Information Loss in Two Dimensional CFT

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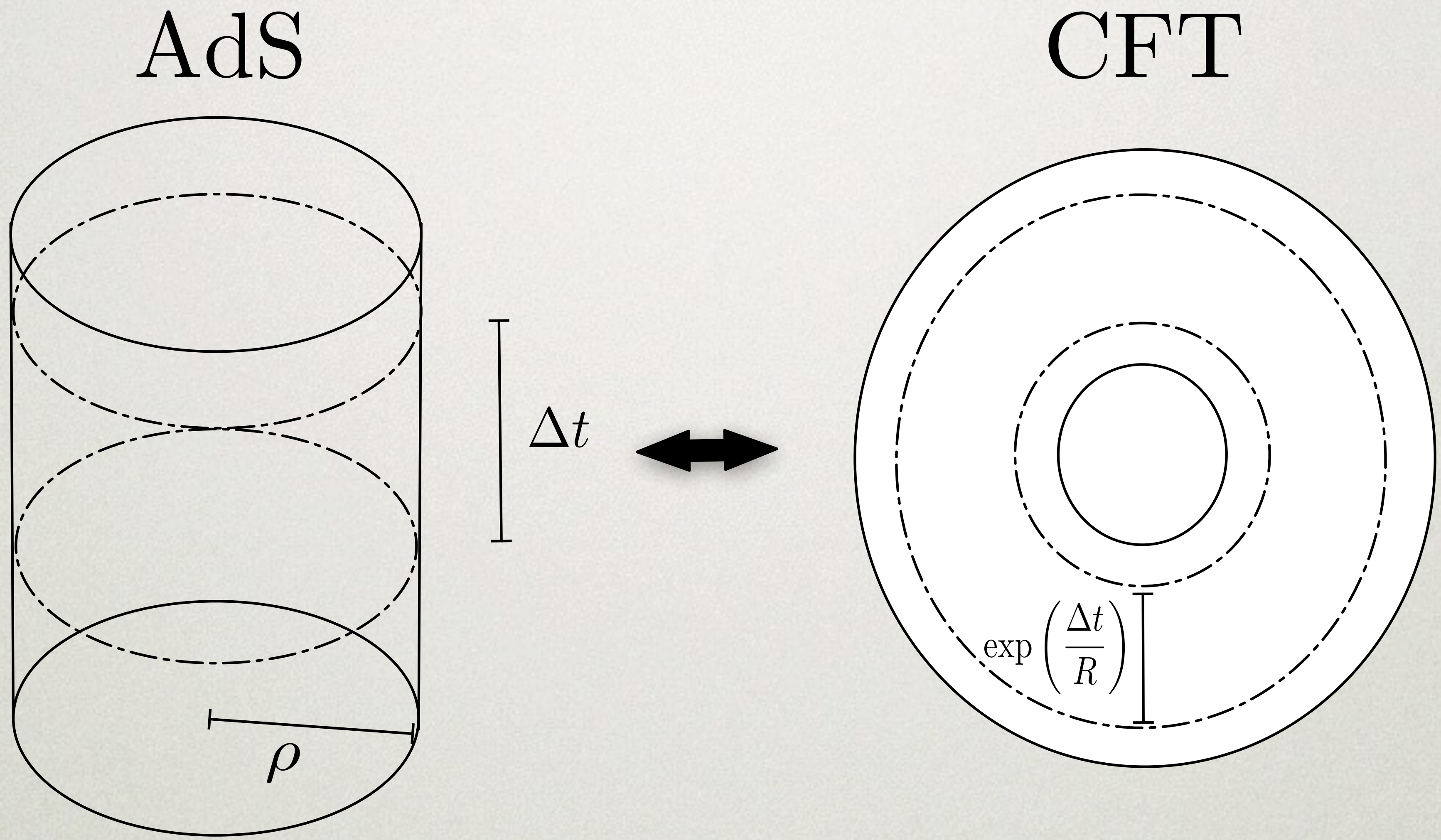
based on work with
Liam Fitzpatrick, Daliang Li, & Junpu Wang
and work with Matthew Walters

AdS/CFT Basics



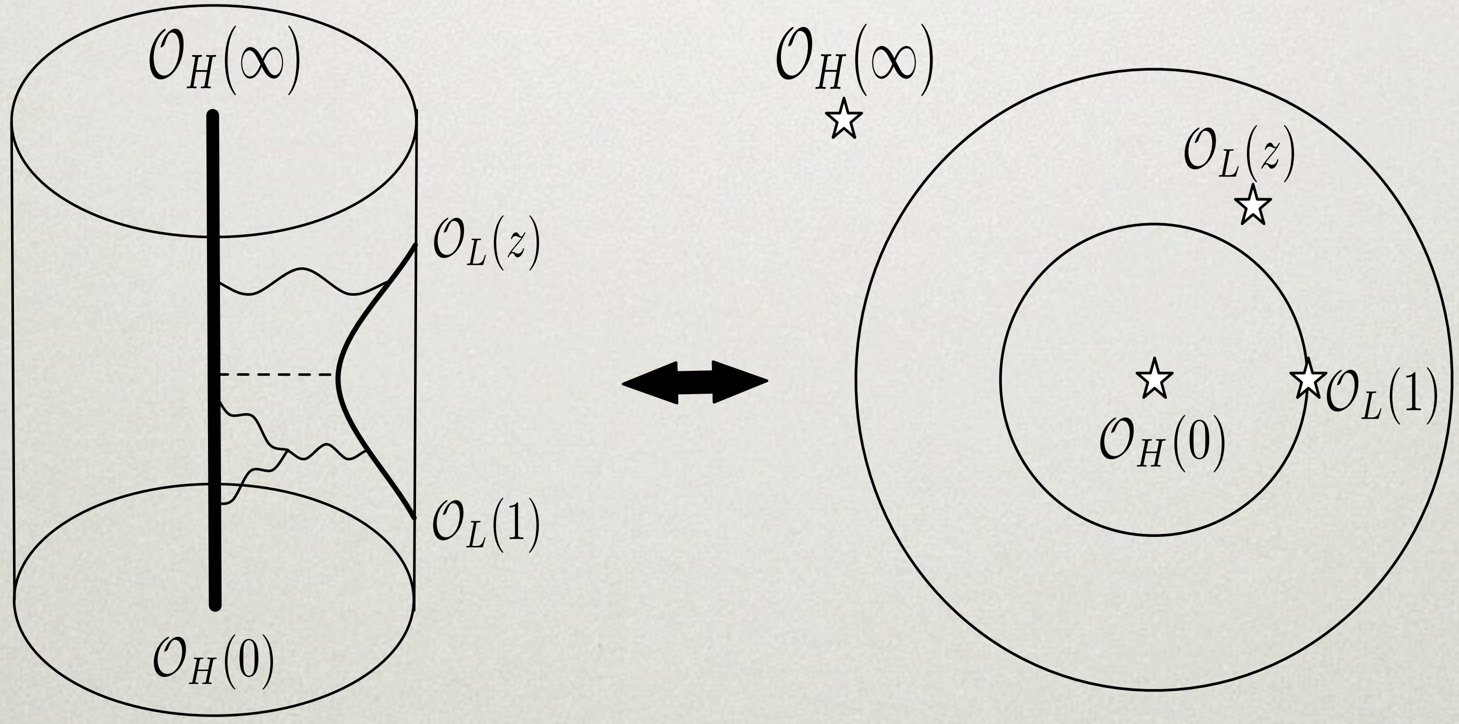
$$H_{AdS} = D_{CFT}$$

Long-range potentials/forces: $V(\rho) \sim e^{-\tau \frac{\rho}{R_{AdS}}}$

Observable: Four-Point CFT Correlator, as a Probe of AdS

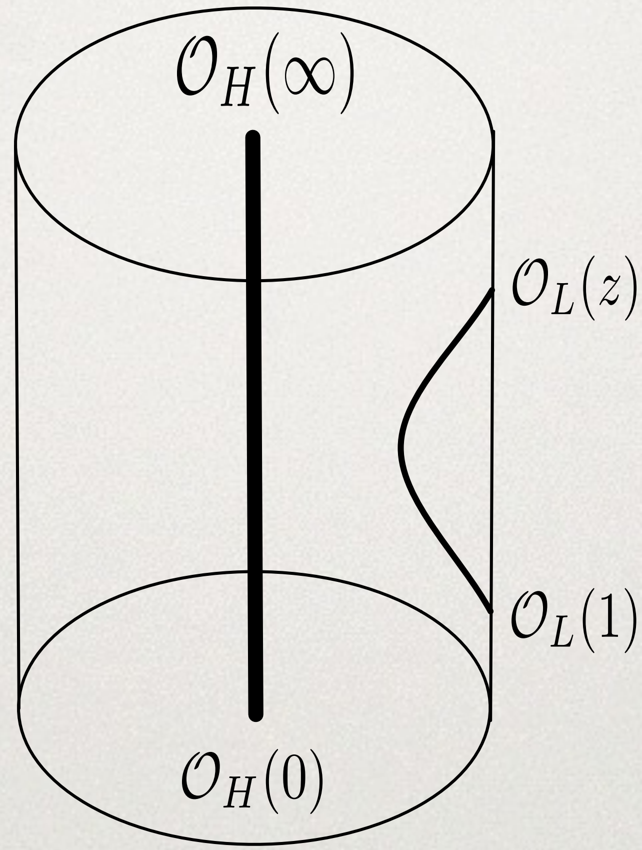
What Observable (in AdS/CFT)?

$$\langle \mathcal{O}_H(\infty)\mathcal{O}_L(1)\mathcal{O}_L(z)\mathcal{O}_H(0)\rangle$$



Some basic facts we can we learn from this…

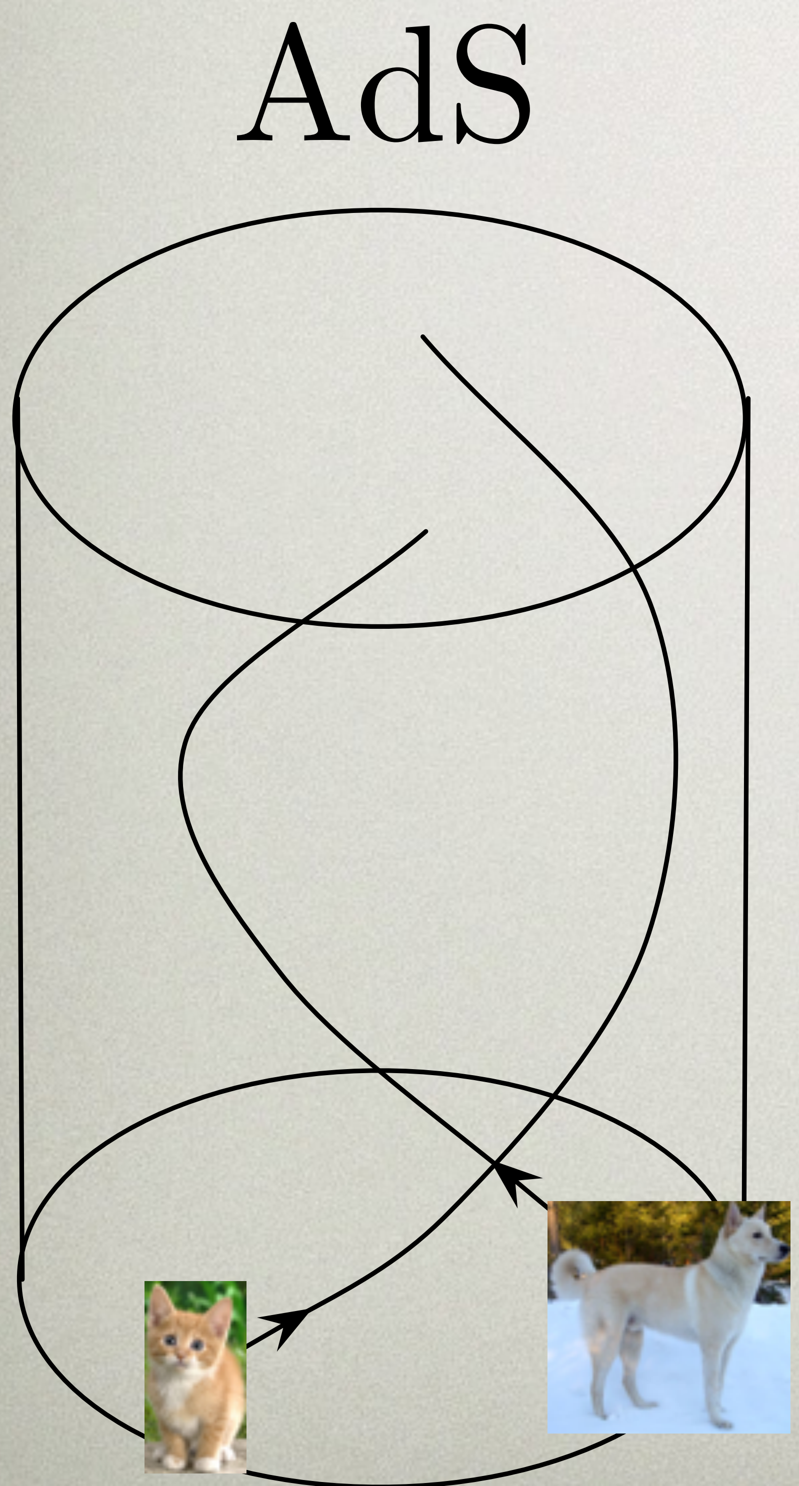
Directly Measure Bulk Geometry



$$\langle \mathcal{O}_H(\infty)\mathcal{O}_L(1)\mathcal{O}_L(z)\mathcal{O}_H(0)\rangle \approx e^{-\Delta_L L_{\text{geod}}(z,1)}$$

Follows at large mass from the geometric optics approximation in AdS.

Energies and Forces in Two-Object Dynamics



Look at states in the OPE:

$$\mathcal{O}_c(x)\mathcal{O}_d(0) \supset \sum_{E,\ell} \mathcal{O}_{E,\ell}$$

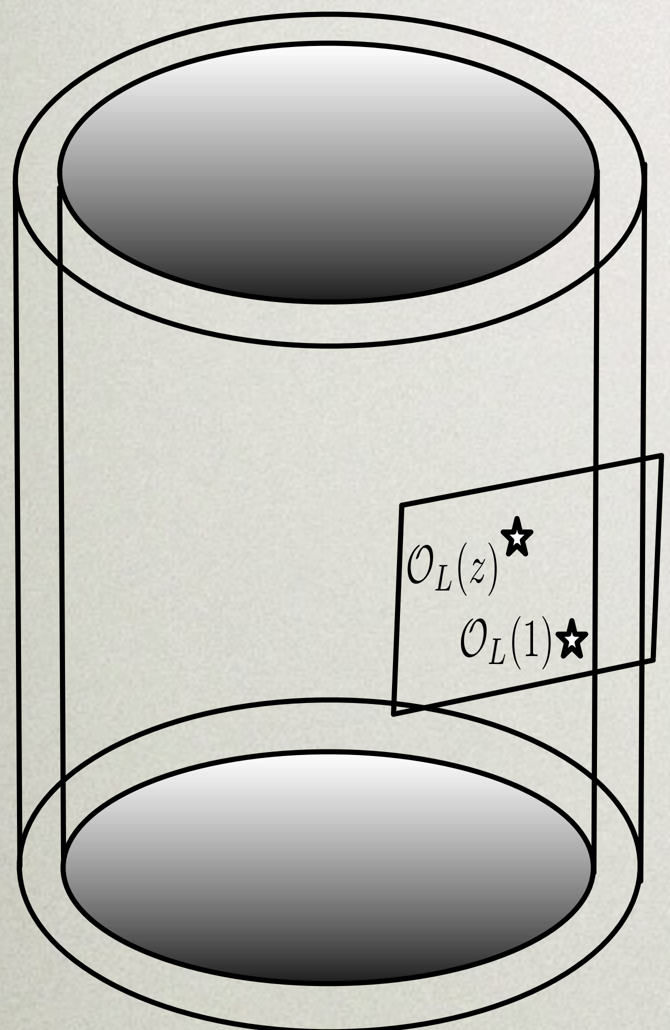
AdS Energy = CFT Dimension

$$E_{n,\ell} = E_c + E_d + E_{KE} + \gamma(n,\ell)$$

Anomalous dimension, $\gamma(n,\ell)$, is a **binding energy**.

Probing Black Holes and the Hawking Temp

$\mathcal{O}_H(\infty)$



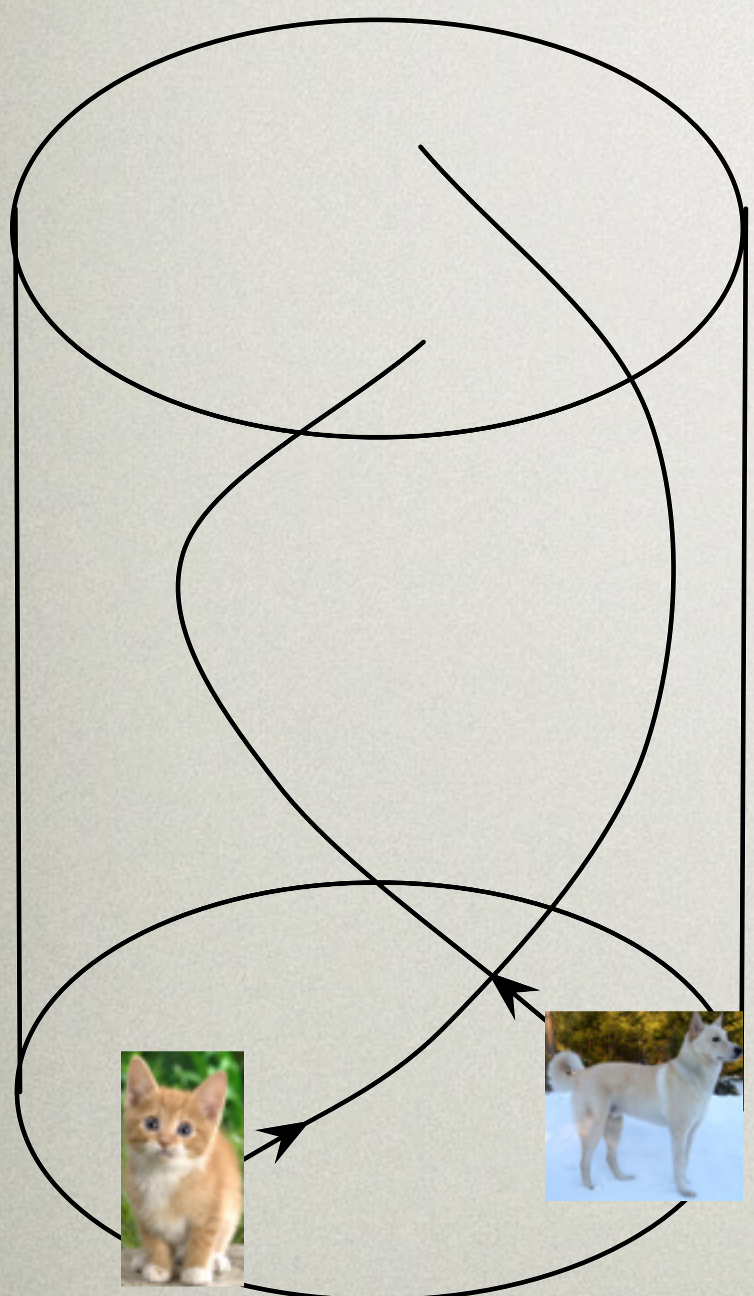
$\mathcal{O}_H(0)$

$$\approx \langle \mathcal{O}_L(1)\mathcal{O}_L(z)\rangle_{T_H} \text{ ?}$$

For example, a correlator in BTZ black hole background:

$$ds^2 = \left(r^2 + 1 - 24\frac{h_H}{c}\right)dt^2 + \frac{dr^2}{r^2 + 1 - 24\frac{h_H}{c}} + r^2 d\phi^2$$

(An Appetizer)



Long Range Locality and Universal Forces

Formal Definition of Long-Distance Locality?

$\psi_c =$ 

$\psi_d =$ 

$\Longrightarrow \; \exists \; \psi_{cd} =$ 

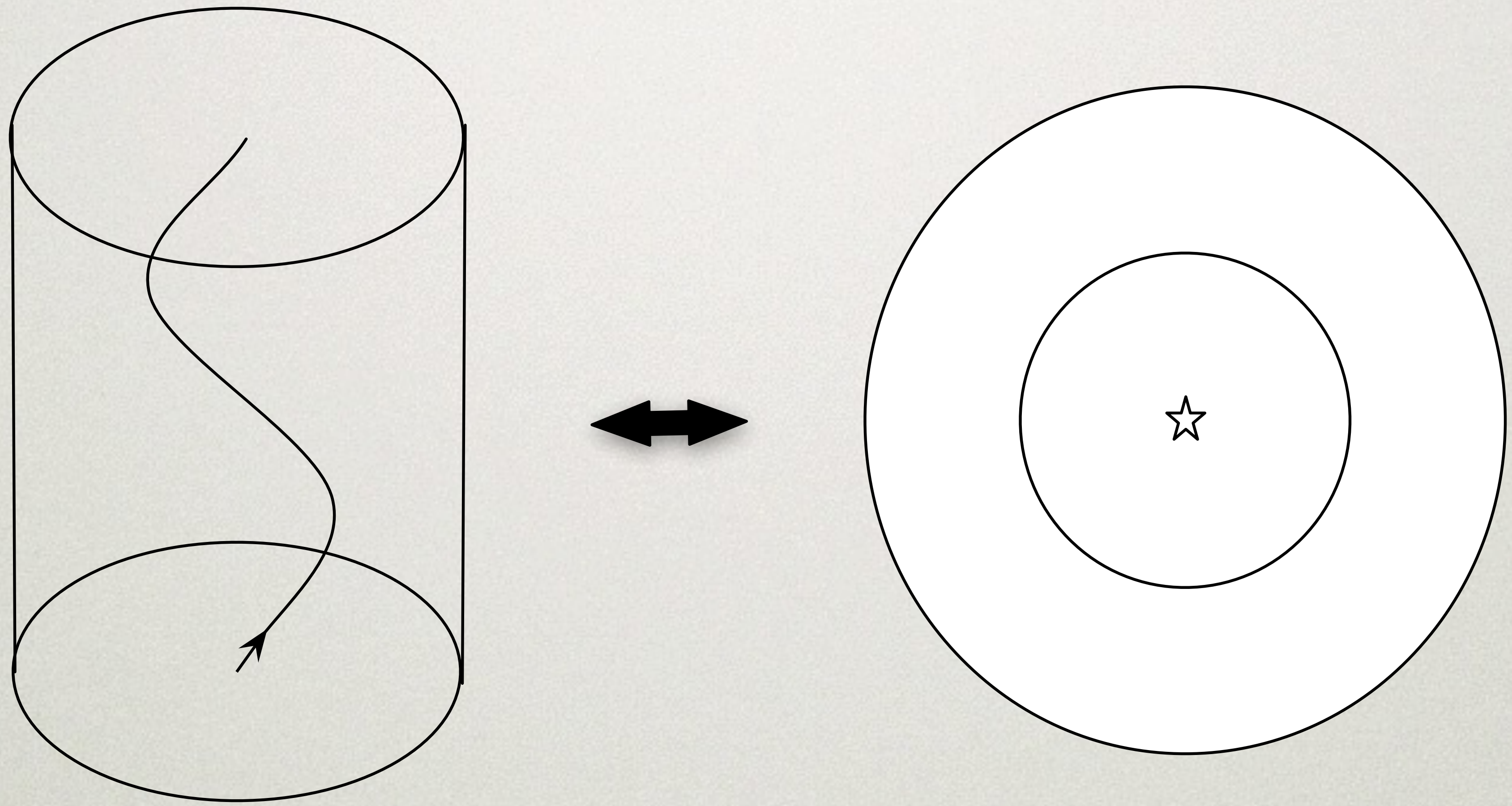
Statement about **structure of the Hilbert Space**:

$$\mathcal{H}_{AdS} = \mathcal{H}_{CFT} \approx \text{Fock Space}$$

AdS Motion = Conformal Representation Theory

AdS

CFT



$\psi_{n,\ell}(t,\rho,\Omega)$

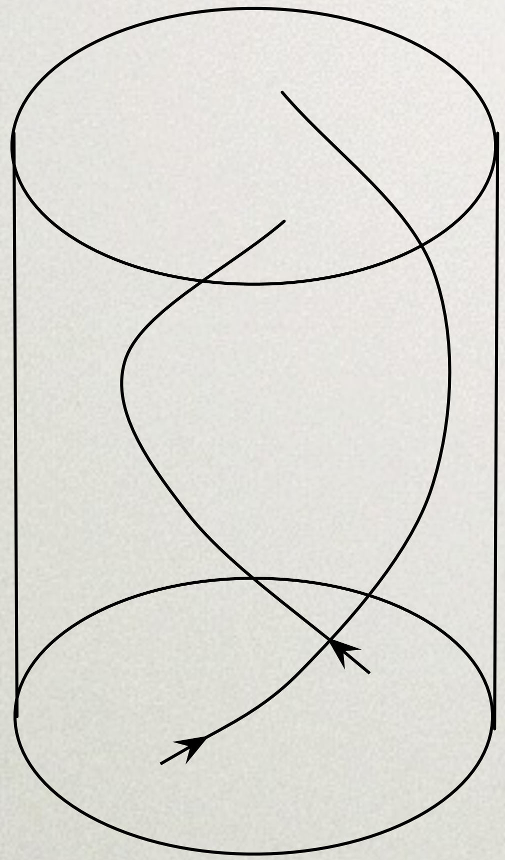
$\left(\partial^{2n}\partial_{\mu_1}\cdots\partial_{\mu_\ell}\mathcal{O}\right)|0\rangle$

Center of Mass for Excited State

Descendant of a Primary

An Example: Two Particle States

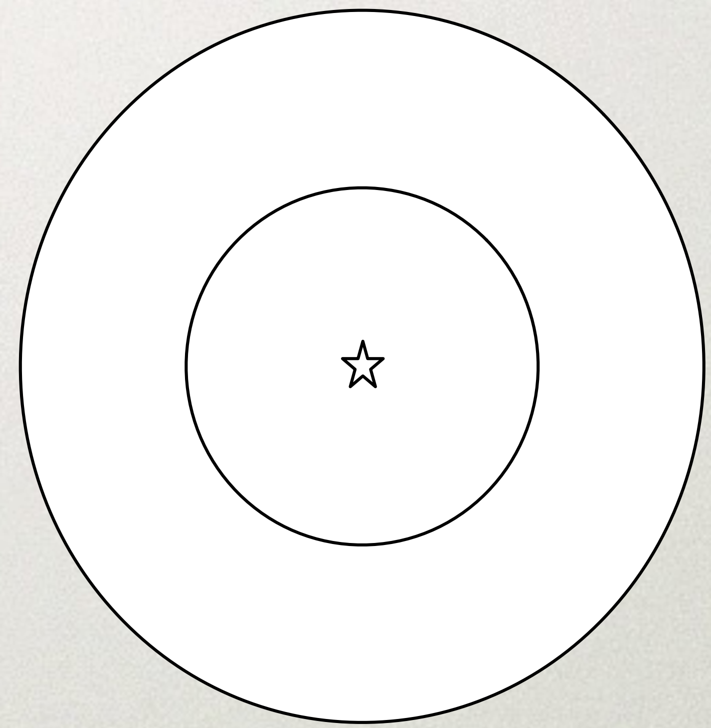
AdS



$\psi_\ell(t, r_i, \theta_i)$

Two Particle States
CoM at Origin

CFT



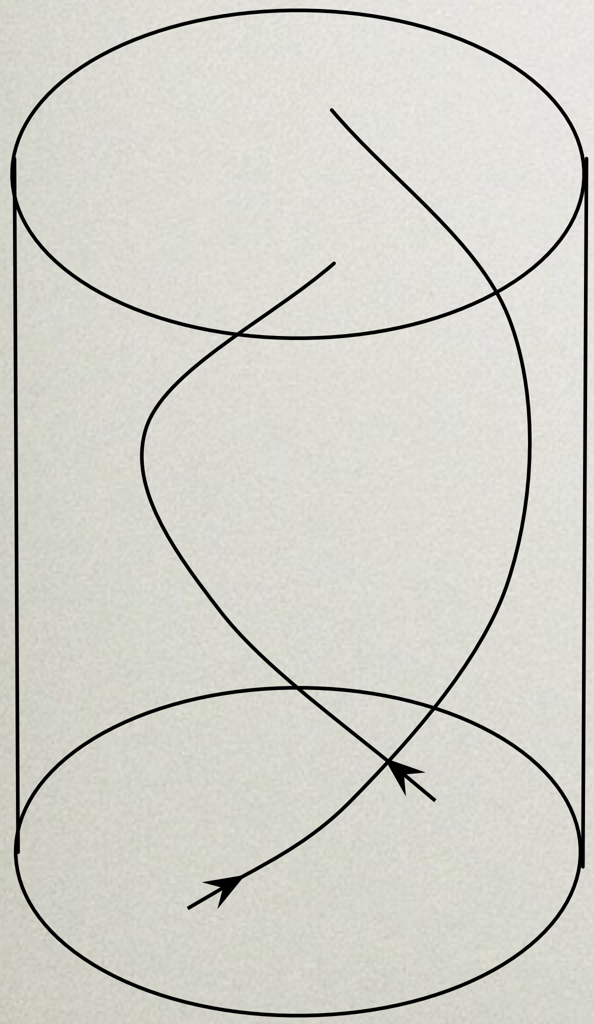
$\mathcal{O}\partial^\ell\mathcal{O}|0\rangle$

`Double-Trace' Primary

How to Define Distant Objects?

Geodesic separation between cat & dog:

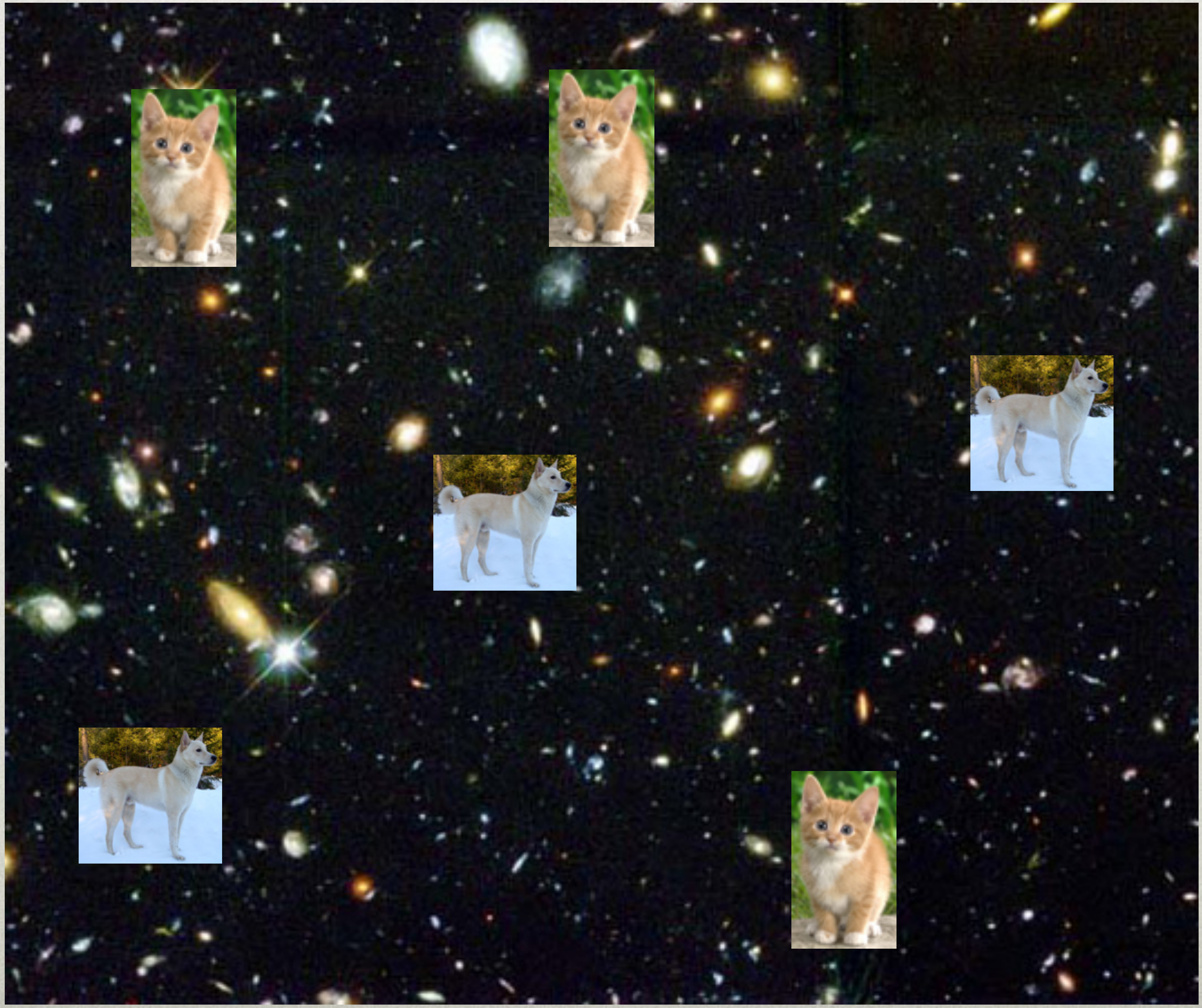
AdS



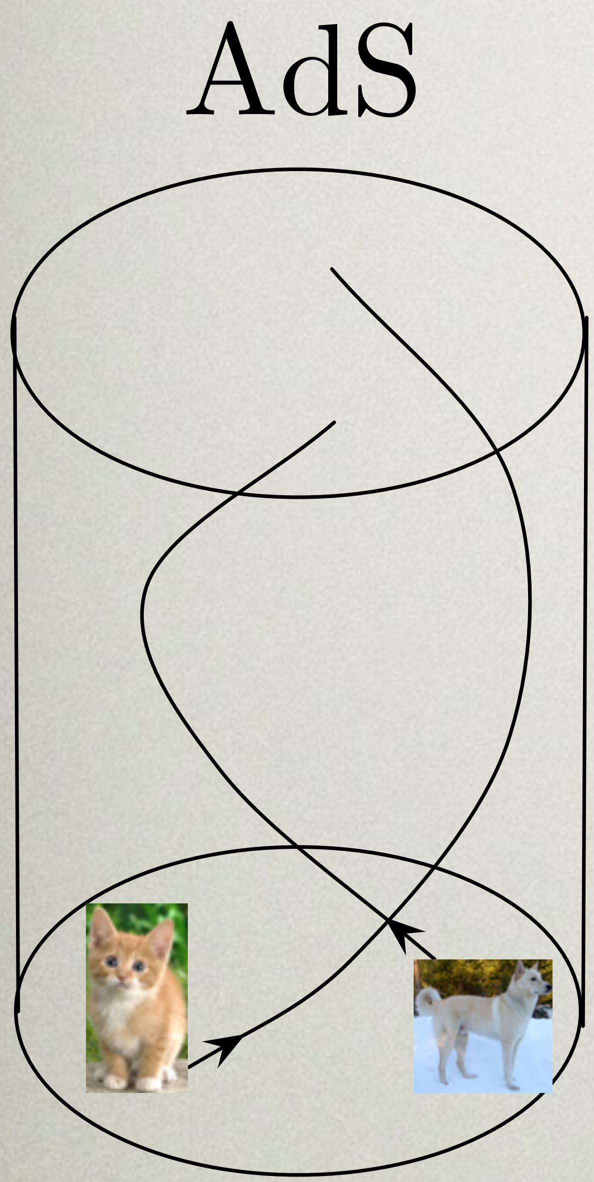
$$\kappa \sim R_{AdS} \log \ell$$

Are there multi-object states at **large angular momentum**???

A Fock Space at Large Separation?



Two Object Dynamics



AdS Energy = CFT Dimension:

$$E_{n,\ell} = E_c + E_d + E_{KE} + \gamma(n,\ell)$$

Anomalous dimension, $\gamma(n,\ell)$,
is a `**binding energy**'.

Existence of states as $\ell \to \infty$ with vanishing $\gamma(n,\ell)$ implies **AdS Cluster Decomposition**.

General Theorem (Any CFT in d>2)

Consider OPE of **any** two scalar primary operators:

$$\mathcal{O}_1(x)\mathcal{O}_2(0) = \sum_{\Delta,\ell} c^{12}_{\Delta,\ell}\mathcal{O}_{\Delta,\ell}(0)$$

For each n there **exists** a tower of operators/states

$$\mathcal{O}_{\Delta,\ell} \quad \text{with} \quad \Delta = \Delta_1 + \Delta_2 + 2n + \ell + \gamma(n,\ell)$$

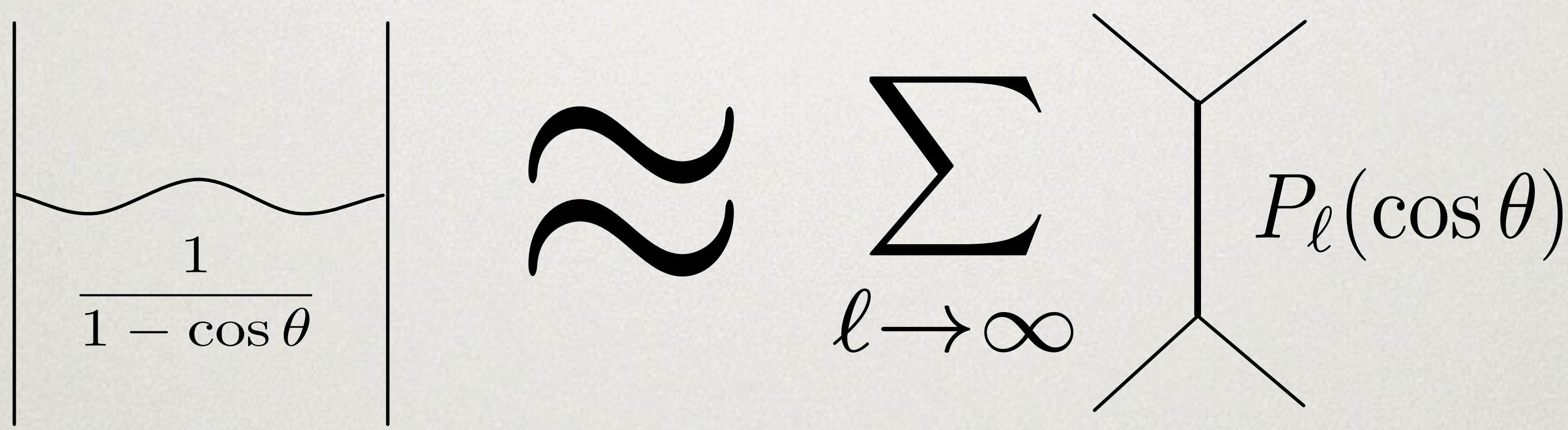
as $\ell \to \infty$, where the anomalous dimensions

$$\gamma(n,\ell) \to \frac{\gamma_n}{\ell^{\tau_m}} \quad \text{or} \quad \gamma_n e^{-\tau_m \kappa}$$

from leading twist exchange, generically $T_{\mu\nu}$

The Idea of the Proof: A Scattering Analogy

Free propagation and massless exchange require large amplitude at large ℓ, e.g.



Completely analogous CFT phenomenon. Implies existence and energy of large ℓ states.

Partial Wave Amplitudes —> Conformal Partial Waves

t-channel singularity —> **lightcone OPE** singularity

Gravity and the CFT Stress Tensor

In AdS/CFT, graviton states created by stress tensor.

$$g_{\mu\nu}(X) \leftrightarrow T_{\mu\nu}(x)$$

Graviton vertices must be universal:

$$\langle \mathcal{O}_H(\infty)\mathcal{O}_H(0)T_{\mu\nu}(1)\rangle \sim \frac{\Delta_H}{\sqrt{c}}$$

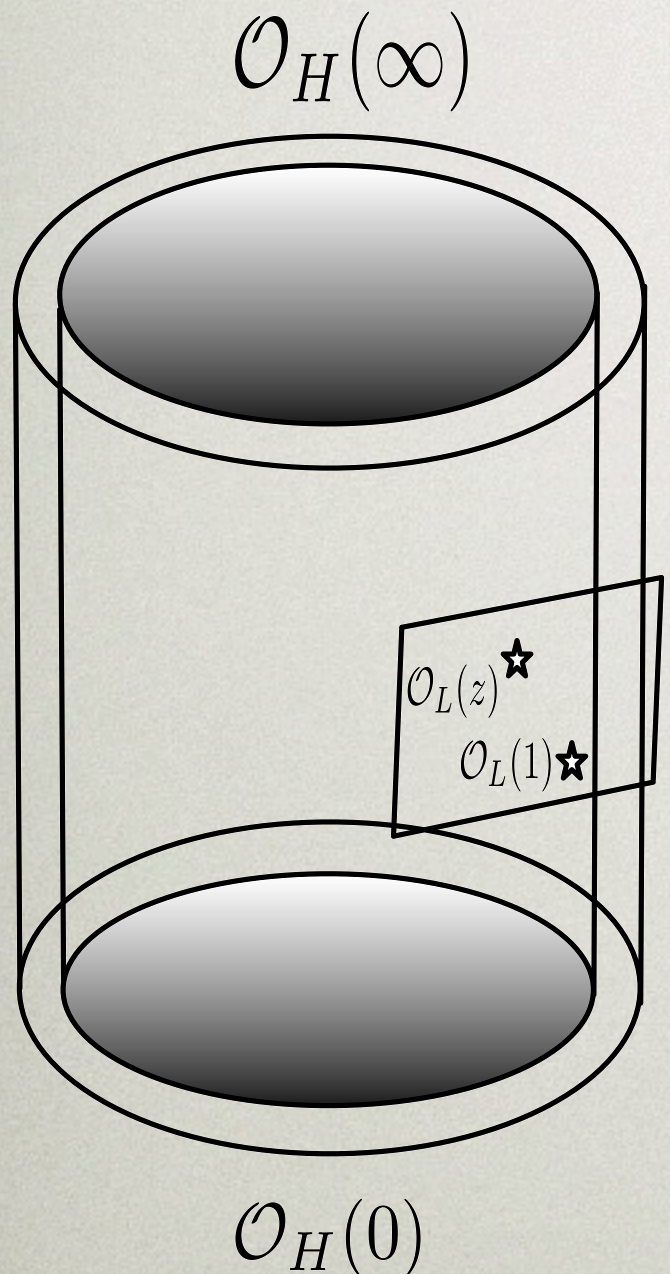
This comes from the stress tensor normalization

$$\langle T_{\mu\nu}T_{\alpha\beta}\rangle \sim c$$

plus Ward identities for conformal symmetries.

Long-range gravity is completely universal.

Probing Black Holes: the Heavy-Light Limit



$$\approx \langle \mathcal{O}_L(1)\mathcal{O}_L(z)\rangle_{T_H} \, ?$$

For example, a correlator in BTZ black hole background:

$$ds^2 = \left(r^2 + 1 - 24\frac{h_H}{c}\right)dt^2 + \frac{dr^2}{r^2 + 1 - 24\frac{h_H}{c}} + r^2 d\phi^2$$

We can! And in a general, theory-independent way…

A Universal Piece of All heavy-Light Correlators in 2D CFTs

The Conformal Partial Wave Decomposition

Natural to **organize amplitudes** into `blocks', ie irreducible representations of the **symmetry**.

In flat space scattering, these are partial waves.

Virasoro conformal blocks encapsulate contributions from all states related by Virasoro:

$$\langle \mathcal{O}_1(\infty)\mathcal{O}_2(1)\mathcal{O}_3(z)\mathcal{O}_4(0)\rangle = \sum_{h,\bar{h}} P_{h,\bar{h}} \mathcal{V}_{h_i,h,c}(z)\mathcal{V}_{\bar{h}_i,\bar{h},c}(\bar{z})$$

More powerful than global conformal—
Encapsulate all quantum gravitational effects!

Universality of the Vacuum Block

Always the case that

$$\langle \mathcal{O}_H \mathcal{O}_H \mathcal{O}_L \mathcal{O}_L \rangle = \langle \mathcal{O}_H \mathcal{O}_H \rangle \times \langle \mathcal{O}_L \mathcal{O}_L \rangle + \cdots$$

Thus the vacuum and its Virasoro descendants, aka the **vacuum block**, always contribute.

All Virasoro blocks, and in particular the vacuum block, have a natural interpretation in AdS/CFT.

‘Gravitons’ and Virasoro

‘Graviton’ states in 3d Gravity or 2d CFT…

$$g_{\mu\nu}(X) \leftrightarrow T_{\mu\nu}(x)$$

$$G_N = \frac{3}{2c}$$

In 2d, the stress tensor is purely (anti-)holomorphic

$$T(z) = \sum_n z^{-2-n} L_n$$

Virasoro generators are modes of stress tensor.

Gravitons and Virasoro

States created by acting at the origin, so

$$|\text{grav}\rangle = T(0)|0\rangle = L_{-2}|0\rangle$$

Global conformal generators are

$$L_1, L_0, L_{-1}$$

and these annihilate the vacuum. Other generators

$$L_{-2}, L_{-3}, \cdots$$

create `**gravitons'**, and do not annihilate the vacuum, as Virasoro is `spontaneously broken'.

Some Intuition for the Virasoro Algebra

Graviton correlators and interactions
are fixed by the Virasoro symmetry algebra:

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}n(n^2-1)\delta_{n,-m}$$

We are interested in large central charge:

$$c = \frac{3}{2G}$$

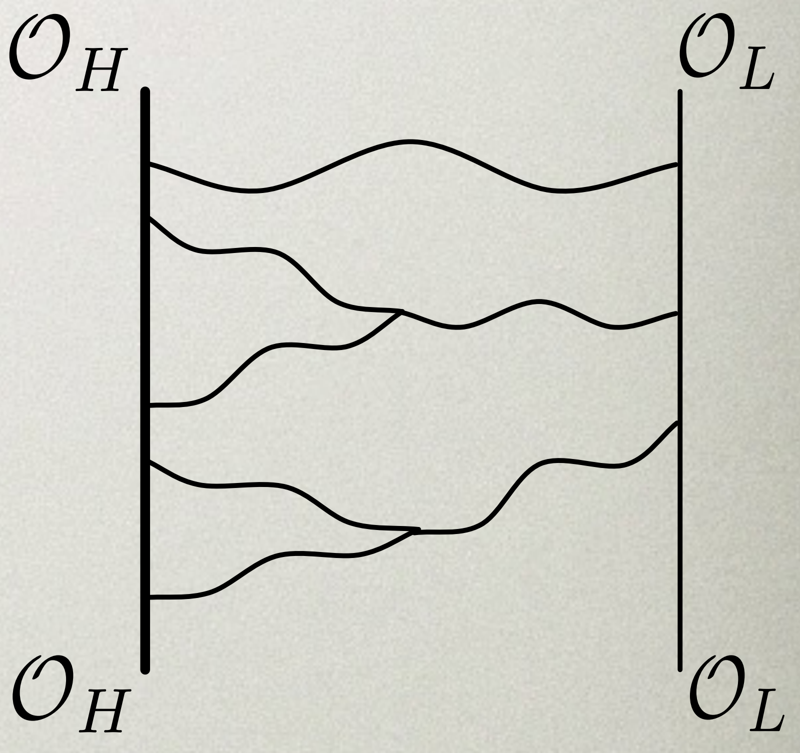
Virasoro approximately just **decoupled oscillators**…
(we will go far beyond this conceptually useful limit)

Virasoro Vacuum Block

Virasoro vacuum block = all graviton exchanges in AdS

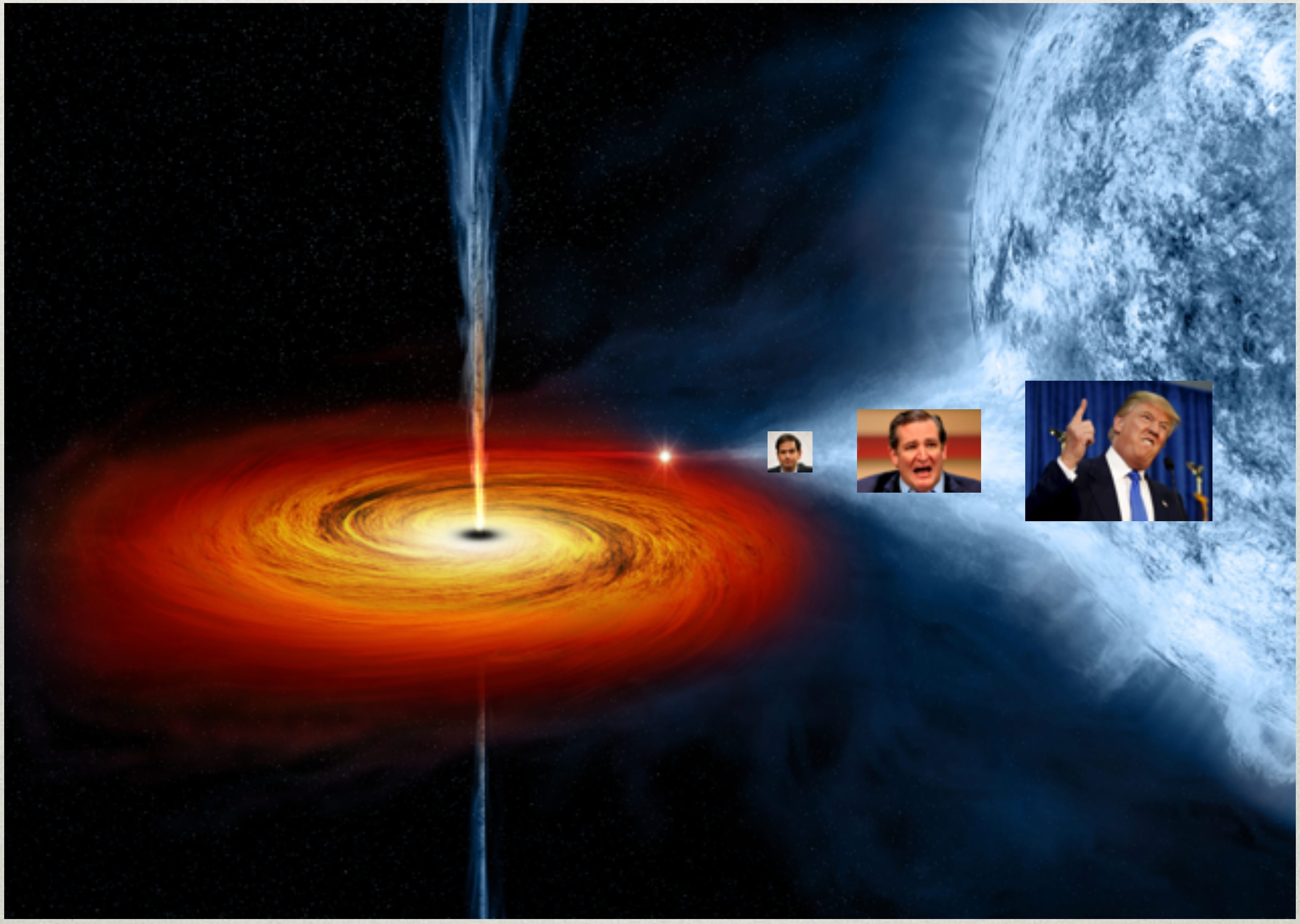
(exact projector)

$$\left\langle \mathcal{O}_H \mathcal{O}_H \left(\sum_{\{m_i,k_i\}} \frac{L_{-m_1}^{k_1} \cdots L_{-m_n}^{k_n} \rangle \langle L_{m'_{n'}}^{k'_{n'}} \cdots L_{m'_1}^{k'_1}}{\mathcal{N}_{\{m_i,k_i,m'_j,k'_j\}}} \right) \mathcal{O}_L \mathcal{O}_L \right\rangle \approx$$



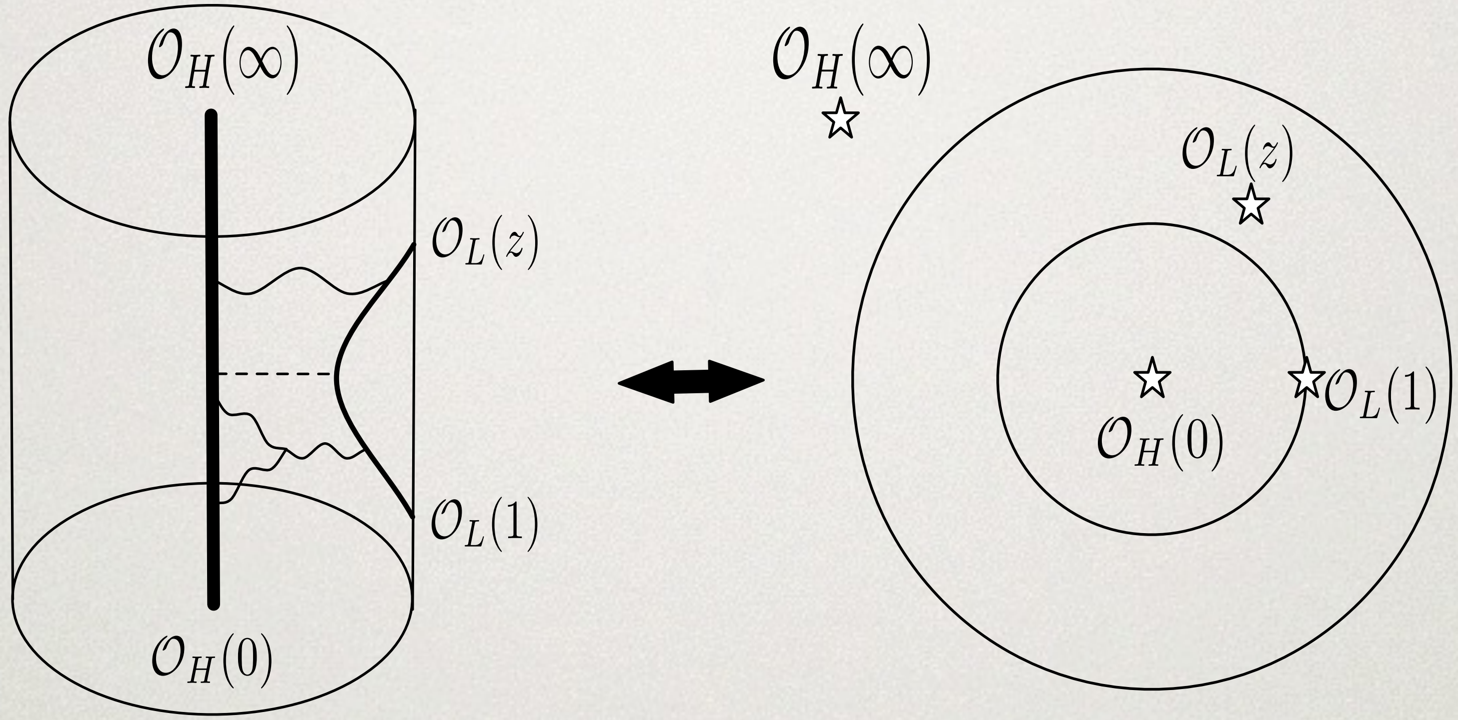
Light object in field of black hole in 3d AdS.

To Study Black Holes: Toss in Light Probes



Politics provides many examples.

Heavy-Light Semi-Classical Limit



$$h_H, \bar{h}_H \propto c \gg h_L, \bar{h}_L$$

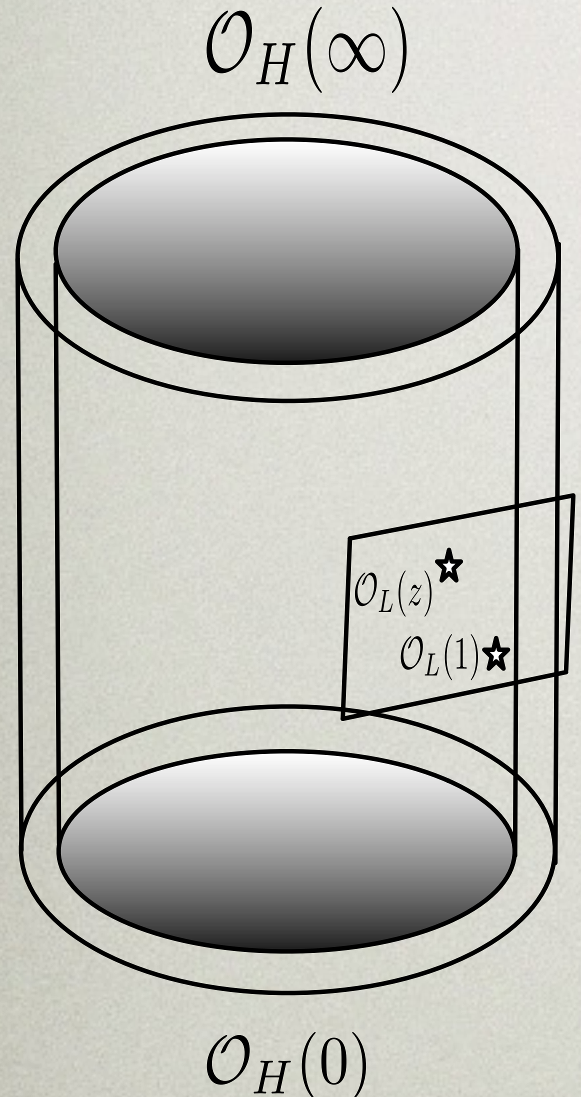
$$c \to \infty$$

How to compute?

Many ways, but no time to explain. On to the result…

The Heavy-Light Vacuum Block at Large C

The heavy-light Virasoro vacuum block is:



$$\mathcal{V}(t) = \left(\frac{\pi T_H}{\sin(\pi T_H t)} \right)^{2h_L}$$

with

$$T_H = \frac{1}{2\pi}\sqrt{24\frac{h_H}{c} - 1}$$

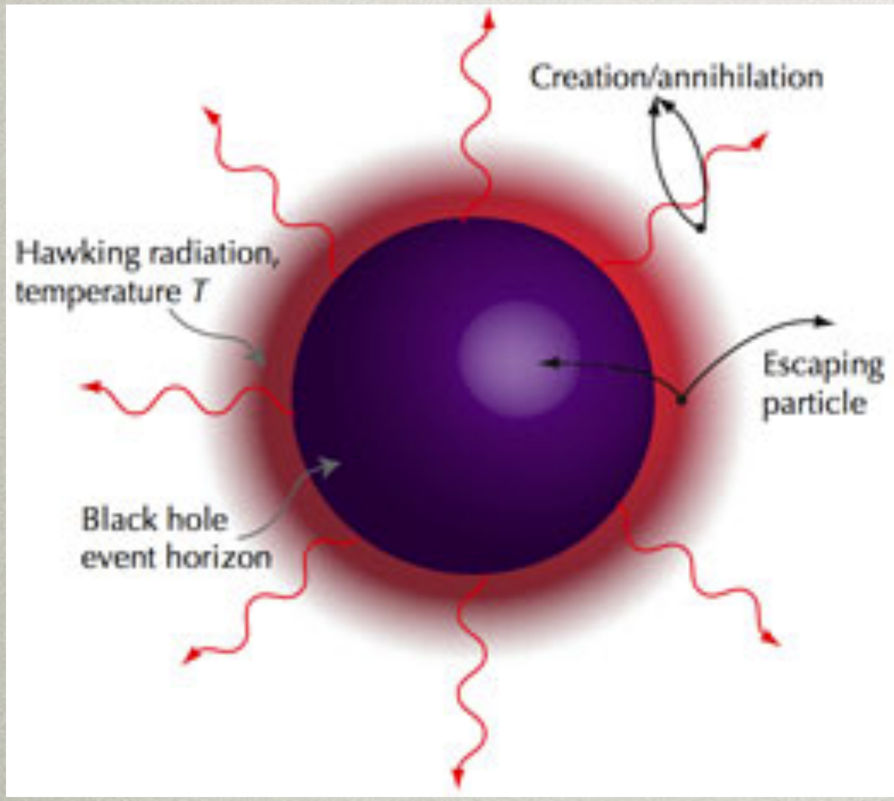
after we transform to the cylinder via

$$t = -\log(1-z)$$

‘Hard’ and ‘Easier’ Information Loss Problems

The 'Hard' Problem (not this talk)

This is the **information paradox**: how do we reconcile local, diff invariant QFT in the bulk with the unitarity of the quantum mechanical description.



versus

Hawking Radiation

Unitary CFT

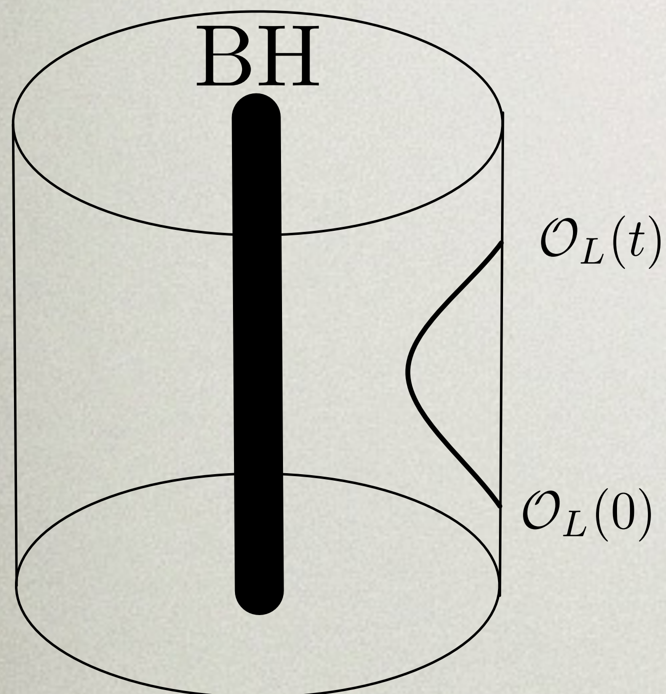
The 'Hard' Problem (not this talk)

From the modern (AdS/CFT) point of view, this is the **problem of bulk reconstruction** near and beyond horizons. **Hard to well-define the question!!**

Most potential resolutions give up local physics near the horizon (e.g. Firewalls) or augment quantum mechanics (e.g. final state or PR mirror operators).

An 'Easier' Problem

Easier because it involves **sharp signatures** in CFT.



Maldacena's version: Probe correlators in a black hole background **decay exponentially** at arbitrarily late times. Violates unitarity for a theory living on a compact space.

Summary: black hole physics is **too thermal**.

Let's think about another feature of thermality…

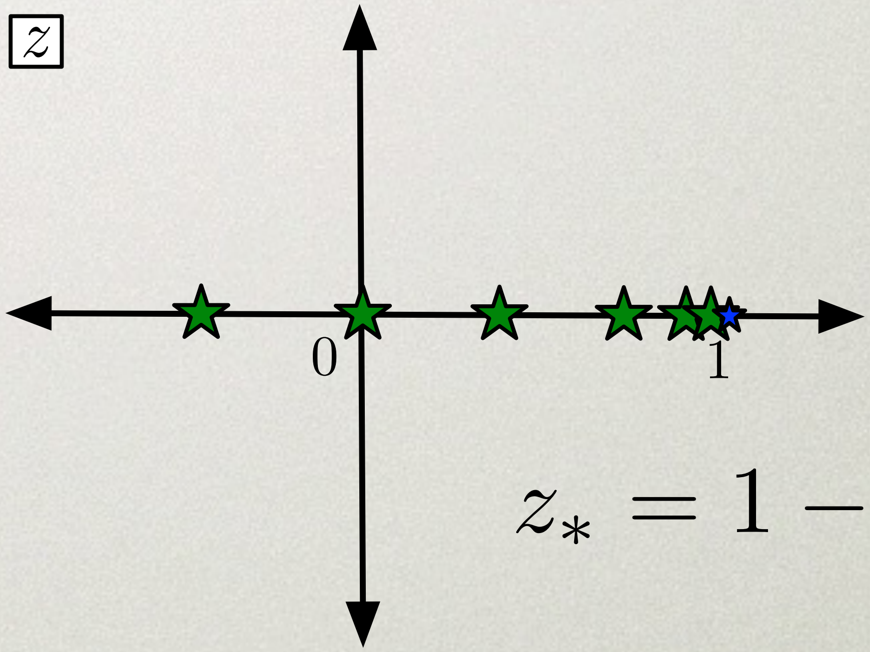
Another Version of the 'Easier' Problem

The KMS condition ~ **periodicity in Euclidean time**.
It arises geometrically from AdS Black Holes.

$$\langle \mathcal{O}(z)\mathcal{O}(0)\rangle_T$$

with

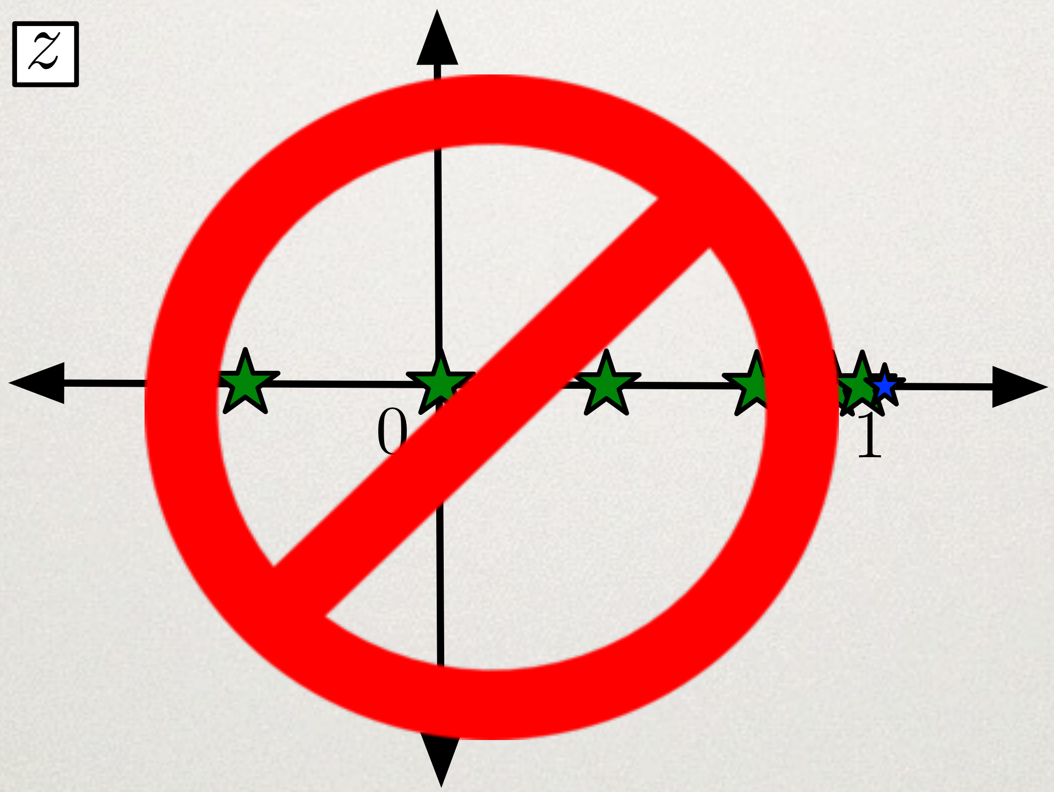
$$z = 1 - e^{-t}$$



$$z_* = 1 - e^{\frac{n}{T}}$$

Leads to **periodic images** of the **OPE singularity** in (the Euclidean region of) CFT correlators.
Allowed for correlators in the **canonical ensemble**.

An 'Easier' Problem: Forbidden Singularities

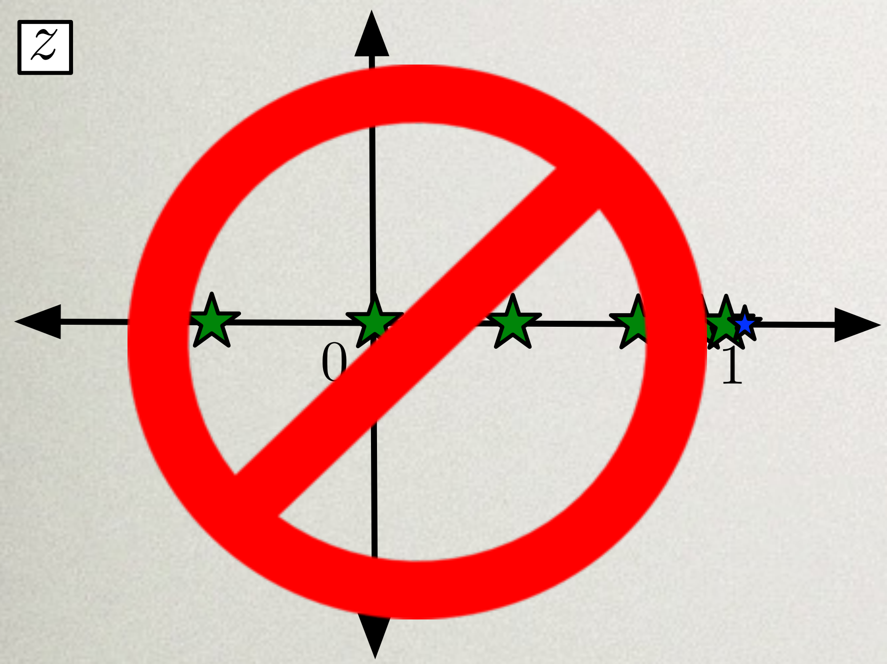


In pure state CFT correlators such as:

$$\langle \mathcal{O}_H(\infty)\mathcal{O}_L(1)\mathcal{O}_L(z)\mathcal{O}_H(0)\rangle$$

Only the true OPE singularities
are allowed in the Euclidean region!

Why Forbidden? Radial Quantization.



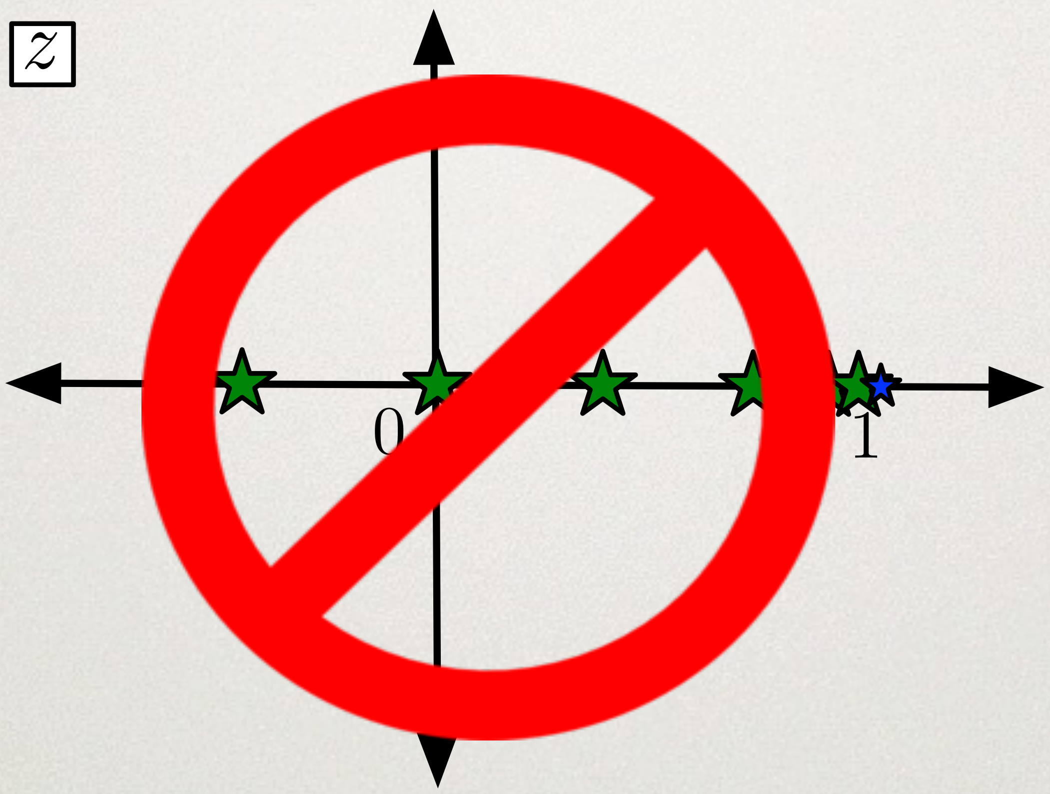
Away from OPE limits
we can view

$$\langle \mathcal{O}_H(\infty)\mathcal{O}_L(1)\mathcal{O}_L(z)\mathcal{O}_H(0)\rangle$$

as the inner product of normalizable states.

A theorem on Hilbert spaces: we can expand
in a basis and obtain a convergent sum.
No non-OPE singularities in Euclidean region!

Our 'Easier' Problem: Forbidden Singularities



Next we recall that at large central charge, correlators

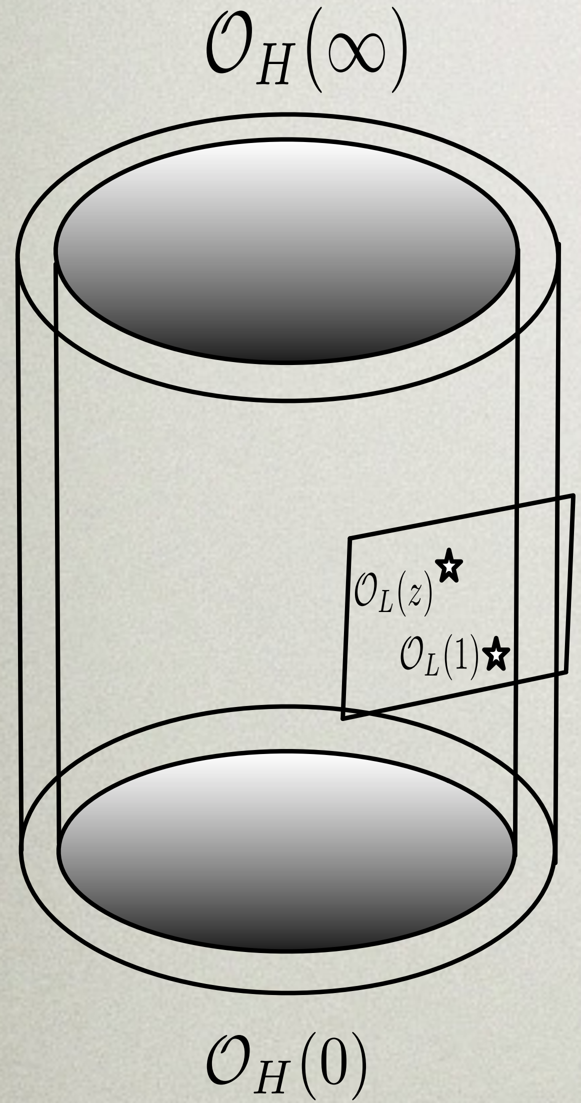
$$\langle \mathcal{O}_H(\infty)\mathcal{O}_L(1)\mathcal{O}_L(z)\mathcal{O}_H(0)\rangle$$

are, in fact, **'too thermal'** = have forbidden singularities.

Information Loss is a Universal Feature of Virasoro Blocks at Large C

The Heavy-Light Vacuum Block at Large C

The heavy-light Virasoro vacuum block is:



$$\mathcal{V}(t) = \left(\frac{\pi T_H}{\sin(\pi T_H t)} \right)^{2h_L}$$

with

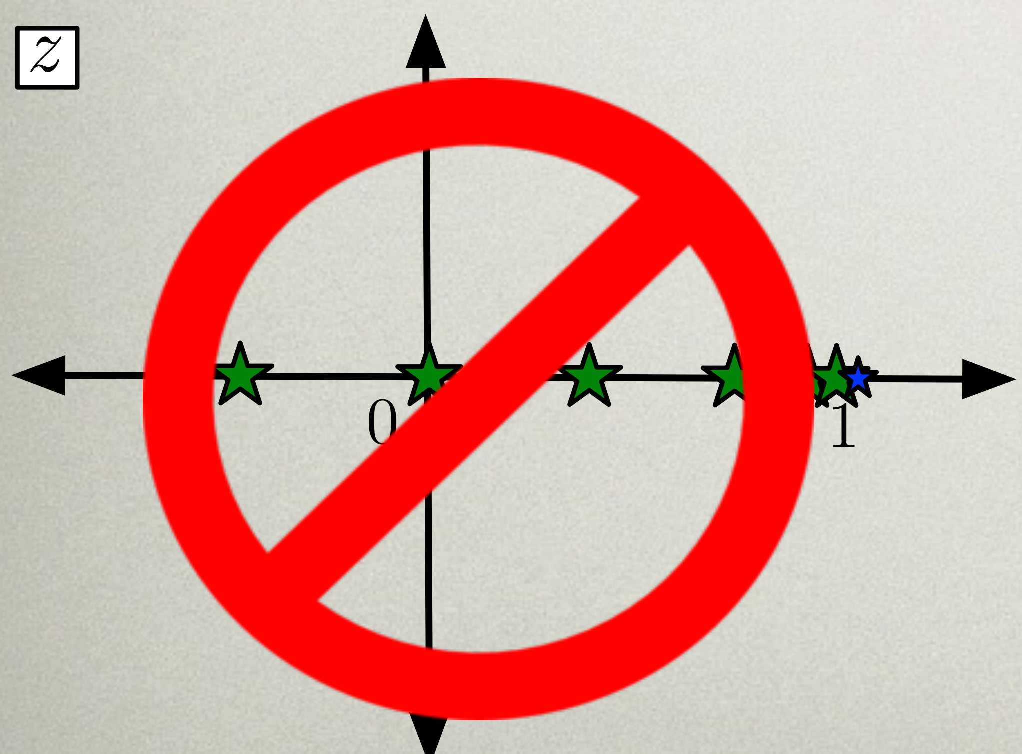
$$T_H = \frac{1}{2\pi}\sqrt{24\frac{h_H}{c} - 1}$$

after we transform to the cylinder via

$$t = -\log(1-z)$$

It's Too Hot For Its Own Good

A '**too thermal**' correlator at the Hawking temperature.
We see explicitly it has **forbidden singularities**.
Must be **resolved** at finite central charge,
and within its own universal structure.



$$\mathcal{V}(t) = \left(\frac{\pi T_H}{\sin(\pi T_H t)} \right)^{2h_L}$$

$$= \langle \mathcal{O}_L(t) \mathcal{O}_L(0) \rangle_{T_H}$$

All Blocks Decay at Large Lorentzian Time

We can also compute general heavy-light blocks (assuming light intermediate dimension):

$$\mathcal{V}_{h_I}(z) \propto \left(\frac{1-w}{1-z}\right)^{h_L} w^{h_I - 2h_L} {}_2F_1(h_I, h_I, 2h_I, w) \quad \text{with} \quad w = 1-(1-z)^{2\pi i T_H}$$

These have forbidden singularities/branch cuts too, but more importantly, at large Lorentzian times:

$$\mathcal{V}_{h_I}(z) \propto e^{-2\pi h_L T_H t_L}$$

So **every block** decays exponentially! Thus Maldacena's problem also occurs due (only) to the Virasoro blocks — it's **universal**.

Information Loss Is Due To The Virasoro Blocks

From CFT point of view, problem was to understand **why information loss ever occurs!**

The answer — it follows from expansion of blocks:

$$h_H \propto c, \quad c \to \infty$$

Equivalent to Perturbation theory in G_N

Doesn't seem to depend on the specific theory, as Virasoro structure is `kinematic'!

How can we obtain exact data on Virasoro Blocks and Resolve Information Loss?

Degenerate Operators

Null Virasoro Descendants? Study the determinant of:

$$\begin{pmatrix} \langle h|L_1^2 L_{-1}^2|h\rangle & \langle h|L_1^2 L_{-2}|h\rangle \\ \langle h|L_2 L_{-1}^2|h\rangle & \langle h|L_2 L_{-2}|h\rangle \end{pmatrix}$$

Indeed, there exist "degenerate states" such as

$$\left(L_{-1}^2 + b^2 L_{-2}\right)|h_{1,2}\rangle = 0$$

where the state must have dimension

$h_{1,2} = -\frac{1}{2} - \frac{3}{4b^2}$ and $c = 1 + 6\left(b + \frac{1}{b}\right)^2$

Degenerate States at Large Central Charge

Let's study the general case:

$$h_{r,s} \stackrel{c\to\infty}{\approx} \frac{c}{24}(1-r^2) + \frac{1-s}{2} + \frac{(r-1)(13+13r-12s)}{24} + \frac{3\left(r^2-s^2\right)}{2c} + \cdots$$

So there are **heavy** and **light** examples:

$$h_{r,1} \approx -\frac{c}{24}(r^2-1) \qquad \text{and} \qquad h_{1,s} \approx \frac{1-s}{2}$$

These are an infinite set of examples where we can obtain **exact** information about the Virasoro vacuum block.

But these Dimensions aren't even Unitary!?

The Virasoro conformal blocks are **analytic functions** of the external dimensions, and have only **simple poles** in the central charge.

Proof: q-expansion absolutely convergent, coefficients just polynomials in external dimensions.

Expect that via **analytic continuation**, degenerate correlators provide a lot of information.

But the real evidence comes from **explicit results**…

Connecting Degenerate State Correlators to Large C Results

Degenerate States

A useful piece of information:

$$\sum_{p_i} \frac{[(r-1)!]^2 \left(b^2\right)^{r-k}}{\prod_{i=1}^{k-1}(p_1+\cdots+p_i)(r-p_1-\cdots-p_i)} L_{-p_1}\cdots L_{-p_k}|h_{r,1}\rangle$$

[Benoit & Saint-Aubin]

this is a general formula for the null descendant, so it vanishes inside of correlators.

It provides an r^{th} **order differential equation** satisfied by the exact vacuum Virasoro block.

Heavy Degenerate Blocks at Large Central Charge

At **large c** we obtain the simpler differential equation:

$$\left(\partial_t - h_L g_r(t)\right)\mathcal{V}(t) = 0$$

$$g_r(t) = \coth\left(\frac{t}{2}\right) - r\coth\left(\frac{rt}{2}\right)$$

An r^{th} order equation has become first order!

Physical case just requires analytic continuation to:

$$r = 2\pi i T_H$$

The heavy-light vacuum block is the solution.

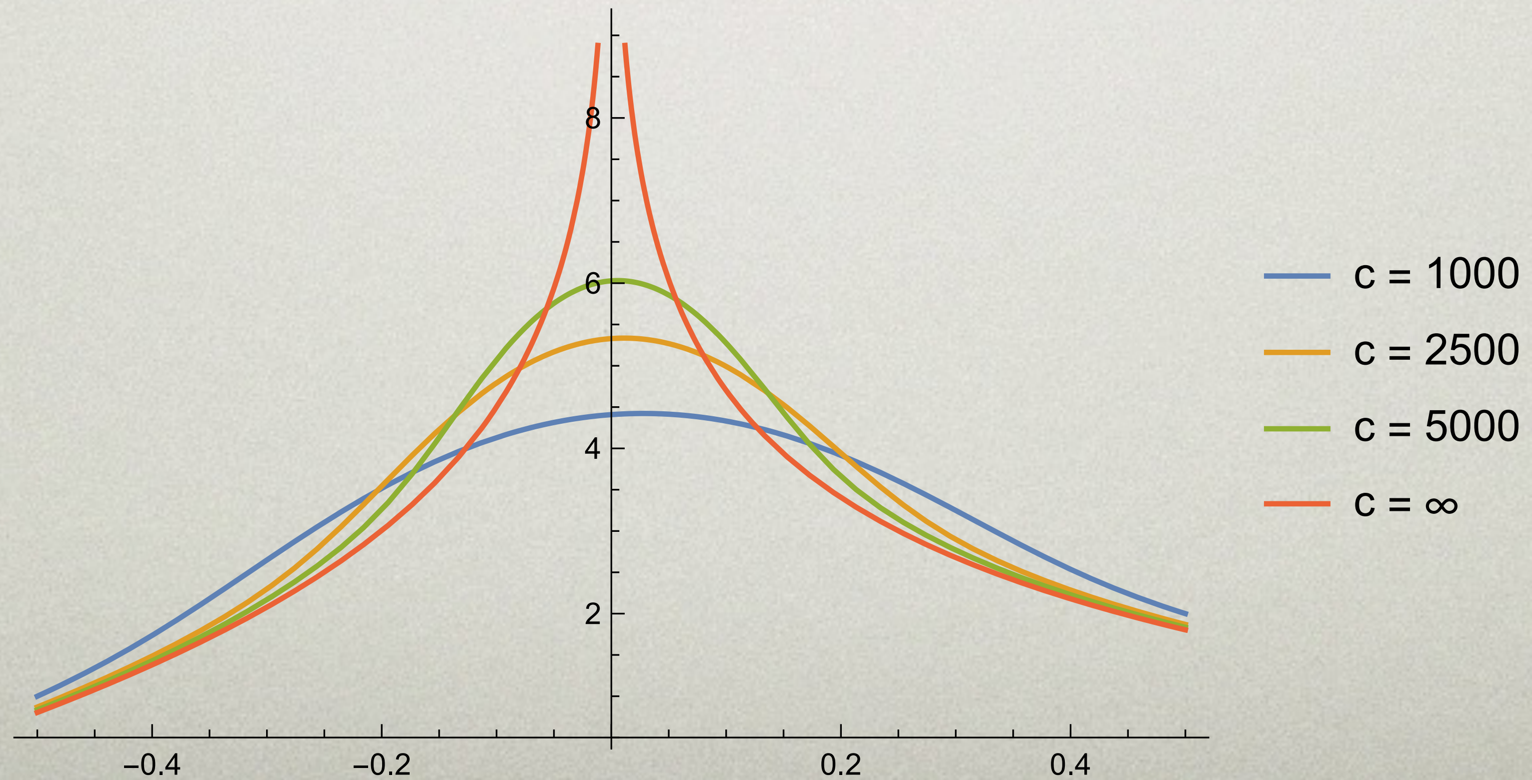
Resolution of CFT Information Loss Problems

Forbidden Singularities: An Explicit Example

Simplest example is the (2,1) case, with exact block:

$$\mathcal{V}_{2,1} = \frac{{}_2F_1(2, b^2+1, 2b^2+2, z)}{z^2} \quad \overset{c\to\infty}{\rightarrow} \quad \frac{1}{\sinh^2(t)}$$

where $h_L = 1$. In the vicinity of z = 2



Forbidden Singularity Resolution, in General

Singularities are **resolved in a universal way**.
Differential equation for correlators near singularities:

$$\sigma_n^2(r)\mathcal{V}''(x) - x\mathcal{V}'(x) - 2h_L\mathcal{V}(x) = 0$$

Putting c back, it can be solved by the function:

$$S(x) = \int_0^\infty dp\, p^{2h_L-1} e^{-px - \frac{\sigma_n^2(r)}{2c}p^2}$$

It's a natural toy model for an entire function that has a singularity at large c… but this is actually the physical result.

A Prediction for 1/c Perturbation Theory

Consider expanding this function in 1/c:

$$S(x) = \int_0^\infty dp\, p^{2h_L - 1} e^{-px - \frac{\sigma_n^2(r)}{2c} p^2}$$

$$\propto \frac{1}{x^{2h_L}} + \frac{\sigma_n^2(r)(2h_L + 1)h_L}{c\, x^{2h_L+2}} + \cdots$$

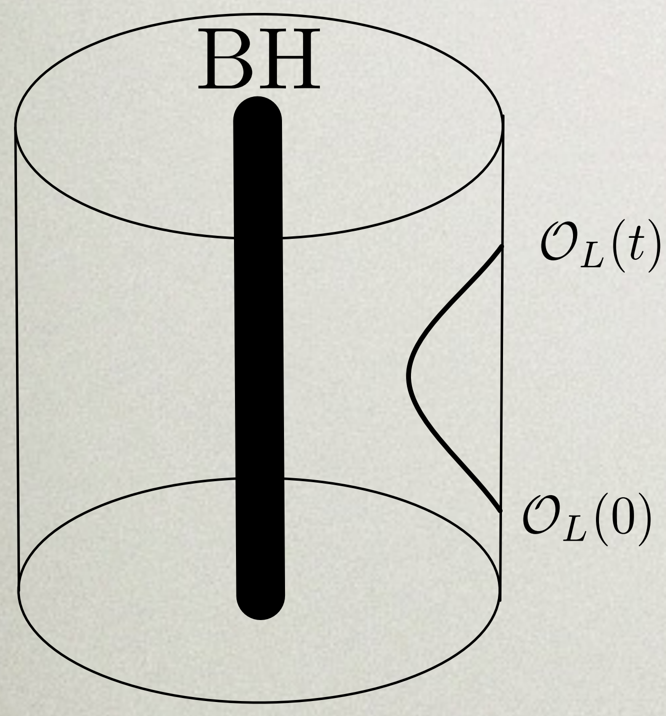
So it makes a **prediction** about form of 1/c corrections to the general heavy-light blocks.

We computed them, and they match.

Late Lorentzian Time Behavior

Late Time (Non-)Decay in the Lorentzian Region?

The large central charge blocks/correlators decay **exponentially**, at a rate we found earlier:



$$\mathcal{V} \sim e^{-2\pi h_L T_H t_L}$$

To avoid information loss this must cease before:

$$|\mathcal{V}| \sim e^{-S_{BH}}$$

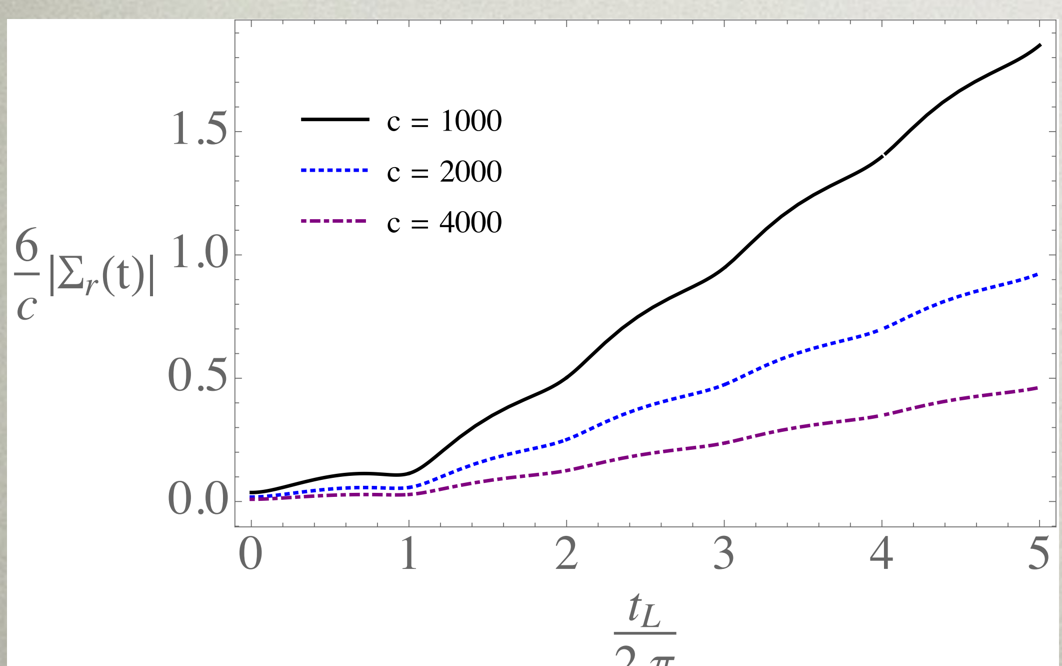
We can study the late time behavior via analytic continuation of our degenerate differential equations.

Late Time (Non-)Decay in the Lorentzian Region?

At NLO at large central charge, we find a general differential equation:

$$-h_L g_r(t)\mathcal{V}(t) + \mathcal{V}'(t) + \frac{\Sigma_r(t) + \Sigma_{-r}(t)}{b^2}\mathcal{V}''(t) = 0$$

The function Σ_{T_H} **grows linearly** at late time:



$$\frac{1}{c}\Sigma_{T_H}(t_L) \approx \frac{t_L}{cT_H} \propto \frac{t_L}{S_{BH}}$$

Late Time (Non-)Decay in the Lorentzian Region?

$$-h_L g_r(t)\mathcal{V}(t) + \mathcal{V}'(t) + \frac{\Sigma_r(t) + \Sigma_{-r}(t)}{b^2}\mathcal{V}''(t) = 0$$

The terms are all of the same order when:

$$\mathcal{V}'(t_L) \sim \frac{t_L}{S_{BH}}\mathcal{V}''(t_L)$$

which occurs precisely when

$$t_L = \frac{S_{BH}}{2\pi h_L T_H} \qquad \text{so that} \qquad \mathcal{V} \sim e^{-S_{BH}}$$

Effects that resolve forbidden singularites also qualitatively change late Lorentzian behavior!

Late Time (Non-)Decay in the Lorentzian Region?

Beyond times of order the BH entropy, we cannot truncate to a 2nd order differential equation — our approximations break down, as formally it seems we need an "infinite order differential equation".

This likely implies very chaotic behavior at even larger times, which is what we expect.

We haven't demonstrated this, but we'd like to!

Non-Perturbative Effects in C or ‘Solitons’ of ‘Gravitons’

Exact Block as a Sum over (AdS?) Solutions

At large c, we might write the asymptotic expansion

$$\mathcal{V}_0(z) = \mathcal{V}_{c=\infty}(z)\left(1 + \frac{f_1(z)}{c} + \cdots\right) + e^{-cs(z)}\left(g_0(z) + \frac{g_1(z)}{c} + \cdots\right) + \cdots$$

`Perturbative AdS Vacuum'

Exchange of `Heavy State'

Can accomplish this via **Borel resummation**.

But what are these heavy states, ie instanton geometries?

What Happens in the Degenerate Case?

The heavy state must make a contribution that solves the null descendant differential equation, so it must be:

$$\phi_{(2,1)} \times \phi_{(2,1)} = \mathbf{1} + \phi_{(3,1)}$$

For other degenerate correlators, find that the heavy states that make non-perturbative contributions obey fusion rules, as expected:

$$\phi_{(r,1)} \times \phi_{(r,1)} = \sum_{k=1}^{r} \phi_{(2k-1,1)}$$

A General Conjecture

We conjecture that the semi-classical `instantons' in the asymptotic expansion of the vacuum block look like

$$\sum_{k=1}^{\infty} \phi_{(2k-1,1)}$$

For general blocks of dimension h, we conjecture instantons have semi-classical dimension:

$$h_{inst}(h,k) \approx h + \frac{1}{6}\pi c k \left(\sqrt{\frac{24h}{c} - 1} + \pi k \right)$$

Infinite discretum of solutions to monodromy problem.

Information Loss Takeaway Points

Information Loss as 'Too Thermal' Correlators

(We study 2d CFTs because they have all the same information loss problems as in higher dimensions, yet we have far more powerful tools.)

Information loss manifests in CFT through correlators that are **too thermal**, with large central charge playing the role of a thermodynamic limit.

We will consider two sharp signatures of information loss — **forbidden singularities** in Euclidean correlators, and **exponential decay** at late times in Lorentzian correlators.

Information Loss is Universal in 2+1 Dims

Information loss from 3d black holes (via AdS/CFT) appears to be universal — independent of CFT data such as the spectrum and OPE coefficients. This might be expected based on the robustness of black hole physics, plus the fact that gravitational physics is **encapsulated by Virasoro symmetry**.

In fact, it seems that information loss can be understood based only on the Virasoro symmetry algebra at large central charge.

Information Loss Can be Resolved Explicitly

Information loss problems can be explicitly resolved in an infinite set of examples, which can be analytically continued in order to compute physical correlators. We find non-perturbative effects that resolve the forbidden singularities and alter the behavior of correlators at late times — specifically, when

$$t \sim S_{BH} \quad \text{and} \quad |\mathcal{V}| \sim e^{-S_{BH}}$$

When expanded at large central charge, the results look like a sum over (weird) geometries, and should tell us more about the gravitational path integral.

Conclusions

- Information loss is a consequence of the large central charge expansion of the Virasoro algebra.
- Possible to obtain non-perturbative results of the form $e^{-c} \sim e^{-1/G_N}$ resolving information loss.
- Need to explicitly derive the "instantons"! Bulk geometric interpretation may be odd. Provides a new, partial definition for the bulk path integral.
- We have "almost thermal" correlators — could provide a more precise picture of Papadodimas-Raju Mirror Operator Story, and its limitations?