Quantum Quenches and Black Hole Formation at Large C

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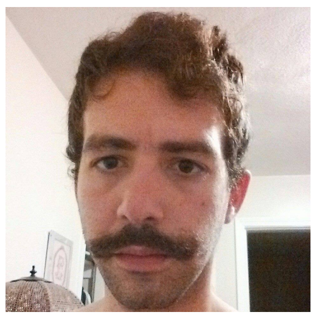
CERN

theory institute on emergent properties of space-time

23 June 2016

work in collaboration with



Tarek Anous (MIT)

Thomas Hartman (Cornell)

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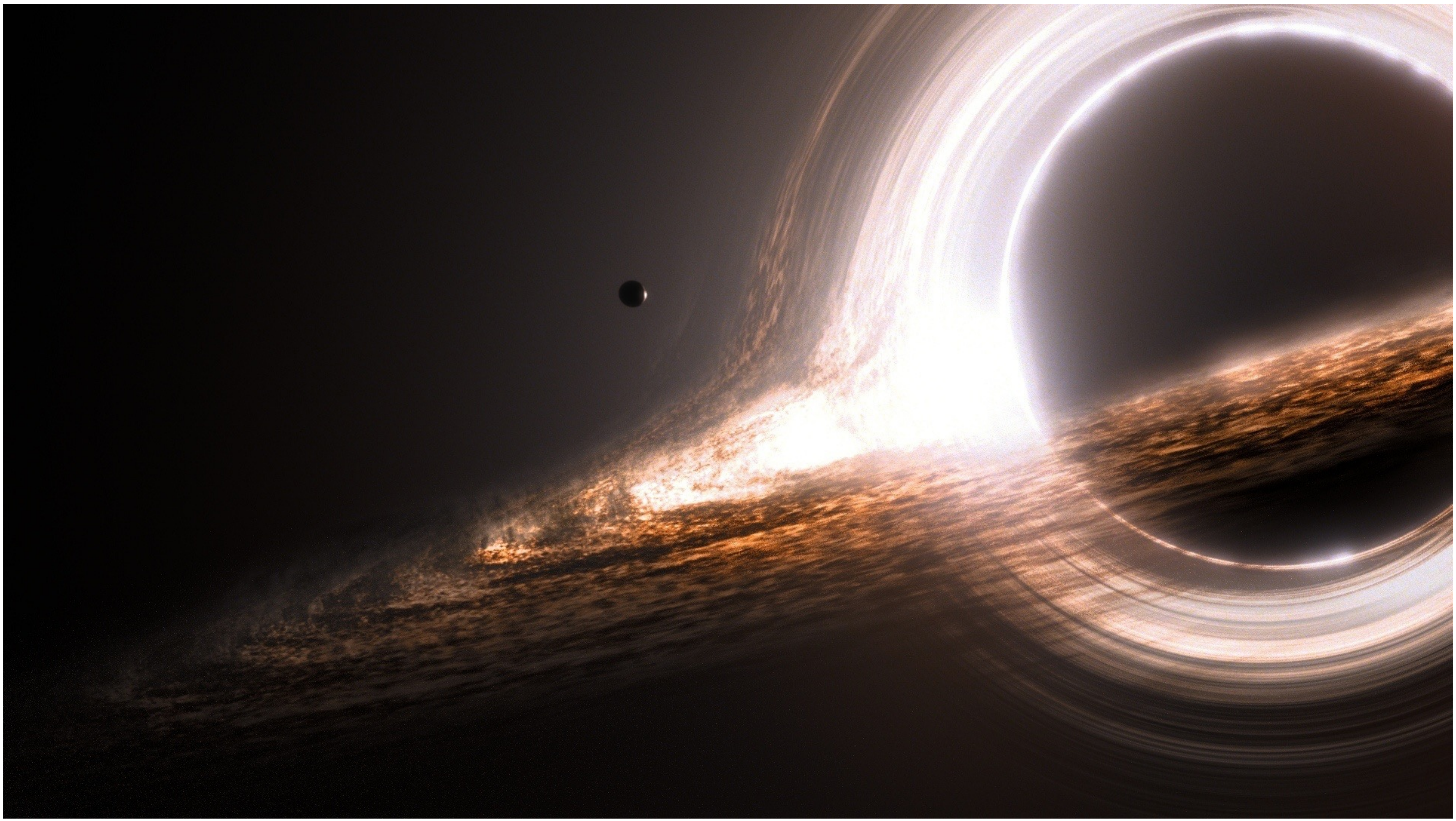
introduction

an illustration of emergent space time



‘Gargantua’, C. Nolan & K. Thorne

emergence & black holes

why do we care about black holes & emergence?

- the classical laws governing the dynamics of space time lead to black holes

  ➛ unless you don't believe in GR you must grapple with their implications; these can be very troubling & constraining.

- it has proven fruitful to confront one's favorite model of spacetime with the rigors of thinking about black holes

if the laws are emergent, can their apparent contradictions be cured by looking at the substrate from which the emerge?

$$S = \frac{k_B c^3 A}{4 G_N \hbar}$$

black holes are thermodynamic systems

$$\rho_{\mathrm{BH}} = \rho_{\mathrm{Gibbs}}(T_H, \Omega)$$

their entropy is proportional to the area of the event horizon

$$S = \frac{k_B c^3 A}{4 G_N \hbar}$$

information loss paradox:

a BH formed from a pure state will evolve into a mixed state (of Hawking radiation)

holography:

a theory of quantum gravity should have information ~ area

this talk

- Everyone’s favorite model of emergent gravity: holography (aka AdS/CFT)
- AdS/CFT relates gravity (often in AdS) to **unitary** field theory (often CFT)
- Lots of interest & progress **gravity ➛ CFT**
- Less is known about **CFT ➛ (quantum) gravity**

  ➛ many interesting ideas & developments in CFT, CMT

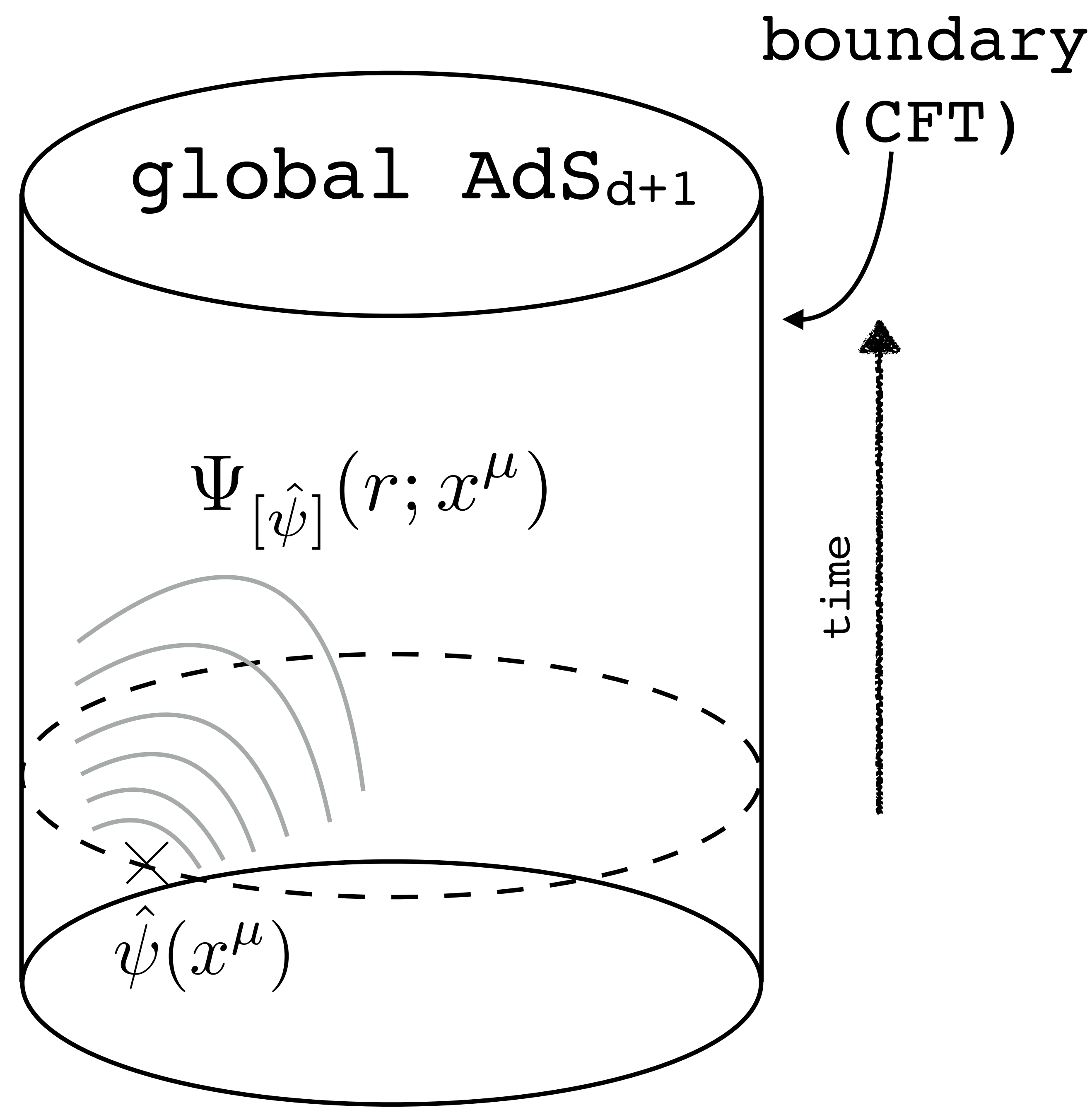
  - time evolution and spread of **entanglement** [Calabrese Cardy,…]
  - **thermalization** of closed quantum systems (e.g. ETH)
  - non-perturbative methods (e.g. bootstrap [see Kaplan,…])

- Thermalization ➛ BH **formation** & evaporation
- To proceed need first-principles model of **BH collapse** in CFT

a few words of reminder on the duality

- field theory lives on D dimensional boundary, while bulk gravity lives in D+1 dimensional interior
- operators in QFT ↔ bulk fields
- there exists semi-classical limit for large no. of local degrees of freedom

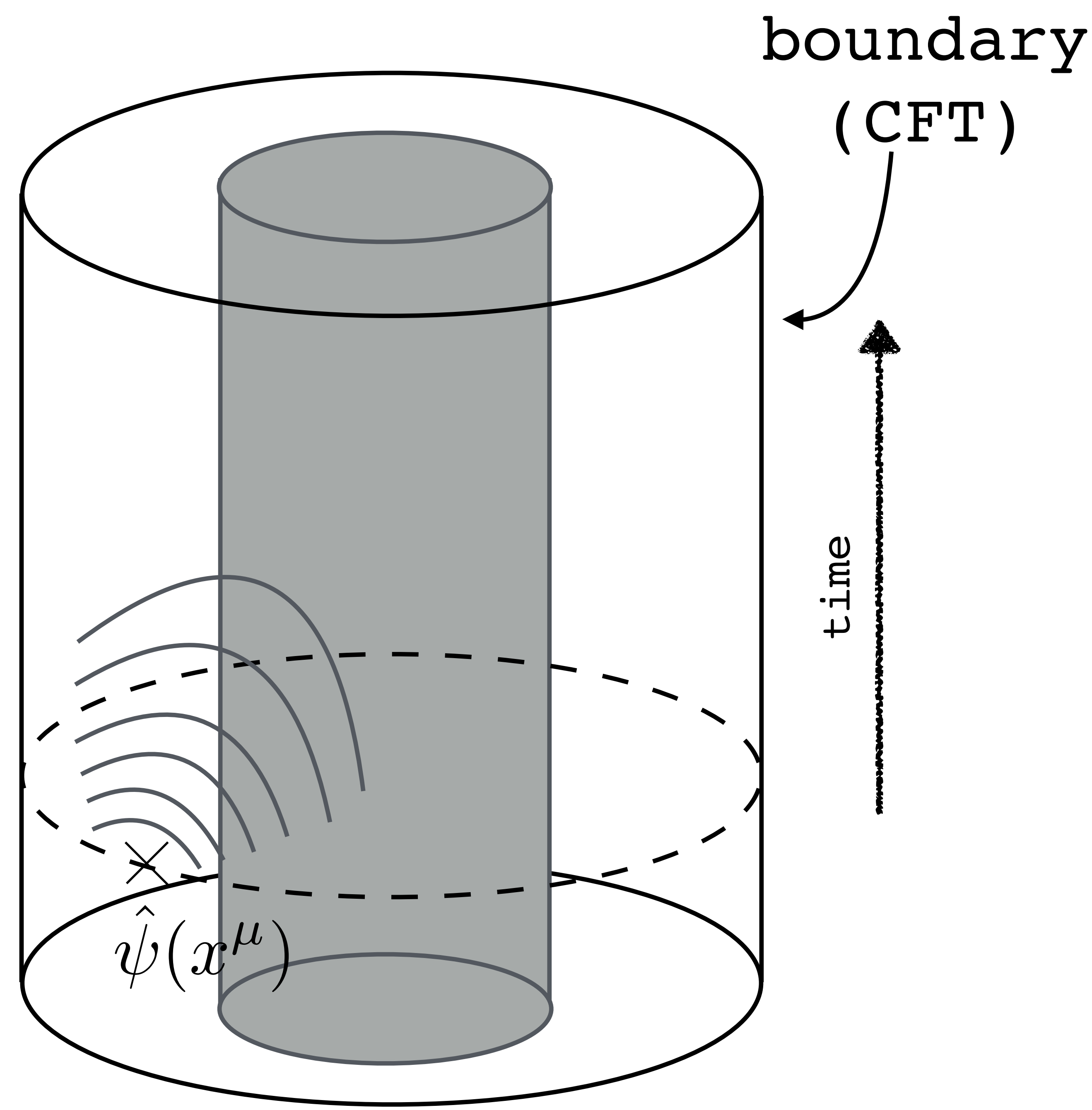
  ➢ large c



QFT in a thermal state = BH in the bulk

- eternal BH in the interior ↔ boundary theory is in thermal ensemble [Witten]
- operator expectation values give thermal results [Gibbons & Hawking]
- BH information paradox in its sharpest form concerns BH formed from collapse

→ need to understand **BH collapse**



the plan of attack

Using holography, we can formulate a more ordinary problem:

1. Define a high-energy non-equilibrium initial state $|\Psi\rangle$

2. Study its time evolution $|\Psi\rangle \to U(t, t_0)|\Psi\rangle$

➙ this is known as a ‘quantum quench’

Questions we want to address:

1. Do observables look thermal and if so in what sense?

2. How is information loss compatible with unitary evolution?

3. Can we probe geometry behind horizon from CFT data?

a few more words on the duality

- in the context of 2D CFT ↔ 3D gravity, focus on a universal sector, by defining a 1/c expansion:
  ➙ any microscopic theory in this class defines some 3D quantum gravity theory

- From bulk point of view this is G_N expansion. Expect non-perturbative corrections to EFT result save the day.

  ➙ this is standard expectation, but hard to substantiate

- Use field theory to give a non-perturbative (emergent) definition of quantum gravity. Should be able to do something

black-hole formation

the arena: 3D gravity / 2D CFT

- Pure 3D gravity is trivial: there are no local dof ➛ BHs do not form [Achucarro & Townsend; Witten]

$$S_{3\mathrm{D}} = S_{\mathrm{CS}}[A] - S_{\mathrm{CS}}[\bar{A}]$$

- 2D CFT puts powerful analytical tools at our disposal

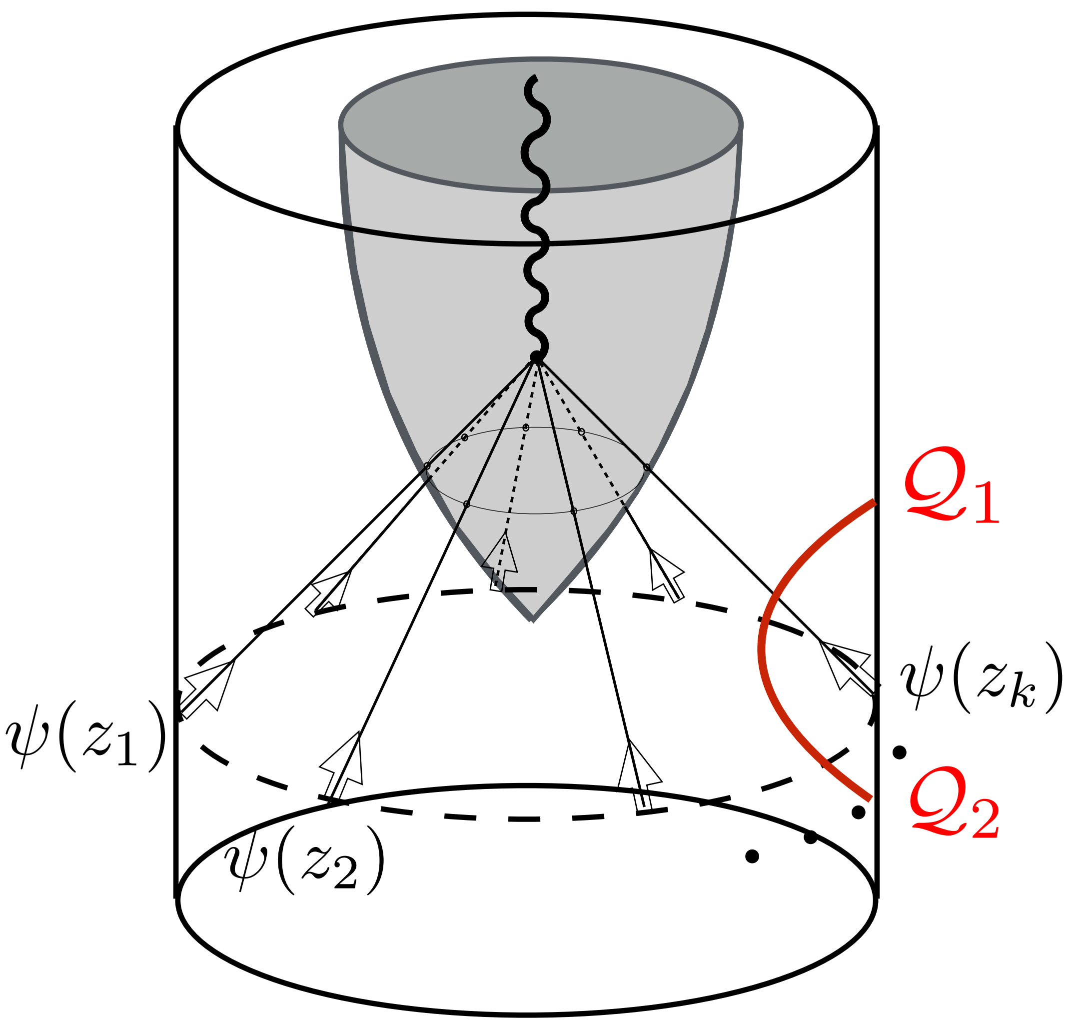
- We add matter (from CFT point of view):

  much richer dynamics, in particular BHs do form
  but must develop new methods to solve

- Results are more widely relevant: 3D gravity is central to microscopics of BH entropy even in higher dimensions [Brown & Henneaux; Strominger;…]

3D gravity + matter non-trivial, but solvable
➛ ideal place to study BH puzzles!

the idea I: gravity motivation



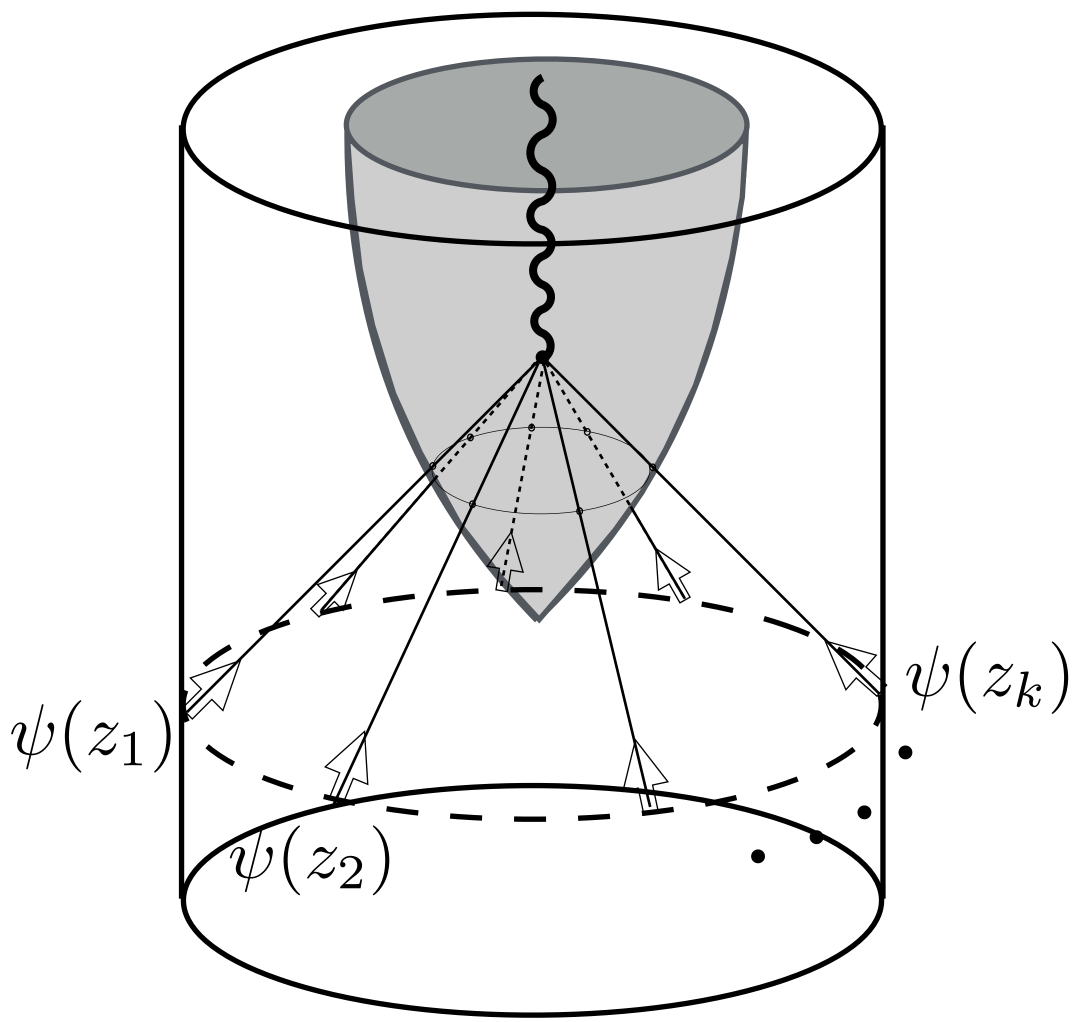
- Throw in a shell of n dust particles

$$T^{\mu\nu} = \rho \bar{U}^{\mu} \bar{U}^{\nu}$$

- BH collapse: Vaidya metric
- Use light operators $\mathcal{Q}$ to probe geometry as function of t

- Certain t-dependent observables such as entanglement entropy are sensitive to behind horizon physics

the idea I: comments



- Natural to think of Vaidya as adding a source: [Bhattacharyya et al.]

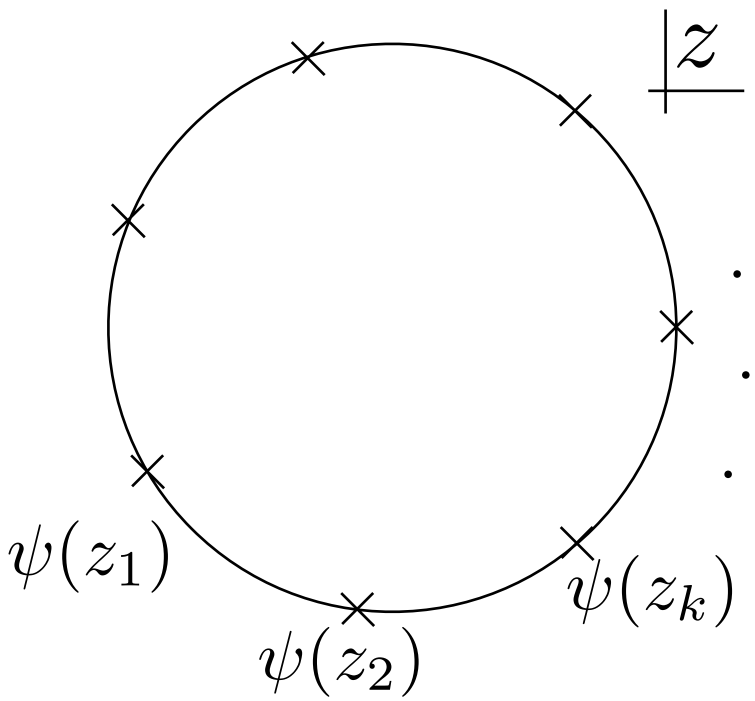
$$S \to S + \int d^2x J(x)\mathcal{O}(x)$$

localized in space and time

- Think of Vaidya instead as a state: shell approaches boundary to within σ & then bounces off

- Can also do source calculation in CFT (for small amplitude)

the idea II: CFT state

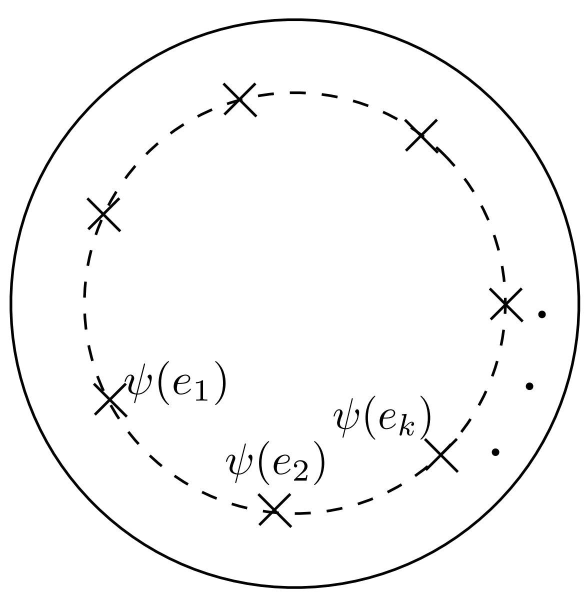


states in CFT are defined by path integral on unit disc

$$|\mathcal{V}\rangle = \frac{1}{\mathcal{N}} \prod_{k=1}^{n} \psi(e_k, \bar{e}_k)|0\rangle$$

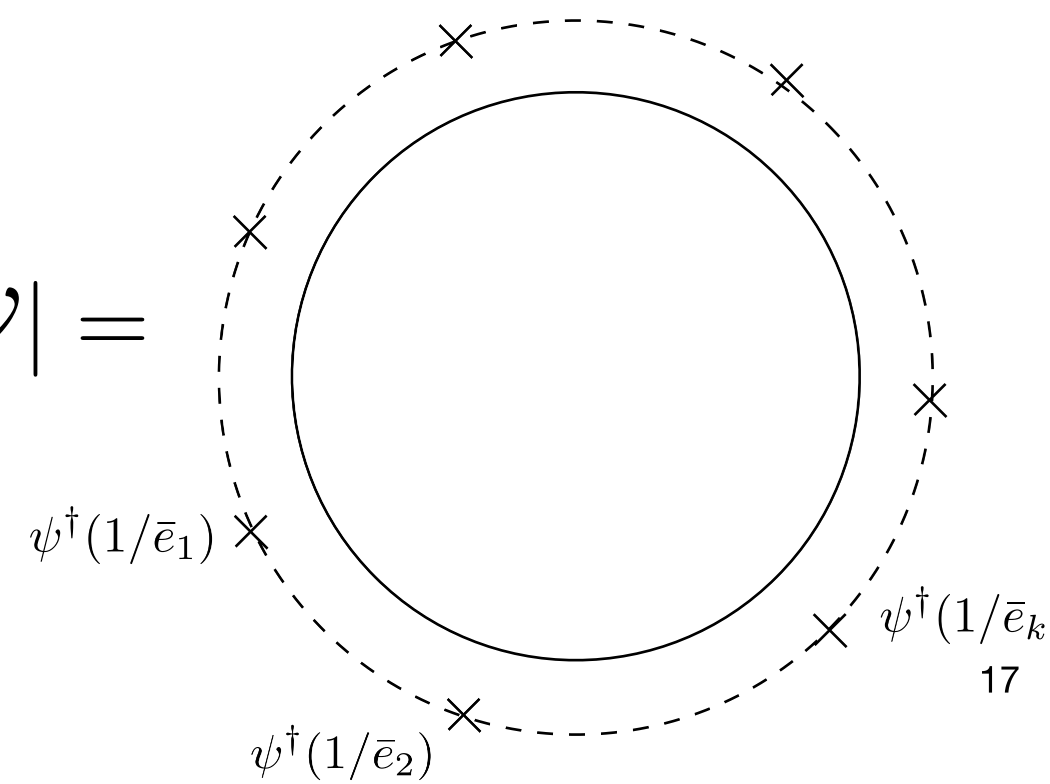
"Ceci n'est pas un état" (not normalizable)

$$|\mathcal{V}\rangle =$$



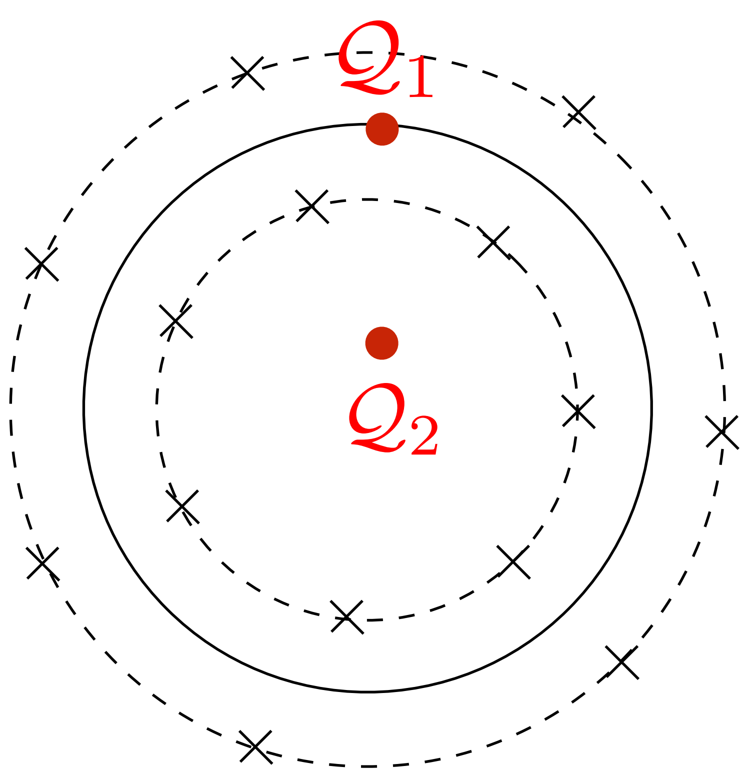
insertion at |z|=1-σ

$$\langle\mathcal{V}| =$$



comments, further details

- insertion of primary at z=0 gives eigenstate on cylinder (no dynamics)
- want to probe this state ➛ insertion of local operators $\mathcal{Q}$



- Limits: $c \to \infty$

$$n \to \infty$$

$$\sigma \to 0$$

$$E \sim nh_\psi/\sigma \to \mathcal{O}(c)$$

actually in CFT can leave σ finite, but need σ ➛ for gravity comparison

probing shell collapse in CFT

We want to compute 2n+p point correlation functions of the form

$$\langle \mathcal{V} | \mathcal{Q}_1(z_1, \bar{z}_1) \cdots \mathcal{Q}_p(z_p, \bar{z}_p) | \mathcal{V} \rangle = \lim_{n \to \infty} \frac{1}{\mathcal{N}^2} \left\langle \prod_{i=1}^{n} e_i^{-2\bar{h}_i} \bar{e}_i^{-2h_i} \psi(1/\bar{e}_i, 1/e_i,)\, \mathcal{Q}_1(z_1, \bar{z}_1) \cdots \mathcal{Q}_p(z_p, \bar{z}_p)\, \psi(e_i, \bar{e}_i) \right\rangle$$

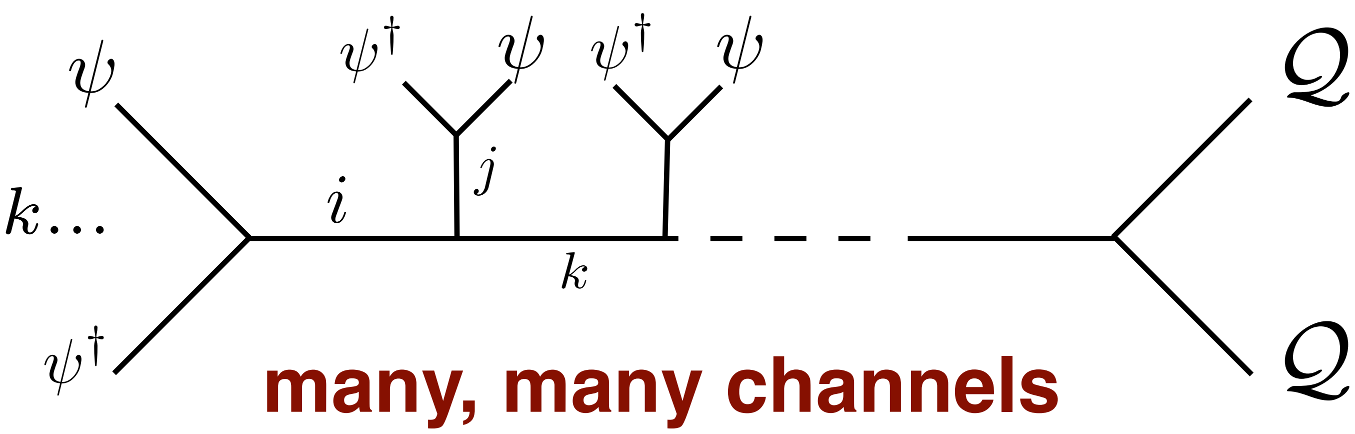
The limit n ➤ ∞ simplifies calculation enormously

Continue Q-insertions to Lorentzian time to study dynamics

Can do this analytically using the conformal block expansion

large-c expansion and conformal blocks

$$\langle \mathcal{V} | \mathcal{Q}_1 \mathcal{Q}_2 | \mathcal{V} \rangle = \sum_{i,j,k,\ldots} c_{ijk\ldots}$$

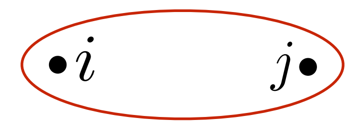


$$= \sum_{p,q,r\ldots} c_{pqr\ldots} \mathcal{F}(h_i/c, h_{p.q,r\ldots}/c, 1-z)$$

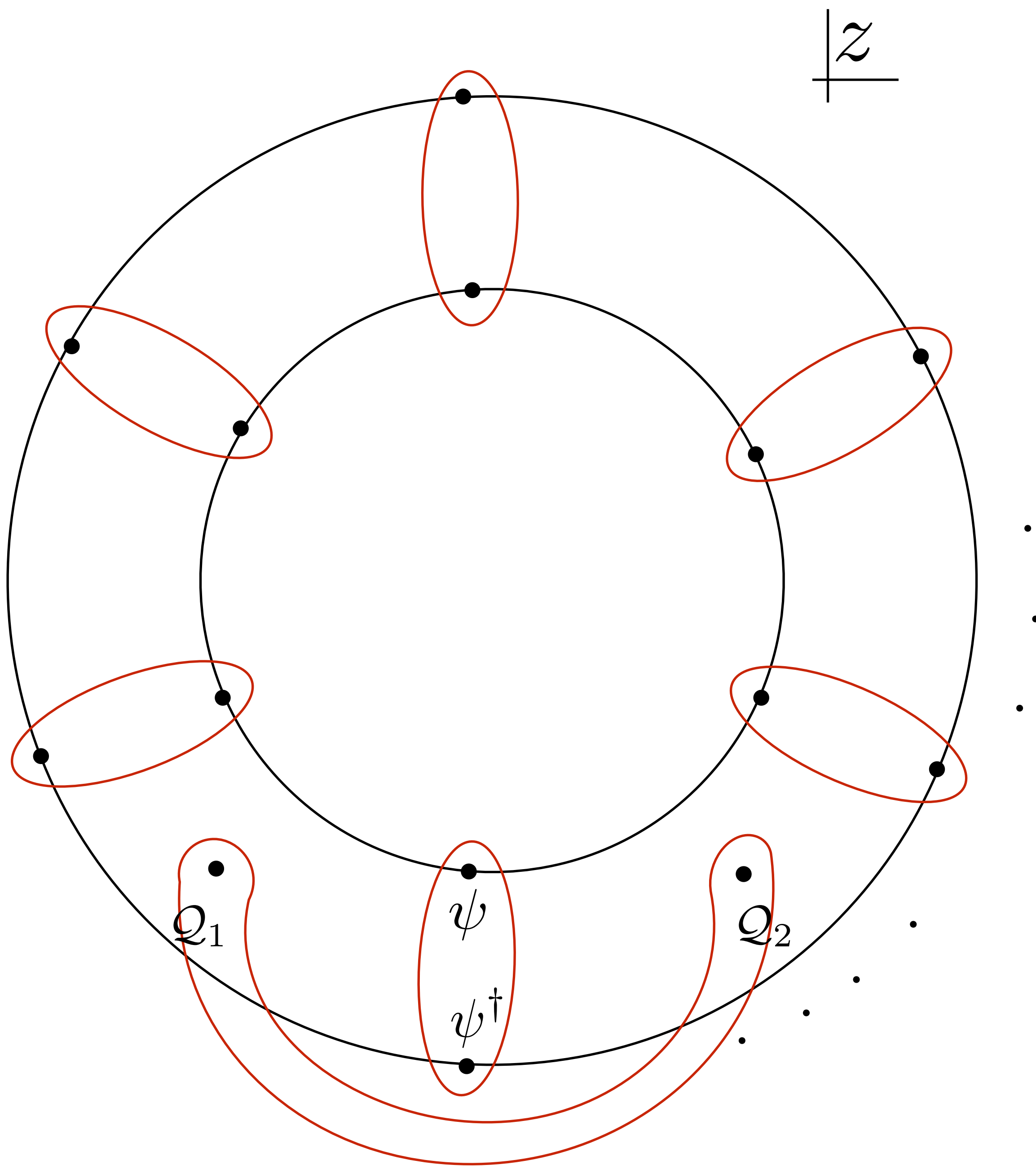
@ large c, the block **exponentiates**: $\mathcal{F} \approx e^{-\frac{c}{6} f(h_i/c, h_p/c, 1-z)}$

need technique to calculate f(h_i,h_p,1-z) in a given channel
➙ [Al. B. Zamolodchikov] "4-spin correlations in Ashkin-Teller model"

monodromy method

for each OPE contraction, draw a cycle 

& consider



$$y''(z) + Ty(z) = 0$$

with

$$T = \sum_{k=1}^{2n+p} \left[\frac{6h/c}{(z - z_k)^2} + \frac{c_k}{z - z_k} \right]$$

$$\frac{\partial f}{\partial z_k} = c_k$$

c_k fixed by monodromy around each cycle

the key assumption

when does a CFT have a holographic dual? We know it must allow/ have

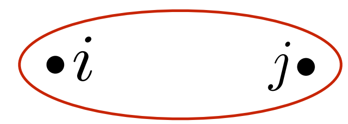
a) a large number of degrees of freedom $c \gg 1$
b) a sparse spectrum of low-lying states

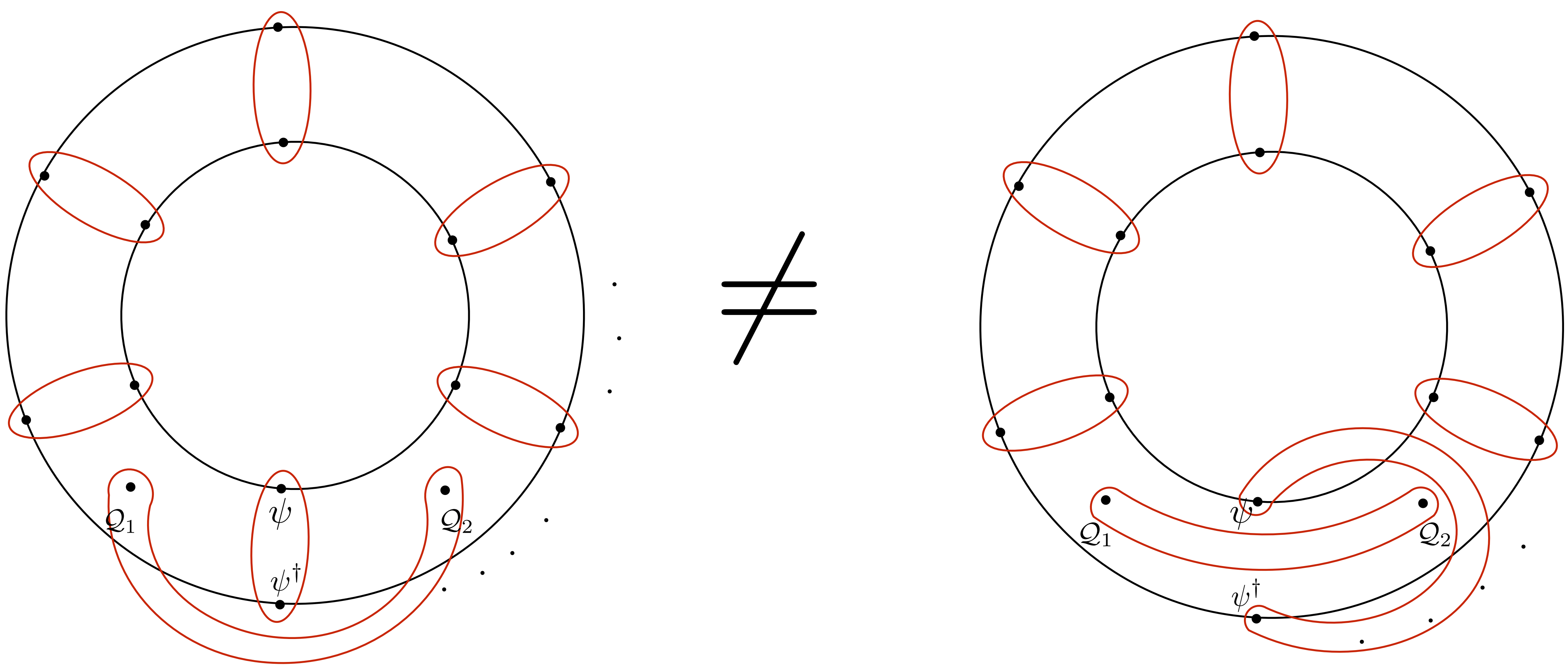
“it from **id**”

In the chosen OPE channel the dominant contribution comes from the identity Virasoro block, that is the unit operator **id** *and all its descendants,* T, ∂T, T^2 $T\partial T\ldots$, *running on the internal lines*

➙ prescribe **trivial monodromy** on all chosen cycles

choice of contraction

for each OPE contraction, draw a cycle 



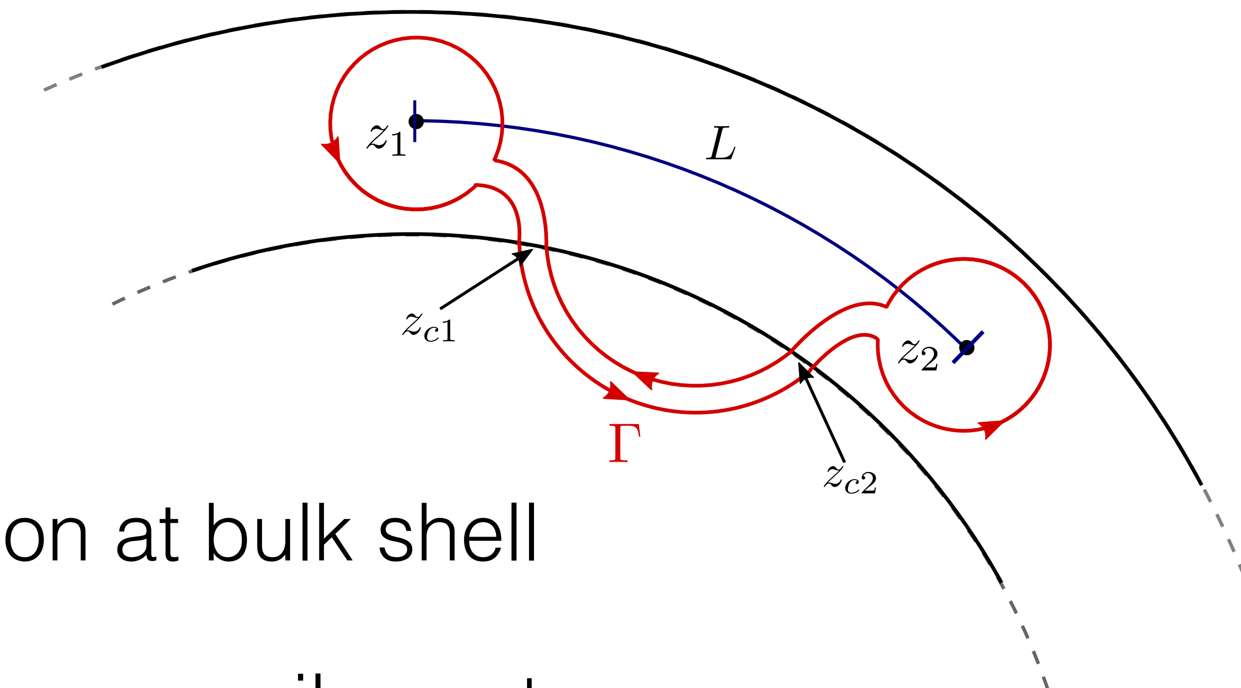
$\mathcal{F}(z)$ is not single valued on z-plane

select **dominant** contribution to correlation function

entanglement entropy

Q-type operators ➞ twist insertions: $G_q(t) = \langle \mathcal{V} | \sigma_q(t, \ell_1) \tilde{\sigma}_q(t, \ell_2) | \mathcal{V} \rangle$

$$S(A) = \lim_{q \to 1} \frac{1}{1-q} G_q(t)$$



crossing points z_{c1} & z_{c2} ↔ refraction at bulk shell

it from id ➞ require trivial monodromy on smile contour

write $z_1 = e^{i\theta_1}, z_2 = e^{i(\theta_1 + L)}$ & continue to Lorentzian time $\theta_1 = t$

maximize S(A) over crossing points ➞ parametric equation for S(t)

entanglement entropy

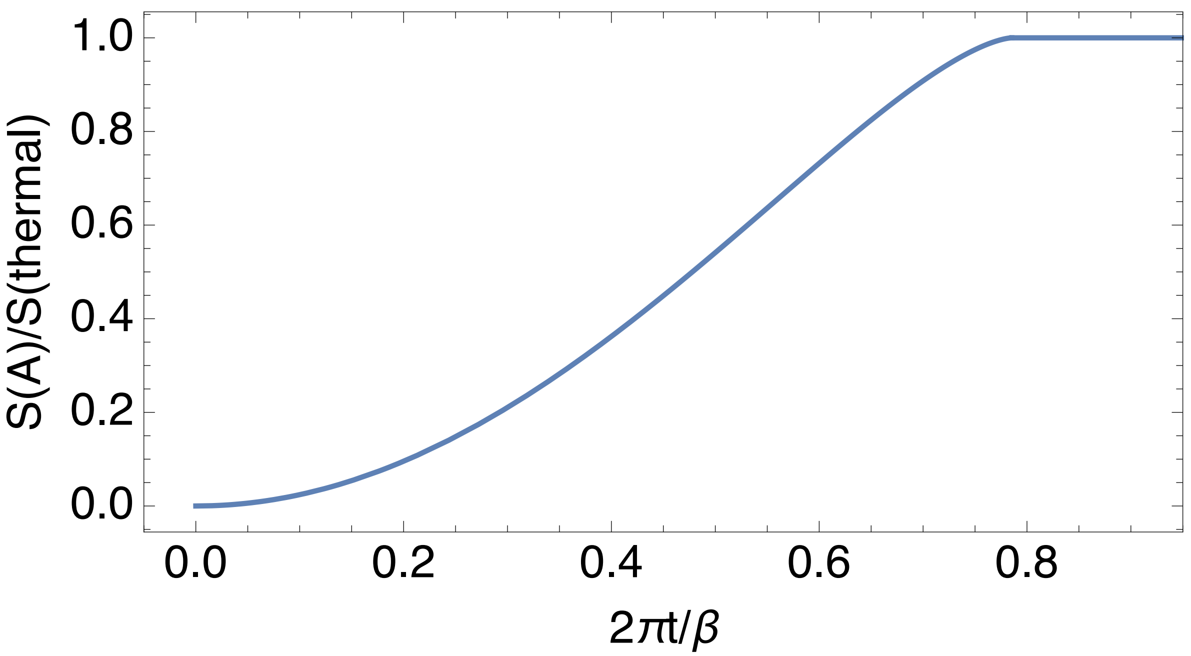
Implicit formula for growth of entanglement entropy:

$$t = \frac{\beta}{2\pi}\cosh^{-1}\left\{\cosh\left(2\pi T q\right) + 2\pi T \tan\left(\frac{L}{2} - q\right)\sinh\left(2\pi T q\right)\right\}$$

$$S_{EE} = \frac{c}{3}\log\left\{\frac{\sin\left(\frac{L}{2}-q\right)\cosh\left(2\pi T q\right) + \frac{1}{2\pi T}\left[1 + \frac{1}{2}\left\{1 + 4\pi^2 T^2\right\}\tan^2\left(\frac{L}{2}-q\right)\right]\cos\left(\frac{L}{2}-q\right)\sinh\left(2\pi T q\right)}{\epsilon_{UV}/2}\right\}$$

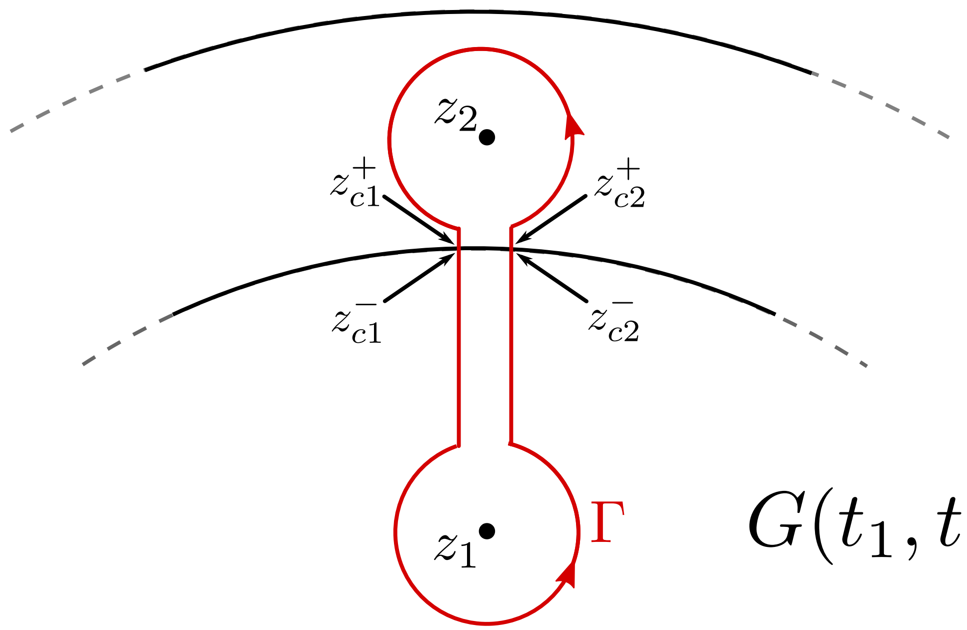
matches **exactly** global AdS_3 Vaidya:

- thermal at late time
- EE growth = change of channel
- sees **beyond horizon**

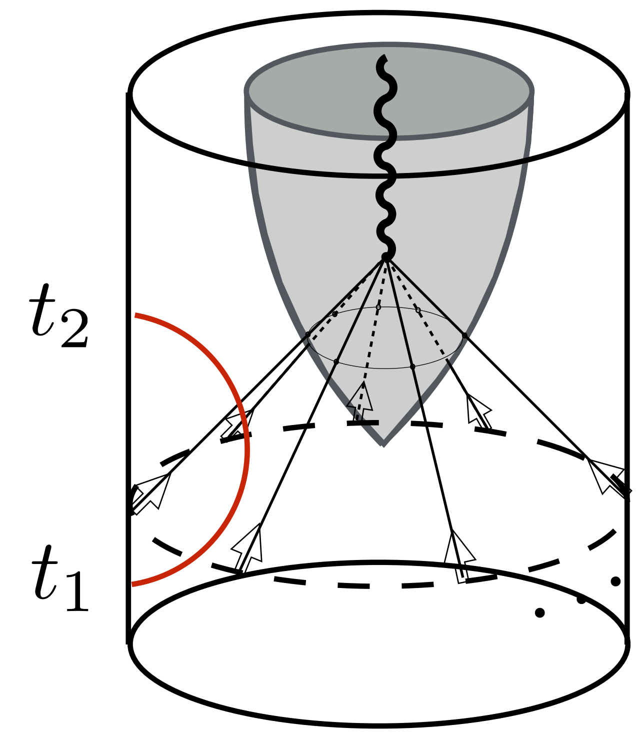


CFT calculation shows that purity of state is preserved: $S(A) = S(A^c)$

two-point correlation (dimension $\Delta^{\mathcal{Q}}$)



$$G(t_1, t_2) \sim \exp(-\frac{2\pi\Delta^{\mathcal{Q}} t}{\beta})$$



$$G(t_1, t_2) = \left(\frac{1}{\pi T}\cos\left(\frac{t_1}{2}\right)\sinh\left(\pi T t_2\right) - 2\sin\left(\frac{t_1}{2}\right)\cosh\left(\pi T t_2\right)\right)^{-2\Delta^{\mathcal{Q}}}$$

not (yet) known from gravity (but limit matches known flat space result)

exponential decay at late times in conflict with unitarity:
CFT loses information!

information loss and retrieval

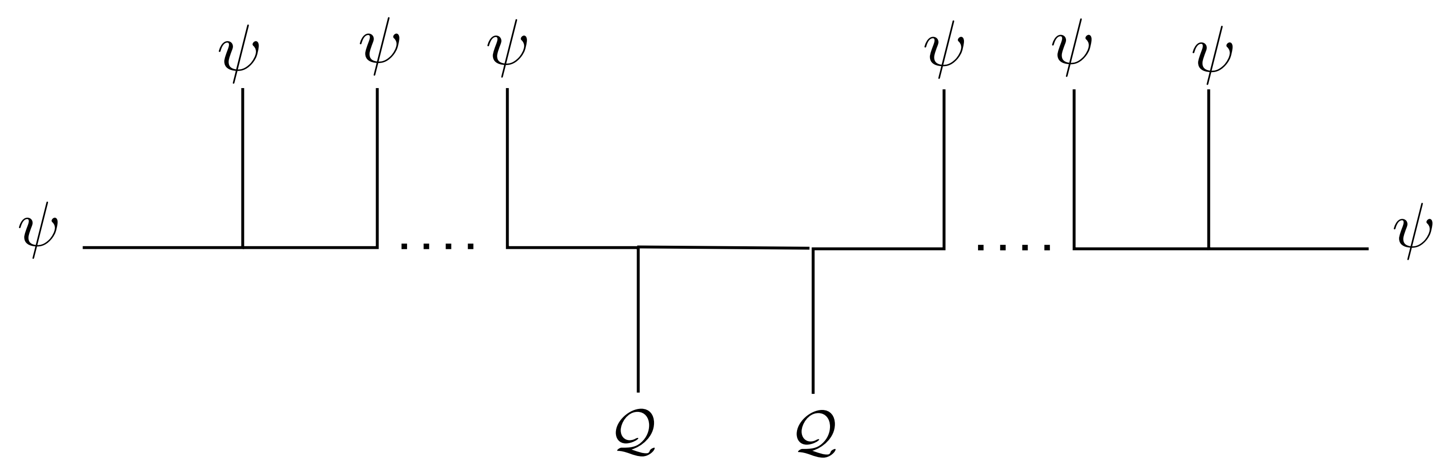
unitarity demands that the long time average of

$$\overline{|G(t)|} = \overline{\left|\sum_{n,k} e^{i(E_n - E_k)t}\,\Psi_n^*(\mathcal{V})\langle n|\mathcal{Q}|k\rangle\langle k|\mathcal{Q}|\mathcal{V}\rangle\right|} \neq 0$$

➙ correlations **cannot become arbitrarily small**

We neglected contributions exponentially suppressed at t=0 (must be present due to crossing symmetry)

$$G(t) = \sum_{\text{primaries}} c_{i,j,k,\ldots}$$



restore unitary at large time ➙ non-perturbative effects in G_N

summary & outlook

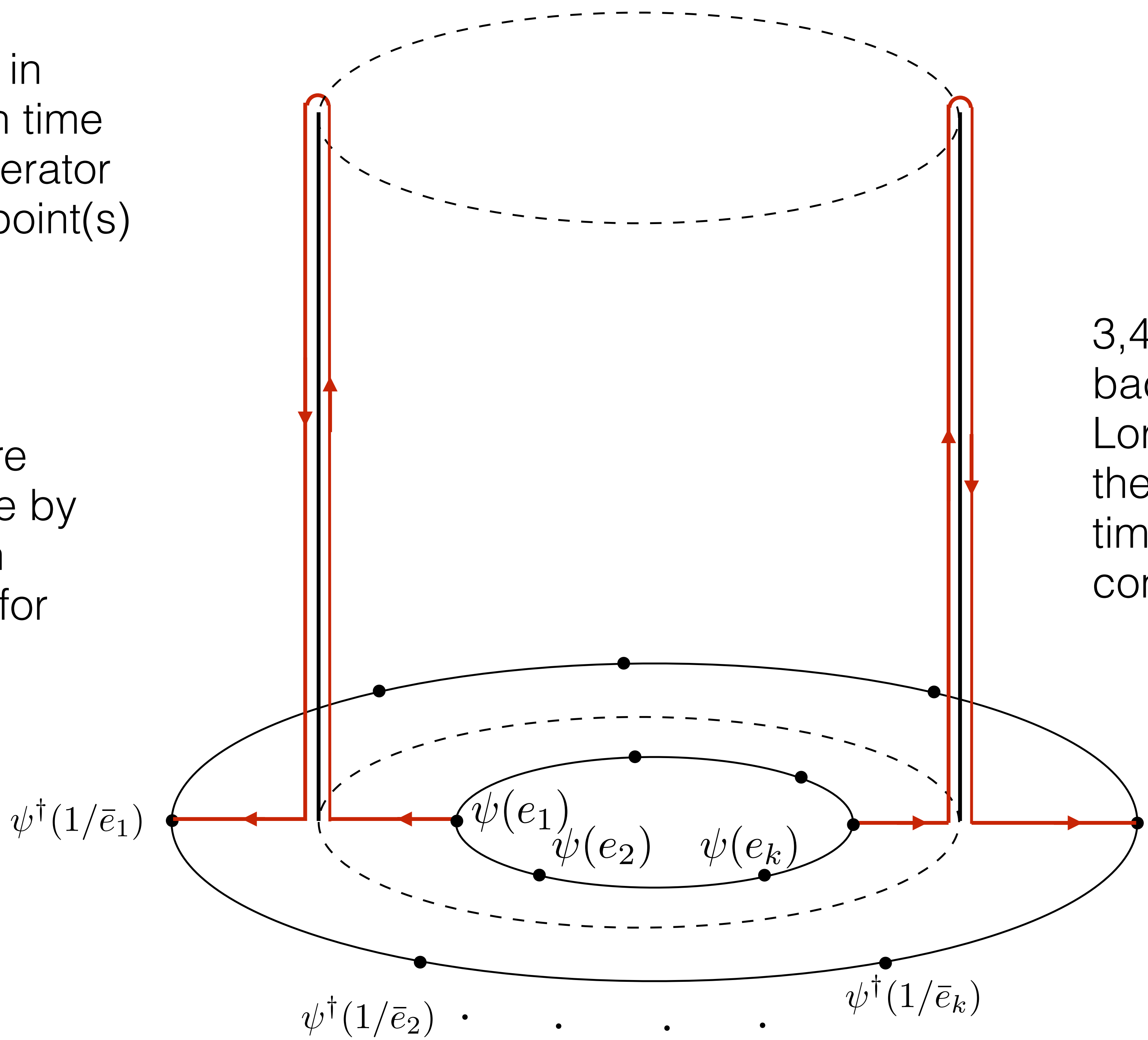
- time-dependent 3D quantum gravity with matter in 1/c expansion
  ‘it from id’ ➙ ideal arena to think about quantum BHs
- first-principles BH collapse via continuum monodromy method
- obtain **time-dependent** prescription for S(A) from CFT
  ➙ thermal (-like) but manifestly preserves purity
- CFT correlation functions seemingly **violate unitarity** (naïve). Non-perturbative corrections restore unitarity
- on gravity side these correspond to non-perturbative effects in G_N. Geometric interpretation of other channels?

thank you

alternative picture: IN-IN computation

2.) evolve in Lorentzian time until Q-operator insertion point(s)

1.) prepare initial state by Euclidean evolution for time σ



3,4.) evolve back in Lorentzian time, then Euclidean time to form conjugate