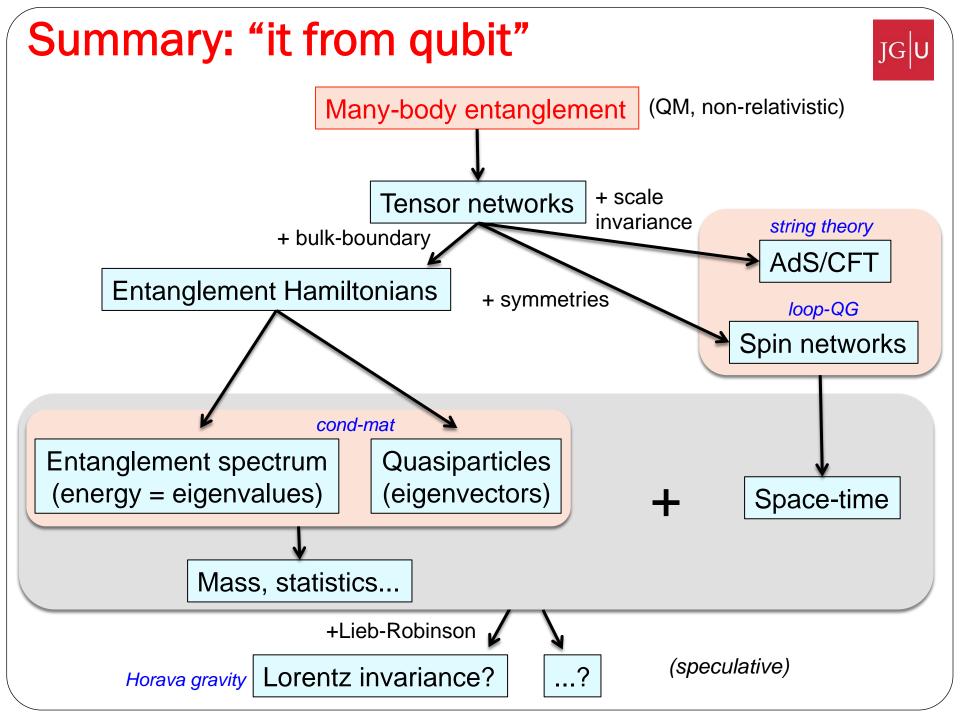












#### **Other developments**



• Exact holographic mapping X.-Liang Qi, arXiv:1309.6282

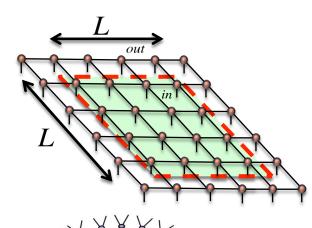
- AdS/CFT as Quantum Error-Correcting Code A. Almheiri, X. Dong, D. Harlow, JHEP 1504:163 (2015)
- Holographic Quantum Error Correcting Codes F. Pastawski, B. Yoshida, D. Harlow, J. Preskill, JHEP 06 149 (2015) J. I. Latorre, G. Sierra, arXiv:1502.06618
- Einstein's equations from Entanglement Entropy T. Faulkner, M. Guica, T. Hartman, R. C. Myers, M van Raamsdonk, JHEP 03 051 (2013); B. Swingle, M. van Raamsdonk, arXiv:1405.2933

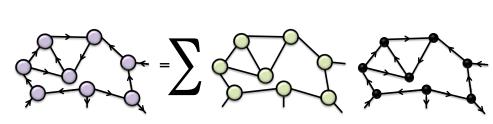
#### Some cross-over open questions

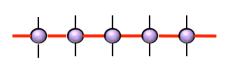
- cMERA, wavelets, and AdS/CFT?
- Superpositions of TTNs? Linear optics?
- Consistent AdS/TN? What about other correspondences?
- TN structure of, e.g., N=4 SYM? (Type-IIB on AdS<sub>5</sub> x S<sup>5</sup>) c.f. talk by Can one derive string theory from entanglement? J. Molina-Vilaplana

C. Papadopoulos, RO work in progress G

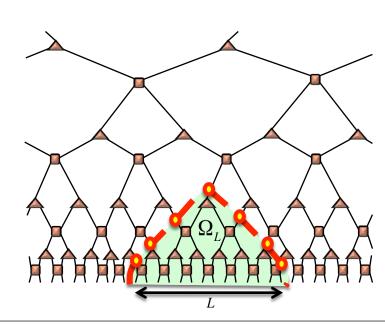
- Non-classical gravity from "exotic" TNs? (topological order & D-branes, TNs with symmetries...)
- Holographic multipartite entanglement? Holographic mixed-state entanglement? *D. Pang, RO, work in progress*
- "Gravitational" interpretation of branching MERA?
- Numerical simulations of gravity with TN methods?
- "entanglement renormalization" > "holographic renormalization"?
- Lorentz invariance from Lieb-Robinson bounds?
- •

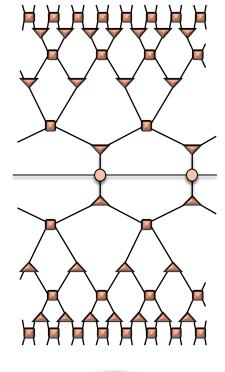






# Thank you!





JGU

