Conformal field theory and energy-momentum tensor on the lattice

Etsuko Itou (KEK)

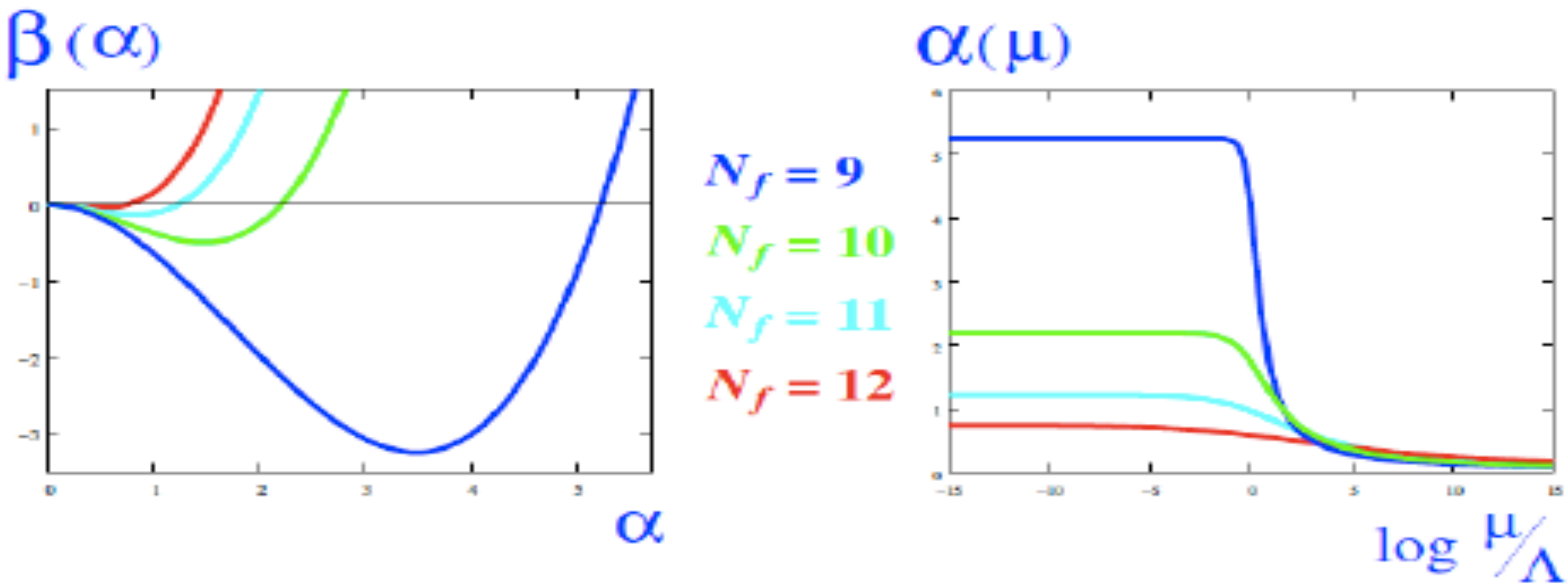
Conformal theory:
E.I. , PTEP (2013)8, 083B01
E.I. , PTEP (2015)4, 043B08
E.I. and A.Tomiya, PoS LATTICE2014 (2014) 252
Energy-momentum tensor:
Asakawa, Hatsuda, E.I., Kitazawa, Suzuki, Phys.Rev. D90 (2014) 1, 011501
E.I.,Suzuki, Taniguchi, Umeda PoS(LATTICE 2015)303
Entanglement entropy:
E.I., K. Nagata, Y. Nakagawa, A. Nakamura and V.I.Zakharov
PTEP 2016 (2016) no.6, 061B01, arXiv: 1512.01334 [hep-th]

Workshop “Emergent property of space-time”, June 30, 2016 @CERN

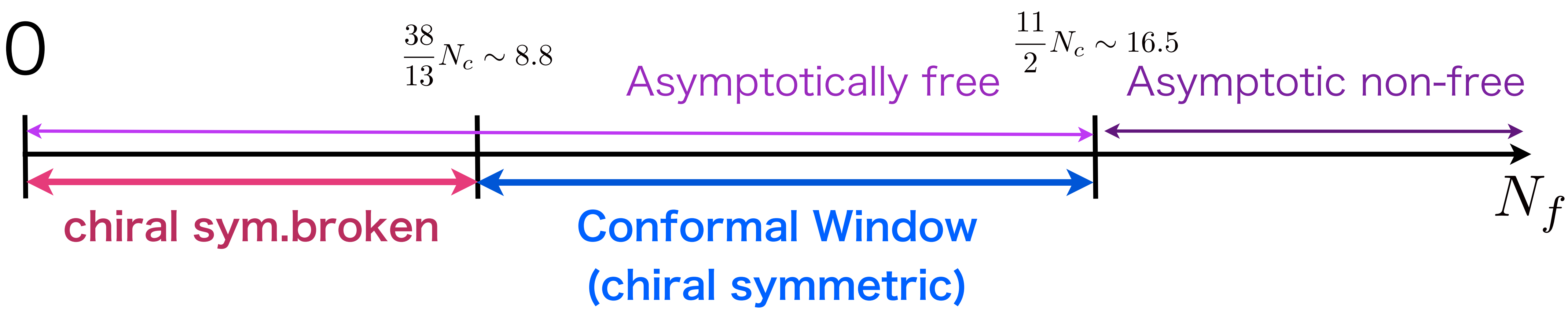
conformal window

SU(3) gauge theory with massless Nf fermions

Two loop analysis $\beta(\alpha) = -b\alpha^2 - c\alpha^3$

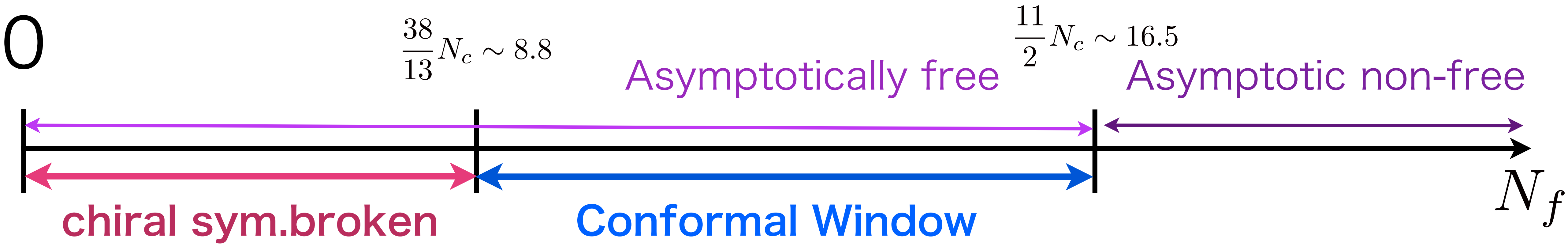


Phase structure based on two loop



Conformal Window of SU(3) gauge

Perturbation



Schwinger-Dyson

Nf*=11.9

Exact RG

Nf*= $10.0^{+1.6}_{-0.7}$

Lattice simulation

only known nonperturbative and gauge invariant regularization

Methods for IRFP study using lattice simulations

- Step scaling method (discretized renormalization group) measurement of the growth ratio of the Z-factor when the IR scale is changed

  Luescher, Weisz and Wolff, NPB 359 (1991) 221

  $$g_R^2(1/sL)/g_R^2(1/L)$$

- Hyperscaling for mass of composite state in mass deformed theory

  Miransky, PRD59(1999)105003
  Luty, JHEP 0904(2009)050
  Del Debbio and Zwicky, PRD82(2010)014502

  $$M_X \sim m_q^{\frac{1}{1+\gamma_m^*}}$$

- Hyperscaling for the Dirac eigenmodes for massless fermions

  Patella,PRD86(2012)025006
  Cheng, Hasenfratz, Petropulos and Schaich, JHEP1307(2013)061

  $$\nu(\lambda) = V \int_{-\lambda}^{\lambda} \rho(\omega) d\omega \propto V \lambda^{\frac{4}{1+\gamma_m(\lambda)}}$$

- Shape of the correlation fn. for the composite op.

  Ishikawa, Iwasaki, Nakayama and Yoshie, PRD87(2013)071503

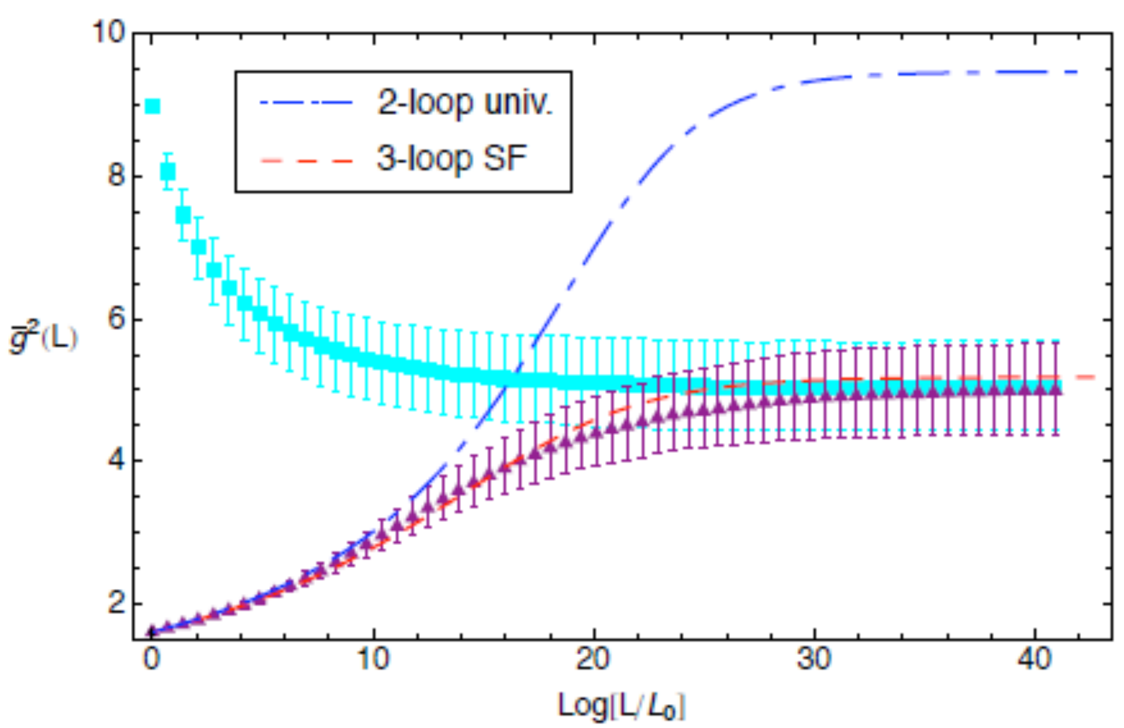
  $$G_H(t) = \tilde{c}_H \frac{\exp(-\tilde{m}_H t)}{t^{\alpha_H}}$$

Step scaling method

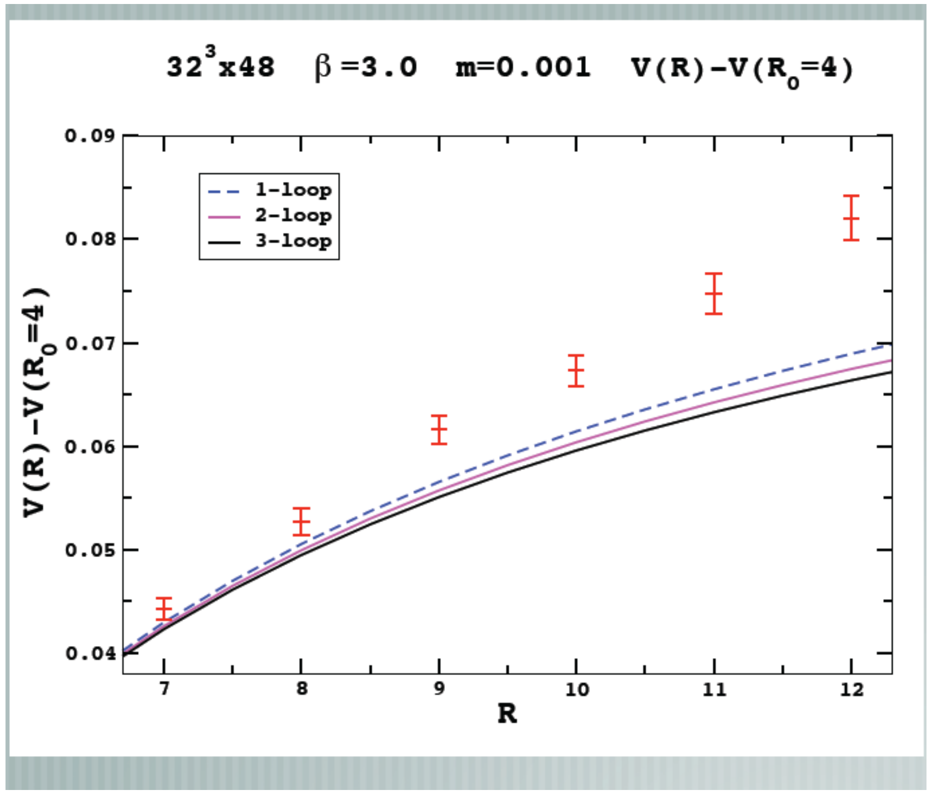
Running coupling constant/ renormalized coupling SU(3) Nf=12 theory

Appelquist et al. (SF scheme)
Phys.Rev.D79:076010,2009

Taking constant extrapolation in (a/L)



Fodor et al. (potential scheme)
PoS LAT2009:055,2009, talk at Lattice2010



Plot: Slide of K.Holland's talk at Lattice2010

The continuum extrapolation was not considered.

(O(a) effects depends on the renormalization scheme)

Renormalization scheme and universality

scheme transformation $g_1 \rightarrow g_2 = f(g_1)$

$f(g_1)$ is an analytic fn. of g_1

beta fn. $\beta(g_2) = \frac{\partial f(g_1)}{\partial g_1} \beta(g_1)$

The existence of the fixed point is scheme independent.

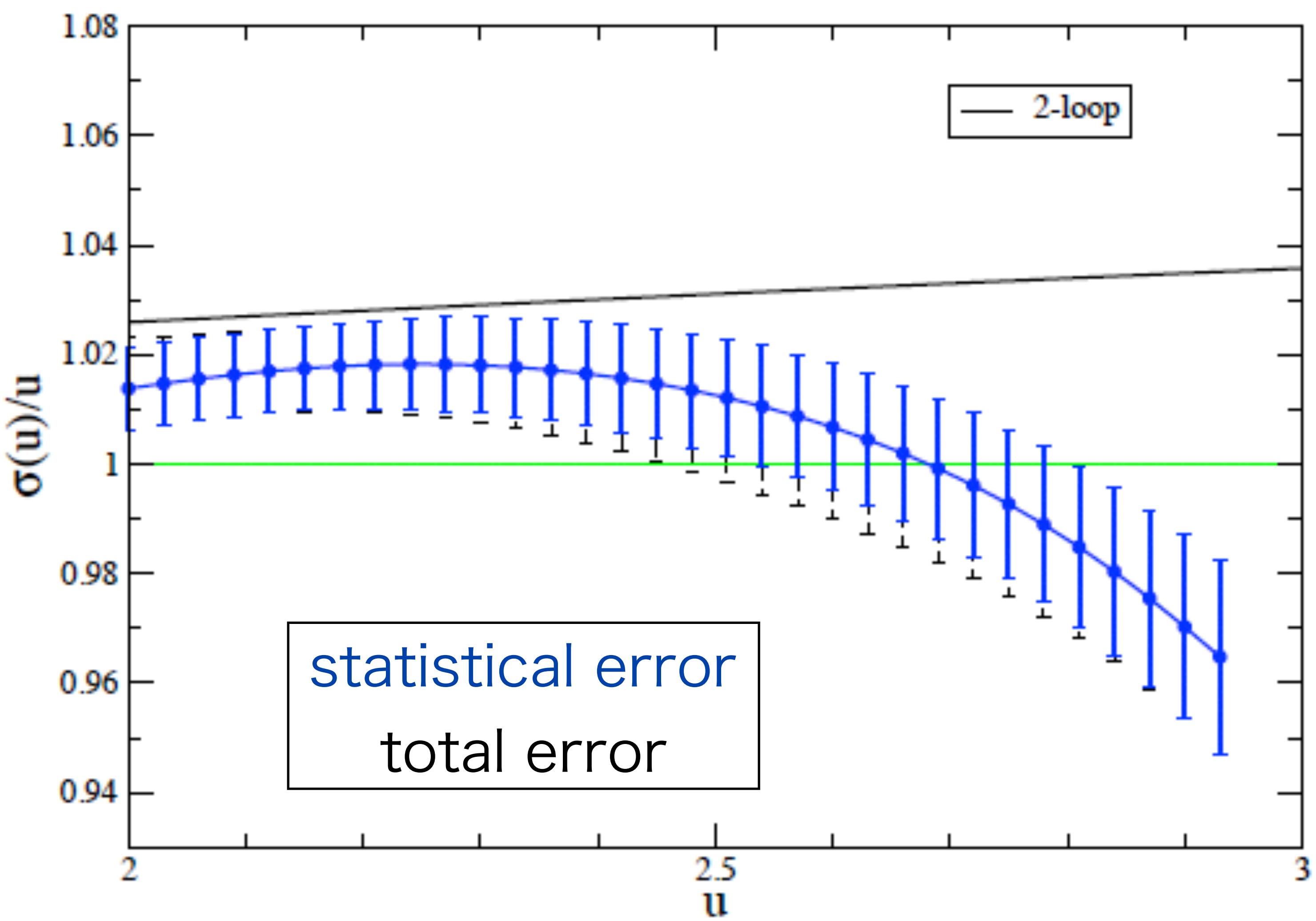
Our work

- ✦ independent renormalization scheme
- ✦ taking continuum limit carefully
- ✦ exact massless simulation using twisted b.c.

Growth ratio of TPL coupling (SU(3) Nf=12)

E.I. PTEP (2013) 083B01

$$\sigma(u)/u = g_R^2(1/sL)/g_R^2(1/L)$$



$$g^{*2}_{\rm TPL} = 2.69 \pm 0.14({\rm stat.})^{0}_{-0.16}({\rm syst.})$$

Critical exponent of the beta fn.

$$\beta(g^2) \sim \gamma_g^*(g^{2*} - g^2)$$

Our result

$$\gamma_g^* = 0.57^{+0.35}_{-0.31}({\rm stat.})^{0}_{-0.16}({\rm syst.})$$

SF scheme	2 loop at $g^{2*} = 9.4$	4 loop (MS bar)
$\gamma_g^* = 0.13 \pm 0.03$	$\gamma_g^* = 0.36$	$\gamma_g^* = 0.28$

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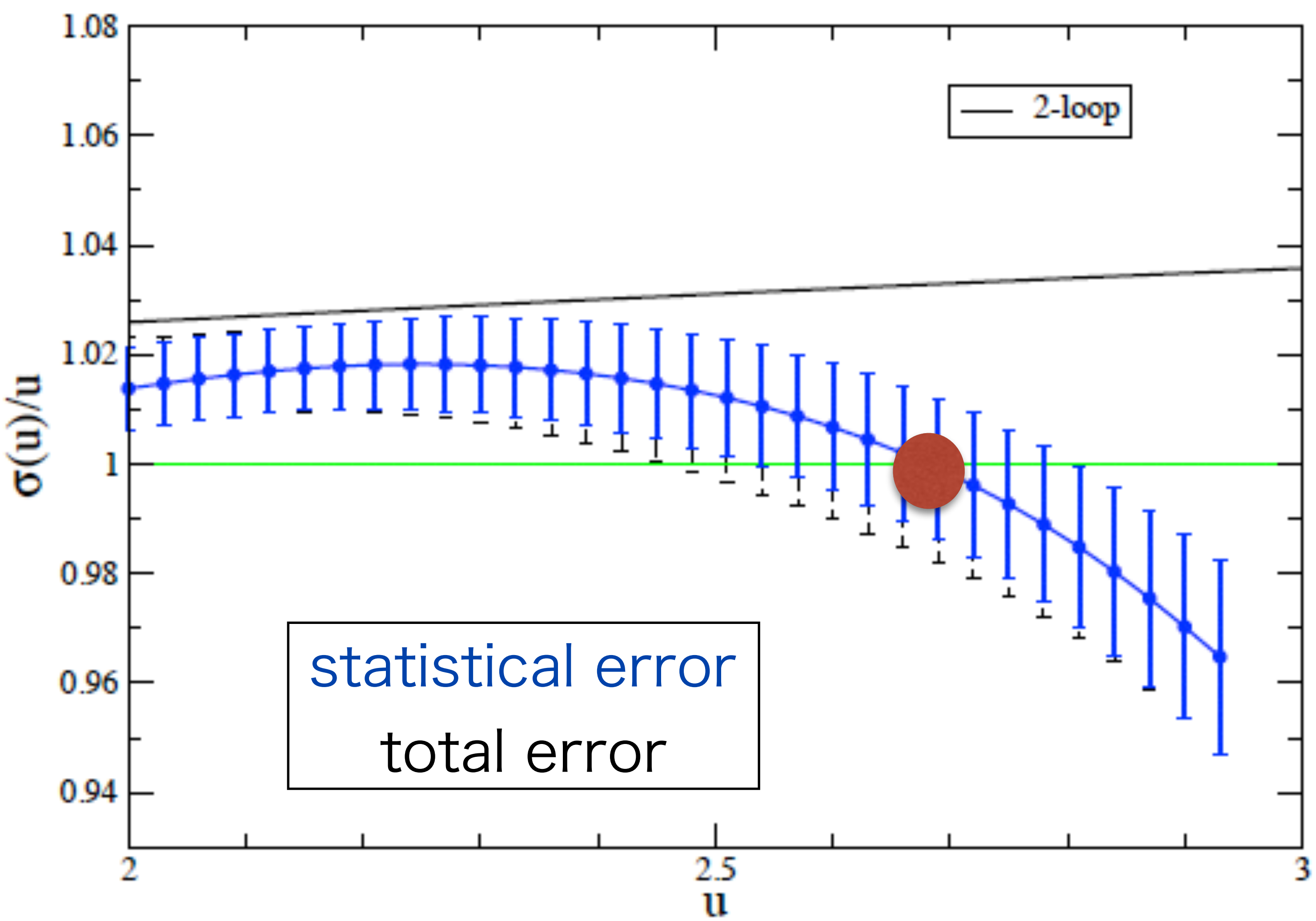

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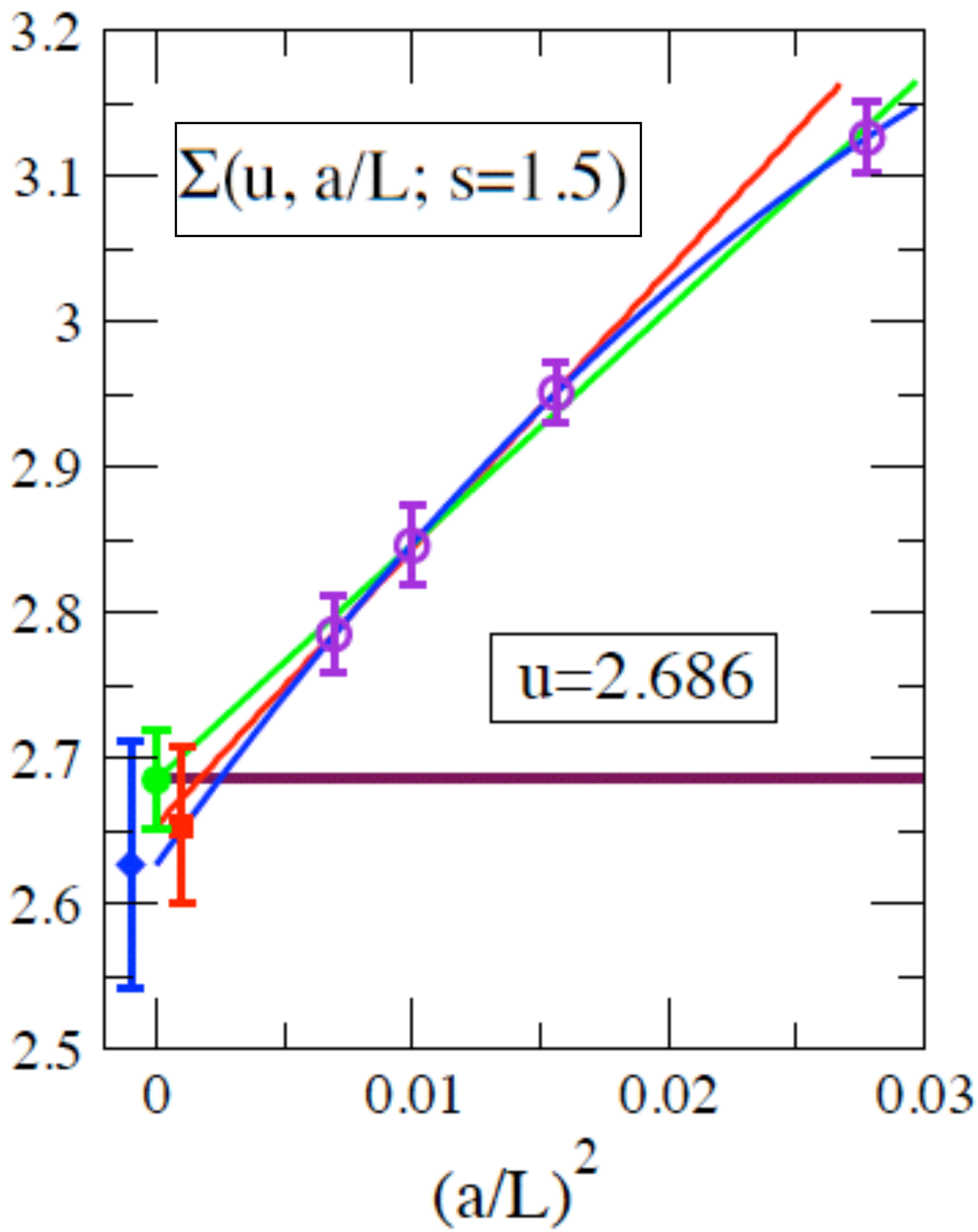
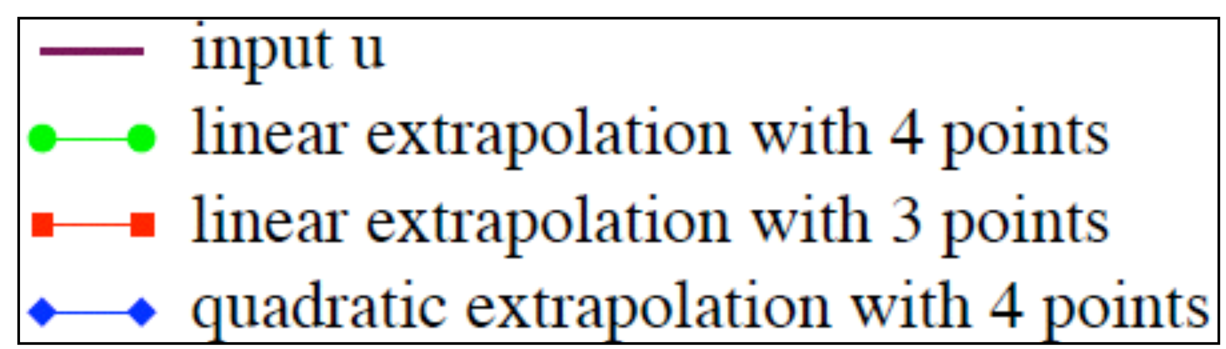
Continuum extrapolation



s=1.5 step scaling

L/a=6 -> L/a=9
L/a=8 -> L/a=12
L/a=10 -> L/a=15
L/a=12 -> L/a=18

2 loop prediction in this region is $\sigma(u=2.69) \sim 2.78$



The systematic error is small in the strong coupling region in this scheme.

(Fit range dependence and "s" (step scaling parameter) dependence are also small.)

Methods to obtain γ_m^* using lattice simulations

* Step scaling method (discretized renormalization group)

Luescher, Weisz and Wolff, NPB 359 (1991) 221

mass anomalous dim <=> anomalous dim. of pseudo-scalar op.

* Hyperscaling for mass of composite state in mass deformed theory

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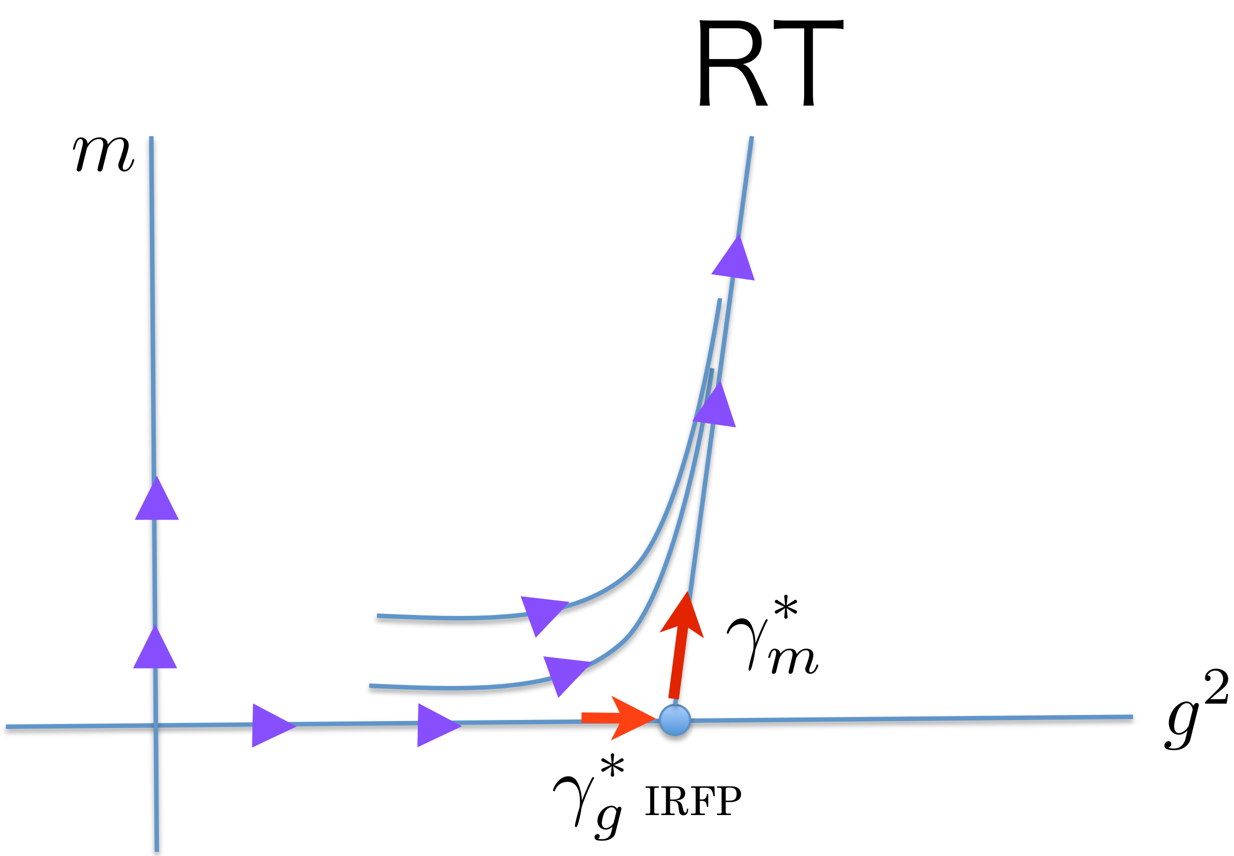
Summary of critical exponents



Further studies are necessary to find the universal quantities.

SF scheme : PRD79 (2009) 076010
Fodor's data: PLB703 (2011) 348-358
Fit(I): PRD84(2011) 054501
Fit(II): PRD84 (2011) 116901
LatKMI : PRD86 (2012) 054506
Cheng et.al : JHEP1307 (2013) 061
Ours : PTEP (2013)083B01
arXiv: 1307.6645

	γ_g^*	γ_m^*
2loop	0.36	0.77
4loop (MS bar)	0.28	0.25
S.-D.		0.80
SF scheme	0.13(3)	
Fodor's data		0.403(13) 0.35(23)
LatKMI		0.4-0.5
Cheng et. al.		0.32(3)
Ours	0.57(35)	$0.081^{+0.03}_{-0.02}$

hyperscaling (mass) $M_X \sim m_q^{\frac{1}{1+\gamma_m^*}}$

Del Debbio and Zwicky: Phys.Rev.D89(2014)014503

$\delta g \equiv g - g*$ corrections (cont. limit)

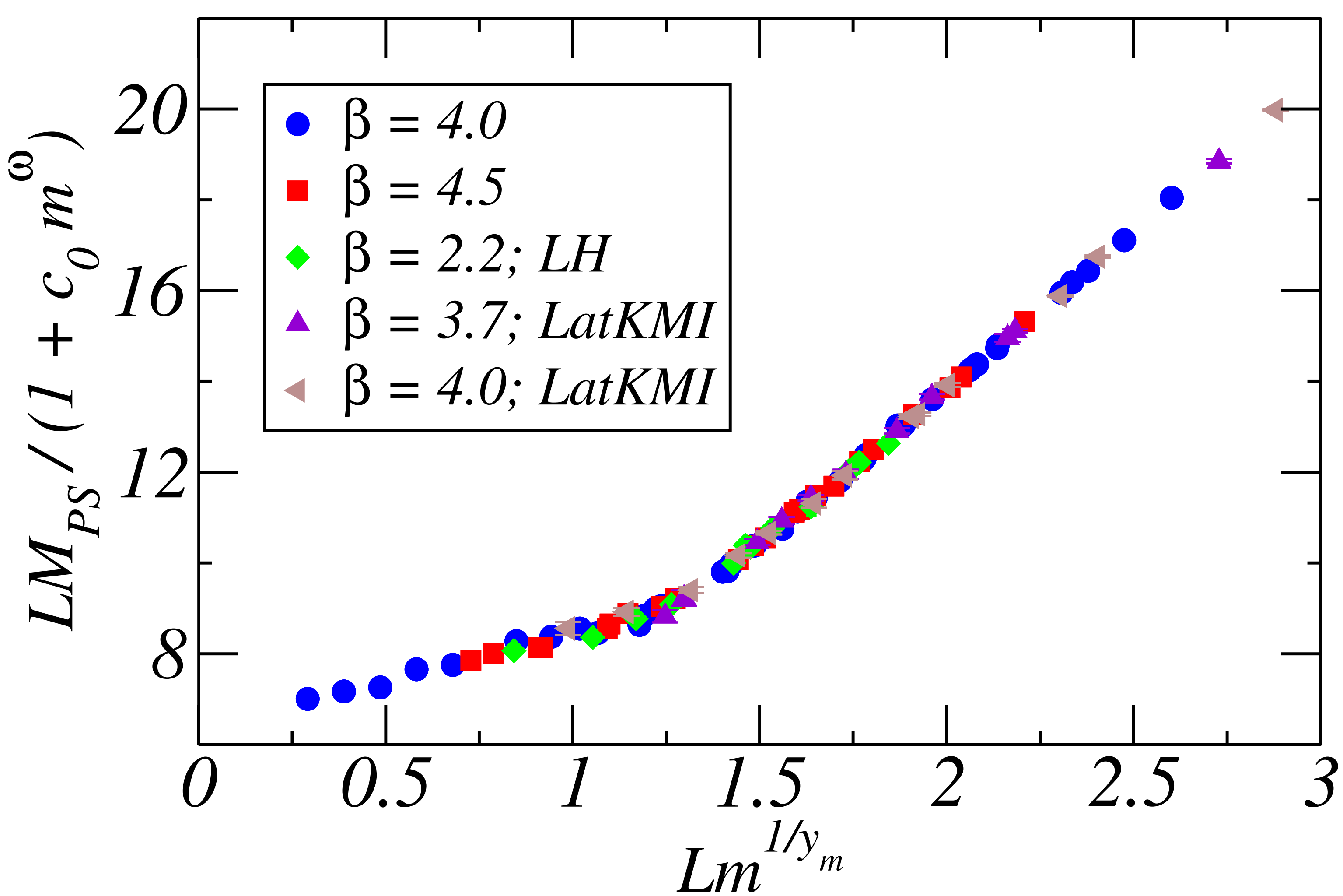
$$m(b) = mb^{\gamma_m^*} \exp\left[-\frac{\gamma_m^{(1)}}{\beta_1}\delta g f(b)\right]$$

A.Hasenfratz

$\delta g \equiv g - g*$ corrections (Finite vol. effect)

$$M_H = L^{-1} f(x, g_0 m^\omega), \quad \omega = -y_0 / y_m$$

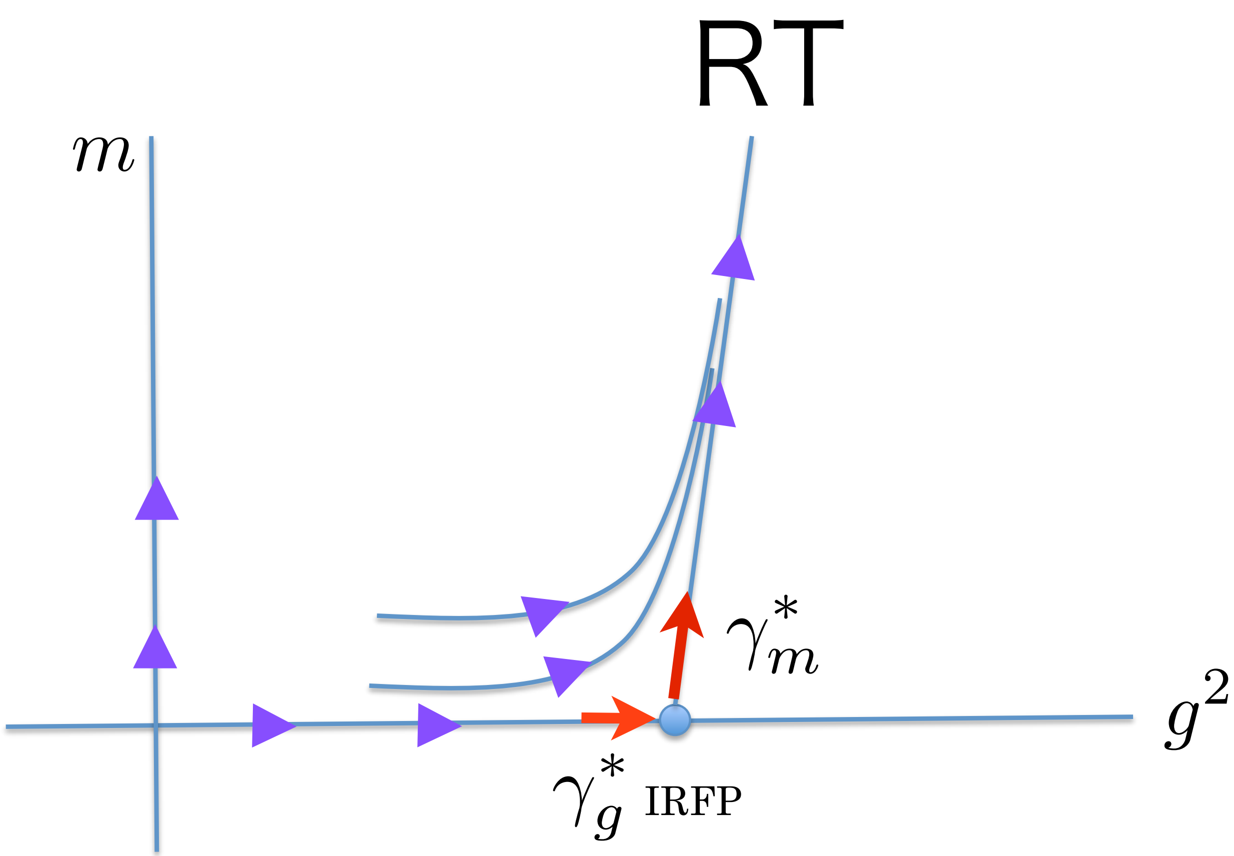
$$LM_H = F(x)\left(1 + g_0 m^\omega G(x)\right)$$



pion at β=4.0,4.5, LH, KMI :
y_m=1.24[1], y_0=-0.51[5] ;
χ^2 /dof =1.4 [95]

Slide of seminar @KEK
by A.Hasenfratz

Summary of critical exponents



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Cheng et. al.		
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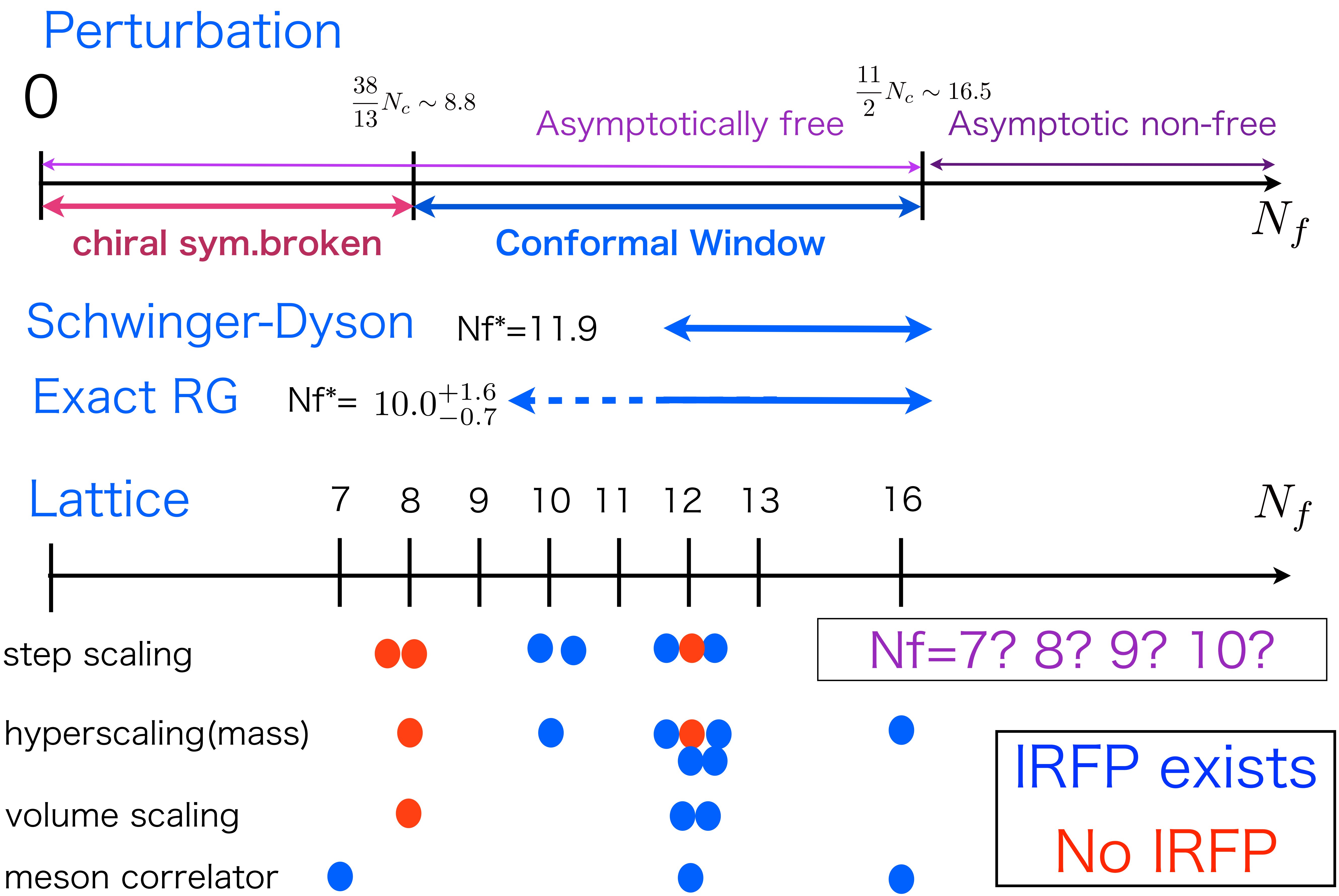
Lowest Nf in the Conformal Window？



Conformal bootstrap

Assume that the absence of a relevant operator around the IRFP for the any reps., the upper bound of the anomalous dimension is determined.

Iha, Makino, Suzuki arXiv:1603.01995

$$SU(N_f)_L \times SU(N_f)_R$$

staggered fermion Nf=12 $\gamma^* \leq 1.29$

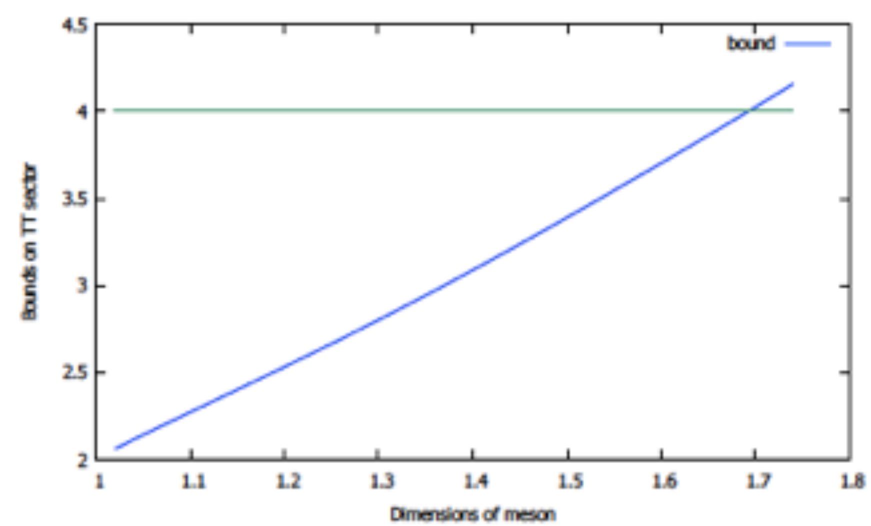
staggered fermion Nf=8 $\gamma^* \leq 1.33$

Y.Nakayama arXiv:1605.04052

Wilson (or Domain-Wall) fermion Nf=8

$$SU(N_f)_V$$

$$SU(N_f)_L \times SU(N_f)_R$$



$\gamma^* \leq 1.31$

Conclusion and Directions (CFT part)

- Methods to find the IRFP in QCD-like theory
- SU(3) Nf=12 has an IRFP
- Two critical exponents for beta-fn. and anomalous dim. are calculated
- The anomalous dim. at IRFP is quite smaller than the prediction by 2-loop or S-D eq.
- Critical Nf* almost determine Nf*~8
- Universality class as a CFT (central charge?)
    - Energy-momentum tensor
    - Entanglement entropy

Plenary talks of Lattice conference

- G. T. Fleming, PoS (Lattice 2008) 021
- E. Pallante, PoS (Lattice 2009) 015
- L. Del Debbio, PoS (Lattice 2010) 004
- E. Neil, PoS (Lattice 2011) 009
- D. Nogradi, PoS (Lattice 2011) 010
- J. Giedt, PoS (Lattice 2012) 006
- J. Kuti, PoS (Lattice 2013) 004
- E. Itou, PoS (Lattice 2013) 005
- Y. Aoki, PoS (LATTICE 2014) 011
- A. Hasenfratz, 2015

EMT on the lattice

EMT on Lattice

Energy-momentum tensor (EMT)

- ✦ generator of general coord. transformation
- ✦ conserved quantity (energy density, momentum, pressure)
- ✦ universal quantity (central charge in conformal theory)

Lattice regularization

- ✦ nonperturbative regularization
- ✦ gauge invariant
- ✦ discretized space-time coordinate

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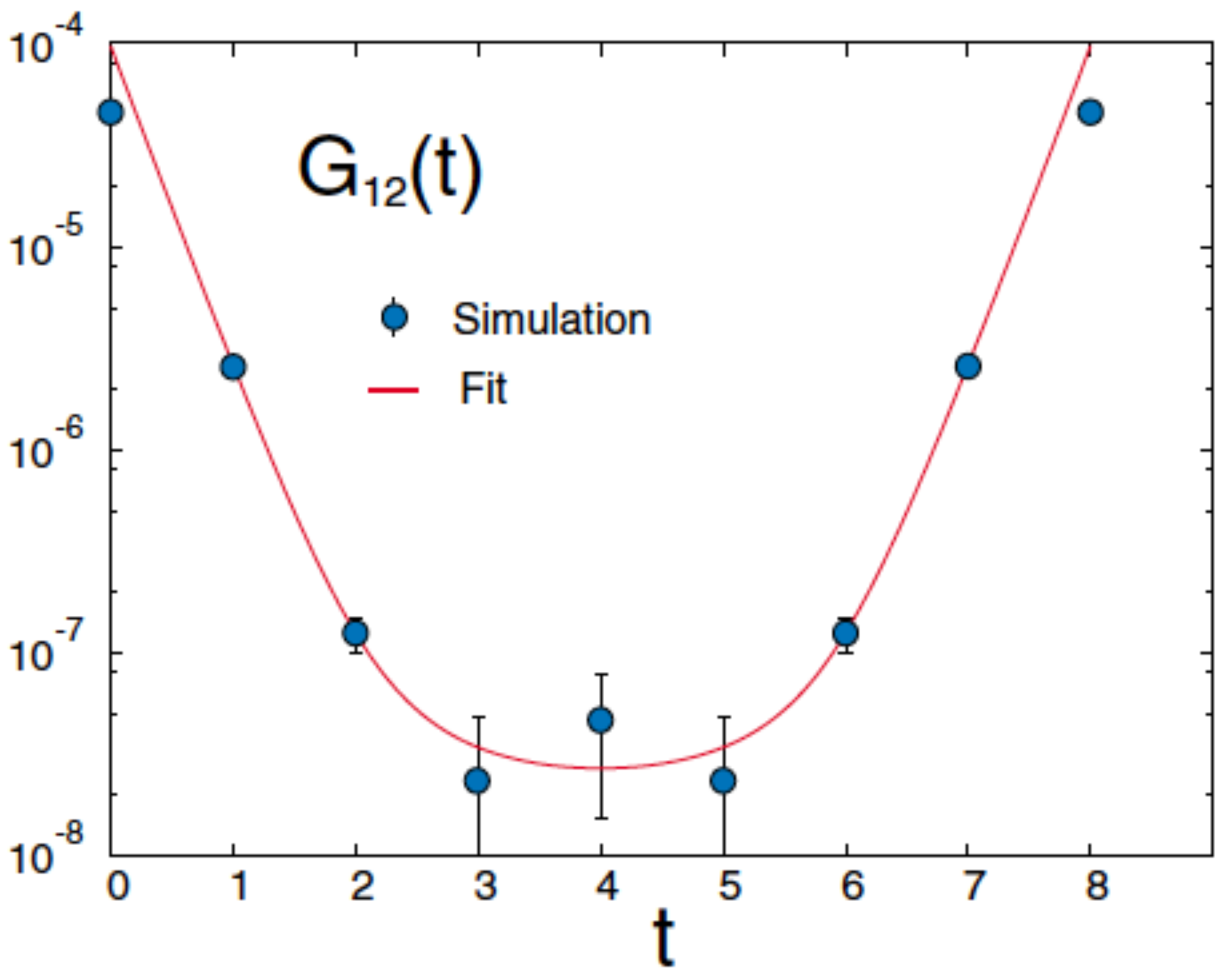
Same quantum number with the vac. (signal is noisy)

How to define the renormalized EMT

Shear viscosity in QGP phase



Matsubara-Green's function G12(t),
Nakamura-Sakai(2005)
800,000 conf.

shear viscosity: retarded Green's fn.

$$\eta = -\int \langle T_{12}(\vec{x},\tau) T_{12}(\vec{x}',0)\rangle_{\mathrm{ret.}}$$

obtained by the analytic continuation
of Matsubara Green's fn.

$$G_\beta(\vec{p},t) = \sum_n e^{i\omega_n t} \int d\omega \frac{\rho(\vec{p},\omega)}{i\omega_n - \omega}$$

quenched QCD
(pure Yang-Mills theory)

Before going to the conformal theory…

- ✦ give a better def. of EMT on the lattice
- ✦ show the result for the finite-T (quenched) QCD

Basic Idea

Quantum field theory

(UV divergence)

perturbation with dim. reg.
+YM gradient flow
(general covariance OK!)

lattice reg.
+Wilson flow
(with a->0 limit)

At finite flow time, UV finite!

Firstly, we obtain the relation between them perturbatively.
Assume that it applies to the nonperturbative regime.

YM gradient flow

Flow equation

Luescher, JHEP 1008, 071 (2010)

Yang–Mills gradient flow (continuum theory)

$$\partial_t B_\mu(t,x) = D_\nu G_{\nu\mu}(t,x) = \Delta B_\mu(t,x) + \cdots , \qquad B_\mu(t=0,x) = A_\mu(x)$$

Wilson flow (lattice theory)

$$\partial_t V(t,x,\mu) V(t,x,\mu)^{-1} = -g_0^2 \partial S_{\text{Wilson}}, \qquad V(t=0,x,\mu) = U(x,\mu)$$

link variable: $U_\mu(x) = e^{ig_0 A_\mu(x)}$

t: fictitious time direction (flow-time)

$x = (\vec{x}, \tau)$

UV finiteness of the gradient flow

Flow equation (continuum)

$$\partial_t B_\mu(t,x) = D_\nu G_{\nu\mu}(t,x)$$ initial condition: $B_\mu(t=0,x) = A_\mu(x)$

perturbative solution in the leading order

$$B_\mu(t,x) = \int d^D y K_t(x-y) A_\mu(y)$$

$$K_t(z) = \int \frac{d^D p}{(2\pi)^D} e^{ipz} e^{-tp^2}$$

$p^2 > 1/t$ modes are suppressed (a smooth UV cutoff)

Smeared in the range $|x| < \sqrt{8t}$

signal becomes clear?

Finiteness is shown perturbatively in all order

Luescher and Weisz, JHEP 1102, 051(2011)

Energy-momentum tensor

$$T_{\mu\nu}(x) = \frac{1}{g_0^2}\left[F^a_{\mu\rho}(x)F^a_{\nu\rho}(x) - \frac{1}{4}\delta_{\mu\nu}F^a_{\rho\sigma}(x)F^a_{\rho\sigma}(x)\right]$$

Renormalized EMT within dim. reg.

$$\{T_{\mu\nu}\}_R(x) = T_{\mu\nu}(x) - \langle T_{\mu\nu}(x)\rangle$$

Dim=4 rank2 symmetric operator

$$U_{\mu\nu}(t,x) \equiv G^a_{\mu\rho}(t,x)G^a_{\nu\rho}(t,x) - \frac{1}{4}\delta_{\mu\nu}G^a_{\rho\sigma}(t,x)G^a_{\rho\sigma}(t,x)$$

$$E(t,x) \equiv \frac{1}{4}G^a_{\mu\nu}(t,x)G^a_{\mu\nu}(t,x)$$

Here, ops. are constructed by flowed field.

``Suzuki method”

- small flow-time expansion -

Suzuki, PTEP 2013, no8, 083B03, [Erratum: PTEP2015,079201(2015)],

relation…dim.=4 op on the lattice vs. renormalized EMT at small flow-time

$$U_{\mu\nu}(t,x) = \alpha_U(t)\left[\{T_{\mu\nu}\}_R(x) - \frac{1}{4}\delta_{\mu\nu}\{T_{\rho\rho}\}_R(x)\right] + O(t),$$

$$E(t,x) = \langle E(t,x)\rangle + \alpha_E(t)\{T_{\rho\rho}\}_R(x) + O(t),$$

coefficients…given by renormalized coupling and coeff. of beta fn.

$$\alpha_U(t)(g;\mu) = g^2\left\{1 + 2b_0\left[\ln(\sqrt{8t}\mu) + s_1\right]g^2 + O(g^4)\right\},$$

$$\alpha_E(t)(g;\mu) = \frac{1}{2b_0}\left\{1 + 2b_0 s_2 g^2 + O(g^4)\right\},$$

b_0 1-loop coeff. of beta fn.   MSbar scheme

$s_1 =$ -0.0863575

$s_2 =$ 0.05578512

cf.) Nonperturbative method:
L.DelDebbio, A.Patella,A.Rogo, JHEP 1311,212(2013)

How to get EMT

for quenched QCD

Step 1

Generate gauge configuration at t=0 (usual process)

Step 2

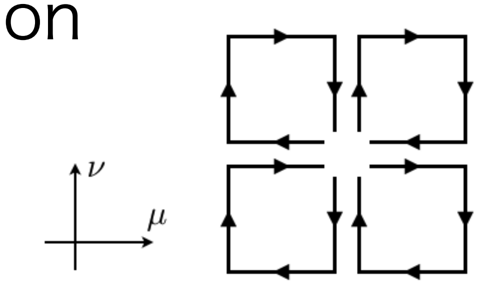
Solve the Wilson flow eq. and generate the gauge configuration at flow time (t)

$$a \ll \sqrt{8t} \ll \Lambda_{QCD}^{-1} \text{ or } T^{-1}$$

Step 3

Measure two dim=4 ops. using flowed gauge configuration

$$U_{\mu\nu}(t,x), E(t,x)$$



Step 4

Take the continuum limit. Then take t->0 limit.
(Take care the feasible window of flow time)

$$T_{\mu\nu}^{R}(x) = \lim_{t\to 0}\left\{ \frac{1}{\alpha_U(t)} U_{\mu\nu}(t,x) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)} [E(t,x) - \langle E(t,x)\rangle_0] \right\}$$

One-point fn. of EMT in finite temperature quenched QCD

Asakawa, Hatsuda, E.I., Kitazawa, Suzuki (FlowQCD coll.)
Phys.Rev. D90 (2014) 1, 011501

Simulation setup

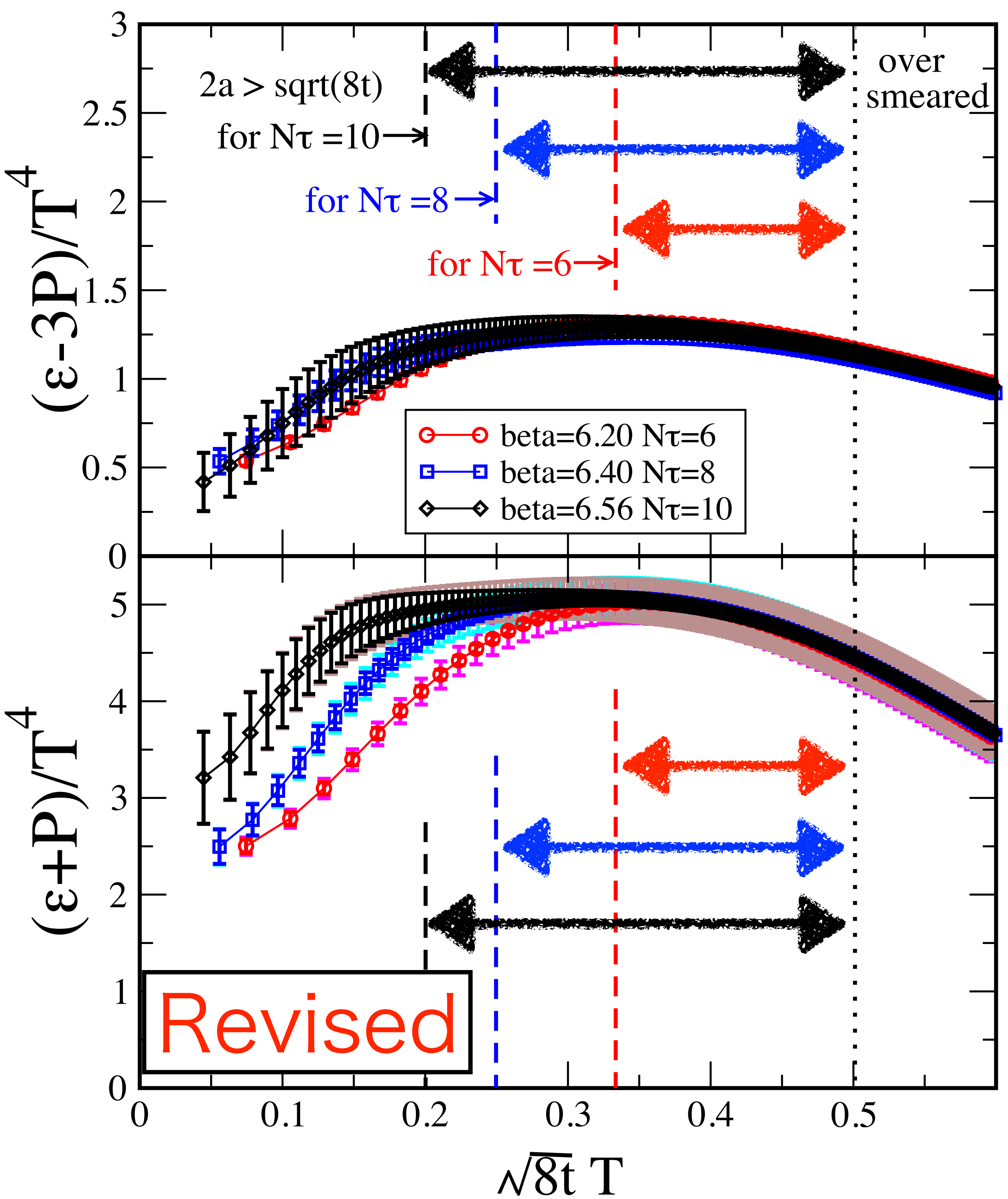
- Wilson plaquette gauge action
- lattice size (Ns=32, Nt=6,8,10,32)
- # of confs. is 100 - 300
- simulation parameters

N_τ	6	8	10	T/T_c
β	6.20	6.40	6.56	1.65
	6.02	6.20	6.36	1.24
	5.89	6.06	6.20	0.99

Temperature is determined by
Boyd et. al. NPB469,419 (1996)

Parametrization is given by
alpha collaboration NPB538,669 (1999)

flow time dependence
(T=1.65Tc)



each dark color shows statistical error

each light color includes systematic error

trace anomaly $\sum_{i=1}^{4} T_{ii} = \frac{\epsilon - 3P}{T^4}$

entropy density $T_{44} - T_{11} = \frac{\epsilon + P}{T^4}$

feasible flow time

longer than lattice cutoff

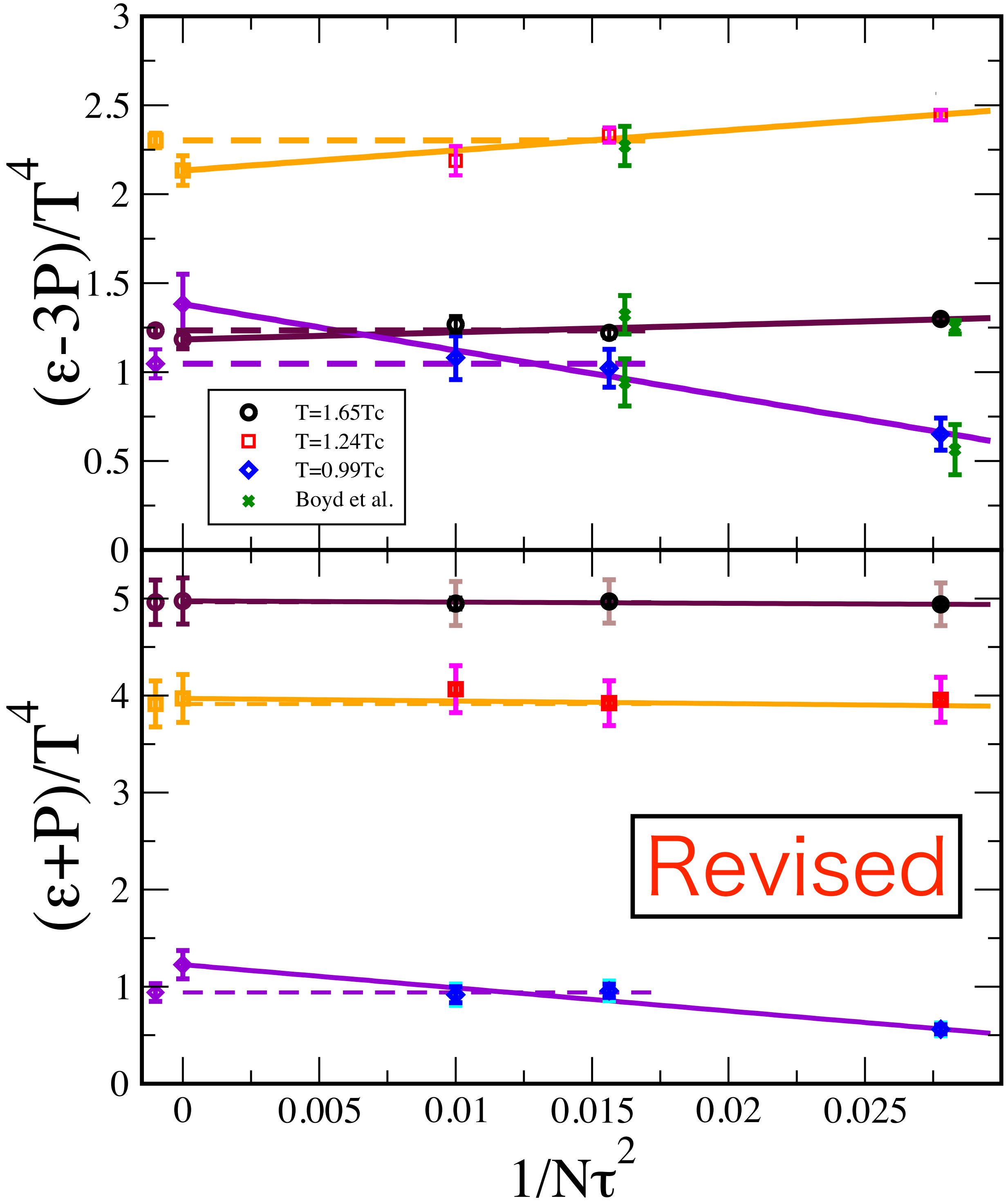
avoid an over-smeared regime

$$2a < \sqrt{8t} < N_\tau a/2$$

- show a plateau
  (small higher dimensional op.)
  Practically, no need t-> 0 limit
  **finer lattice simulation shows a slope

- systematic error coming from scale setting is dominated in entropy density

Continuum extrapolation



$\sqrt{8t}T = 0.40$

3point linear extrap.
(2pt. const. extrap.)

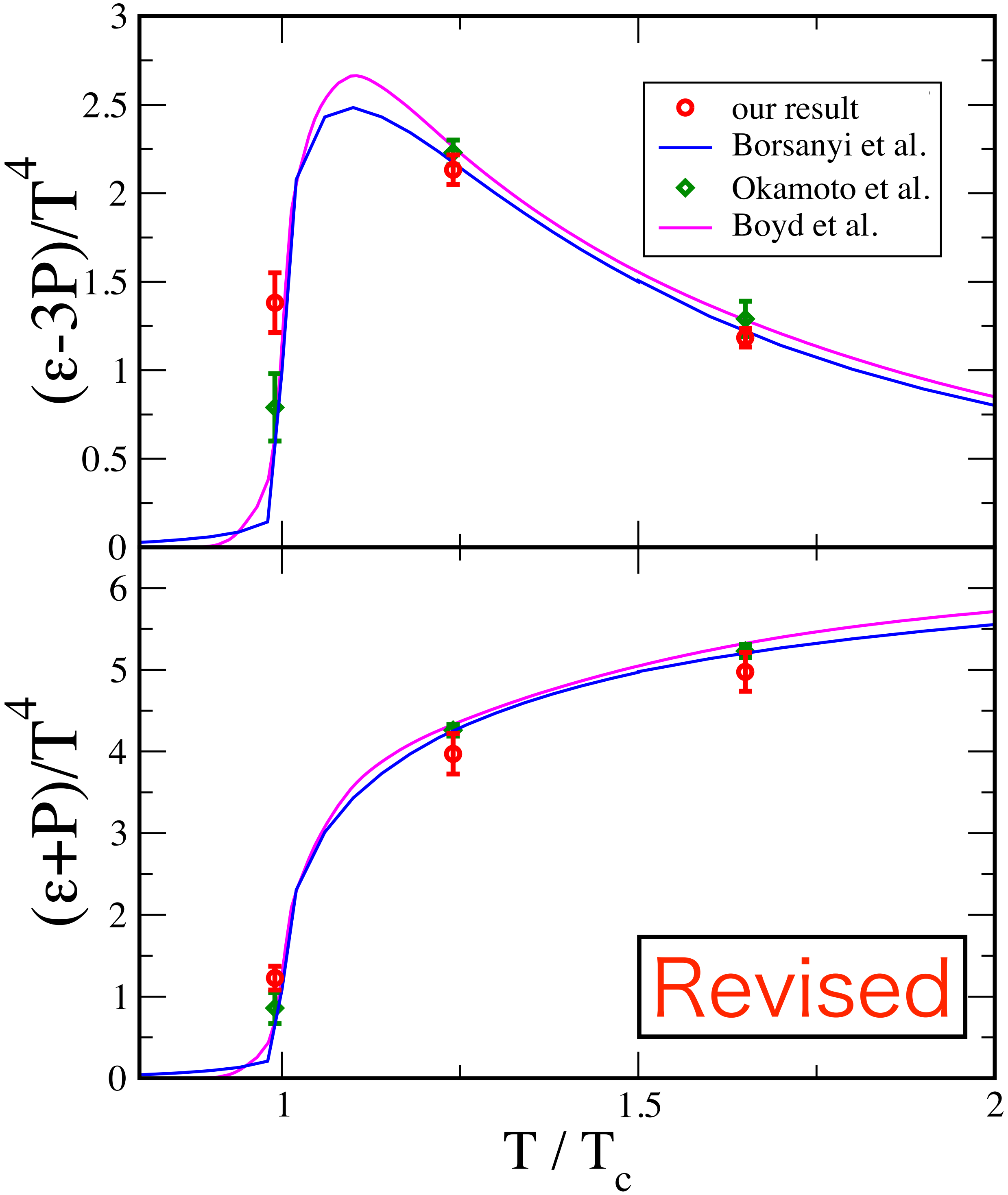
We also see the data at $\sqrt{8t}T = 0.35$

In cont.lim. the result is consistent.

t->0 limit is not needed in this case

Comparison with the results given by integration method

Phys.Rev. D90 (2014) 1, 011501, arXiv:1312.7492v3[hep-lat]



Boyd et. al. NPB469,419 (1996)

Okamoto et. al. (CP-PACS) PRD60, 094510 (1999)

Borsanyi et. al. JHEP 1207, 056 (2012)

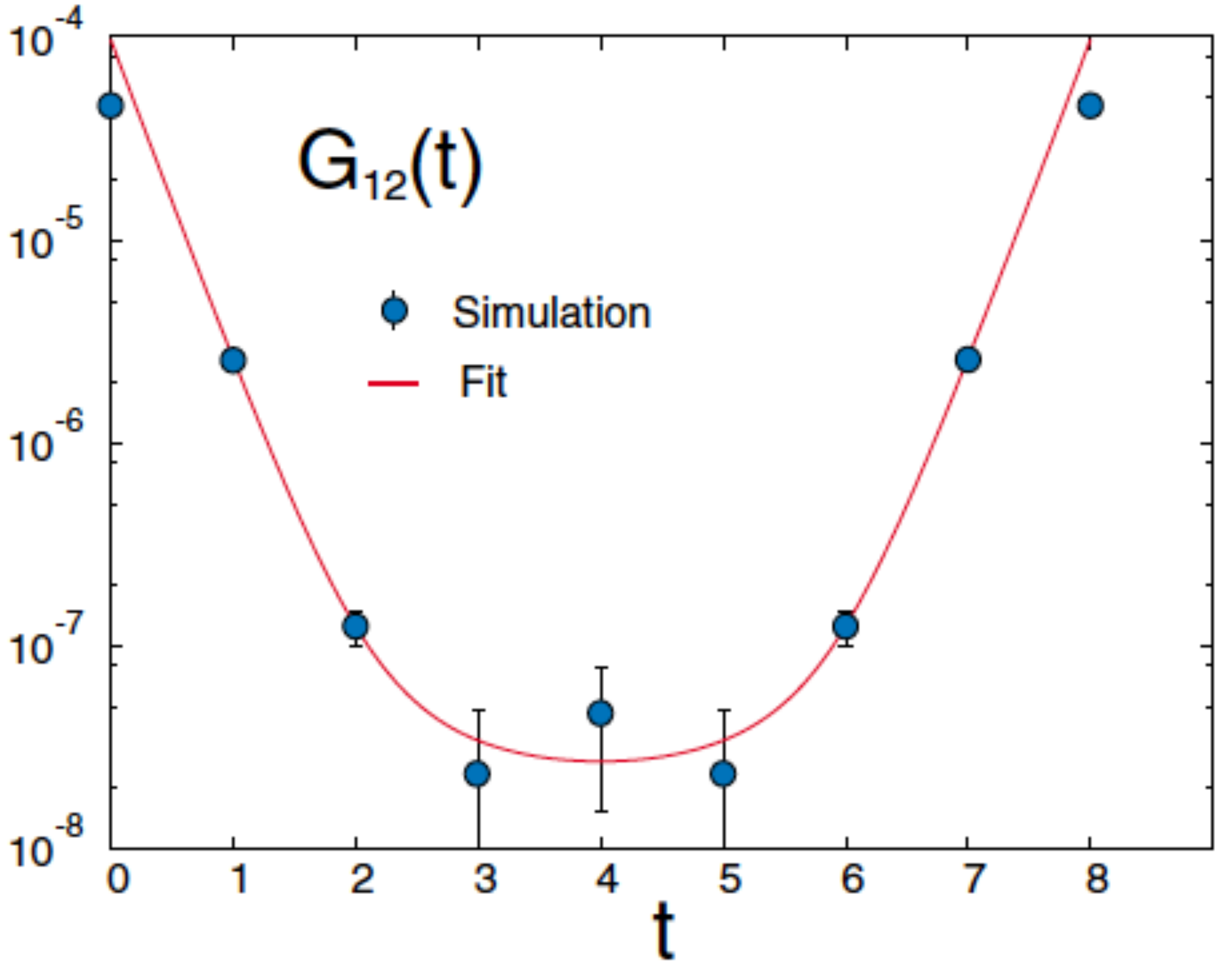
Integration method is based on (macroscopic) thermodynamics.

Our method is based on the (microscopic) quantum field theory.

two-point fn. of EMT

preliminary

Shear viscosity in QGP phase



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quenched QCD
(pure Yang-Mills theory)

Renormalization

$$T_{\mu\nu}^{(R)}(g_0) = Z(g_0) T_{\mu\nu}^{(bare)}$$

Meyer (2007)…1loop approximation

Fodor et al. (2013)…calculate Z-factor from entropy density

This work … Not necessary

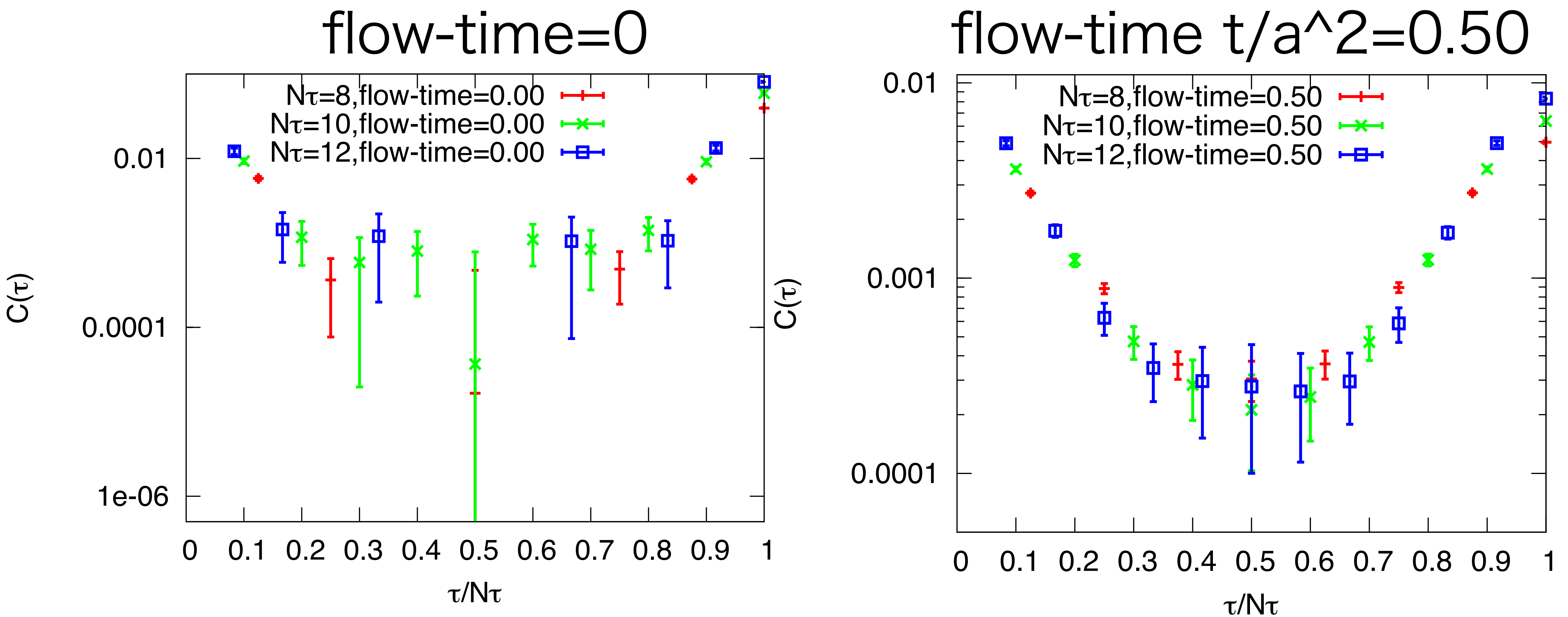
(usage of Suzuki coefficient and MSbar coupling)

cf.) $\frac{sT}{4} = \langle T_{11}^{(R)} \rangle$

$$\langle T_{12} T_{12} \rangle = \frac{1}{4} \langle (T_{11} - T_{22})(T_{11} - T_{22}) \rangle$$

lattice raw data

$$C(\tau) = \langle \frac{1}{N_s^3} \sum_{\vec{x}} U_{12}(\vec{x},\tau) \frac{1}{N_s^3} \sum_{\vec{y}} U_{12}(\vec{y},0) \rangle$$



fixed smeared length in lattice unit

beta=6.40,Nt=8, 2,000 conf.

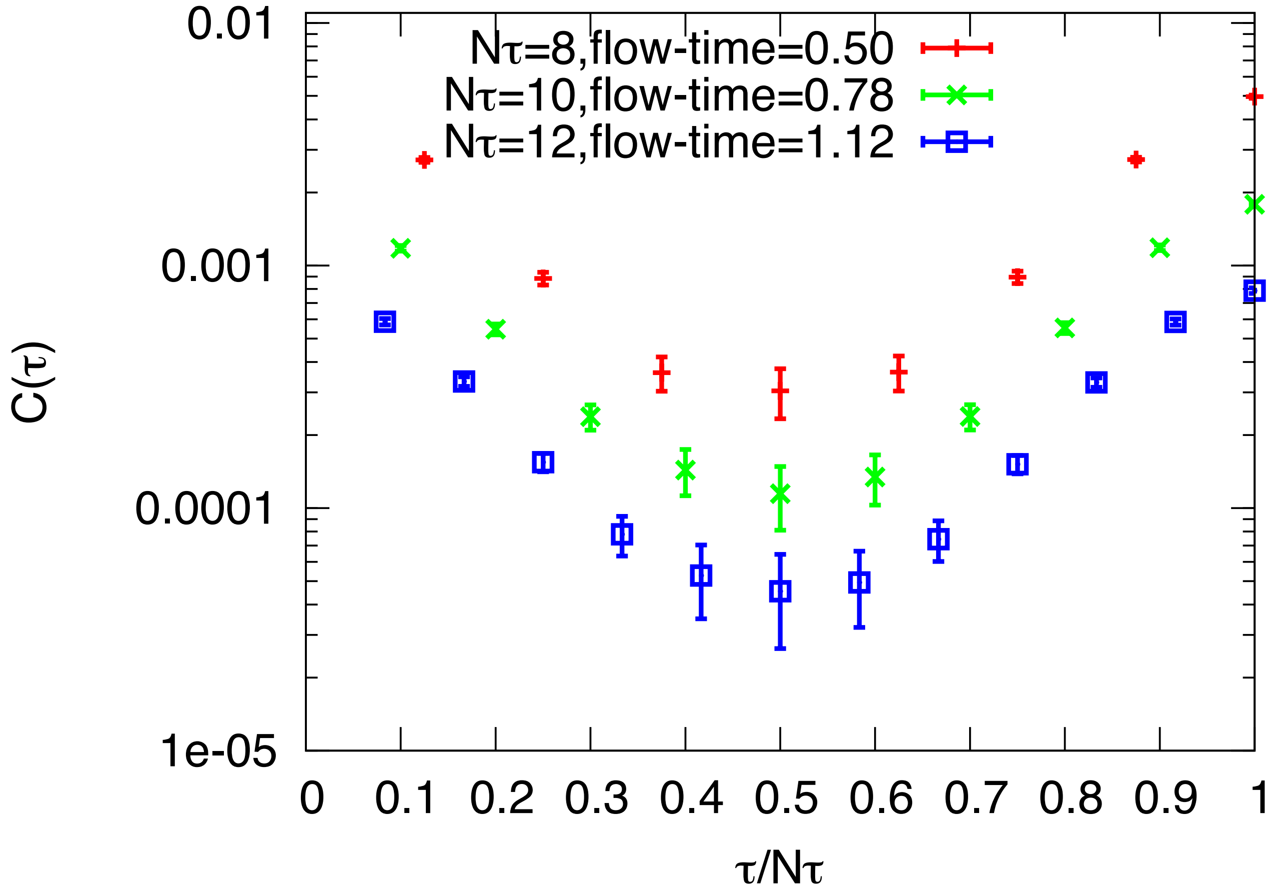
beta=6.57,Nt=10, 1,100 conf.

beta=6.72,Nt=12, 650 conf.

EMT correlator

$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left\{\frac{1}{\alpha_U(t)}U_{\mu\nu}(t,x) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)}[E(t,x) - \langle E(t,x)\rangle_0]\right\}$$

$$C(\tau) = \langle \frac{1}{N_s^3}\sum_{\vec{x}} U_{12}(\vec{x},\tau)\frac{1}{N_s^3}\sum_{\vec{y}} U_{12}(\vec{y},0)\rangle$$



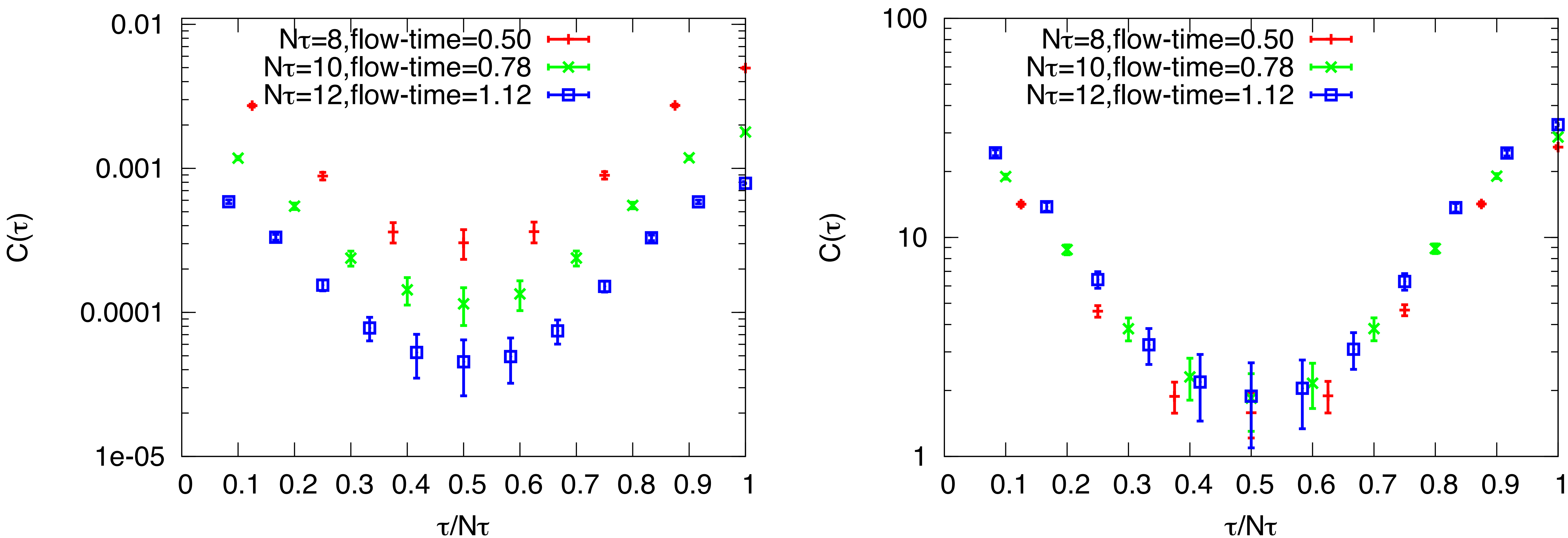
fixed smeared length in physical unit $\sqrt{8t}T = 0.25$

EMT correlator

$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left\{\frac{1}{\alpha_U(t)}U_{\mu\nu}(t,x) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)}\left[E(t,x) - \langle E(t,x)\rangle_0\right]\right\}$$

$$C(\tau) = \langle \frac{1}{N_s^3}\sum_{\vec{x}} U_{12}(\vec{x},\tau)\frac{1}{N_s^3}\sum_{\vec{y}} U_{12}(\vec{y},0)\rangle$$

$$C(\tau) = \frac{1}{T^5}\langle \sum_{\vec{x}} T_{12}(\vec{x},\tau)\sum_{\vec{y}} T_{12}(\vec{y},0)\rangle$$



fixed smeared length in physical unit $\sqrt{8t}T = 0.25$

Nf=12 case

(conformal theory)

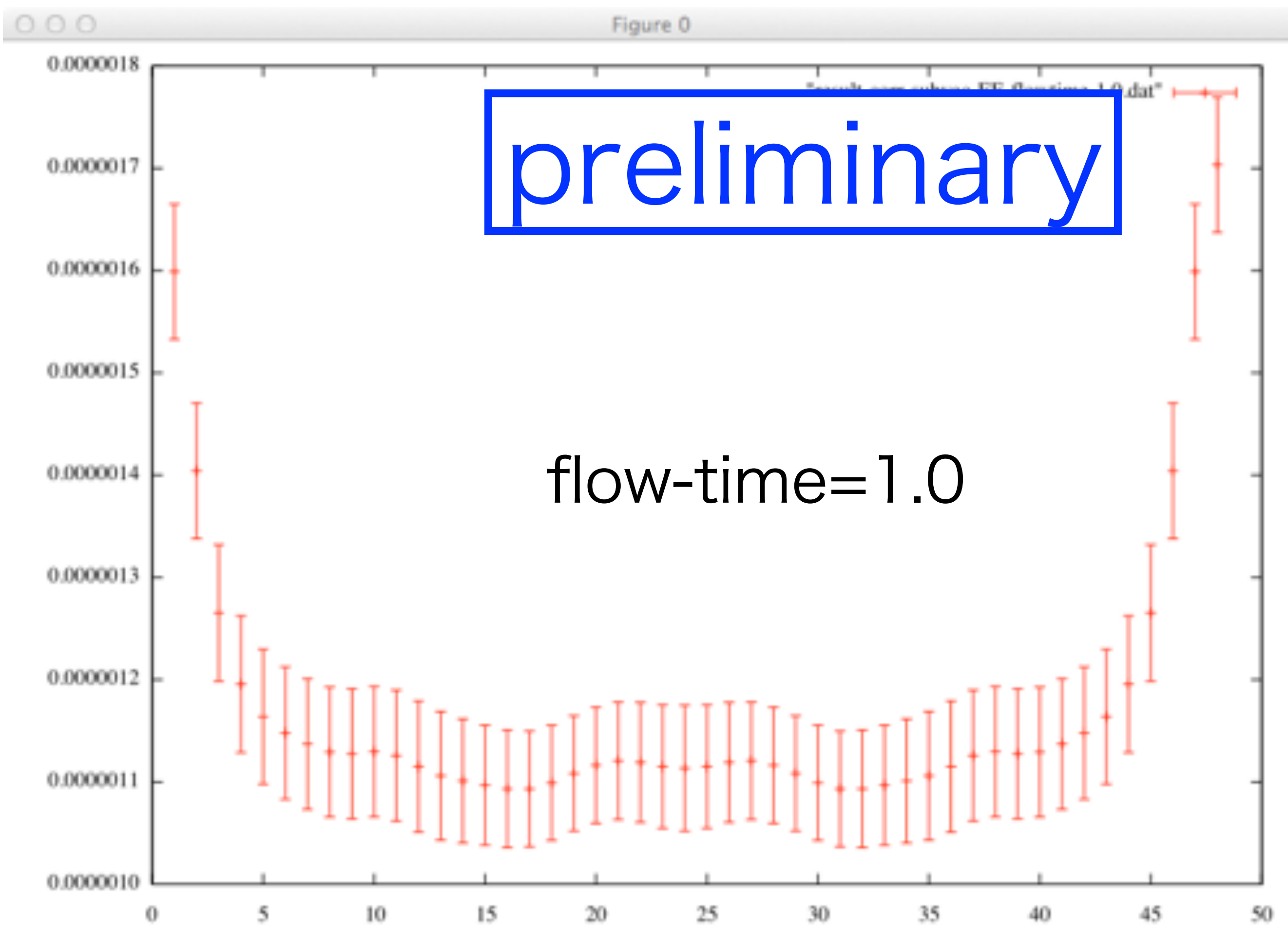
very preliminary

Correlation fn. of the fluctuation

massless Nf=12 at IRFP

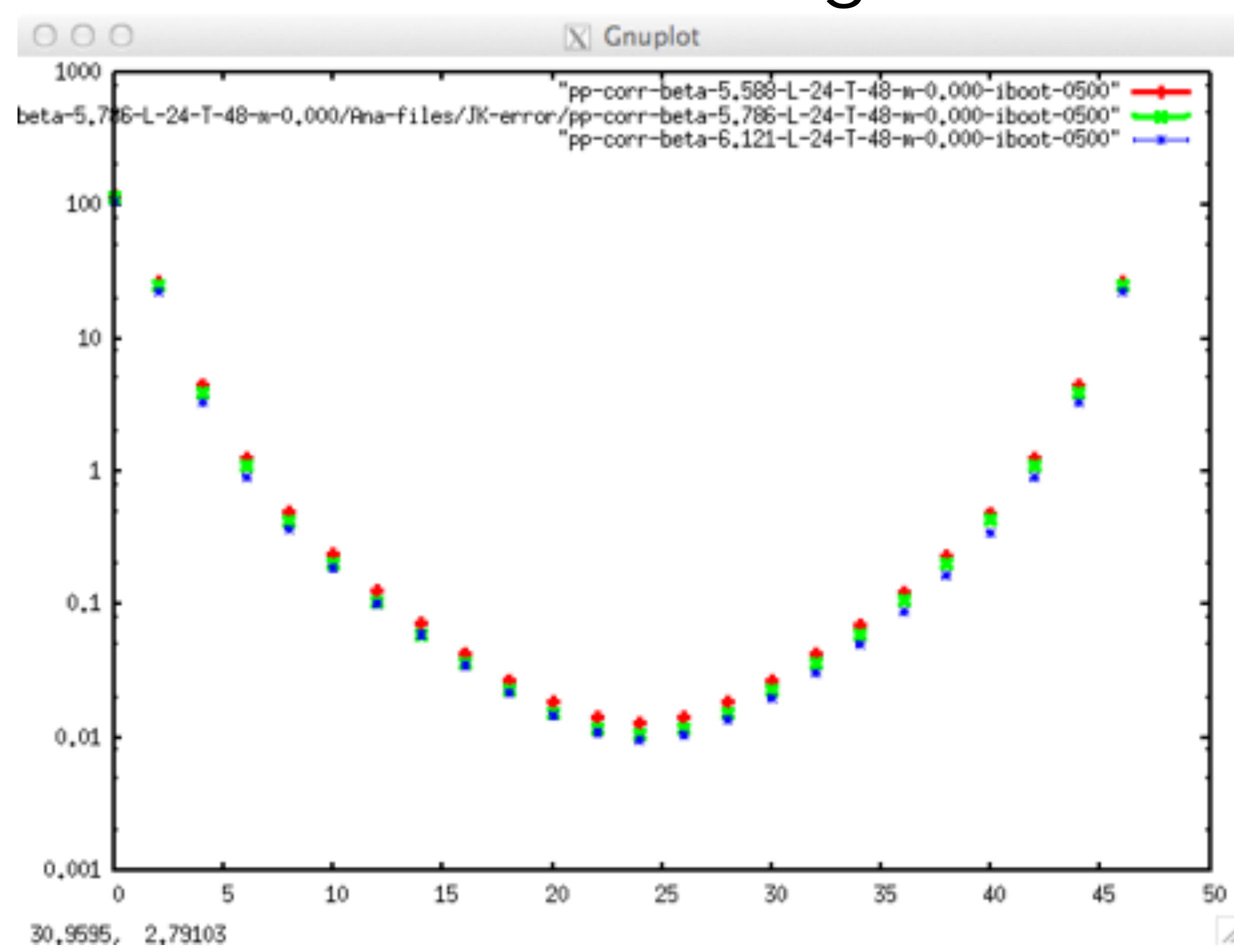
Trace part of the EMT

vertical axis: linear scale



pseudo scalar op.

vertical axis: log scale



Conservation law of energy-momentum tensor?

$$\partial_\tau T^\mu_\mu = 0$$

Conclusion
(EMT part)

- ✦ Novel method to obtain EMT using the lattice simulation
- ✦ quenched results (1pt.fn) show that the small flow time expansion is promising
- ✦ clear statistical signal, small systematic error
- ✦ 2pt. fn. calculation is also doable!!

Future directions

- ✦ two-point function of EMT (shear and bulk viscosity, heat capacity)
- ✦ include dynamical fermion fields (need the calculation of Z-factor)
- ✦ application to conformal field theory (central charge, dilation physics)
- ✦ supersymmetry on the lattice