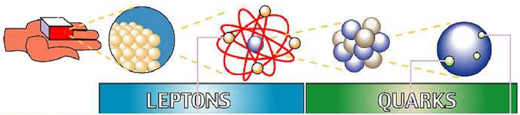
A unification of information and matter
A solution of chiral fermion problem

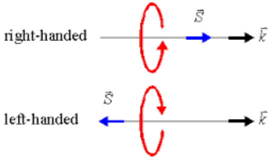
Xiao-Gang Wen MIT (2016)

How to gain a deeper understanding of our world?

A reductionist approach: Elementary particles

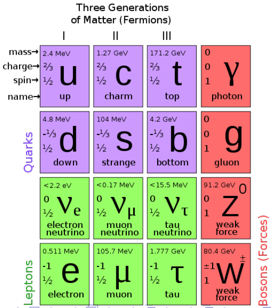
A deeper and more systematic understanding is gained by **dividing things into smaller parts**
until we reach the indivisible elements – *elementary particles*





- **Matter particles** (spin-$\frac{1}{2}$ fermions)
- leptons (electron-e, neutrino-ν, ...)
- quarks (up-u,down-d, ... with "color")

- **Force particles** (spin-1 gauge bosons)
- photon-γ: electromagnetic interaction
- gluon-g (colored): strong interaction
- Z, W: weak interaction



A unification between matter and information

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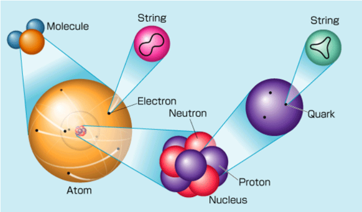
- **Information**: Changing information (qubits) → frequency
  According to quantum physics: frequency → energy $\hbar\omega = E$
  According to relativity: energy → mass → **Matter**

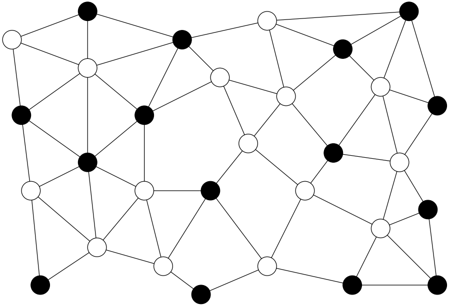
=



How does matter come from information?

- **Reductionist approach** assumes that space is EMPTY, and things placed in space are divisible.
- **Emergence approach** assume that space is a dynamical medium – an ocean of **qubits**, 0's and 1's
  Elementary particles are the motions, the defects, the "whirlpools", etc in the ocean of qubits.

  Space ∼ Ocean Elementary particles ∼ Bubbles in ocean



Do we really believe space to be an ocean of qubits

Can ocean of simple qubits (quantum information) really produce all kinds of matter and our rich world?

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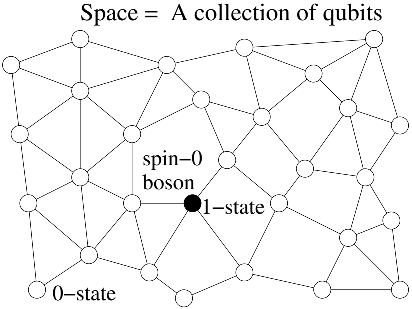
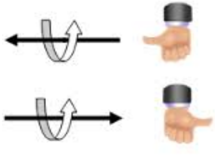
If all matter was formed by one kind of spin-0 bosons, then the ocean of simple qubits could indeed produce such bosonic elementary particle, and all kinds of matter.



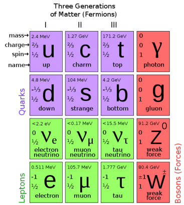
But our elementary particles are very complicated

Seven wonders of our universe:

1. Identical particles
2. Spin-1 bosons with only two-components
3. Particles with Fermi statistics
4. Fractional angular momentum (spin-1/2)



5. Only right-hand fermions couple the $SU(2)$ spin-1–bosons
6. Lorentz symmetry
7. Spin-2 bosons with only two-components (gravitons)



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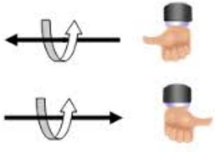

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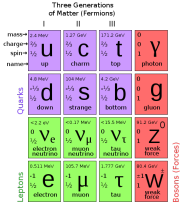

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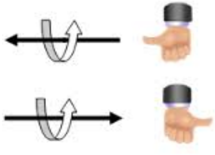

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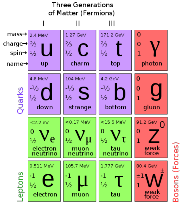

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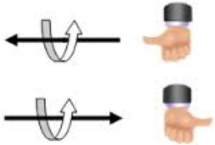

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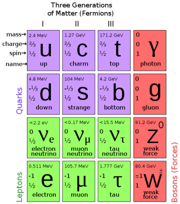
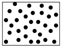
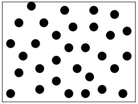


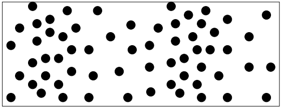
- **A "great unification"** Qubits unify gauge boson and fermion
- **A new view**: Our world is made of quantum information!

The magic of long-range entangled qubits $\to$ emergence of electromagnetic waves (photons)

- Wave in superfluid state $|\Phi_{\text{SF}}\rangle = \sum_{\text{all position conf.}} \big|$  $\big\rangle$:

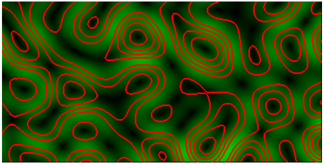


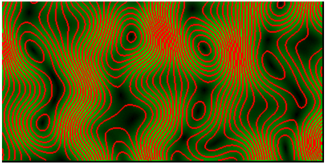
density fluctuations:
$\partial_t^2 \rho - \partial_x^2 \rho = 0$
$\to$ Longitudinal wave



Qubit-1's in the qubit ocean form closed strings

- Wave in closed-string liquid $|\Phi_{\text{string}}\rangle = \sum_{\text{closed strings}} \big|$  $\big\rangle$:

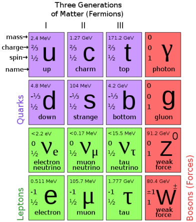


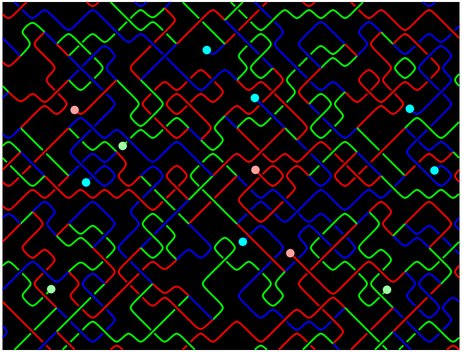


String density $\boldsymbol{E}(\boldsymbol{x})$ fluctuations $\to$ waves in string liquid. Closed strings $\to$ $\boldsymbol{\partial} \cdot \boldsymbol{E} = 0$ $\to$ **only two transverse modes**. $\to$
$\dot{\boldsymbol{E}} - \boldsymbol{\partial} \times \boldsymbol{B} = \dot{\boldsymbol{B}} + \boldsymbol{\partial} \times \boldsymbol{E} = \boldsymbol{\partial} \cdot \boldsymbol{B} = \boldsymbol{\partial} \cdot \boldsymbol{E} = 0$. ($\boldsymbol{E}$ electric field)

The magic of long-range entangled qubits
$\to$ Emergence of Yang-Mills theory (gluons)

- If string has different types and can branch
  $\to$ string-net liquid $\to$ Yang-Mills theory
- Different ways that strings join $\to$ different gauge groups
- String types $\to$ representations of gauge group.





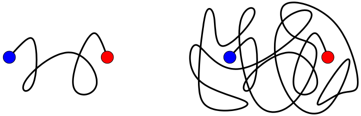
A picture of our vacuum

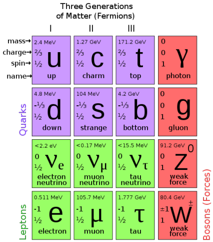
A string–net theory of light and electrons

- **String-loops $\to$ electromagnetic int.** γ
- **String-nets $\to$ Strong** g **and weak** W, Z **interactions**

Wen hep-th/01090120, hep-th/0302201; Levin-Wen cond-mat/0404617

The magic of long-range entangled qubits
→ Emergence of Fermi statistics





- In string liquids, the ends of string behave like point particles (gauge charges).
- String attached to the particle does not cost energy, but can give Fermi statistics to the particle
  End of string = fermion (electron & quark).
- **A unification of gauge interactions and Fermi statistics**

Levin-Wen cond-mat/0302460, cond-mat/0407140

The magic of long-range entangled qubits → emergence of (linearized) Einstein gravity

- In Gu-Wen arXiv:gr-qc/0606100; arXiv:0907.1203,
  we designed a lattice model with field $h_{ij} = h_{ji}$
  → helicity $0; 0, \pm 1, \pm 2$.
- The helicity $0; 0, \pm 1$ modes are strongly fluctuating, *and strongly interacting with helicity ± 2 modes*.
- The helicity ± 2 modes are weakly fluctuating.
- The helicity $0; 0, \pm 1$ modes aquire a large mass gap, and only the helicity ± 2 modes survive as low energy mode with $\omega \sim k$ dispersion(?).
- The gaplessness of the helicity ± 2 modes is robust against any lattice perturbation. There is an emergent (linearized) diffeomorphism gauge symmetry in the low energy effective theory of the helicity ± 2 modes → linearized Einstein gravity(?).
  Weinberg-Witten theorem → no emergent Einstein gravity?

The magic of long-range entangled qubits $\to$ emergence of $\omega \sim k^3$ gravitons

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Xu arXiv:cond-mat/0602443

Lattice model

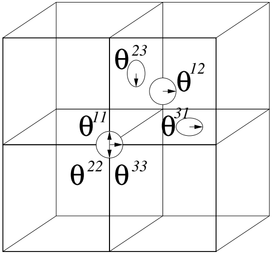
Each vertex: three rotors $(\theta^{aa}, L_{aa}), aa = 11, 22, 33$.
Each face: one rotor (θ^{ab}, L_{ab}), $ab = 12, 23, 31$.
$\mathcal{L} = \sum L_{ab}\dot{\theta}^{ab} - \text{Complicated } H$
Total six modes (spin waves) with helicity $0, 0, \pm 1, \pm 2$

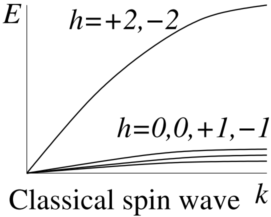
$$\mathcal{L}_I = L_{ab}\dot{\theta}^{ab} - \left[(L_{ab})^2 - \frac{(L_{aa})^2}{2}\right] - \theta^{ab}R^{ab} - U(\partial_a L_{ab})^2 - U(R^{aa})^2 + \cdots$$

$$\mathcal{L}_{II} = L_{ab}\dot{\theta}^{ab} - \left[\epsilon^{imn}\partial_m(L_{nj} - \frac{\delta_{nj}L_{aa})}{2})\right]^2 - (R^{ab})^2 - U(\partial_a L_{ab})^2 - U(R^{aa})^2 + \cdots$$

where $R^{ab} = \epsilon^{ahc}\epsilon^{bdg}\partial_h\partial_d\theta^{gc}$.

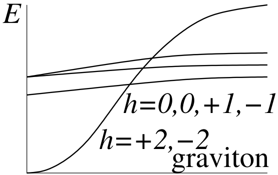
- The helicity ± 2 modes are classical and the classical picture is valid.





Classical spin wave

- A $h=0$ mode is described by (θ, L):
  $L_{ab} = (\delta_{ab}\partial^2 - \partial_a\partial_b)L$,
  $\theta = (\delta_{ab}\partial^2 - \partial_a\partial_b)\theta^{ab} = R^{aa}$
  Quantum fluctuations: $\delta\theta = 0, \quad \delta L = \infty$
  To have gap:
  θ^{ab} must be discretized $\Delta\theta^{ab} = 2\pi/n_G$
  L_{ab} must be compactified $L \sim L + n_G$
  Constraint and gauge transformation:
  $(\delta_{ab}\partial^2 - \partial_a\partial_b)\theta^{ab} = 0$,
  $L_{ab} \to L_{ab} + (\delta_{ab}\partial^2 - \partial_a\partial_b)L$
- $h = 0, \pm 1$ modes are described by (θ^a, L_a):
  $\theta^{ab} = \partial_a\theta^b + \partial_b\theta^a$, $L_a = \partial_b L_{ab}$
  Quantum fluctuations: $\delta L_a = 0, \quad \delta\theta^a = \infty$
  L_a is discrete $\to$ gap.
  Constraint and gauge transformation:
  $L_a = \partial_b L_{ab} = 0$, $\theta^{ab} \to \theta^{ab} + \partial_a\theta^b + \partial_b\theta^a$



Partial quantum freeze

$\mathcal{L}_{I,II}$ + Constraint and gauge trans. = linearized Einstein or Horava-Lifshitz gravity with $\theta^{ij} \sim g^{ij} - \delta^{ij}$

Emergence of chiral fermions

The standard model is not well defined theoretically

- Calculations of the standard model are based on perturbation $\rightarrow$ the standard model is only defined perturbatively.
- But the perturbative expension does not converge $\rightarrow$ the standard model is not well defined (non-perturbatively)

Chiral fermion problem:

How to put the standard model on lattice.

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If we achieve that, then we show **matter = information**

- Each lattice point has finite degrees of freemdom = a few qubits.
- The dynamics of the quibts is described by our lattice model
  $\to$ low energy excitations of lattice qubit model will be the elementary particles described by the standard model.

Qubit $\to$ matter (elementary particles)

Chiral $SO(10)$ GUT on lattice

- We like construct a lattice fermion model with $SO(10)$ symmetry, such that its low energy excitations are described by R-hand massless Weyl fermions that form the 16-dim representation of the $SO(10)$: $\mathcal{H} = \psi^\dagger_{R\alpha}\sigma^i\partial_i\psi_{R\alpha}$ where σ^i are Pauli matrices.

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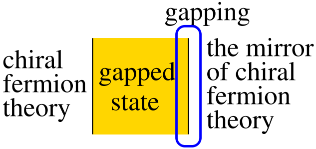
- There is a non-interacting lattice fermion model with $SO(10)$ symmetry, such that its low energy excitations are described by R-hand + L-rand massless Weyl fermions which both form the 16-dim representation of the $SO(10)$: $\mathcal{H} = \psi^\dagger_{R\alpha}\sigma^i\partial_i\psi_{R\alpha} + \psi^\dagger_{L\alpha}(-)\sigma^i\partial_i\psi_{L\alpha}$.

Mirror fermion construction



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- The R-fermions are on one surface, and the L-fermions are on the other surface.

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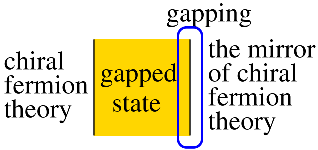

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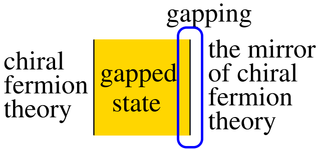

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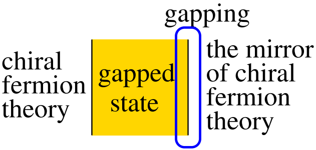

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Composite fermion in 1+1D

- Consider 3-4-5-2 chiral fermion model + mirror sector in 1+1D: two right-moving mirror fermions (spin $1/2$) ψ_3 and ψ_4 of $U(1)$ charge 3 and 4
two left-moving mirror fermions (spin $-1/2$) $\bar\psi_5$ and $\bar\psi_2$ of $U(1)$ charge -5 and -2.
- The composite fermions are

$$\bar\chi_3 = \bar\psi_2(\psi_4\bar\psi_5), \qquad \bar\chi_4 = \bar\psi_2(\psi_3\bar\psi_5), \qquad \text{spin } -\frac{1}{2}$$

$$\chi_5 = \psi_4(\psi_4^*\bar\psi_5^*), \qquad \chi_2 = \psi_3(\psi_4\bar\psi_5), \qquad \text{spin } \frac{1}{2}$$

- The composite fermions and the mirror fermions can be fully gapped by the mass term $\bar\chi_3\psi_3 + \bar\chi_4\psi_4 + \bar\psi_5\chi_5 + \bar\psi_2\chi_2$
$\to$ the chiral 3-4-5-2 model in 1+1D can be defined on lattice.
This result is incorrect since the 3-4-5-2 model in 1+1D has an $U(1)$ gauge anomaly.
- Interaction can gap out the mirror sector **???**

Mirror fermion construction II

- Here I like to argue that the idea of using interaction to gap out the mirror sector still works, but the condition from the composite fermion construction:
  "A mirror sector with symmetry G can be gapped out without breaking the symmetry if there exists composite fermions such that we can find symmetric mass terms of composite/mirror fermions to gap out all the composite/mirror fermions."
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  Wen arXiv:1305.1045

  A mirror sector with symmetry G can be gapped out without breaking the symmetry if (1) there exists a (possiblly symmetry breaking) mass term to give all mirror fermions a mass, and (2) $\pi_n(G/G_m) = 0$ for $n = 0, 1, 2, 3, 4, 5$ where G_m is the group of unbroken symmetry.

- For our example $G = SO(10)$, $\delta\mathcal{H} = \psi^T_{L\alpha} \mathrm{i}\sigma^2 h^a \Gamma^{\alpha\beta}_a \psi_{L\beta} + h.c.$, and $G_m = SO(9)$. $SO(10)/SO(9) = S^9$. The $SO(10)$ mirror sector can be gapped out.

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- The above arguement has a loophole. A random configuration of $h^a(x^\mu)$ in 3+1D space-time may trap point, line,... topological defects. $h^a(x^\mu) = 0$ at the center of defects, which may trap fermion zero modes. Those fermion zero modes may become the low energy excitations in the disordered phase.

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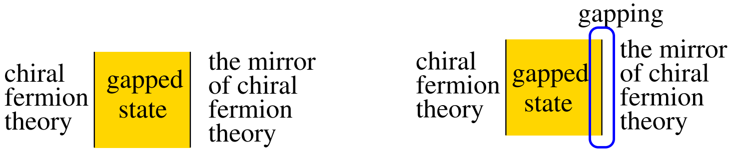
- But for our case, h^a lives on S^9 and $\pi_n(S^9) = 0$ for $n < 9$. Thus there is no point defect since $\pi_3(S^9) = 0$,
  there is no line defect since $\pi_2(S^9) = 0$,
  there is no sheet defect since $\pi_1(S^9) = 0$,
  there is no domain-wall defect since $\pi_0(S^9) = 0$.

An arguement for the conjecture (continued)



- After integrating out the massive mirror fermions, the effective S^9 non-linear σ-model for $h^a(x^\mu)$ at the 3+1D boundary may contain θ-topological term and/or WZW-topological term. But since $\pi_4(S^9) = 0$ and $\pi_5(S^9) = 0$, both topological terms do not exist. The S^9 non-linear σ-model for $h^a(x^\mu)$ can have a disordered phase that do not break the $SO(10)$ symmetry.

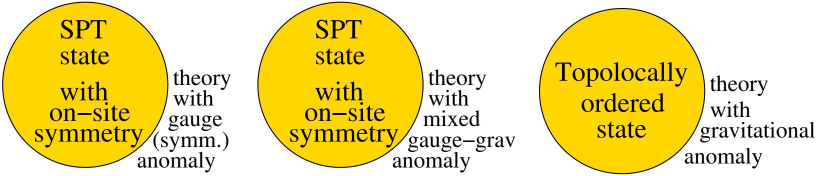
The Weyl fermions in 16-dim rep. of $SO(10)$ can be defined on lattice. It does not have any known and unknown anomalies.

Anomaly and gapped quantum phases

- If the chiral fermion theory on the boundary has anomaly, the gapped bulk state will be in a non-trivial topologial phase.

- This picture is very general and applies to both fermionic and bosonic systems

Wen arXiv:1303.1803



- If the bulk phase is trivial, then its boundary can be gapped without breaking the symmetry.
- If a chiral theory is free of all anomalies, then its can always be put on lattice. $\rightarrow$ **The solution of chiral fermion problem.**