Remarks on Emergence
and
Emergent conformal symmetry in quantum mechanics
and
near extremal black holes

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Emergence

- Some property of the system that is not obvious from the fundamental laws.

- Some property we hope we can explain with the fundamental laws but we do not have a clear idea on how to do it…

Examples

- Thermodynamics. 2nd Law of thermodynamics. Irreversibility.
- Hydrodynamics.
- Phase transitions.
- 2nd order phase transitions and critical behavior → conformal symmetry.

- In many cases we find new symmetries that were not obvious in the original laws. E.g. Conformal symmetry in 2nd order phase transitions.

- In condensed matter physics or biology → we think we know the basic laws. So emergence is key.

More is different

Anderson

Fundamental physics

- The fundamental theory could be very different from what we observe at low energies.
- Eg QCD is very different from nuclear physics or hadron physics. Confinement → emergent phenomenon.

- As we go to shorter distances, do we search for more symmetries ?

- Electroweak unification → grand unification → some overarching natural symmetry ?

- These are gauge symmetries, not fundamental symmetries. At short distances quantum gravity renders the short distance concept invalid.

Emergent gauge symmetries

- In condensed matter systems it is common to have emergent gauge symmetries. Fractional quantum Hall effect → emergent Chern Simons gauge theories.
- The original system does not have a global symmetry associated to the emergent symmetry.
- E.g. CP^N model → emergent U(1) gauge field.

- Higgs ? Hierarchy problem ?

- How do strongly coupled field theories behave ?
- CFT → bootstrap → concrete results for the simplest theories.

Polyakov
Belavin, Zamolodchikov, Polyakov
Rattazzi, Rychkov, Tonni, Vichi
El Showk, Paulos, Simmons-Duffin,
Costa, Penedones, Poland, Kos,
Fitzpatrick, Kaplan, ….

Gauge gravity duality and emergent spacetime

- Start form a quantum field theory without gravity in d dimensions.
- Get a gravity theory in d+1 dimensions.

- Emergent gauge symmetry: reparametrizations in higher dimensions, and gravity. Spacetime emerges together with the associated gauge symmetry.

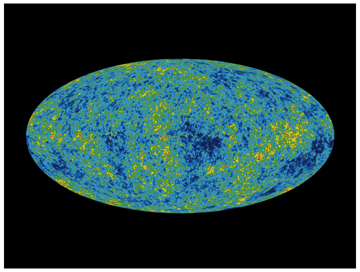
- Bulk duals are a weird phenomenon that can arise in very strongly coupled theories

- There are probably many weird quantum phases to be discovered!

- The fundamental laws of our universe can have an alternative formulation where the usual concepts of space-time emerge. Might be crucial for solving singularities.

Emergent symmetries in gravity

- We can have solutions that preserve some interesting parts of the full reparametrization symmetry.
- Flat space → Lorentz
- De Sitter. Inflation → approximate scale invariance → scale invariance in the sky!



- Near extremal black holes → also a scaling symmetry

Importance of quantum mechanics

- For many properties quantum mechanics is crucial.
- It is useful to think about entanglement and other notions from quantum information theory.
- ~~Follow the money~~
- → Follow the qubit

Entanglement and the gauge gravity duality

- Entanglement entropy of subregions $\rightarrow$ geometric property of the bulk.
- Entanglement and geometry

Local boundary quantum bits are highly interacting and very entangled

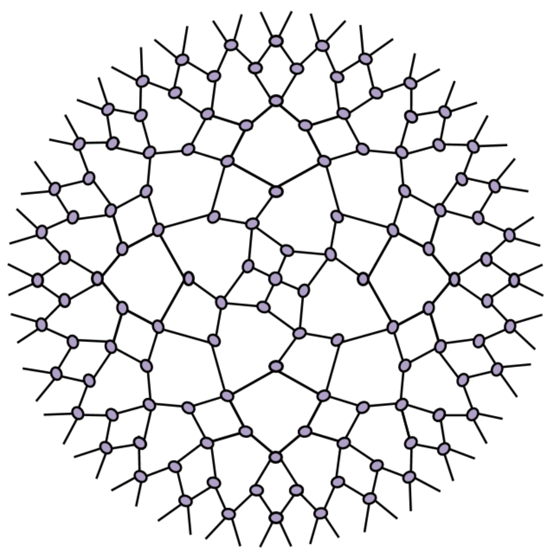
Ryu, Takayanagi, Hubbeny, Rangamani

$$S(R) = \frac{A_{\rm min}}{4G_N}$$

Generalization of the Black hole entropy formula.

Tensor networks and the emergence of space

- Tensor networks: practical technique to write numerical wavefunctions of manybody systems.



White
Vidal
Swingle
Qi
Harlow, Pastawski,
Preskill, Yoshida
....

- The network has a structure or ``geometry'' that is constrained by the entanglement properties of the system.

Black hole information problem

- We think that black holes as seen from the outside can be described as ordinary quantum systems with a finite number of qubits = Area in Planck units. This is consistent with unitarity in its evolution.

- Gravity says that the interior is smoothly connected to the exterior. This is the equivalence principle.

- How is the interior described in the same variables where the entropy (or unitarity) is manifest ?

Hawking
Mathur,
Almheiri, Marolf, Polchinski,
Sully, Stanford, Wall

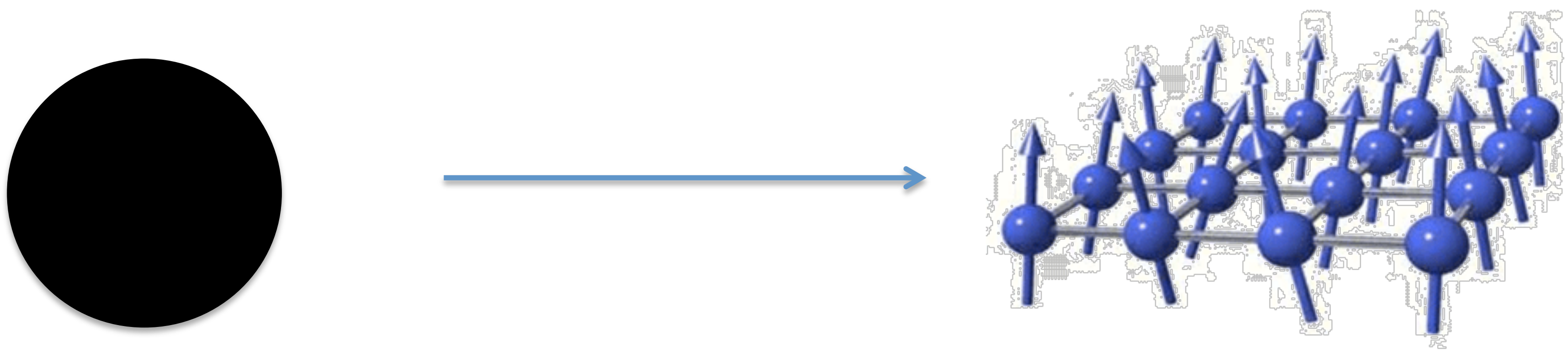
Does the interior <u>emerge</u> ? How ?

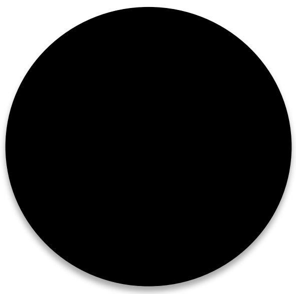
Now to the more specific topic

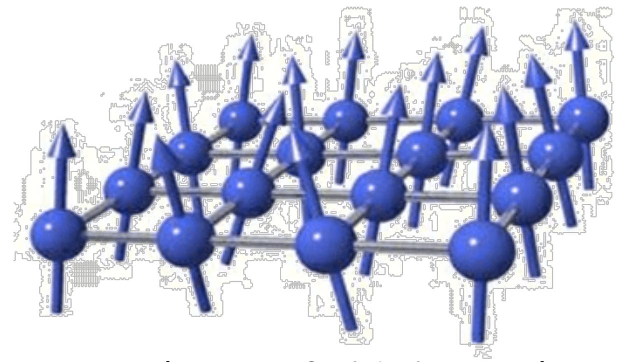
Emergent conformal symmetry
in
quantum mechanics
and
near extremal black holes

Black holes from outside

- Black hole seen from the outside = thermal quantum mechanical system with a finite, but very large, number of qubits.







(Not a field theory.)

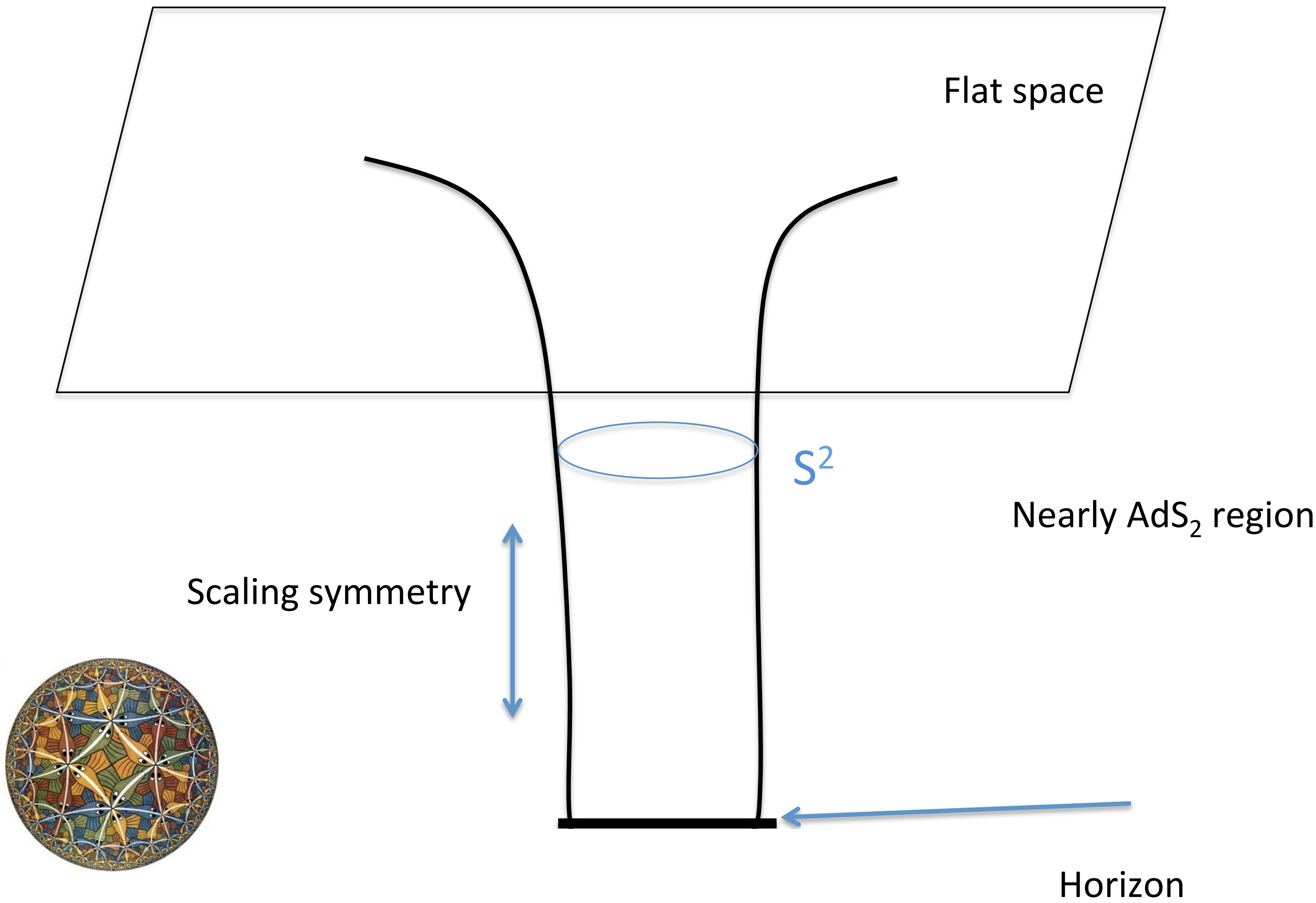
Extremal black hole $M \geq Q$ $M^2 \geq J$

Low energies, near horizon $\frac{M-Q}{M} \ll 1$

AdS_2 region

?

Near extremal black hole



Scale invariance in quantum mechanics

- No go:

Density of states consistent with scale invariance:

$$\rho(E) \propto \frac{1}{E} \; , \quad \text{or} \quad \delta(E) \qquad ?$$

Either divergent in IR or no dynamics.

Gravity in two dimensions

- No go:

Naïve two dimensional gravity :

$$\int \sqrt{g}(R - 2\Lambda) + S_M$$

Einstein term topological → no contribution to equations of motion.
Equations of motion → set important part of the stress tensor to zero

No dynamics !

Extremal Entropy

- The black holes have non-zero entropy at extremality.
- This has been matched to the ground states of many quantum mechanical configurations in string theory.

Strominger, Vafa, …. Sen, Dabholkar, Murthy….

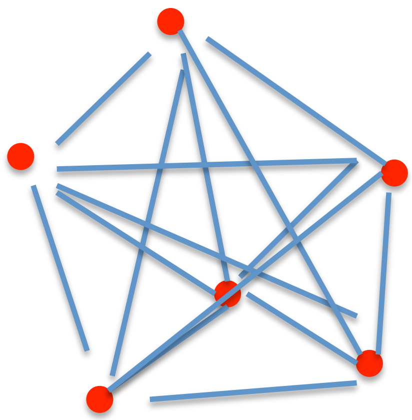
- What about leaving extremality ?

An interesting quantum mechanical model

- Interesting and simple quantum mechanical model that displays an emergent conformal symmetry at low energies.

- Was inspired by condensed matter physics problems and was introduced by Sachdev, Ye and Kitaev.

Fermions with random interactions



Sachdev, Yee, Kitaev
Georges, Parcollet

Polchinski, Rosenhaus,
Anninos, Anous, Denef

Kitaev, unpublished

Douglas Stanford, JM + Yang

Sachdev, Yee, Kitaev model

Quantum mechanical model, only time.

$$\{\psi_i, \psi_j\} = \delta_{ij}$$

N Majorana fermions or Gamma matrices.

$$H = \sum_{i_1, \cdots, i_4} j_{i_1 i_2 i_3 i_4} \psi_{i_1} \psi_{i_2} \psi_{i_3} \psi_{i_4}$$

js → either random or slowly varying

$$\langle j^2_{i_1 i_2 i_3 i_4} \rangle = J^2 / N^3$$

J = single dimension one coupling.

N fermions , N large

- Theory is trivial in the UV.
- The Hamiltonian is a relevant deformation and the theory flows to an interacting theory in the IR.

- Model is solvable in the large N limit.

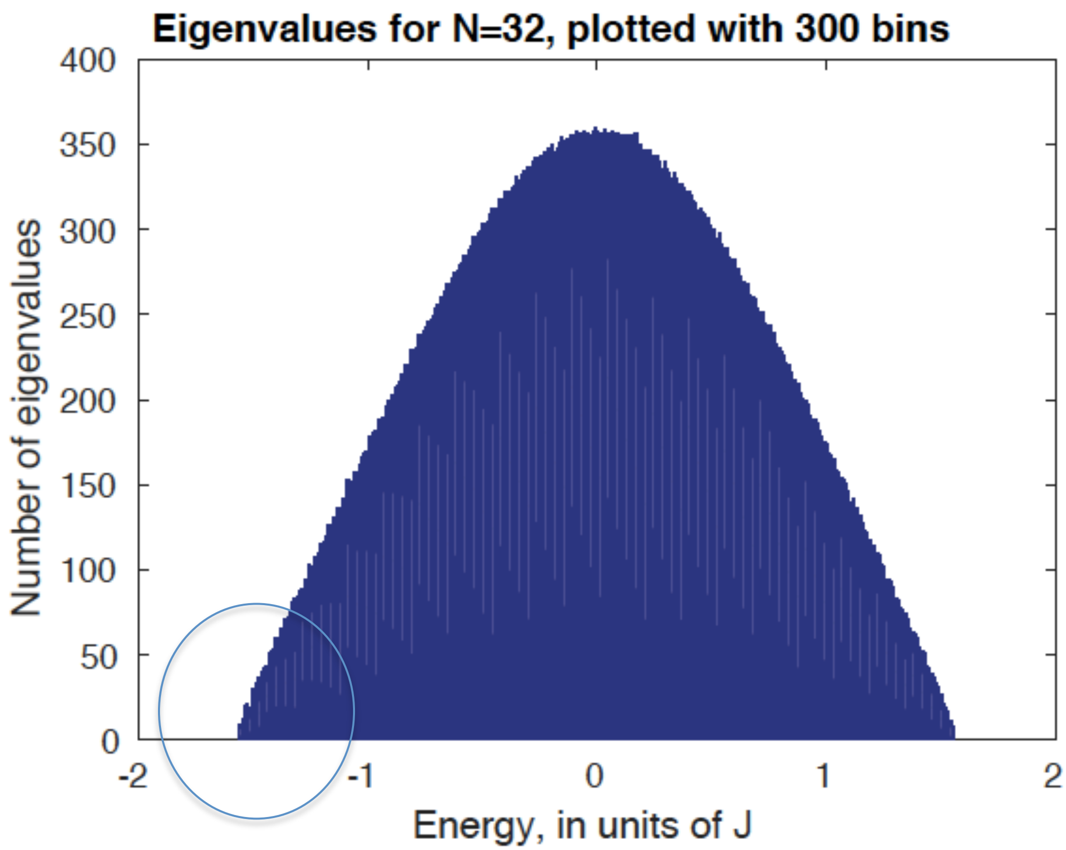
- It flows to an IR almost conformal fixed point

$$\frac{1}{J} \ll t, \ \beta \ll \frac{N^{\text{Power}}}{J}$$

- It is scale invariant to leading order in N.
- There are universal violations of scale invariance to subleading orders in the 1/N expansion.

Spectrum

D. Stanford

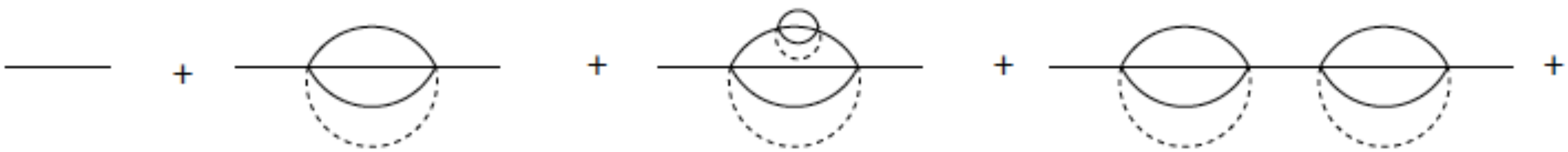


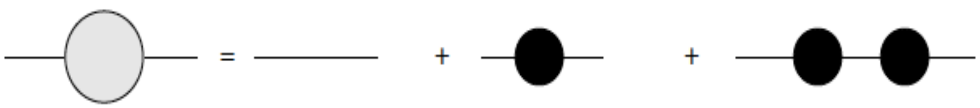
(specific, but random J's)

$$\dim_H = 2^{\frac{N}{2}}$$

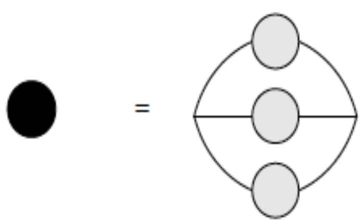
Number of random couplings $\propto N^4 \ll 2^N$

Solvable thanks to the simple structure of diagrams





$$= G(\tau,\tau') = (\partial_\tau - \Sigma)^{-1}$$



$$= \Sigma(\tau,\tau') = J^2 G(\tau,\tau')^3$$

Large N effective action

Integrate out the fermions and the couplings to obtain an effective action for fermion bilinears.

$$S = \frac{N}{2}\left[\log\det(\partial_t - \Sigma) - \int d\tau d\tau' \Sigma(\tau,\tau')G(\tau,\tau') + \frac{J}{4}G(\tau,\tau')^4\right]$$

Equations of motion from this action give the same as the Schwinger Dyson equations above.

Can be solved numerically, or analytically if we replace 4 → large number .

It is non-local. The bilocal terms come from the integral over the couplings.

This effective action is correct to leading orders, where we can ignore the replicas.

In the IR → Conformal symmetry

$$G = (\partial_\tau - \Sigma)^{-1} \longrightarrow G * \Sigma = 1$$

$$\Sigma(\tau, \tau') = J^2 G(\tau, \tau')^3$$

$$G(\tau, \tau') \propto \frac{1}{(\tau - \tau')^{2\Delta}}$$ Is a solution

If G is a solution, and we are given an arbitrary function f(τ), we can generate another solution:

$$G \longrightarrow G_f(\tau, \tau') = [f'(\tau) f'(\tau')]^\Delta G(f(\tau), f(\tau'))$$

We get an emergent reparametrization symmetry

Use: Go from zero temperature to finite temperature solution

$$G(\tau, \tau') \propto \frac{1}{(\tau - \tau')^{2\Delta}}$$

$$f(\tau) = \frac{\beta}{\pi} \tan \frac{\pi \tau}{\beta}$$

$$G_f = \left[\frac{\pi}{\beta \sin \frac{\pi \tau}{\beta}} \right]^{2\Delta}$$

- Is nice!
- Problem → Infinite number of solutions.
- f → like a Nambu-Goldstone boson.

- Fix: Remember that the symmetry is also explicitly broken (like the pion mass).

$$S = -\frac{N\#}{J}\int dt\, Sch(f,t)\ , \qquad Sch(f,t) = \left(\frac{f''}{f'}\right)' - \frac{1}{2}\frac{f''^2}{f'^2}$$

- The overall coefficient is small → Gives rise to large effects when we integrate over it.
- Leading term in derivative expansion with a global SL(2,R) symmetry. (This is a gauge symmetry, those do not change the IR solution of the model).

Thermal free energy

$$-\beta F = N\left[c_1 \beta J + s_0 + \# \frac{2\pi^2}{\beta J}\right]$$

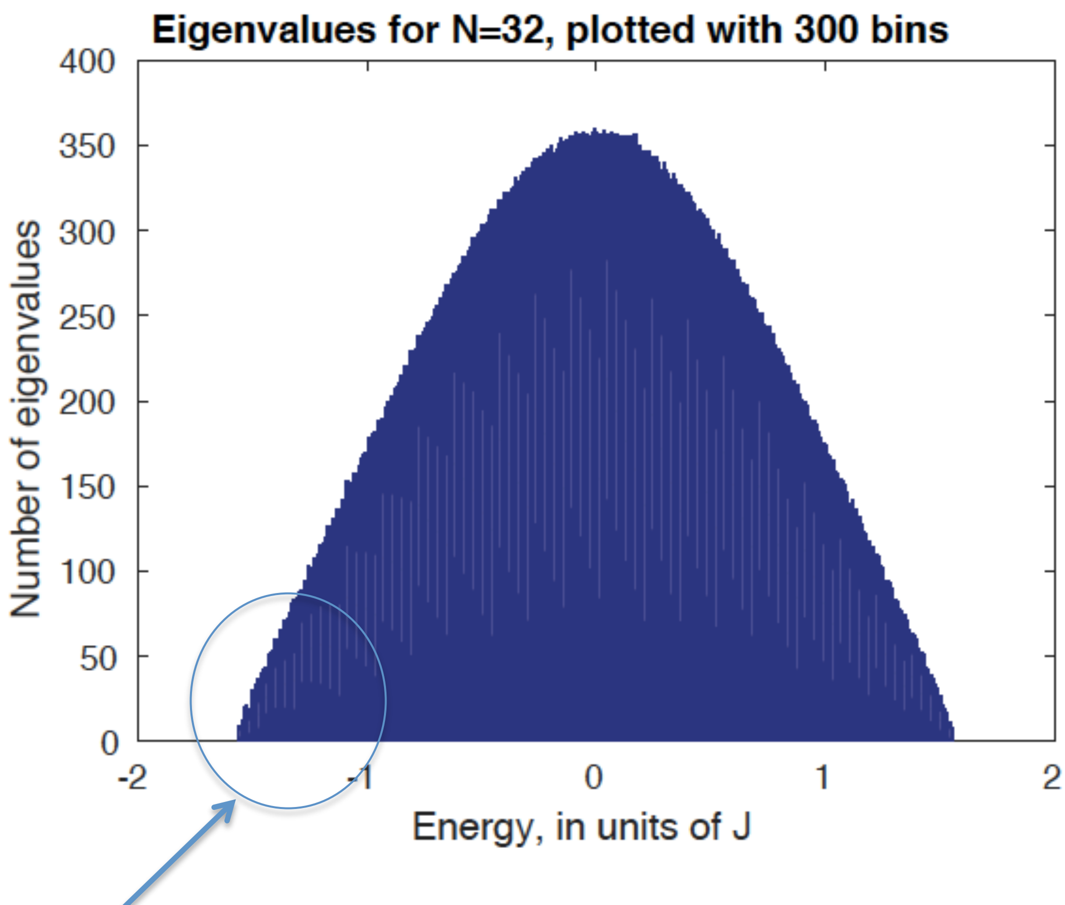
Ground state energy
(not important)

Extremal entropy

Near extermal entropy → linear in T

From Nambu-Goldstone mechanism.

Ground state entropy ?



$$\rho(E) \sim e^{Ns_0} F(E)$$

$$\rho(E) \sim e^{Ns_0 + N\sqrt{(E-E_g)/E_g}} F(E)$$

$$F(E) \sim \text{constant}$$

Four point function

- We expected a conformal invariant answer.
- But, due to the reparametrization zero modes → infinity.
- Adding the Nambu-Goldstone (euclidean) action → get a finite answer. But is not conformal.
- Still the reparametrization symmetry and its slight breaking are running the show!
- $NCFT = NCFT_1$ → how conformal symmetry is realized in QM.

N-AdS$_2$/N-CFT$_1$

- Gravity in AdS$_2$ does not make sense, when we add finite energy excitations.
- Slightly break the symmetry.
- Simplest model:

Teitelboim Jackiw
Almheiri Polchinski

$$\text{Area}_{S^2} = \phi_0 + \phi$$

$$\int d^2x\sqrt{g}\phi(R+2) + \phi_0 \int d^2x\sqrt{g}R$$

Ground state entropy. Topological term.

$$\int \sqrt{g}\phi(R+2)$$

Equation of motion for ϕ → metric is AdS_2

Equation of motion for the metric → ϕ is almost completely fixed

$$ds^2 = d\rho^2 + \sinh^2 \rho d\tau^2$$

$$\phi = \phi_h \cosh \rho$$

Value at the horizon

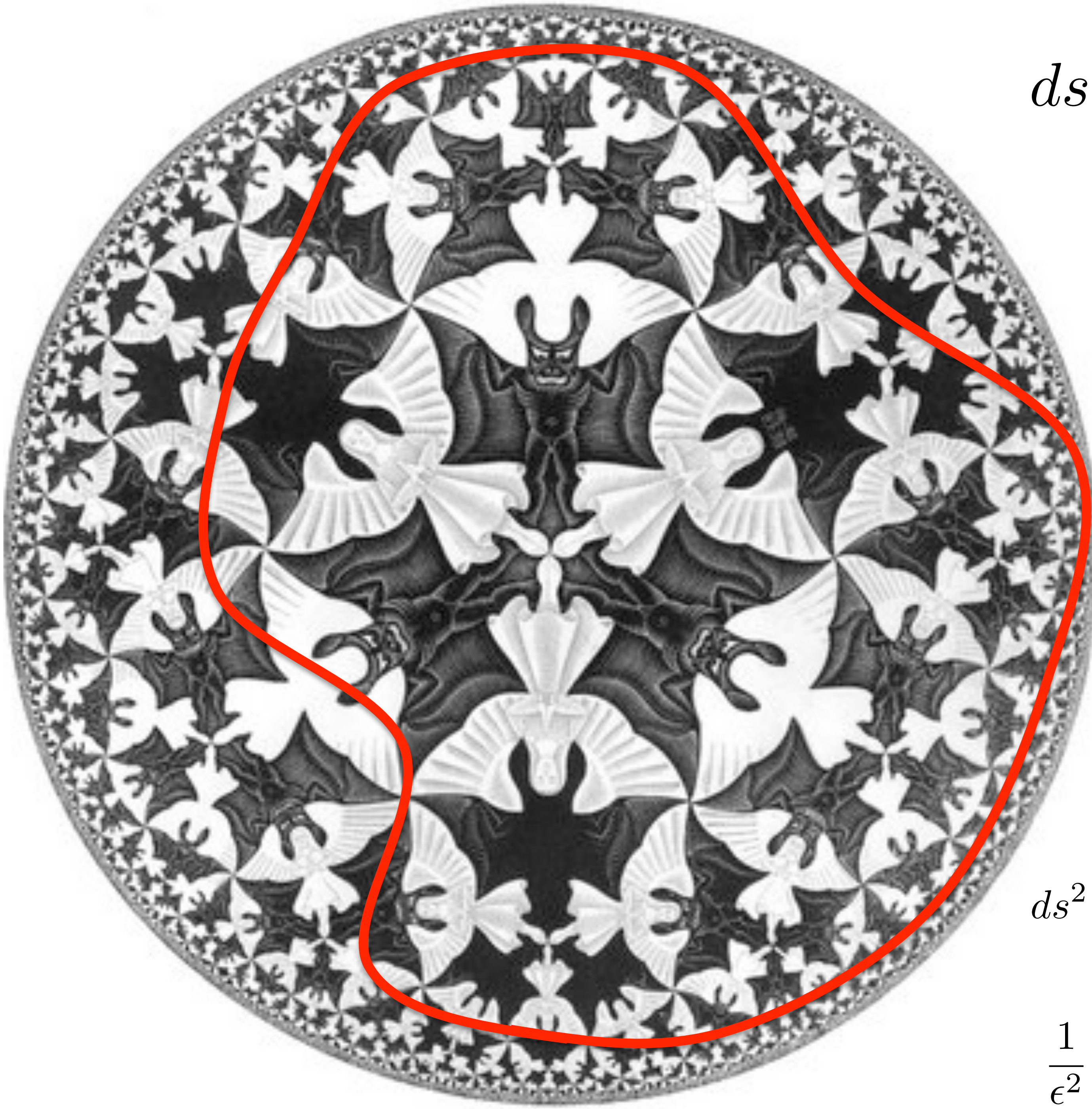
Position of the horizon.

In the full theory: when ϕ is sufficiently large → change to a new UV theory

Asymptotic boundary conditions:

$$ds^2|_{Bdy} = \frac{1}{\epsilon^2} du^2$$ ← Fixed proper length

$$\phi|_{Bdy} = \frac{1}{\epsilon} \phi_r(u)$$



$$ds^2|_{Bdy} = \frac{1}{\epsilon^2} du^2$$

Infinite number of solutions.

$$ds^2 = d\rho^2 + \sinh^2\rho d\tau^2$$

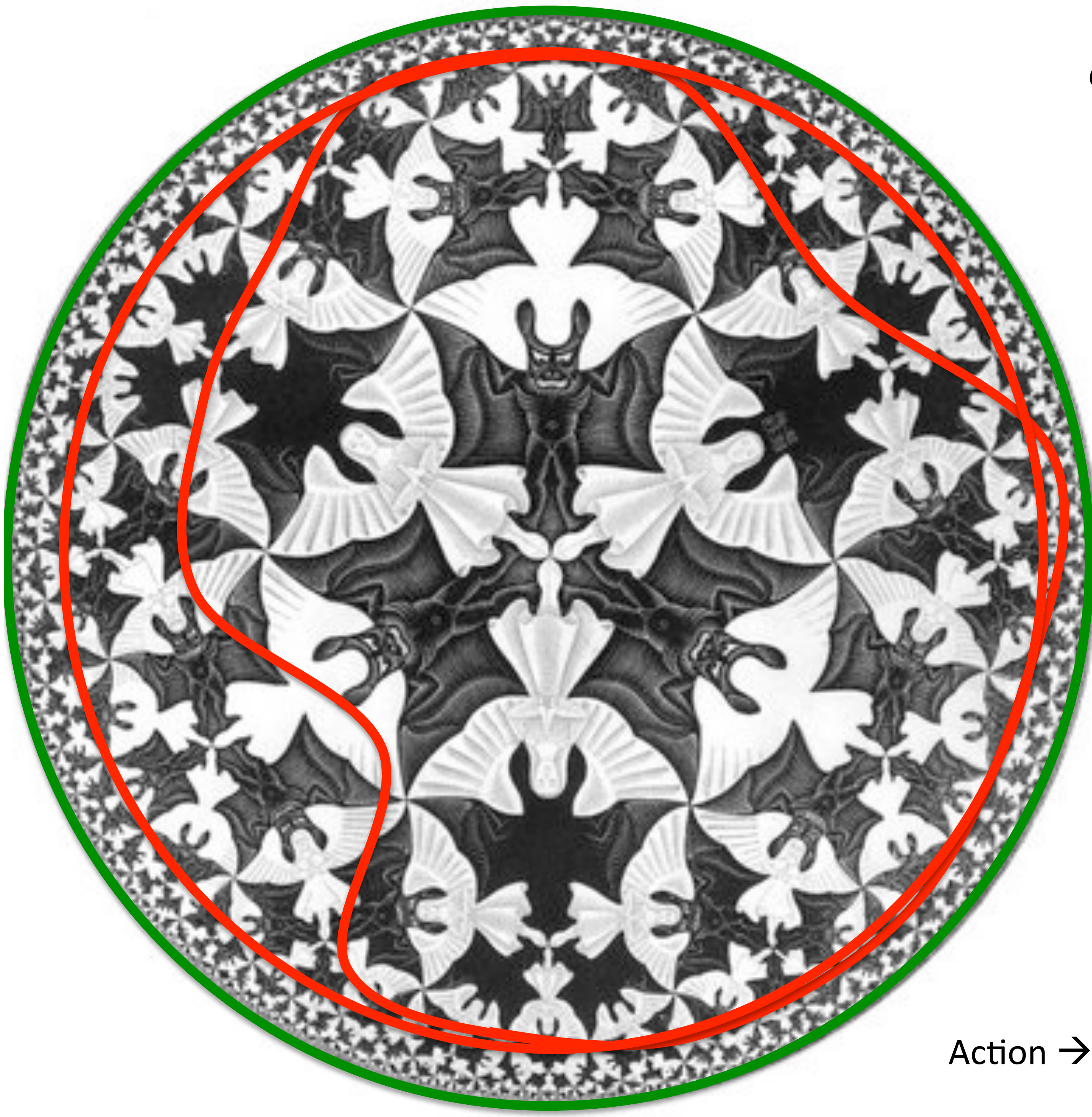
$$\rho(\tau)$$

$$\frac{1}{\epsilon^2} = (\rho'^2 + \sinh^2\rho)\left(\frac{d\tau}{du}\right)^2$$

- Similar to the boundary gravitons of AdS_3

Brown, Henneaux,
Strominger,
Turiaci Verlinde

- Here one must break the symmetry.



$$ds^2|_{Bdy} = \frac{1}{\epsilon^2} du^2$$

$$\phi|_{Bdy} = \frac{1}{\epsilon}\phi_r(u)$$

One one solution

Action → related to Schwarzian

$$S = \int d^2x \sqrt{g}\phi(R+2) - 2\int \frac{\phi_r(u)}{\epsilon^2} du K \to$$

$$S = \frac{1}{\epsilon^2} - \int du \phi_r(u) Sch(t, u)$$

$$t(u)$$

t = Usual AdS_2 time coordinate → Emergent time coordinate.
u = Boundary system (quantum mechanical) time coordinate

Properties fixed by the Schwarzian

- Free energy
- Part of the four point function that comes from the explicit conformal symmetry breaking. This part leads to a correlators with maximal growth in the commutator.

Kitaev

$$\text{growth of commutators} \sim \frac{1}{N}(\beta J)e^{2\pi t/\beta}$$

- Both agree with the NAdS2 problem.

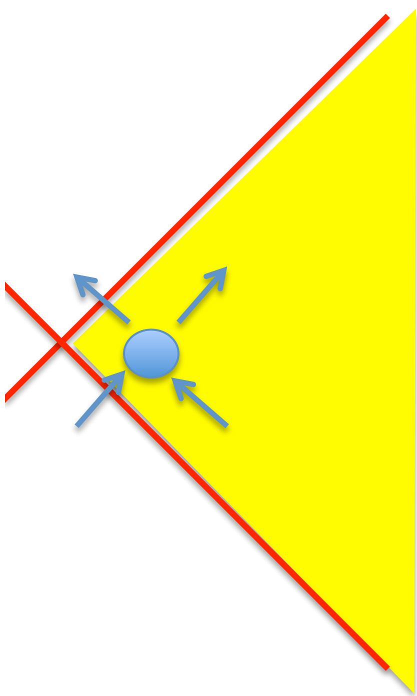
- We have done more computations that depend on the details of the model and can be thought of as coming from additional fields in $NAdS_2$.

Growth of commutators

$$\langle [V(t), W(0)]^2 \rangle \leq \frac{1}{N} e^{2\pi t/\beta}$$

Chaos bound

Shenker Stanford
Shenker Stanford, JM
(Sekino Susskind)



Leading correction given by a scattering amplitude in the near horizon region.

time translations → boosts

Growth of the graviton contribution with energy, fixed by the spin of the graviton.
Saturates the chaos bound.

Growth of commutators

$$\langle [V(t), W(0)]^2 \rangle \leq \frac{1}{N} e^{2\pi t/\beta}$$

Chaos bound

Shenker Stanford
Shenker Stanford, JM

- Gravity saturates it.
- Stringy corrections generically lower it.
- This model saturates it , does it have a gravity dual ? → we think that the answer is no.

$$\lambda = \frac{2\pi}{\beta}\left(1 - \frac{S - S_0}{S}\frac{l_s^2}{R_{AdS}^2}\right)$$

Correction is small for near extremal black holes → Universality of Schwarzian action.

- This notion of $NAdS_2$ is similar to:

- Inflation = NdS = almost de-Sitter. We need a scalar field to have inflation end and to lead to the observable universe.

- In fact NdS_2 is a type of inflationary theory, except that the inflaton is not a dynamical field.

Models of holography

Large N quantum system

Harder

- Free boundary theories.
- O(N) interacting theories.
- Sachdev Ye Kitaev
- Maximally supersymmetric Yang mills at very strong t'Hooft coupling, $g^2 N >> 1$

Gravity/string dual

Easier

- Bulk theories with massless higher spin fields.
- Very slighly massive higher spins
- O(1) masses for the higher spin fields.
- Gravity theory. Higher spin particles are very massive.

Conclusions

- There are simple models in quantum mechanics with an interesting emergent reparametrization symmetry.
- We have studied one model in detail.
- Near extremal black holes exhibit the same symmetries.
- The emergence of the symmetry and its slight breaking should be valid in any model with scale invariance…
- The proper systematic way to understand the symmetry remains to be explored. (The symmetry is approximate, it arises only in the large N limit, etc).

Message

- We are exploring the emergence of time, and its reparametrization transformations.