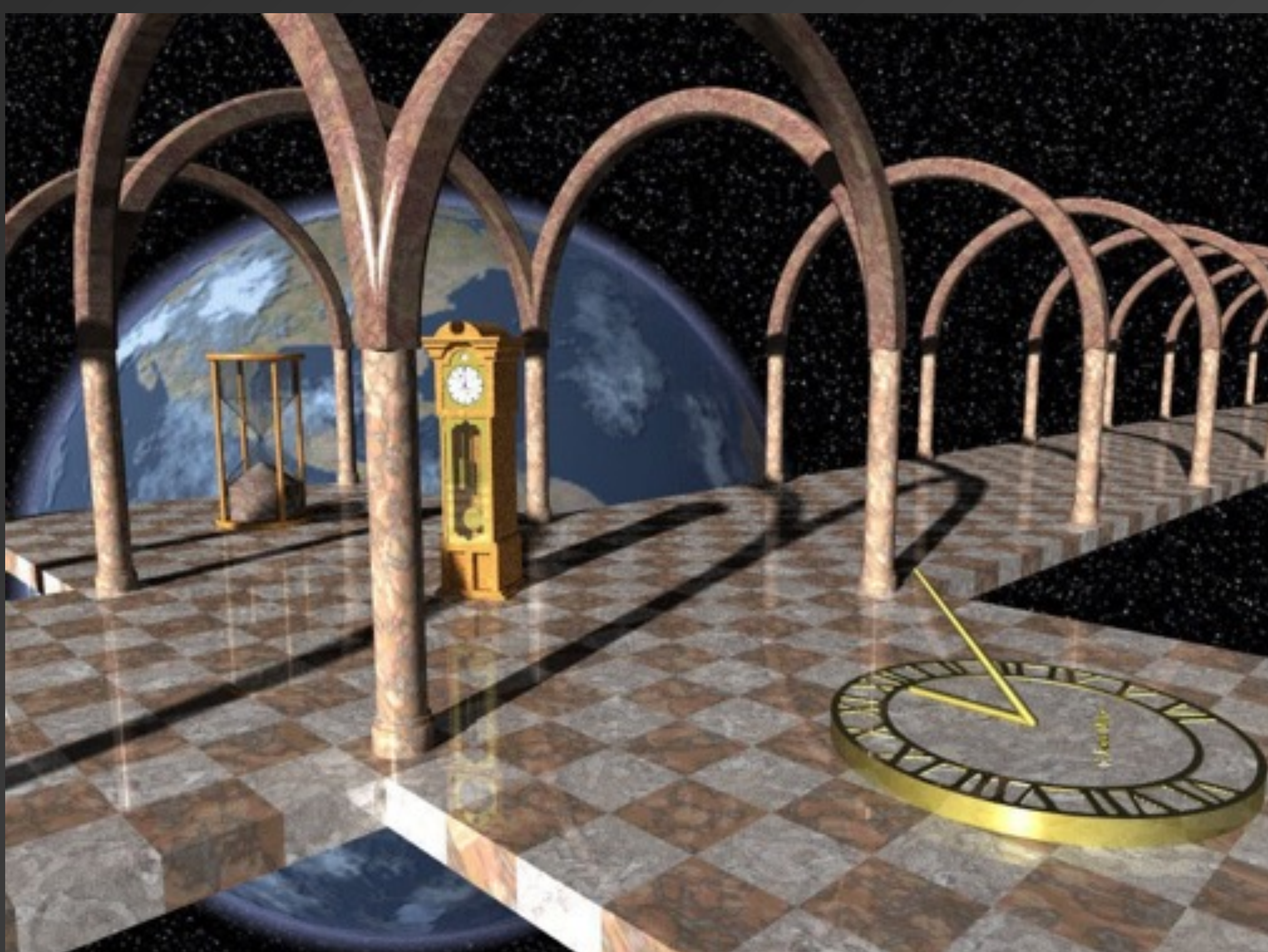
Emergent gravity: From Condensed matter analogues to Phenomenology.

Emergent properties of space-time. CERN.



Stefano Liberati
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GR, a beautiful but weird theory...

Some tantalising features of General Relativity

- Singularities
- Critical phenomena in gravitational collapse
- Horizon thermodynamics
- Spacetime thermodynamics: Einstein equations as equations of state.
- The "dark ingredients" of our universe?
- Faster than light and Time travel solutions
- AdS/CFT duality, holographic behaviour
- Gravity/fluid duality

Gravity as an emergent phenomenon?

Emergent gravity idea: quantizing the metric or the connections does not help because perhaps these are not fundamental objects but collective variables of more fundamental structures.

- GR ⇒ Hydrodynamics
- Metric as a collective variable
- All the sub-Planckian physics is low energy physics
- Spacetime as a condensate of some more fundamental objects
- Spacetime symmetries as emergent symmetries
- Singularities as phase transitions (big bang as geometrogenesis)
- Cosmological constant as deviation from the real ground state

- Many models are nowadays resorting to emergent gravity scenarios
  - Causal sets
  - Quantum graphity models
  - Group field theories condensates scenarios
  - AdS/CFT scenarios where the CFT is considered primary
  - Gravity as an entropic force ideas
  - Condensed matter analogues of gravity

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C.Barcelo, S.L and M.Visser,
“Analogue gravity”
Living Rev.Rel.8,12 (2005-2011).

The Analogue gravity pathway
Emergent spacetimes

And I cherish more than anything else the Analogies, my most trustworthy masters. They know all the secrets of Nature, and they ought least to be neglected in Geometry.
Johannes Kepler

Unruh '81, Visser '93
But see also White '73

A paradigmatic example: Acoustic Gravity

Continuity $\frac{\partial\rho}{\partial t}+\vec{\nabla}\cdot(\rho\vec{v})=0$ Euler $\rho\frac{d\vec{v}}{dt}\equiv\rho\left[\frac{\partial\vec{v}}{\partial t}+(\vec{v}\cdot\nabla)\vec{v}\right]=-\vec{\nabla}p-\rho\vec{\nabla}\Phi+\vec{f}_{\text{viscosity}}$

External Forces $\vec{f}_{\text{viscosity}}=+\eta\nabla^2\vec{v}+\left(\zeta+\frac{1}{3}\eta\right)\vec{\nabla}\left(\vec{\nabla}\cdot\vec{v}\right)$

$p=$ pressure, $\eta=$ dynamic viscosity, $\zeta=$ bulk viscosity,

$\Phi=$ potential of external driving force (gravity included)

Basic Assumptions

$\vec{\nabla}\times\vec{v}=\vec{0}$ $\vec{v}=\vec{\nabla}\psi$ $\rho=\rho(p)$ $c_s^2=\frac{dp}{d\rho}$

Irrotational Flow
Barotropic
Viscosity free flow

Linearize the above Eq.s around some background

$$\rho(t,x)=\rho_0(t,x)+\varepsilon\rho_1(t,x)$$
$$p(t,x)=p_0(t,x)+\varepsilon p_1(t,x)$$
$$\psi(t,x)=\psi_0(t,x)+\varepsilon\psi_1(t,x)$$

And combine then so to get a second order field equation

$$\frac{\partial}{\partial t}\left(c_s^{-2}\rho_0\left(\frac{\partial\psi_1}{\partial t}+\vec{v}_0\cdot\nabla\psi_1\right)\right)=\nabla\cdot\left(\rho_0\nabla\psi_1-c_s^{-2}\rho_0\vec{v}_0\left(\frac{\partial\psi_1}{\partial t}+\vec{v}_0\cdot\nabla\psi_1\right)\right)$$

This looks messy but if we introduce the "acoustic metric"

$$g_{\mu\nu}\equiv\frac{\rho_0}{c_s}\begin{bmatrix}-\left(c_s^2-v_0^2\right) & -v_0^j\\ -v_0^i & \delta_{ij}\end{bmatrix}$$

We get

$$\Delta\psi_1\equiv\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\,g^{\mu\nu}\partial_\nu\psi_1\right)=0$$

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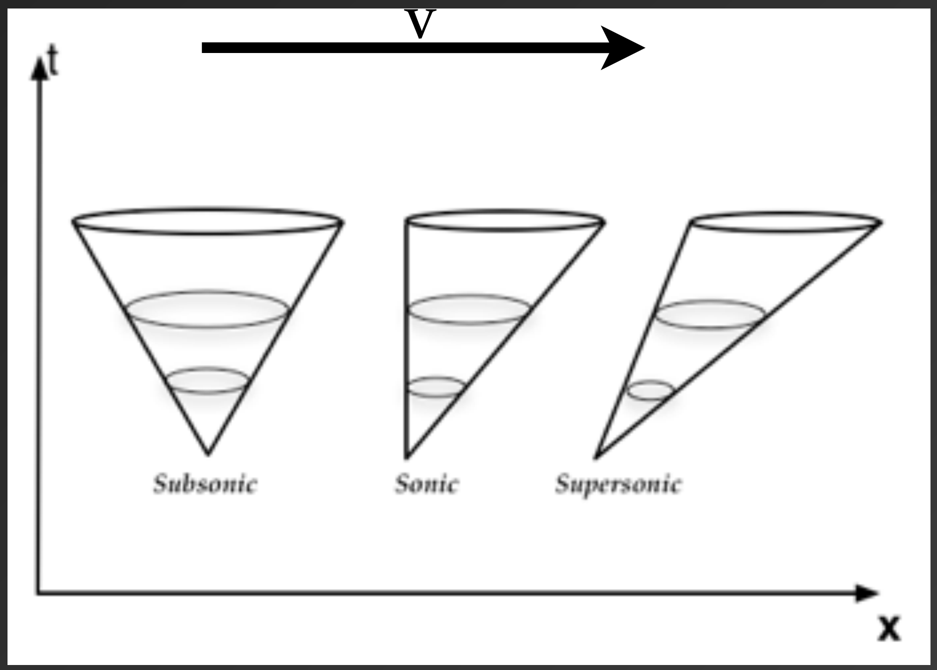
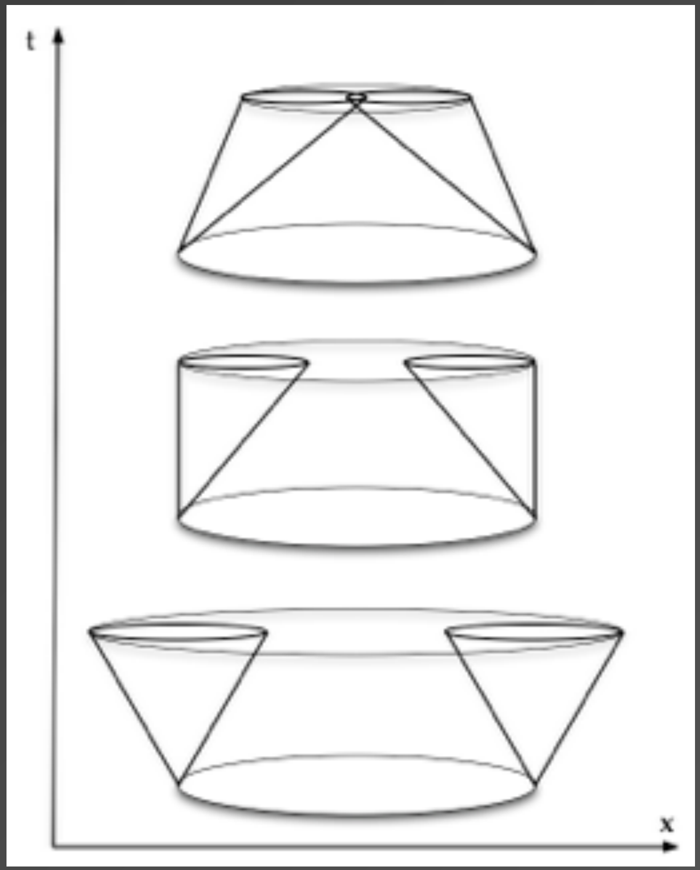
$$\Delta\psi_1\equiv\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\,g^{\mu\nu}\partial_\nu\psi_1\right)=0$$

This is the same equation as for a scalar field moving in curved spacetime, possibility to simulate FRW and Black Holes!

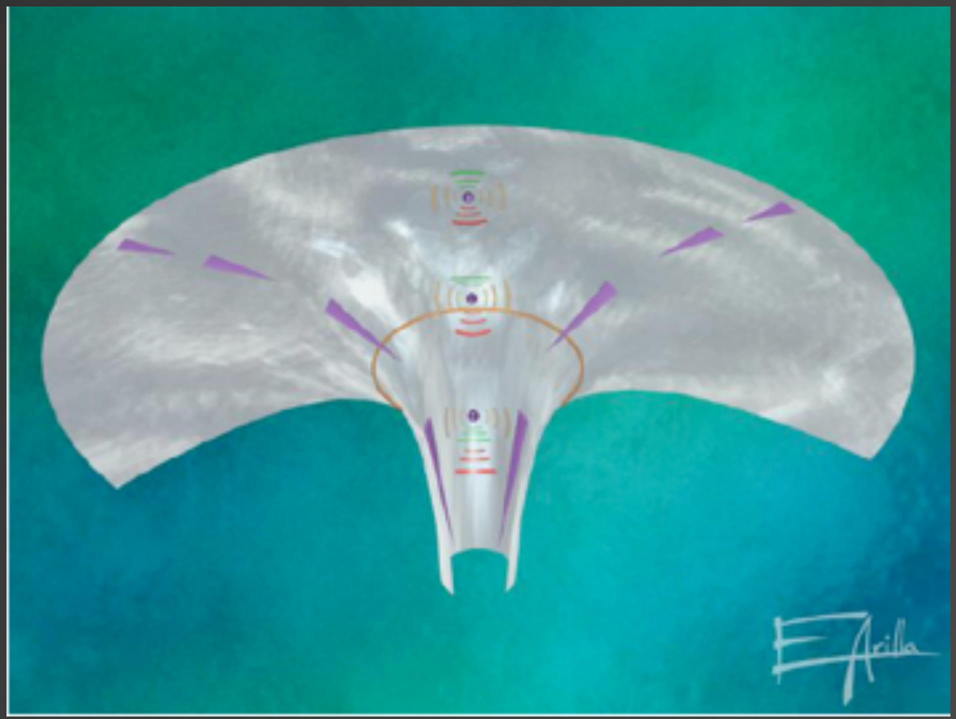
Analogue Black holes

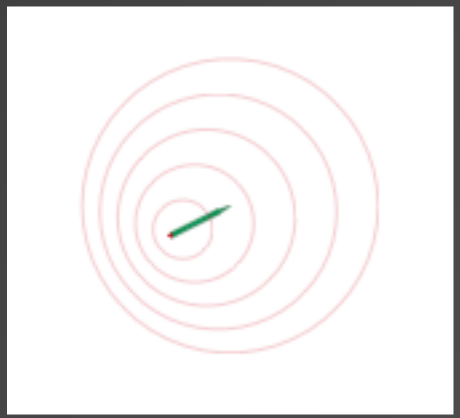


A moving fluid will tip the "sound cones" as it moves. Supersonic flow will tip the cone past the vertical.

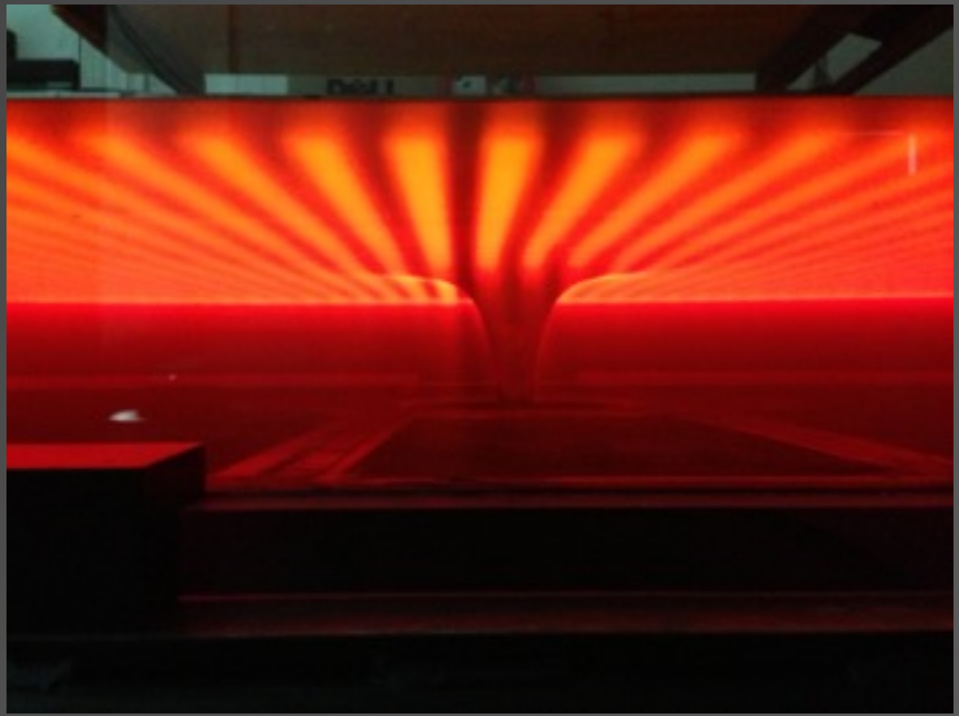


A moving fluid can form "trapped regions"" when supersonic flow will tip the cone past the vertical.





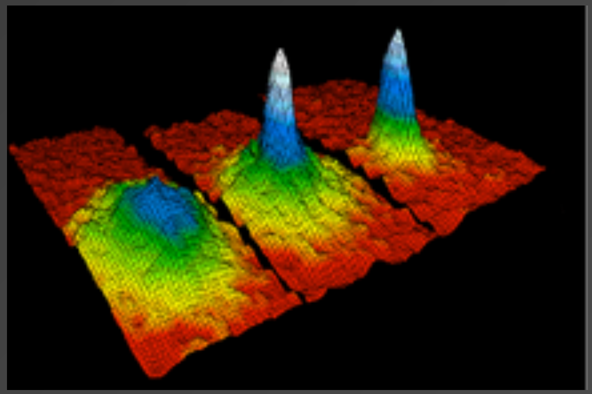
A moving fluid will drag sound pulses along with it



SISSA-Nottingham experiment: an analogue of superradiant scattering of gravity waves (PI: S. Weinfurtner)

A prototype quantum analogue model: BEC

A BEC is quantum system of N interacting bosons in which most of them lie in the same single-particle quantum state
($T<T_c \sim 100$ nK, $N_{atoms} \sim 10^5 \div 10^6$)



It is described by a many-body Hamiltonian which in the limit of dilute condensates gives a non-linear Schrödinger equation

(a=s-wave scattering length)

$$i\hbar \frac{\partial}{\partial t}\hat{\Psi} = -\frac{\hbar^2}{2m}\nabla^2\hat{\Psi} - \mu\hat{\Psi} + \kappa|\hat{\Psi}|^2\hat{\Psi}.$$

$$\kappa(a) = \frac{4\pi a\hbar^2}{m}.$$

This is still a very complicate system, so let's adopt a mean field approximation

$$\text{Mean field approximation: } \hat{\Psi}(t,\mathbf{x}) = \psi(t,\mathbf{x}) + \hat{\chi}(t,\mathbf{x}) \quad \text{where} \quad |\psi(t,\mathbf{x})|^2 = n_c(t,\mathbf{x}) = N/V$$

$$\psi(t,\mathbf{x}) = \langle \hat{\Psi}(t,\mathbf{x}) \rangle = \text{classical wave function of the BEC }, \qquad \hat{\chi}(t,\mathbf{x}) = \text{excited atoms}$$

Note that: $\hat{\Psi}|0\rangle = 0$ (atomic Fock vacuum) $\qquad \hat{\Psi}|\Omega\rangle \neq 0$ (ground state)

The ground state is the vacuum for the collective excitations of the condensate (quasi-particles) but this an inequivalent state w.r.t. the atomic vacuum. They are linked by Bogolyubov transformations.

Bose-Einstein condensate: an example of analogue emergent spacetime

By direct substitution of the mean field ansatz in the non-linear Schrödinger equation gives

$$\mathrm{i}\hbar\,\frac{\partial}{\partial t}\psi(t,\mathbf{x}) = \left(-\frac{\hbar^2}{2m}\nabla^2 - \mu + \kappa|\psi|^2\right)\psi + 2\kappa\left(\tilde{n}\psi + \tilde{m}\psi^*\right)$$

$$\mathrm{i}\hbar\,\frac{\partial}{\partial t}\widehat{\chi} = \left(-\frac{\hbar^2}{2m}\nabla^2 - \mu + 2\kappa n_T\right)\widehat{\chi} + \kappa m_T\,\widehat{\chi}^\dagger$$

Background dynamics

$$n_c \equiv |\psi(t,\mathbf{x})|^2\,; \qquad m_c \equiv \psi^2(t,\mathbf{x});$$

$$\tilde{n} \equiv \langle\widehat{\chi}^\dagger\widehat{\chi}\rangle; \qquad \tilde{m} \equiv \langle\widehat{\chi}\widehat{\chi}\rangle;$$

$$n_T = n_c + \tilde{n}; \qquad m_T = m_c + \tilde{m}.$$

Excitations dynamics

These are the so called Bogoliubov-de Gennes equations
The first one encodes the BEC background dynamics
The second one encodes the dynamics for the quantum excitations

The equations are coupled via the so called anomalous mass m̃ and density ñ. Which we shall neglect for the moment...

Let's consider quantum perturbations over the BEC background and adopt the "quantum acoustic representation" (Bogoliubov transformation)

$$\widehat{\chi}(t,\mathbf{x}) = e^{-\mathrm{i}\theta/\hbar}\left(\frac{1}{2\sqrt{n_c}}\,\widehat{n}_1 - \mathrm{i}\frac{\sqrt{n_c}}{\hbar}\,\widehat{\theta}_1\right)$$

for the perturbations one gets the system of equations

$$\partial_t\widehat{n}_1 + \frac{1}{m}\nabla\cdot\left(\widehat{n}_1\,\nabla\theta + n_c\,\nabla\widehat{\theta}_1\right) = 0,$$

$$\partial_t\widehat{\theta}_1 + \frac{1}{m}\nabla\theta\cdot\nabla\widehat{\theta}_1 + \kappa(a)\,n_1 - \frac{\hbar^2}{2m}\,D_2\widehat{n}_1 = 0.$$

Where D_2 is a represents a second-order differential operator: the linearized quantum potential

$$D_2\,\widehat{n}_1 \equiv -\frac{1}{2}n_c^{-3/2}\left[\nabla^2(n_c^{+1/2})\right]\widehat{n}_1 + \frac{1}{2}n_c^{-1/2}\,\nabla^2(n_c^{-1/2}\,\widehat{n}_1).$$

Acoustic metric

For very long wavelengths the terms coming from the linearized quantum potential D_2 can be neglected.

$$\Delta\theta_1 \equiv \frac{1}{\sqrt{-g}}\,\partial_\mu\left(\sqrt{-g}\,g^{\mu\nu}\,\partial_\nu\right)\widehat{\theta}_1 = 0,$$

The so obtained metric is again the acoustic metric

$$g_{\mu\nu}(t,\mathbf{x}) \equiv \frac{c_s}{\lambda}\begin{bmatrix} -\left(c_s^{\,2}-\mathrm{v}_0^2\right) & \vdots & -(\mathrm{v}_0)_j \\ \cdots & \cdot & \cdots \\ -(\mathrm{v}_0)_i & \vdots & \delta_{ij} \end{bmatrix} = \frac{n_0}{c_s m}\begin{bmatrix} -\left(c_s^{\,2}-\mathrm{v}_0^2\right) & \vdots & -(\mathrm{v}_0)_j \\ \cdots & \cdot & \cdots \\ -(\mathrm{v}_0)_i & \vdots & \delta_{ij} \end{bmatrix}$$

$$c_s = \frac{\hbar}{m}\sqrt{4\pi\rho a}$$

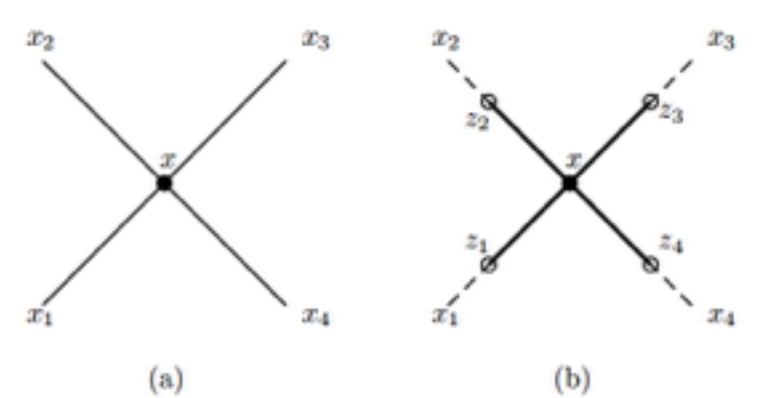


Figure 7.2: (a) Feynman diagram of a four-particles vertex. (b) Lowest order Feynman diagram for the interaction of four quasi-particles (dashed lines), induced by the interactions of particles (solid line). The circles denote the conversion of particles into quasi-particles at intermediate points. They represent the action of the Bogoliubov transformations.

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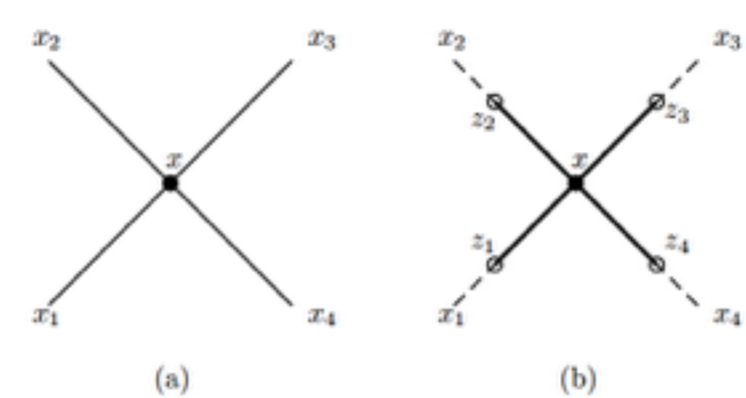
Lessons

The collective excitations (phonons) of the BEC propagate on a Lorentzian spacetime determined by the background velocity and density and they can be meaningfully quantised

- The atomic physics enters only in determining the fundamental constants of the low energy phenomenology (e.g. speed of sound c_s=analogue speed of light)
- The spacetime are stably causal as they inherit the causal structure from lab system: no-time machines possible.
- Lorentz symmetry of the phononic theory is an low energy symmetry
- Non Local quasi-particles interactions



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Acoustic Lorentz invariance breaking in BEC analogue gravity

If instead of neglecting the quantum potential we adopt the eikonal approximation (high-momentum approximation) we find, as expected, deviations from the Lorentz

E.g. the dispersion relation for the BEC quasi-particles is

$$\omega^2 = c_s^2 k^2 + \left(\frac{\hbar}{2m}\right)^2 k^4$$

This (Bogoliubov) dispersion relation (experimentally observed) actually interpolates between two different regimes depending on the value of the fluctuations wavelength
$\lambda=2\pi/|k|$ with respect to
the "acoustic Planck wavelength"
$\lambda_C=h/(2m_{cs})=\pi\xi$ with ξ=healing length of BEC=$1/(8\pi\varrho a)1/2$

For $\lambda \gg \lambda C$ one gets the standard phonon dispersion relation $\omega \approx c|k|$
For $\lambda \ll \lambda C$ one gets instead the dispersion relation for an individual gas particle (breakdown of the continuous medium approximation) $\omega \approx (\hbar^2 k^2)/(2m)$

Robustness and Detection of Hawking radiation in Black hole analogues

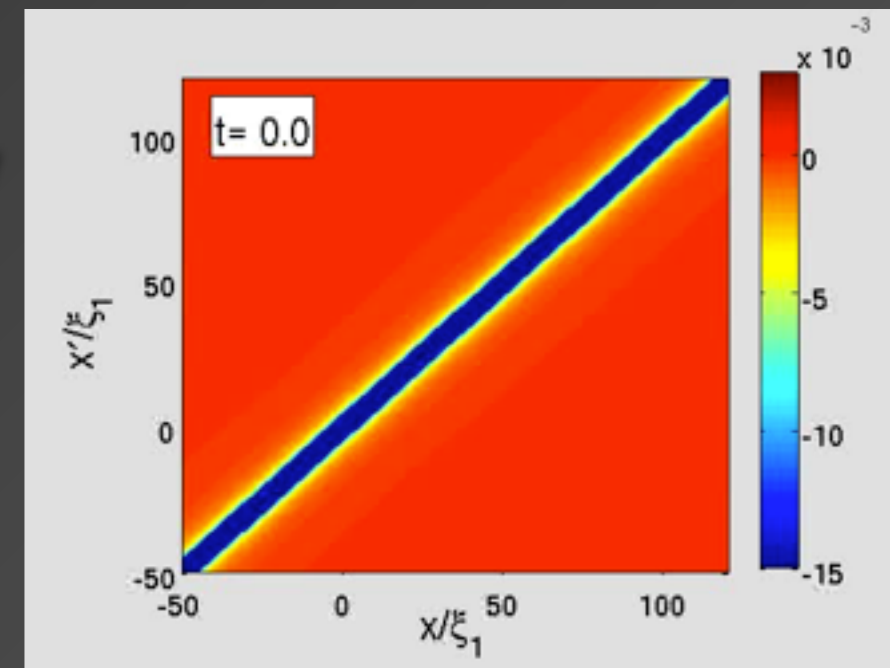
It turned out that Hawking Radiation is robust against LIV (see e.g. Parentani and collaborators recent papers), however you might get (controllable) instabilities such as “black hole laser effect” (superluminal relation in compact supersonic region).

Some facts:

In static spacetimes Hawking radiation robustness is generally assured if there is a separation of scales: $\kappa_{BH} << \Lambda$ where Λ is parametrically related to the UV LIV scale.

Tentative detection via density-density correlations

Recent observation of a characteristic instability for compact ergo regions J. Steinhauer. Nature Physics (2014). Even more recently first claim of Hawking detection (J. Steinhauer. 2015)!
Open debate on quantum vs classical excitations



Hawking Radiation detection via density-density correlation. BEC simulation. Carusotto et al.

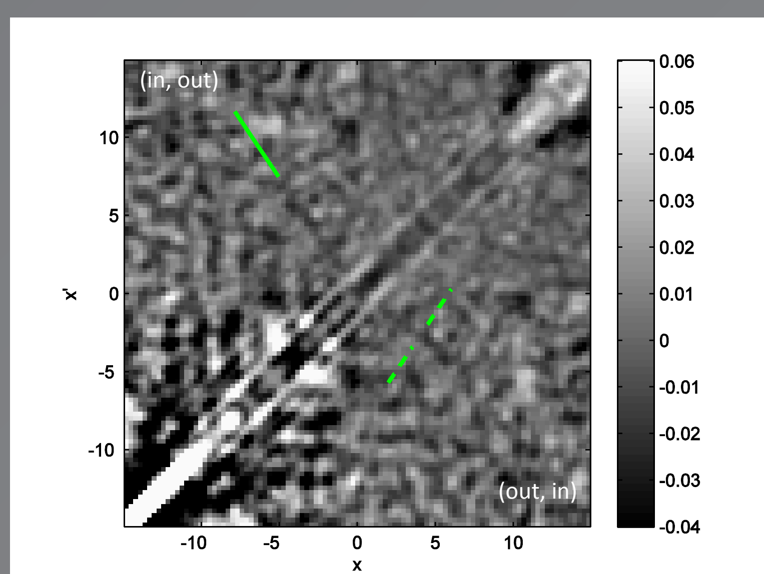


Fig. 3. Observation of Hawking/partner pairs. The horizon is at the origin. The dark bands emanating from the horizon are the correlations between the Hawking and partner particles. The solid line shows the angle of equal times from the horizon, found in Fig. 4. The Fourier transform along the dashed line measures the entanglement of the Hawking pairs.

So does BH thermodynamics survive without Lorentz invariance?

This is interesting even if you do not believe Lorentz invariance can be broken in the UV. Where thermodynamics comes from in Gravitational Theories? More soon...

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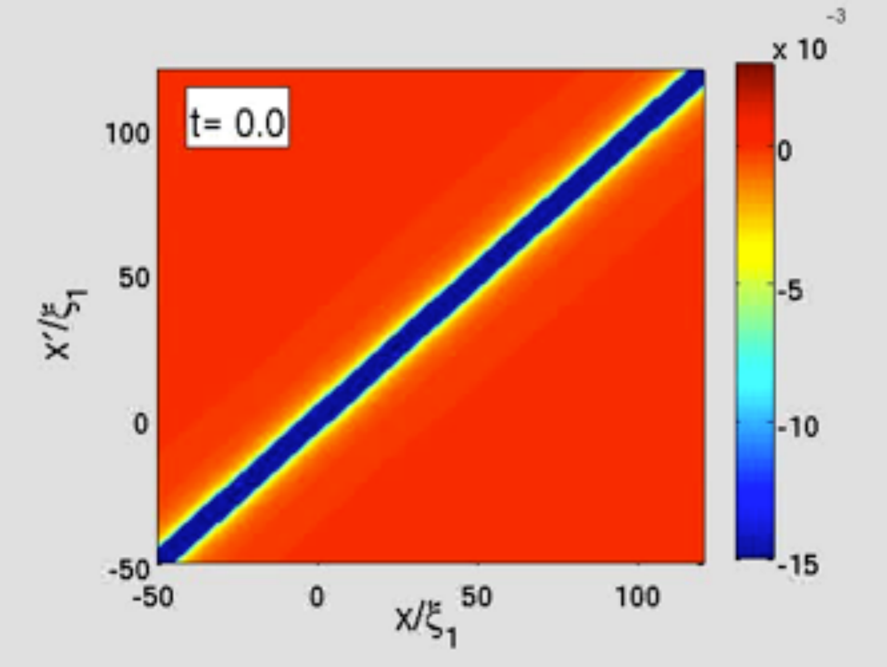

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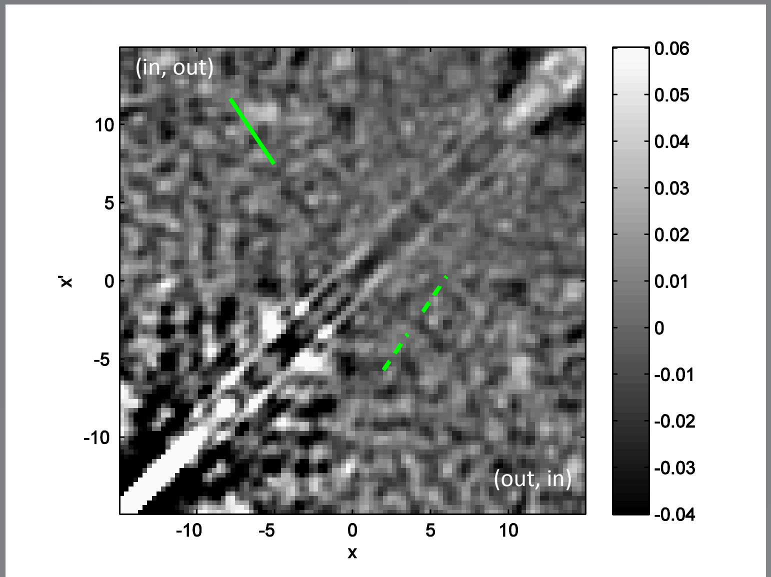

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Where thermodynamics comes from in Gravitational Theories?

More soon...

The Analogue gravity pathway
From emergent spacetimes to emergent gravity

Emergent gravity scenarios

Can we emerge a relativistic theory for Spin 2?
Obstruction: the Weinberg-Witten theorem

"No spin 2 particle can be emergent if you have Lorentz invariance and local Gauge invariant currents or conserved SET"

Hence potential ways out are:

One step emergence

"Spacetime Atoms"+QM → NO MESOSCOPIC PHYSICS ON A SPACETIME → Manifold, Metric, LLI, Diffeo Gravity+SM, Locality

From Manifold to Gravity with Lorentz breaking or non-locality

"Spacetime Atoms" in flat spacetime +QM → Phase transition or Sum over microscopic configurations → Differentiable Manifold, Fields, Locality, Lorentz breaking or non-locality → Low energies → LLI, Diffeo, Gravity+Matter

Induced gravity a la Sakarov

"Spacetime Atoms" +QM → ? → Differentiable Manifold, Fields + Lorentz+ Metric but no geometrodynamics → LLI, Diffeo, Gravity+Matter, Locality a la Induced Gravity

While the first "extreme emergence" route might seem the more fundamental it is also the hardest.

Let's see how far we can go with the Analogue Gravity framework…

A TOY MODEL FOR EMERGENT GRAVITY: NON-RELATIVISTIC BEC

So let's go back to the mean field approximation of BEC and focus on the equation for the background:

F. Girelli, S.L.,L.Sindoni
Phys.Rev.D78:084013,2008

$$i\hbar \frac{\partial}{\partial t}\psi(t,\mathbf{x}) = \left(-\frac{\hbar^2}{2m}\nabla^2 + V_{\rm ext}(\mathbf{x}) + \kappa n_c\right)\psi(t,\mathbf{x}) + \kappa\left(2\tilde{n}\psi(t,\mathbf{x}) + \tilde{m}\psi(t,\mathbf{x})\right)$$

- Can this be encoding some form of gravitational dynamics?
- If yes it must be some form of Newtonian gravity (non relativistic equation)
- But, in order to have any chance to see this, we need to have some massive field

One way to get this is to introduce a soft U(1) breaking term
(i.e. pass from massless Goldstone bosons top massive pseudo-Goldstone bosons)

Note: this kind of symm breaking is actually experimentally realized in magnon (quantized spin wave) BEC in 3He-B (see e.g. related work by G.Volovik)

$$i\hbar\frac{\partial}{\partial t}\hat{\Psi} = -\frac{\hbar^2}{2m}\nabla^2\hat{\Psi} - \mu\hat{\Psi} + \kappa|\hat{\Psi}|^2\hat{\Psi} - \lambda\hat{\Psi}^\dagger$$

We assume $-\mu<\lambda<<\mu$ (soft breaking).

It can be checked that the extra term gives massive phonons which at low momenta propagate on the standard acoustic geometry of BEC

$$\mathcal{E}^2(p) = \frac{p^4}{4m^2} + c_s^2p^2 + \mathcal{M}^2c_s^4$$

$$c_s^2 = \frac{\mu+2\lambda}{m}, \qquad \mathcal{M}^2 = 4\frac{\lambda(\mu+\lambda)}{(\mu+2\lambda)^2}m^2$$

$$\mathcal{M} \ll m \quad \text{if} \quad \lambda \ll \mu$$

Non-relativistic BEC gravitational potential

So we would now like to cast the equation for the a stationary, homogeneous, condensate background in a Poisson-like form with the quasi-particles moving accordingly to the analogue gravitational potential.

$$\vec{F} = \vec{a} = -\mathcal{M}\vec{\nabla}\Phi_{\rm grav}$$
$$\left(\nabla^2 - \frac{1}{L^2}\right)\Phi_{\rm grav} = 4\pi G_{\rm N}\rho + \Lambda$$

where L=> range of the gravitational interaction, G_N => analogue G Newton,
Λ => analogue cosmological constant

Results

1. It is possible to show by looking at the Newtonian limit of the acoustic geometry that the gravitational potential is encoded in density perturbations

2. By adopting the ansatz $\psi = \left(\frac{\mu+\lambda}{\kappa}\right)^{1/2}(1+u({\rm x}))$ $\qquad \eta_{\mu\nu} + h_{\mu\nu}, \qquad h_{00} \propto u(x)$

and looking at the Hamiltonian for the quasi-particles in the non relativistic limit, one can actually show that the analogue of the gravitational potential is

$$\Phi_{grav}({\rm x}) = \frac{(\mu+4\lambda)(\mu+2\lambda)}{2\lambda m}u({\rm x})$$

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This is the form the gravitational potential affecting the quasi-particle motion for a slightly inhomogeneous BEC.
We now want to see if it satisfies some modified Poisson equation…

NON-RELATIVISTIC BEC: EMERGENT NEWTONIAN GRAVITY

Let's consider the equation for a static background with a source term.
The latter is given partly by a localized quasi-particle plus a vacuum contribution due to the unavoidable presence/ backreaction of excited atoms above the condensate

$$\left(\frac{\hbar^2}{2m}\nabla^2 - 2(\mu+\lambda)\right) u(\mathrm{x}) = 2\kappa\left(\bar{n}(\mathrm{x}) + \frac{1}{2}\bar{m}(\mathrm{x})\right) + 2\kappa\left(\tilde{n}_0 + \frac{1}{2}\tilde{m}_0\right)$$

where $\bar{n}(\mathrm{x}) = \tilde{n}(\mathrm{x}) - \tilde{n}_0, \qquad \bar{m}(\mathrm{x}) = \tilde{m}(\mathrm{x}) - \tilde{m}_0$

and $\tilde{n}_0 = \langle 0|\hat{\chi}^\dagger(\mathrm{x})\hat{\chi}(\mathrm{x})|0\rangle, \qquad \tilde{m}_0 = \langle 0|\hat{\chi}(\mathrm{x})\hat{\chi}(\mathrm{x})|0\rangle$

are the quasi-particle vacuum backreaction terms

Now, knowing what is the analogue gravitational potential, this can be cast in the form of a generalized Poisson equation with a (negative) cosmological constant.

NON-RELATIVISTIC BEC: EMERGENT NEWTONIAN GRAVITY

Let's consider the equation for a static background with a source term.
The latter is given partly by a localized quasi-particle plus a vacuum contribution due to the unavoidable presence/backreaction of excited atoms above the condensate

$$\left(\frac{\hbar^2}{2m}\nabla^2 - 2(\mu+\lambda)\right) u(\mathrm{x}) = 2\kappa\left(\bar{n}(\mathrm{x}) + \frac{1}{2}\bar{m}(\mathrm{x})\right) + 2\kappa\left(\tilde{n}_0 + \frac{1}{2}\tilde{m}_0\right)$$

where $\bar{n}(\mathrm{x}) = \tilde{n}(\mathrm{x}) - \tilde{n}_0, \qquad \bar{m}(\mathrm{x}) = \tilde{m}(\mathrm{x}) - \tilde{m}_0$

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where

$$\rho_{\mathrm{matter}}(\mathrm{x}) = \left(\bar{n}(\mathrm{x}) + \frac{1}{2}\bar{m}(\mathrm{x})\right) \qquad G_N \equiv \frac{\kappa(\mu+4\lambda)(\mu+2\lambda)^2}{4\pi\hbar^2 m\lambda^{3/2}(\mu+\lambda)^{1/2}}$$

$$\Lambda \equiv \frac{2\kappa(\mu+4\lambda)(\mu+2\lambda)}{\hbar^2\lambda}\left(\tilde{n}_0 + \frac{1}{2}\tilde{m}_0\right), \qquad L^2 \equiv \frac{\hbar^2}{4m(\mu+\lambda)}$$

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WEIGHTING LAMBDA...

$$\mathcal{E}_\Lambda = \frac{\Lambda c_s^4}{4\pi G_{\mathrm{N}}}, \qquad \mathcal{E}_{\mathrm{P}} = \frac{c_s^7}{\hbar G_{\mathrm{N}}^2}, \qquad \Longrightarrow \qquad \frac{\mathcal{E}_\Lambda}{\mathcal{E}_{\mathrm{P}}} \propto \rho_0 a^3 \left(\frac{\lambda}{g\rho_0}\right)^{-5/2}.$$

The cosmological constant scale is suppressed by a small number (the dilution factor $\rho a^3 \ll 1$) w.r.t. the analogue/emergent Planck scale!

The cosmological constant is not the phonons ground state energy,
neither it is the atoms grand canonical energy density *h*, or energy density *ε=h+μρ*
It is just related to the subdominat second order correction to these latter quantities due to quantum depletion (the part related to the excitations) and its scale is the healing scale.

Cosmological constant in emergent gravity: lessons from BEC

$$\Lambda = -\frac{20 m g \rho_0 (g\rho_0 + 3\lambda)}{3\sqrt{\pi}\hbar^2 \lambda}\sqrt{\rho_0 a^3} F_\Lambda\left(\frac{\lambda}{g\rho_0}\right),$$

S. Finazzi, S. Liberati and L. Sindoni, Phys. Rev. Lett. 108, 071101 (2012)

This model is too simple to be realistic but still teaches us some lessons

- The analogue cosmological constant that we have discussed cannot be computed as the total zero-point energy of the condensed matter system, even when taking into account the natural cutoff coming from the knowledge of the microphysics
- In fact the value of Λ is related only to the (subleading) part of the zero-point energy proportional to the quantum depletion of the condensate.

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This implies for emergent gravity scenarios that

- There could be no a priori reason why the cosmological constant should be computed as the zero-point energy of the system. More properly, its computation must inevitably pass through the derivation of Einstein equations emerging from the underlying microscopic system.
- the energy scale of Λ can be several orders of magnitude smaller than all the other energy scales for the presence of a very small number, nonperturbative in origin, which cannot be computed within the framework of an EFT dealing only with the emergent degrees of freedom (i.e., semiclassical gravity).
- From an emergent gravity approach, the low energy effective action (and its renormalization group flow) is computed within a framework that has nothing to do with quantum field theories in curved spacetime. Indeed, the explanation of its value (and of its properties under renormalization) would naturally sit outside the domain of semiclassical gravity.

Relativistic BEC and emergent LLI

Of course the BEC model is not Lorentz invariant at all scales as it is fundamentally non relativistic.
But even breaking Lorentz invariance can be done in different ways

S.Fagnocchi, S. Finazzi, SL, M. Kormos, A. Trombettoni: New. J. Phys.

Bose–Einstein condensation, may occur also for relativistic bosons. So far only theoretical model.

$$\hat{\mathcal{L}} = \frac{1}{c^2}\frac{\partial\hat{\phi}^\dagger}{\partial t}\frac{\partial\hat{\phi}}{\partial t} - \nabla\hat{\phi}^\dagger\cdot\nabla\hat{\phi} - \left(\frac{m^2c^2}{\hbar^2} + V(t,\mathbf{x})\right)\hat{\phi}^\dagger\hat{\phi} - U(\hat{\phi}^\dagger\hat{\phi};\lambda_i)\ ,$$

$$\text{with}\quad U(\hat{\phi}^\dagger\hat{\phi};\lambda_i) = \frac{\lambda_2}{2}\hat{\rho}^2 + \frac{\lambda_3}{6}\hat{\rho}^3 + \cdots \quad \text{where}\quad \hat{\rho} = \hat{\phi}^\dagger\hat{\phi}$$

The associated dispersion relation has a gapped and gapless mode

$$\omega_\pm^2 = c^2\left\{k^2 + 2\left(\frac{\mu}{c\hbar}\right)^2\left[1 + \left(\frac{mcc_0}{\mu}\right)^2\right] \pm 2\left(\frac{\mu}{c\hbar}\right)\sqrt{k^2 + \left(\frac{\mu}{c\hbar}\right)^2\left[1 + \left(\frac{mcc_0}{\mu}\right)^2\right]^2}\right\}.$$

$$\text{where}\quad c_0 = \frac{\hbar^2}{2m}\rho U"(\rho,\lambda)\ ,\quad \mu = \text{relativistic chemical potential.}$$

The gapless/massless mode is the interesting one as it admits an effective metric in the phononic regime

In the limit of very relativistic atoms $b \equiv \frac{mcc_0}{\mu} \gg 1$

Long wavelengths limit $k \ll \frac{2mc_0}{\hbar} \implies \omega_-^2 = c_s^2k^2$ with $g_{\mu\nu} = \rho\frac{c}{c_s}\left[\eta_{\mu\nu} + \left(1 - \frac{c_s^2}{c^2}\right)\frac{v_\mu v_\nu}{c^2}\right]$

$c_s^2 = c^2 b/(1+b)$

LLI+Gravity ←

SR ← Short wavelengths limit $k \gg \frac{2mc_0}{\hbar} \implies \omega_-^2 = c^2k^2$ with $g_{\mu\nu} = \eta_{\mu\nu}$

No LLI+Raibow metric/Finsler ? ← Intermediate $\omega_-^2 = c_s^2\left[k^2 + \frac{k^4}{4b\left(\frac{\mu}{\hbar c}\right)^2(1+b)^2}\right]$

Lesson from rBEC: one can have IR and UV relativistic physics but nonetheless Lorentz violation at intermediate scales.
You have to understand the classical/continuous limit to be sure about LI.

Emergent gravity in relativistic BEC

Belenchia, SL, Mohd: arXiv:1407.7896
Phys.Rev. D90 (2014) 10, 104015

$$\mathcal{L}_{\rm eff} = -\eta^{\mu\nu}\partial_\mu\phi^*\partial_\nu\phi - m^2\phi^*\phi - \lambda(\phi^*\phi)^2 + \mu^2\phi^*\phi + i\mu(\phi^*\partial_t\phi - \phi\partial_t\phi^*)$$

Let us again decompose Φ as $\Phi = \Phi_0(1 + \Psi)$, where Φ_0 is the condensed part of the field ($\langle\Phi\rangle = \Phi_0$) and Ψ is the fractional fluctuation which can be written in terms of its real and imaginary parts $\Psi = \Psi_1 + i\Psi_2$

Crucial point: in some suitable regime (neutral background field, $c_s = c$) you can completely mask the Lorentz breaking. In this regime one finds

Excitations Eq.

$$\Box_g\psi_1 - 4\lambda\psi_1 = 0,$$

$g_{\mu\nu} = \phi^2{}_0\eta_{\mu\nu}$ $$\Box_g\psi_2 = 0.$$

Background Eq.

$$R_g - \frac{m^2}{\phi^2} + \Lambda = \langle T_{\rm qp}\rangle, \qquad R_g = -6\,\frac{\Box\varphi_0}{\varphi_0^3}$$

$$\langle T_{\rm qp}\rangle := -12\lambda\left[3\,\langle\psi_1^2\rangle + \langle\psi_2^2\rangle\right],$$

For m->0 (allowed by non zero chemical potential μ) Equivalent to Einstein-Fokker equation of Nordström gravity! (but not truly background independent)

$$R + \Lambda = 24\pi G_{\rm N}\, T,$$

Nordström gravity (1913) is the only other theory in 3+1 dimensions which satisfies the Strong Equivalence principle. However, it is not truly background independent (fixed Minkowski causal structure)

$$\langle T\rangle = -\frac{\Lambda}{6}\left[3\langle\psi_1^2\rangle + \langle\psi_2^2\rangle\right] = -2\lambda\left[3\langle\psi_1^2\rangle + \langle\psi_2^2\rangle\right] = 6\langle T_{qp}\rangle. \qquad \Lambda \equiv 12\lambda\frac{\mu^2}{c^2\hbar} = 12\lambda\frac{M_{\rm P}^2c^2}{4\pi\hbar}. \qquad G_{\rm N}^{\rm eft} = G/4\pi = \hbar c^5/(4\pi\mu^2)$$

$$M_{\rm Pl} = \mu\sqrt{4\pi}/c^2 \qquad \frac{\epsilon_\Lambda}{\epsilon_P} \simeq \frac{\lambda\hbar^2c^2}{\mu^2}$$ small

The "bare" Λ is in this case small and positive but it will generically receives a (negative) correction from the fraction of atoms in the non-condensate phase, the depletion factor.

What next?

This is all interesting but is clearly open to several criticisms:

1. In the non-relativistic BEC you get only Newtonian gravity +Lorentz breaking
2. In the relativistic case, fine tuning of the system gives you Nordstrom gravity which is remarkable but it is still "just" scalar gravity+it is not really background independent

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gravitational theories with universal coupling to energy are characterized by Hamiltonians that are pure boundary terms on shell. In order for this to be the low energy effective description of a field theory with local kinematics, all bulk dynamics must be frozen and thus irrelevant to the construction.

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Emergence of Gravity a la Analogue Model+ background independence requires kinematical non-locality (different microcausality) to start with...

Ok, but can we test some of these ideas?

From Analogue Models to Phenomenology (bite the bullet)

QG phenomenology a la carte

ex pluribus quattuor

Broken or deformed Symmetries

- Lorentz
- Translations
- SUSY (LHC searches but well below QG scale)
- Diffeomorphism (strong bounds from pulsar timing Donoghue et al. PhysRevD.81.084059)

Locality

- QG induced non-locality
- Uncertainty Principle->GUP (no strong constraints)
- Non-commutative geometries

Dimensions

- Extra dimensions (still missing obs. evidence so far)
- Dimensional reduction in QG (early universe?)

QG Modified gravitational dynamics

- E.g. Bouncing Universes
- Regular Black holes, Fuzballs

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Let's start with the PULP stuff..

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Lorentz violation: a possible first glimpse of QG?

Suggestions for Lorentz violation searches (at low or high energies) were not inspired only by Analogue models of emergent gravity.
They came also from several QG models

- String theory tensor VEVs (Kostelecky-Samuel 1989, ...)
- Cosmological varying moduli (Damour-Polyakov 1994)
- Spacetime foam scenarios (Ellis, Mavromatos, Nanopoulos 1992, Amelino-Camelia et al. 1997-1998)
- Some semiclassical spin-network calculations in Loop QG (Gambini-Pullin 1999)
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- Some brane-world backgrounds (Burgess et al. 2002)
- Ghost condensate in EFT (Cheng, Luty, Mukohyama, Thaler 2006)
- Horava-Lifshiftz Gravity (Horava 2009, ...)

Quote: "How you dare to Violate Lorenz Invariance?"

- Lorentz invariance is rooted via Einstein equivalence principle in GR and it is a fundamental pillar of the SM. The more fundamental is an ingredient of your theory the more needs to be tested observationally!
- You do not need Planck scale observations to constraint Planck suppressed Lorentz violations.
- In any quantum/Discrete gravity model it is a non-trivial task to recover exact Local Lorentz Invariance and/or background independence. Hence it is very important to understand what is needed in order to conciliate LLI and forms of hard or quantum discreteness at the Planck scale.

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But what we mean by Lorentz Invariance violation?

Breaking of Local Lorentz Invariance

von Ignatowsky theorem (1911): Axiomatic Special Relativity

Principle of relativity → group structure
Homogeneity → linearity of the transformations
Isotropy → rotational invariance and Riemannian structure
Precausality → observer independence of co-local time ordering

→

Lorentz transformations with unfixed limit speed c
c=∞ → Galileo
$c=c_{light}$ → Lorentz
Experiments determine c!

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Lorentz breaking does not necessarily mean to have a preferred frame!

Break Precausality → Hell breaks loose, better not!

Break Principle of relativity → Preferred frame effects

Break Isotropy → Finsler geometries. E.g. Very Special Relativity (Glashow, Gibbons et al.).

Break Homogeneity → no more linear transformations → No Locally Euclidean Space. → tantamount to give up operative meaning of coordinates

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Let's relax the Relativity Principle first and study the phenomenology. To do this we need a framework…

Dynamical frameworks for LIV

Frameworks for preferred frame effects

See e.g. SL. CQG Topic Review (2013)

EFT+LV

See e.g. Amelino-Camelia. Living Reviews of Relativity

Non EFT proposals:
Spacetime foam models
DSR/Relative Locality

Minimal Standard Model Extension
Renormalizable ops.
(IR LIV - LI SSB)

local EFT with LIV
Non-renormalizable ops,
CPT ever or odd
(no anisotropic scaling),
(UV LIV – QG inspired LIV)

NOTE: CPT violation implies Lorentz violation but LV does not imply CPT violation in local EFT.
"Anti-CPT" theorem (Greenberg 2002).
So one can catalogue LIV by behaviour under CPT

E.g. QED, rot. Inv. dim 3,4 operators

electrons $E^2 = m^2 + p^2 + f_e^{(1)} p + f_e^{(2)} p^2$

photons $\omega^2 = \left(1 + f_\gamma^{(2)}\right) k^2$

(Colladay-Kosteleky 1998, Colemann-Glashow 1998)

E.g. QED, dim 5 operators

electrons $E^2 = m^2 + p^2 + \eta_\pm^{(3)} (E^3/M_{\rm Pl})$

photons $\omega^2 = k^2 \pm \xi(\omega^3/M_{\rm Pl})$

(Myers-Pospelov 2003)

Constraints on QED dim 5 CPT Odd QED extension

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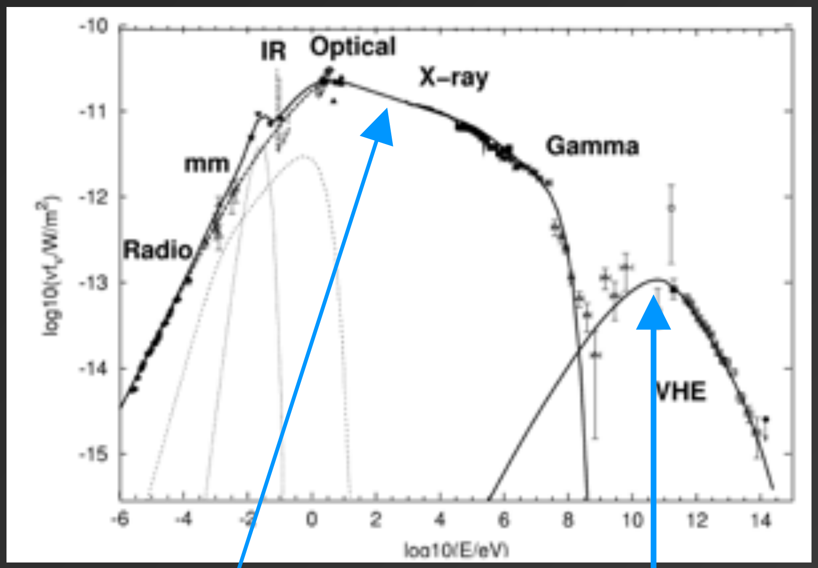
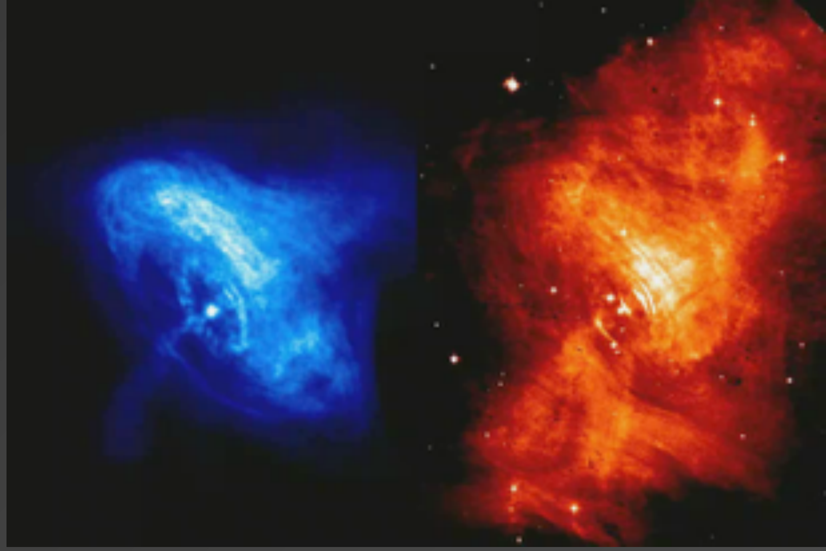


Synchrotron

Inverse Compton



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The Crab nebula a supernova remnant (1054 A.D.) distance ~1.9 kpc from Earth.
Spectrum (and other SNR) well explained by synchrotron self-Compton (SSC)
Electrons are accelerated to very high energies at pulsar: in LI QED $\gamma_e \approx 10^9 \div 10^{10}$
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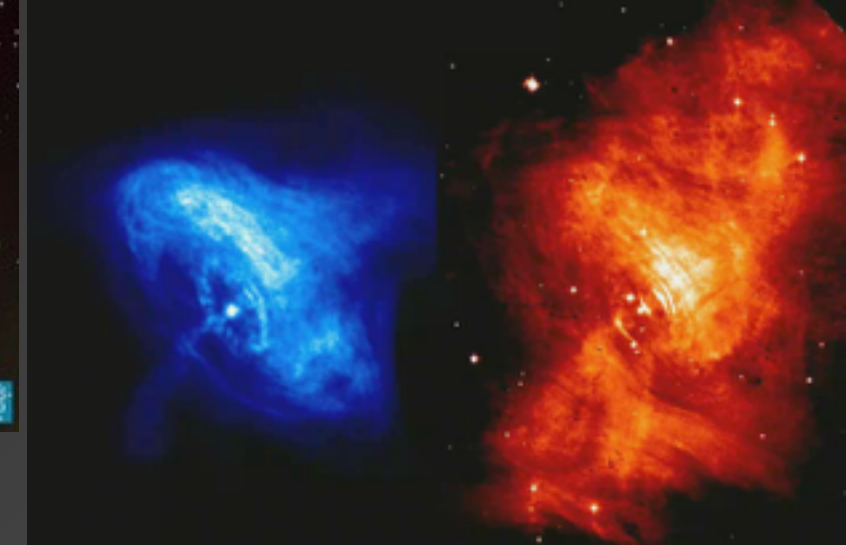

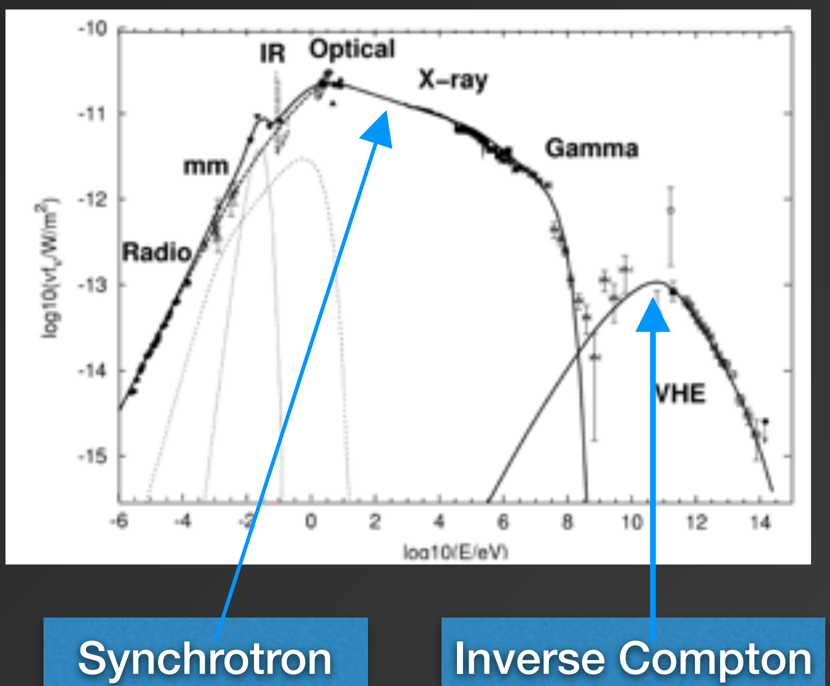

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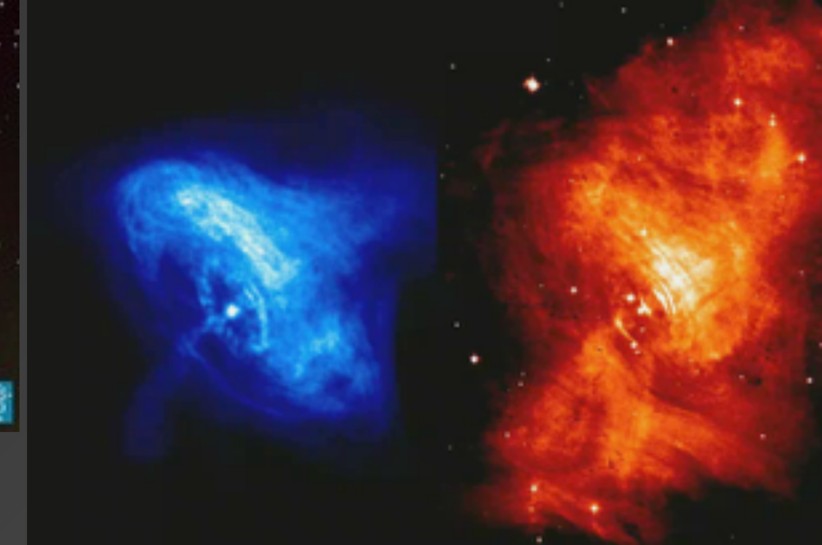

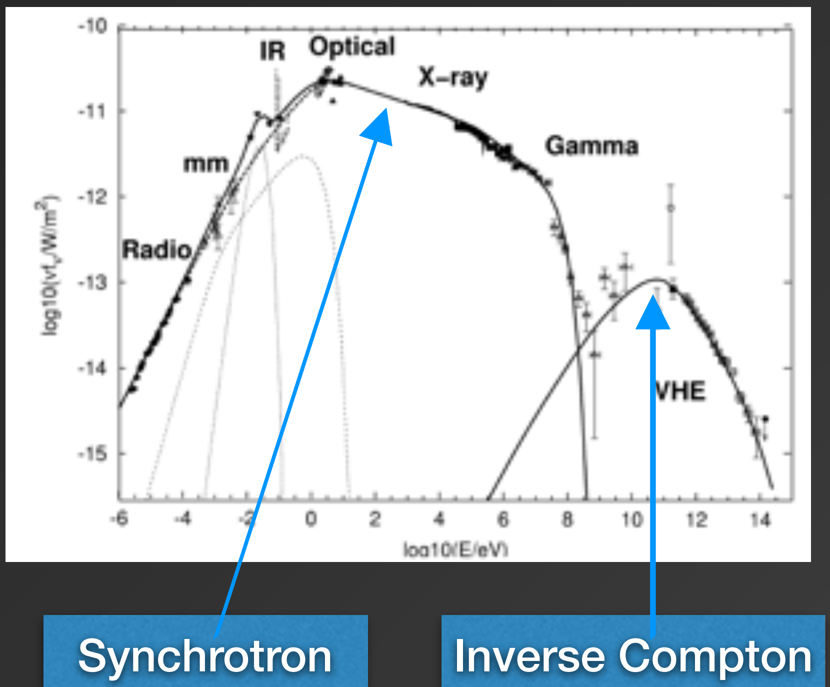

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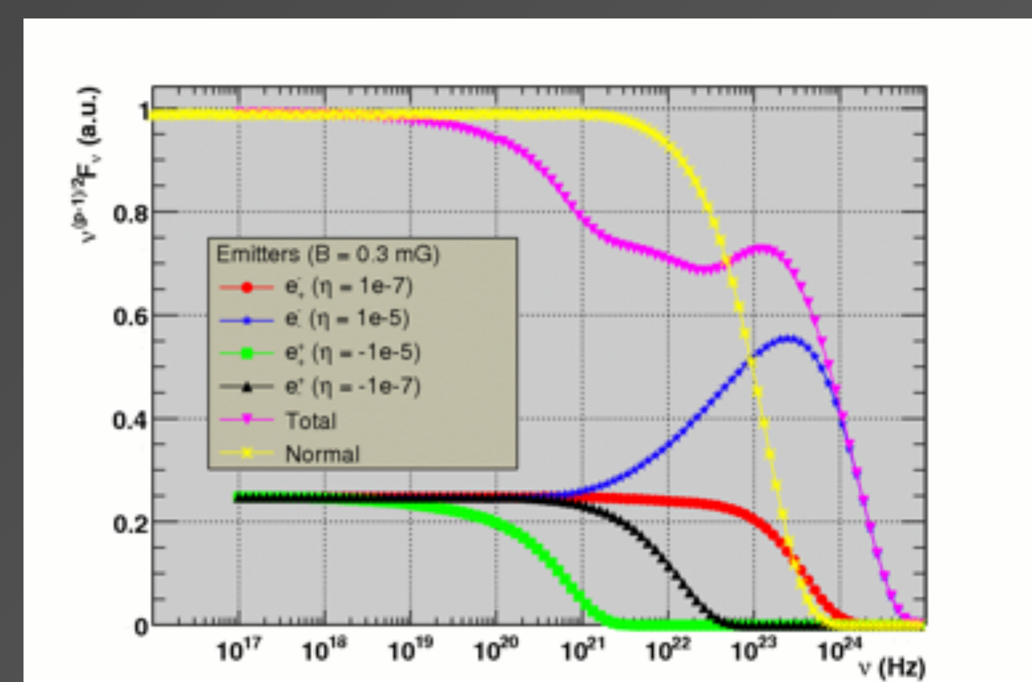

Constraints on QED dim 5 CPT Odd QED extension

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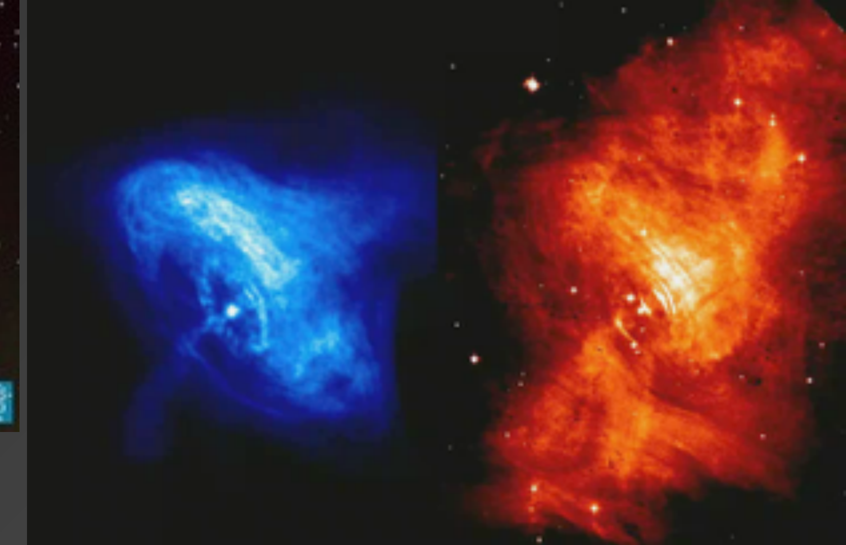

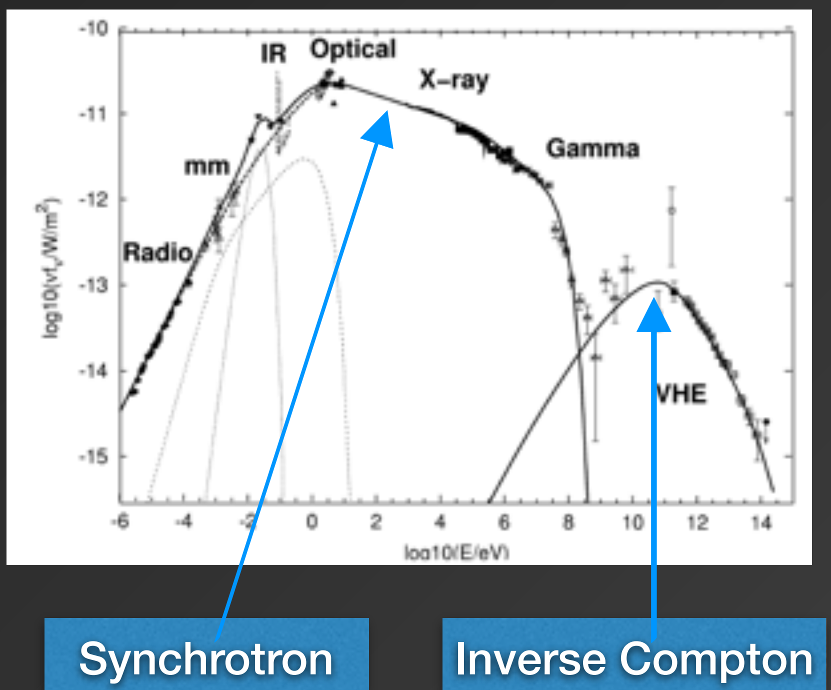

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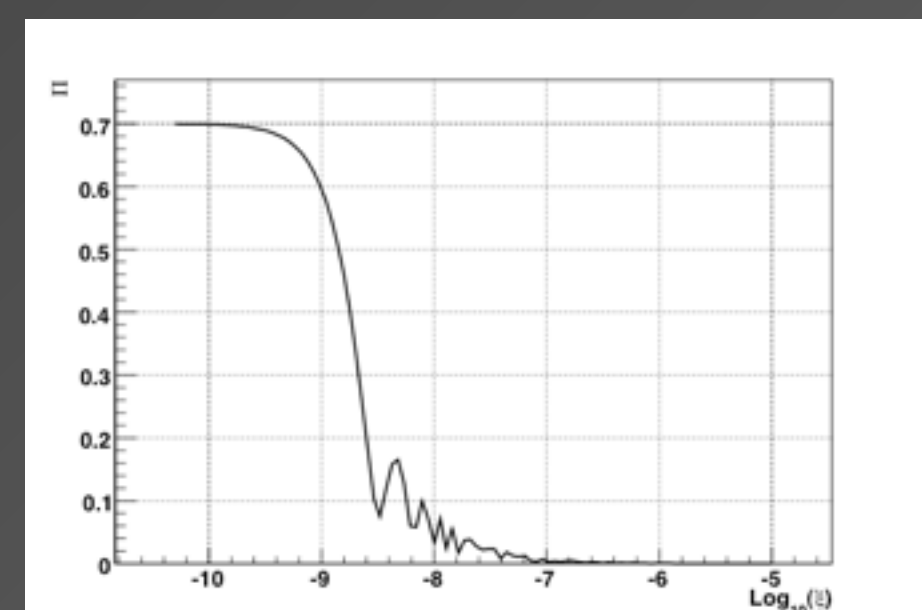

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$$\Delta\theta = \xi\,(k_2^2 - k_1^2)\,d/2M, \quad (\text{where } d = \text{distance source-detector})$$

Polarization recently accurately measured by INTEGRAL mission: 40±3% linear polarization in the 100 keV - 1 MeV band + angle θobs= (123±1.5)° from the North

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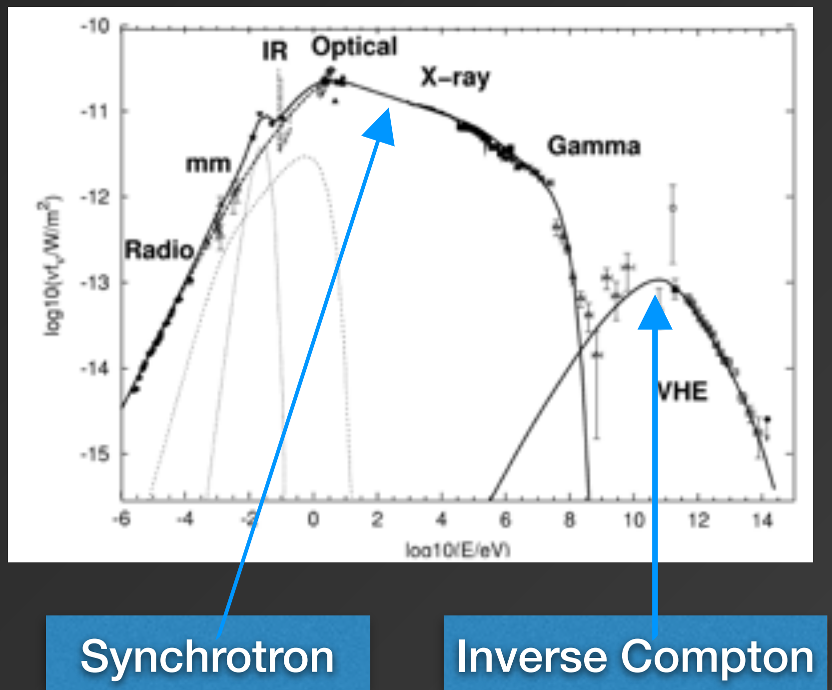

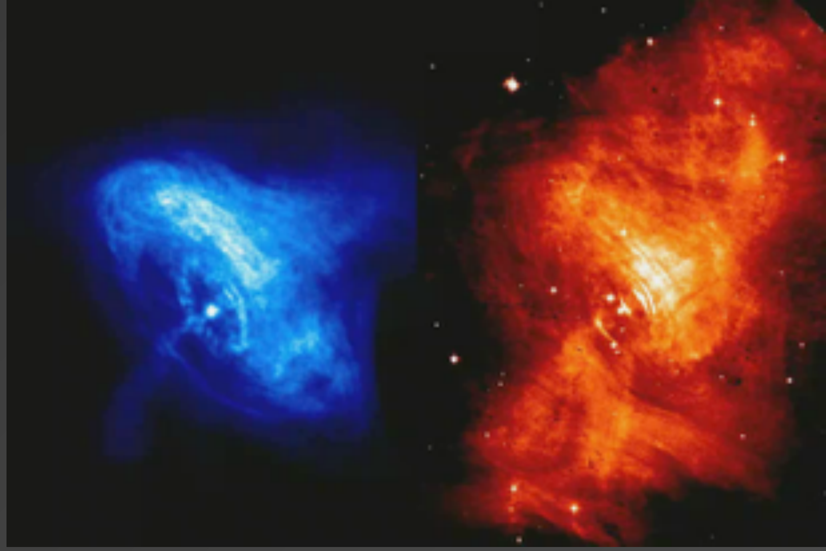

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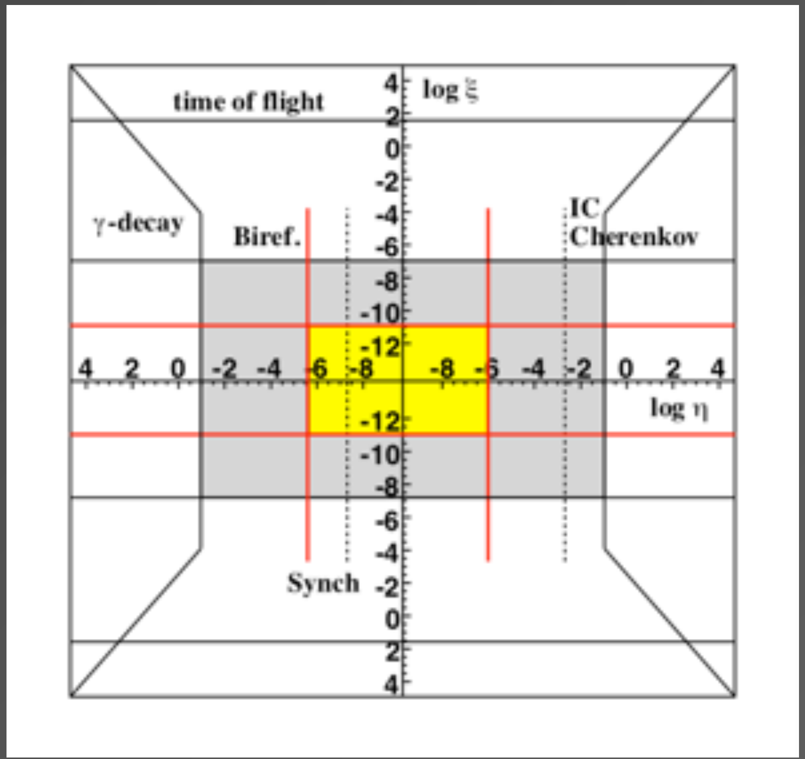

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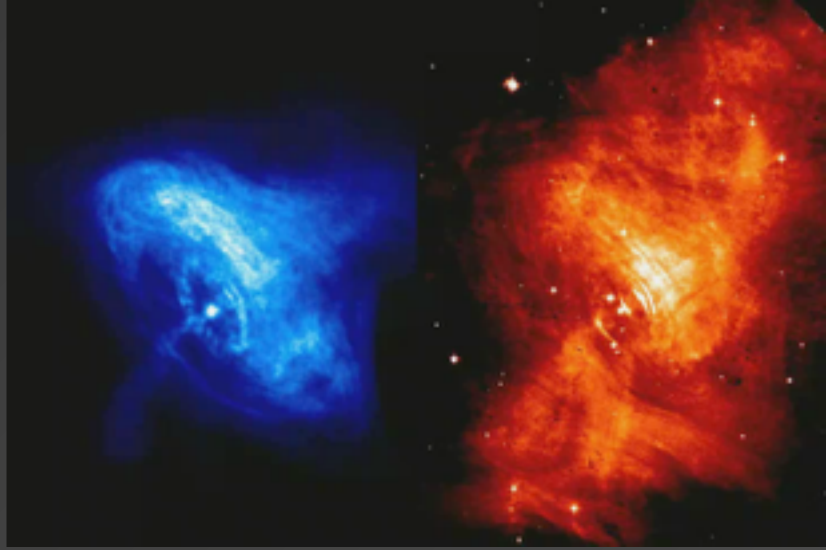

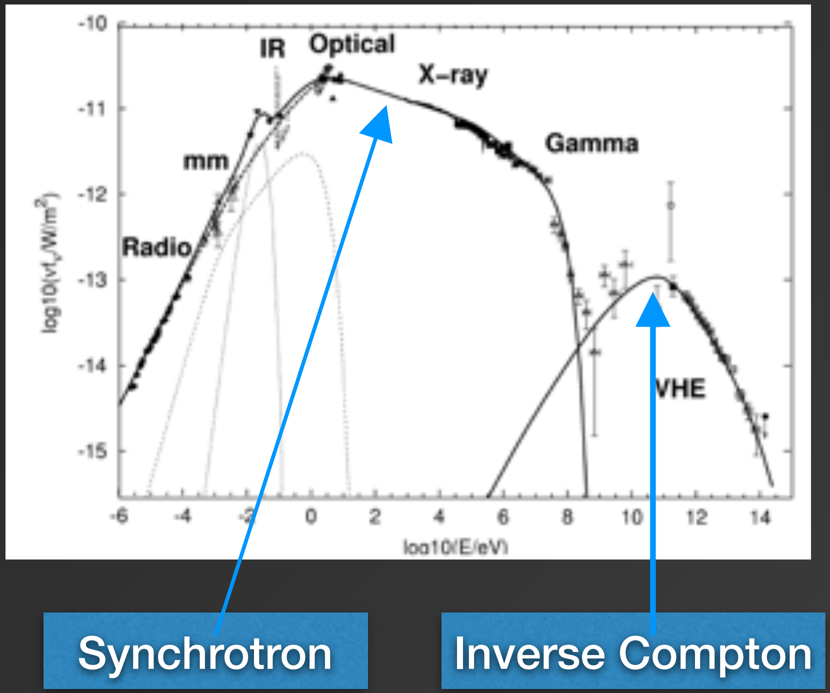

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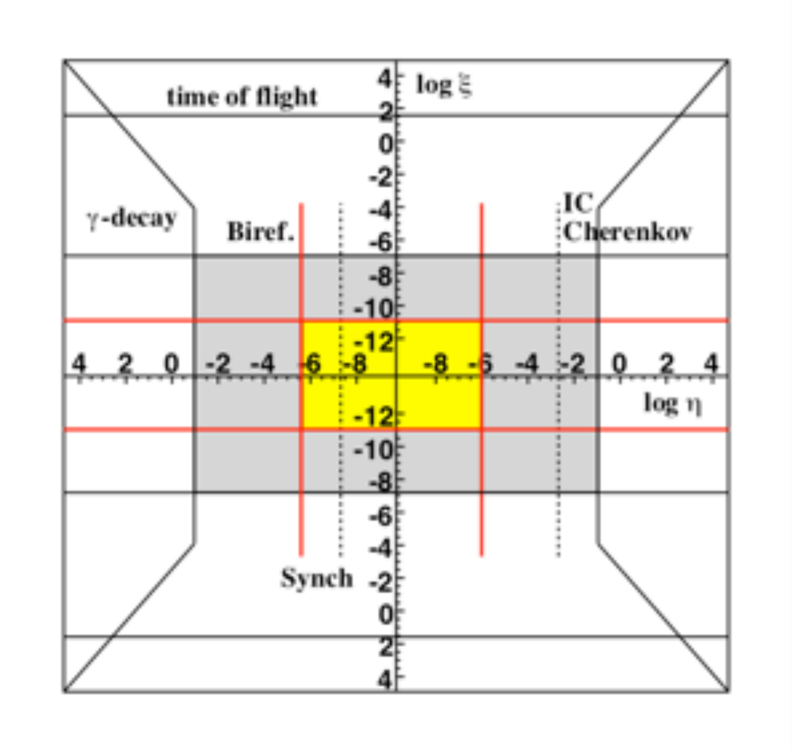

EFT with Lorentz breaking Ops. Matter Sector Constraints

Terrestrial tests:

Penning traps
Clock comparison experiments
Cavity experiments
Spin polarised torsion balance
Neutral mesons
Slow atoms recoils

Astrophysical tests:

Cosmological variation of couplings, CMB
Cumulative effects in astrophysics
Anomalous threshold reactions
Shift of standard thresholds reactions with new threshold phenomenology
LV induced decays not characterised by a threshold
Reactions affected by "speeds limits"

$$
\begin{aligned}
E_\gamma^2 &= k^2 + \xi_\pm^{(n)} \frac{k^n}{M_{pl}^{n-2}} \qquad \text{photons} \\
E_{matter}^2 &= m^2 + p^2 + \eta_\pm^{(n)} \frac{p^n}{M_{pl}^{n-2}} \qquad \text{leptons/hadrons}\ ,
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where, in EFT, $\xi^{(n)} \equiv \xi_+^{(n)} = (-)^n \xi_-^{(n)}$ and $\eta^{(n)} \equiv \eta_+^{(n)} = (-)^n \eta_-^{(n)}$.

Table 2 Summary of typical strengths of the available constrains on the SME at different orders.

Order	photon	e^-/e^+	Hadrons	Neutrinos[a]
n=2	N.A.	$O(10^{-13})$	$O(10^{-27})$	$O(10^{-8})$
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n=4	$O(10^{-8})$ (CR)	$O(10^{-8})$ (CR)	$O(10^{-6})$ (CR)	$O(10^{-4})$* (CR)

GRB=gamma rays burst, CR=cosmic rays
[a] From neutrino oscillations we have constraints on the difference of LV coefficients of different flavors up to $O(10^{-28})$ on dim 4, $O(10^{-8})$ and expected up to $O(10^{-14})$ on dim 5 (ICE3), expected up to $O(10^{-4})$ on dim 6 op. * Expected constraint from future experiments.

SL, CQG Topic Review 2013

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Warning
GZK ISSUE!
$p+\gamma_{CMB} \to p+\pi^0$
$p+\gamma_{CMB} \to n+\pi^+$

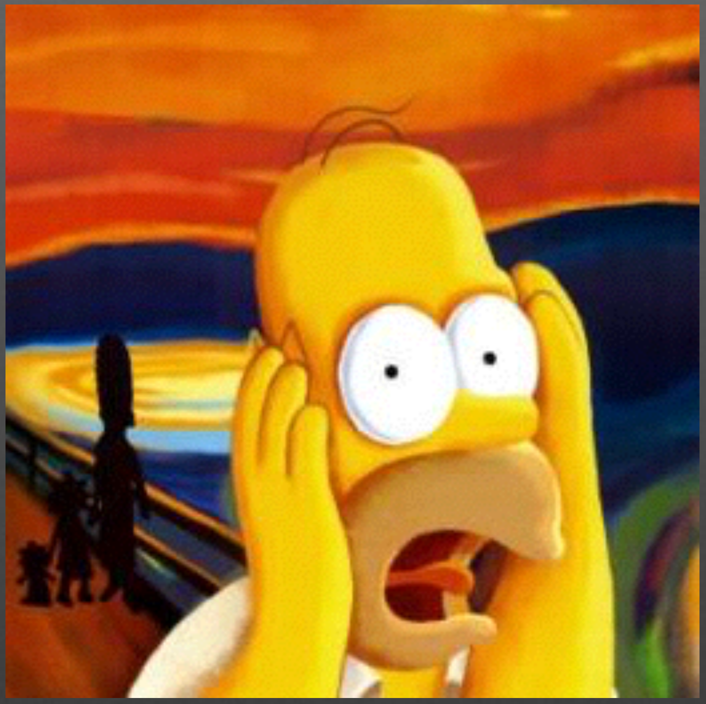
SL, CQG Topic Review 2013

Caveat: A potential problem with the UHECR data?

- With increased statistics the composition of UHECR beyond 10^{19} eV seems more and more dominated by iron ions rather than protons at AUGER. But Telescope Array (TA) in Utah is instead Ok with purely proton composition. Confusion.
- With improved statistic the correlated AUGER UHECR-AGN events have decreased from 70% to 40%: large deflections? i.e. heavy (high Z) ions?
- Also no evidence at the TA for AGN correlation. But some hint of correlation with LLS for E>57 EeV
- Ions do photodisintegration rather than the GZK reaction, this may generate much less protons which are able to create pions via GZK and hence UHE photons.
- Have we really seen the GZK cutoff or sources exhaustion? See e.g. arXiv:1408.5213.
- If not all the constraints on dim 6 CPT even operators would not be robust…
- Furthermore puzzling cut off above 2 PeV in UHE neutrinos at IceCube maybe consistent with p^4 LIV at M_{LIV}~10^{15} GeV. F.W. Stecker, S.T. Scully, SL, D. Mattingly. JCAP 2015

More bad news?
The flies in the Ointment...

Lorentz breaking theories suffers two main theoretical problems



- Naturalness problem
- Possible breakdown of black hole thermodynamics

The "un-naturalness" of small LV in EFT

[Collins et al. PRL93 (2004), Lifshitz theories (anisotropic scaling): Iengo, Russo, Serone (2009)]
Gambini, Rastgoo, Pullin Class.Quant.Grav. 28 (2011) 155005 . Polcinski (2011)

Dim>4 LiV Operators + **No new physics up to E_{LIV}** + **Radiative Corrections** = **Unsuppressed corrections to dim 4 operators (low energy propagators)**

Given the strong constraints on dim 3,4 LIV operators (low energy effects) this is BAD

Note: some interesting exceptions apply.
See Belenchia, SL, Gambassi: JHEP 2016

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Custodial symmetry

One needs EFT range to be bonded by $E_{EFT}<E_{LIV}$
[which we have so far assumed O(M_{Pl})].
So far main candidate SUSY but needs E_{SUSY} not too high.

E.g. gr-qc/0402028 (Myers-Pospelov) or hep-ph/0404271 (Nibblink-Pospelov) or gr-qc/0504019 (Jain-Ralston),
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Assume only gravity LIV with $M_* << M_{PL}$, then percolation into the (constrained) matter sector is suppressed by smallness of coupling constant G_N.

E.g. Horava gravity coupled to LI Standard Model:
Pospelov & Shang arXiv.org/1010.5249v3

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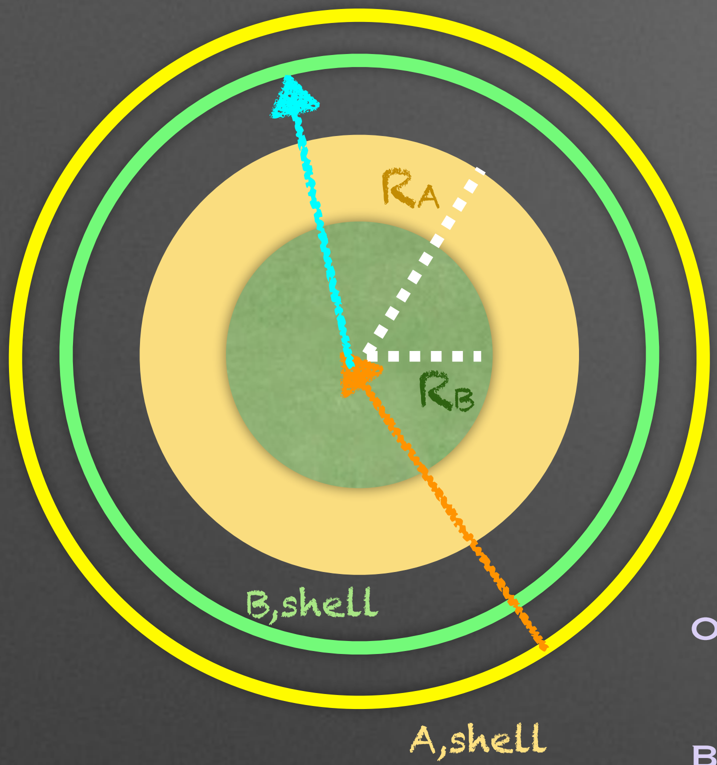
Improved RG flow at HE

Models with strong coupling at high energies improving RG flow a la Nielsen
[G.Bednik, O.Pujolàs, S.Sibiryakov, JHEP 1311 (2013) 064]

Violations of the Generalised Second Law in Lorentz breaking scenarios



A and B fields interacts only gravitationally

$c_B > c_A$ ⟶ $R_B < R_A$ ⟶ $T_{B,Haw} > T_{A,Haw}$

Surround the BH with two shells of A and B fields
It is possible to choose the temperatures of the shells such that

$$T_{B,Haw} > T_{B,shell} > T_{A,shell} > T_{A,Haw}$$

Then $T_{A,shell} > T_{A,Haw}$ implies flux from Shell A to BH
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One can choose the temperatures of the shells in such a way that the two energy fluxes compensate each other.
So BH mass, radius, entropy stay constant.
But $T_{B,shell} > T_{A,shell}$ hence the net effect is to bring heat from cooler shell to hotter one!

Note: split in horizons can be used also to generate classical violation of GSL (region between radii is like ergoregion for B field: possible energy extraction)
Conclusion: Violation of LLI seems to lead to violation of the Generalized Second Law (GSL).

S.L.Dubovsky, S.M.Sibiryakov, Phys. Lett. B 638 (2006) 509.
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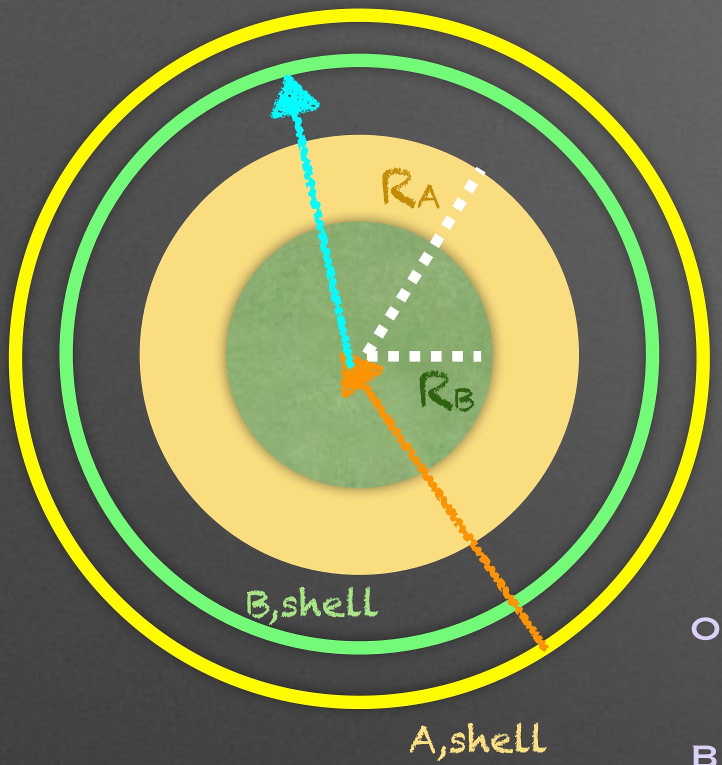

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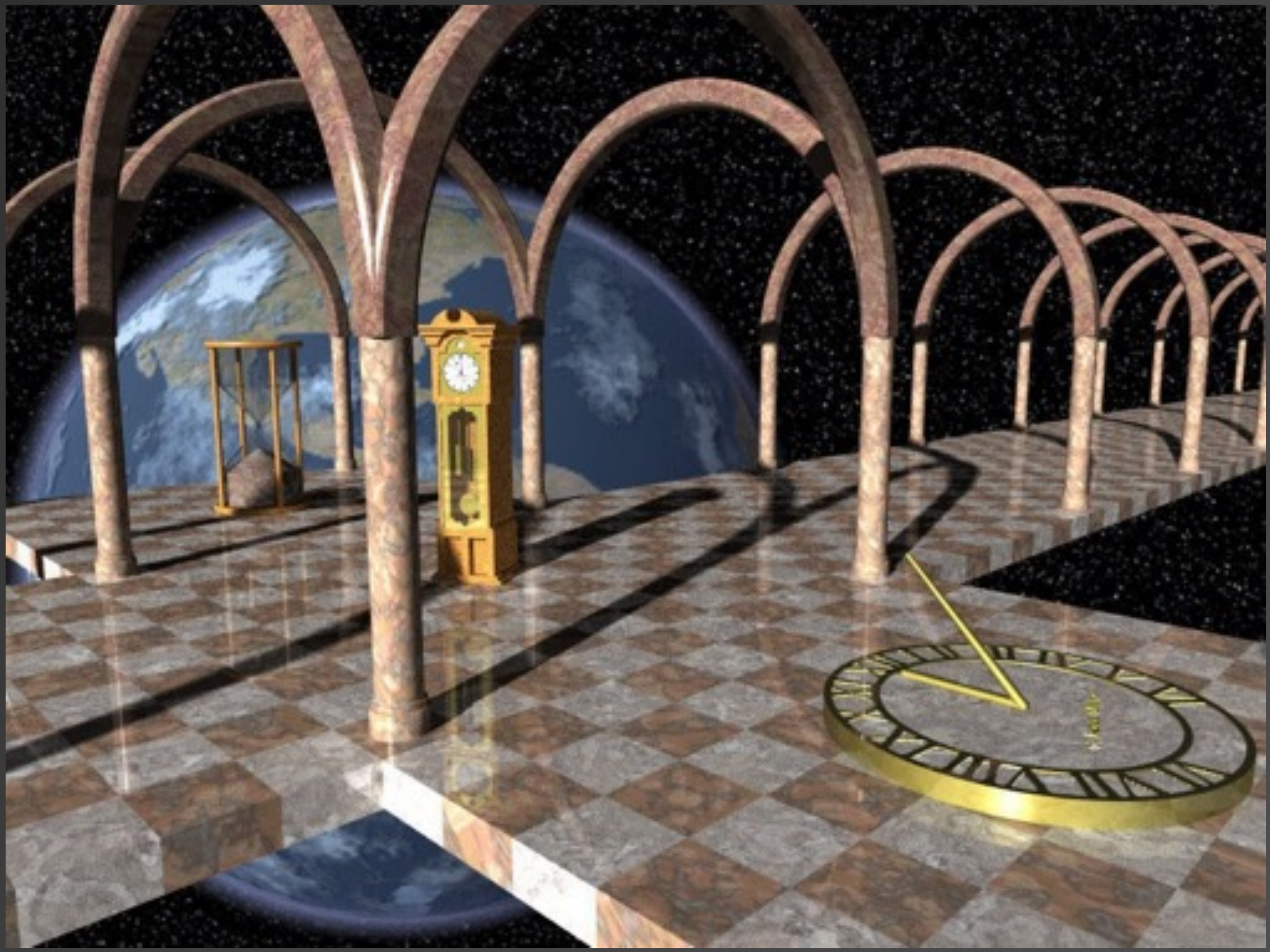
However all of this was not taking into account LIV in gravity and UV completion...



Gravity VS Local Lorentz invariance
(what does not kill you makes you stronger)

Lorentz breaking gravity

Einstein-Aether (Jacobson-Mattingly 2000)

Rotationally invariant Lorentz violation in the gravity sector via a vector field. Take the most general theory for a unit timelike vector field coupled to gravity which is second order in derivatives.

$$\mathcal{S} = \mathcal{S}_{EH} + \mathcal{S}_u = \frac{1}{16\pi G_{\rm ae}} \int dx4\sqrt{-g}\ (R + \mathcal{L}_u)\,.$$

$$\mathcal{L}_u = -Z^{\alpha\beta}_{\gamma\delta}(\nabla_\alpha u^\gamma)(\nabla_\beta u^\delta) + \lambda(u^2+1). \qquad Z^{\alpha\beta}_{\gamma\delta} = c_1 g^{\alpha\beta}g_{\gamma\delta} + c_2\delta^\alpha_\gamma\delta^\beta_\delta + c_3\delta^\alpha_\delta\delta^\beta_\gamma - c_4 u^\alpha u^\beta g_{\gamma\delta}\,,$$

Horava-Lifshitz (Horava 2009)

$$S_{HL} = \frac{M_{\rm Pl}^2}{2}\int dt d^3x\, N\sqrt{h}\left(L_2 + \frac{1}{M_\star^2}L_4 + \frac{1}{M_\star^4}L_6\right),$$

where h is the determinant of the induced metric h_{ij} on the spacelike hypersurfaces, and $L_2 = K_{ij}K^{ij} - \lambda K^2 + \xi\,{}^{(3)}\!R + \eta a_i a^i$ with K is the trace of the extrinsic curvature. K_{ij}, ${}^{(3)}\!R$ is the Ricci scalar of h_{ij}. N is the lapse function, and $a_i = \partial_i \ln N$.

λ=1, ξ=1, η=0 in General Relativity (GR).

IR limit L_2 is Einstein-Aether with hypersurface orthogonal aether field.

Observationally constrained but not ruled out arXiv:1311.7144 [gr-] Yagi et al

$$M_{\rm obs} < M_\star < 10^{16}\ \text{GeV} \qquad M_{\rm obs} \approx \text{few meV} \quad (\text{from sub mm tests})$$

Blas,Pujolas,Sibiryakov, Phys. Lett. B 688, 350 (2010).

The condition $M_* < 10^{16}$ GeV is a consequence of the need to protect perturbative renormalizability w.r.t. the mass scale of the Horava scalar mode

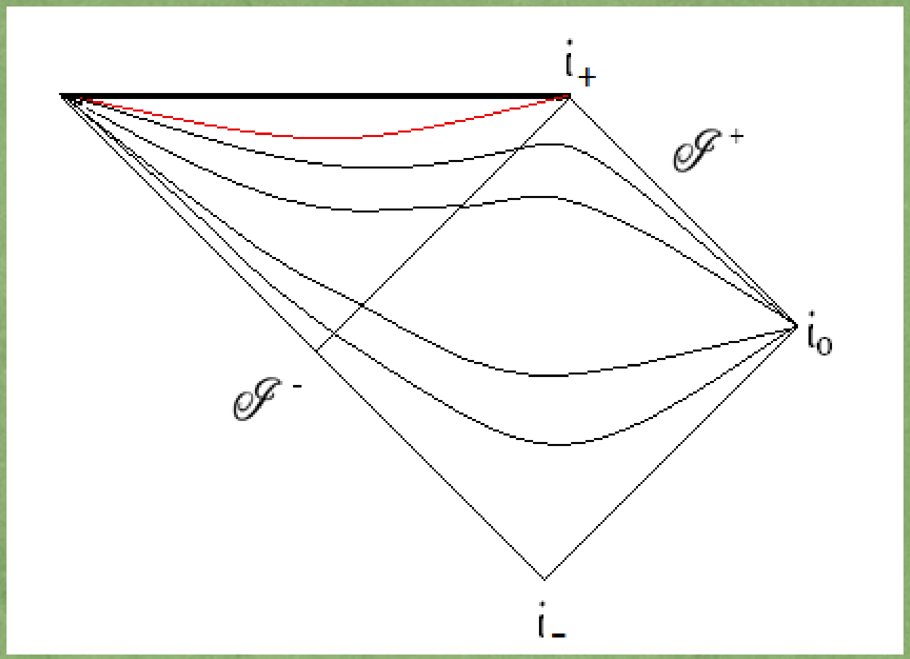
A new hope? Universal horizons

BH, spherically symmetric solution are the same in AE and Horava gravity.
The aether is hypersurface orthogonal and there is a compact constant Kronon hypersurface from which even infinite velocity signals cannot escape.

Alternatively the UH occurs when the Killing field χ associated to energy at infinity becomes orthogonal to the aether field: (χu)=0.
(note: Open issue with stability)

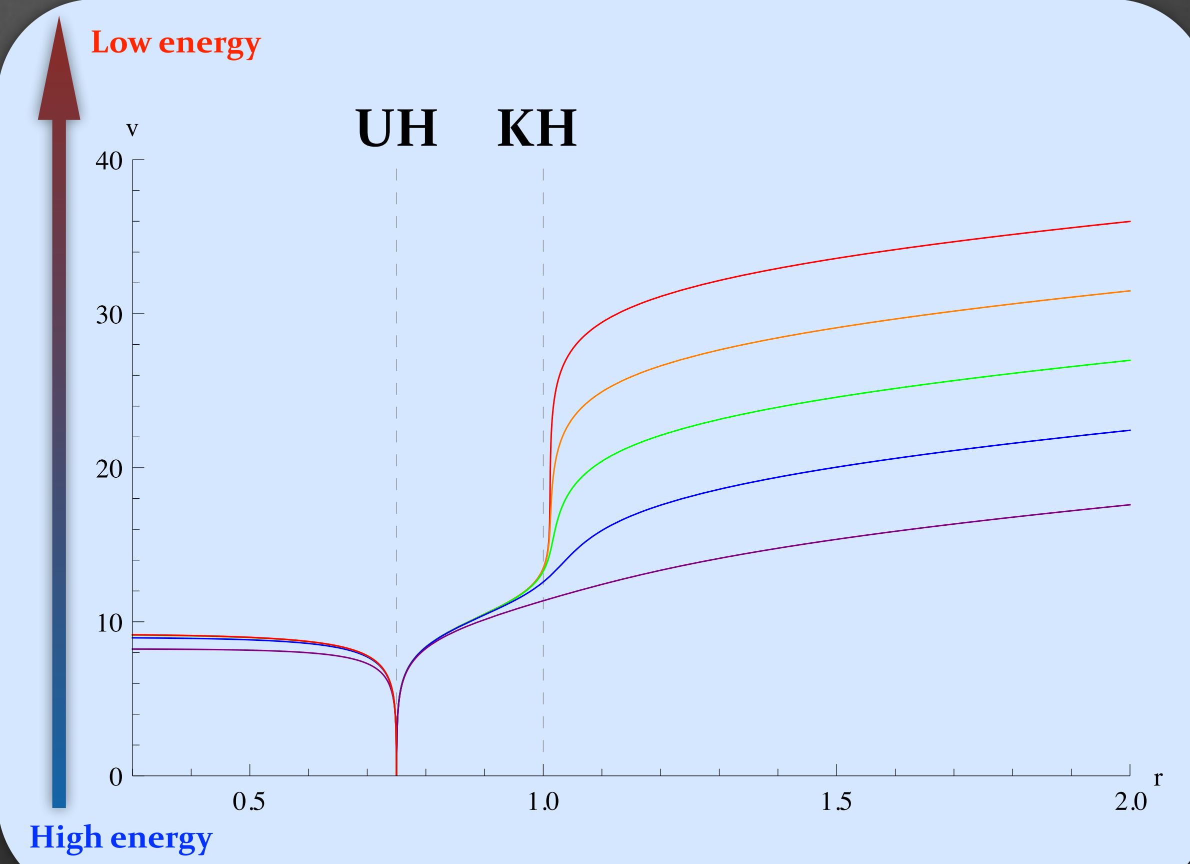
Eternal: D. Blas and S. Sibiryakov (2011), E. Barausse, T. Jacobson, T. P. Sotiriou (2011) and Collapse solutions: M.Saravani, N. Afshordi, Robert B. Mann., (2014).

Analogue model of AE-BH with UH: B.Cropp, S.L and R. Turcati, arXiv:1606.01044 [gr-qc].



Conformal diagram of black hole with Universal horizon, showing lines of constant khronon field, with the Universal horizon shown in red.

Raytracing of of superluminal matter



Cropp, SL, Mohd, Visser.
Phys.Rev. D89 (2014) no.6, 064061

Universal Horizon Thermodynamics?

UH thermodynamics Laws			
	Gist	Status	Math
0th	surface gravity is constant on UH	✔ (but trivial in spherically symm. sols)	$\chi^\mu \nabla_\mu \kappa\vert_{UH} = 0$
1st	Energy conservation	✔	$\delta M_{æ} = \frac{q_{\rm UH}\delta A_{\rm UH}}{8\pi G_{æ}}$
2nd	Non decreasing entropy	✔? (GSL?)	$\delta A_{\rm UH} \geq 0$
3rd	Unattainability of T=0 state	?	?

Berglund, Bhattacharyya, Mattingly
Phys.Rev. D85 (2012) 124019
Arif Mohd. e-Print: arXiv:1309.0907

$$q_{\rm UH} = (1 - c_{13})\kappa_{\rm UH} + \frac{c_{123}}{2} K_{\rm UH}|\chi|_{\rm UH}$$

$$c_{13} = c_1 + c_3 \qquad c_{123} = c_1 + c_2 + c_3$$

$$\kappa_{\rm UH} = \sqrt{-\nabla_\mu \chi_\nu \nabla^\mu \chi^\nu}$$

$$K_{\rm UH} = \nabla_\mu u^\mu$$

κ peeling instead is

$$\bar{\kappa}_{\rm UH} \equiv \frac{1}{2}\frac{\rm d}{{\rm d}r}\frac{{\rm d}r}{{\rm d}v}\bigg|_{\rm UH}$$

$$\bar{\kappa}_{\rm UH} \equiv \frac{1}{2} u^a \nabla_a (u\cdot\chi)\bigg|_{\rm UH}$$

Does the UH radiate? Berglund, Bhattacharyya, Mattingly, Phys.Rev.Lett. 110 (2013) 7, 071301

Tunneling method a la Parikh—Wilczekc leads to predict a thermal spectrum with temperature

$$\boxed{T_{\rm UH} = \frac{\kappa_{\rm UH}}{4\pi c_{æ}}}$$

from this and 1st law

$$S_{\rm UH} = \frac{(1 - c_{13}) c_{æ} A_{\rm UH}}{2G_{æ}}$$

Notes

1. The calculation assumes vacuum at UH for infalling observers (like Unruh for UH)
2. The above temperature obtained is not $\kappa_{\rm UH}/2\pi$ but it is instead $\kappa({\rm peeilng})_{\rm UH}/2\pi$.
3. F. Michel and R.Parentani, Phys. Rev. D91, no. 12, 124049 (2015) get from shell collapse different vacuum state (Unruh for KH) and find radiation at KH temperature.

Lorentz regained: Searching Non-locality

Non-locality as an alternative to symmetry breaking?

What about other mesoscopic physics without Lorentz violation?

- We do have concrete QG models of emergent gravity like Causal Sets or String Field Theory or Loop Quantum Gravity which generically seem to predict exact Lorentz invariance below the Planck scale in spite of (fundamental or quantum) discreteness at the price to introduce non-local EFT.
  Conjecture: Discreetness + Lorentz Invariance = Non-Locality

Note also Marolf's theorem: Emergence of Gravity a la Analogue Model+ background independence requires different notion of locality between fundamental and emergent dof.

Several forms of non-locality

- Non-local kinetic terms
- Non-local interactions
- DSR-like non-locality
- ...

These theories involve a very subtle phenomenology very hard to constraint, still they do show novelties.
Differently from UV Lorentz breaking physics it will be here a matter of PRECISION instead of HIGH ENERGIES...

Non-local D'Alambertians

$$\Box \to f(\Box)$$

Generic expectation if you want to introduce length or energy scale in flat spacetime KG equation without giving up Lorentz invariance.

Concrete examples of kinematical non-locality respectively with non-analytic or analytic function

Causal Set Theory $$\Box_\rho \approx \Box + \frac{\alpha}{\sqrt{\rho}}\Box^2 + \frac{\beta}{\sqrt{\rho}}\Box^2 \ln\left(\frac{\gamma}{\rho}\Box^2\right) + \dots$$

String Field Theory $$\Box \to (\Box + m^2)\exp^{\frac{\Box + m^2}{\Lambda^2}}$$ $$\Lambda = 1/\ell_{\rm nl}$$

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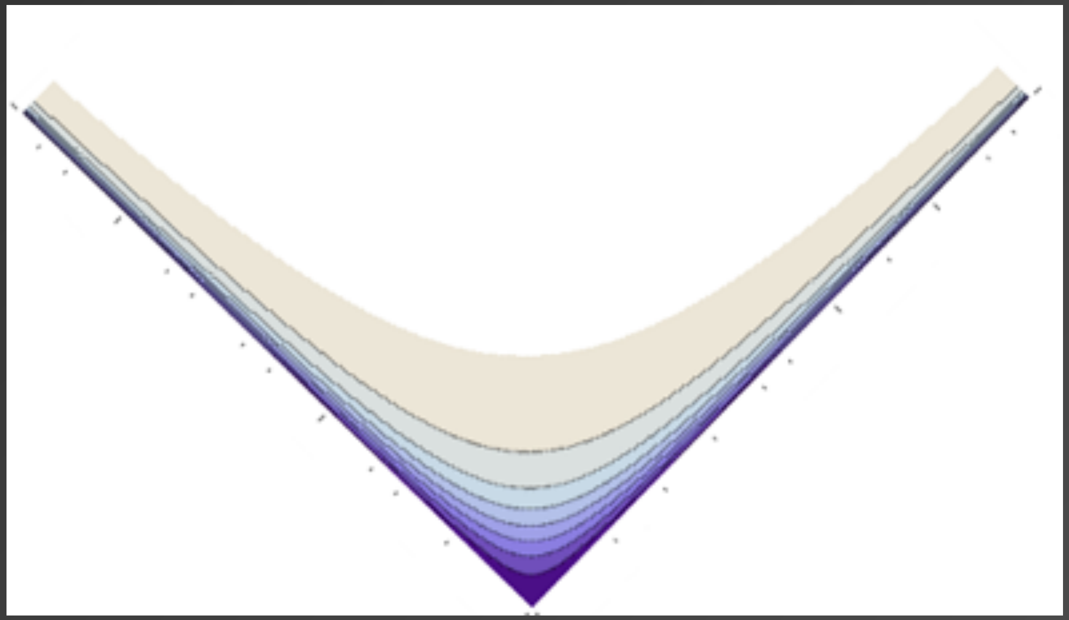
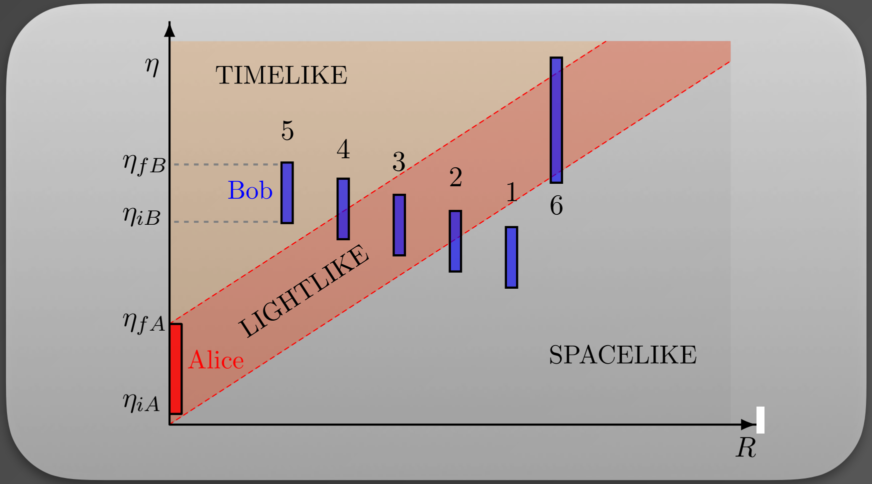
A typical signature of <u>non-analytical</u> non-local propagators are violations of the Huygen principle: The propagator of massless particles can have support inside the light cone in 3+1





Possibly very relevant for relativistic quantum information tests as detectors can influence each other at timelike separations

Opportunity for Phenomenology?

R. H. Jonsson, E. Martin-Martinez, and A. Kempf, Phys.Rev.Lett. 114, 110505 (2015). Ana Blasco, Luis J. Garay, Mercedes Martin-Benito, Eduardo Martin-Martinez. Phys.Rev.Lett. 114 (2015) 14, 141103

Testing non-local EFT with optomechanical oscillators

A. Belenchia, D. Benincasa, SL, F. Marin, F. Marino, A. Ortolan.
Phys.Rev.Lett. 116 (2016) no.16, 161303

E.g. let's consider its non-relativistic limit of a non-local KG with analytic f(□).

$$\sum_{n=0}^{\infty} \underbrace{\frac{1}{n!}\left(-\frac{2m}{\hbar^2}\right)^n \frac{1}{\Lambda^{2(n-1)}}}_{a_n} \frac{1}{\Lambda^2}\mathcal{S}^{n+1} \equiv \mathcal{S}_{NL}.$$

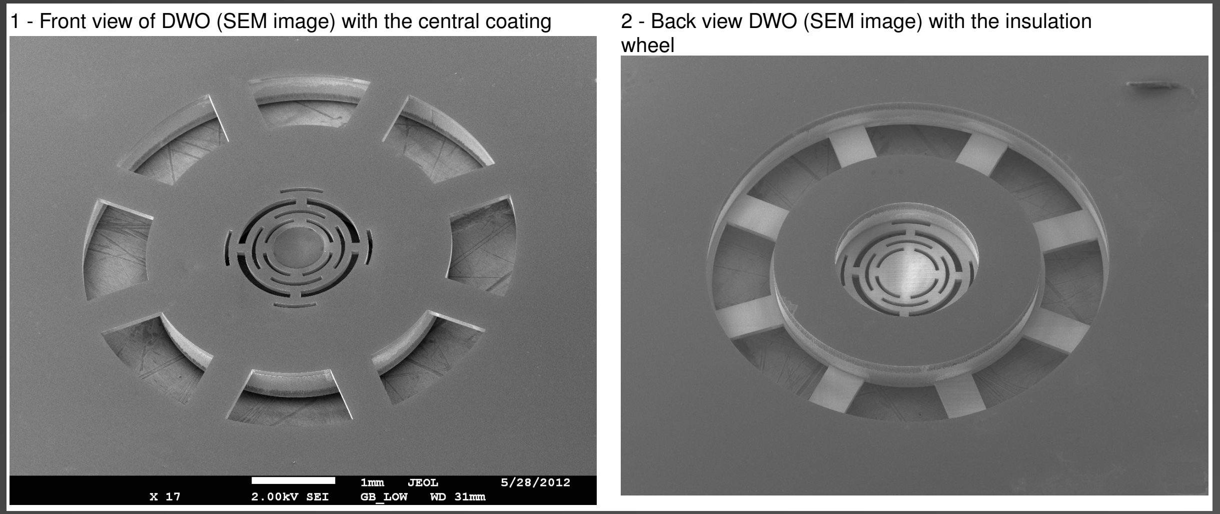
So we get $$(\mathcal{S}_{NL} - V)\,\phi(t,x) = 0.$$

Where can be test this?

HUMOR

Heisenberg Uncertainty Measured with Opto-mechanical Resonators (LENS - Florence, Italy)

Designed to test generalised uncertainty principle
Macroscopic harmonic oscillator.
$m \sim 10^{-11} / 10^{-5}$ Kg $\omega \sim 10^{+5} / 10^{+3}$ Hz



In order to solve the non-local Schroedinger, one needs to adopt a perturbative expansion around a "local" Sch. solution

$$\phi = \phi_0 + \sum_{n=1}^{\infty} \epsilon^n \psi_n$$

With ϵ the small dimensionless parameter for this problem.

$$\epsilon = \frac{m\omega}{\hbar\Lambda^2}$$

And at the lowest order we can solve

$$\left(i\hbar\partial_t + \frac{\hbar^2}{2m}\partial^2_{xx}\right)\psi + \epsilon a_2 \left(\frac{-2}{\hbar\omega}\right)\mathcal{S}^2\psi = \frac{1}{2}m\omega^2 x^2\psi.$$

Spontaneous squeezing from non-locality

Results

$$\langle x\rangle = \sqrt{2}\alpha\cos(t)\left(1+\frac{1}{4}\epsilon\alpha^2 a_2\left[\cos(2t)-1\right]\right)+\mathcal{O}(\epsilon^2),$$

$$\langle p\rangle = \sqrt{2}\alpha\sin(t)\left(1+\frac{1}{4}\epsilon\, a_2\;\left[\alpha^2(7+3\cos(2t))-2\right]\right)+\mathcal{O}(\epsilon^2),$$

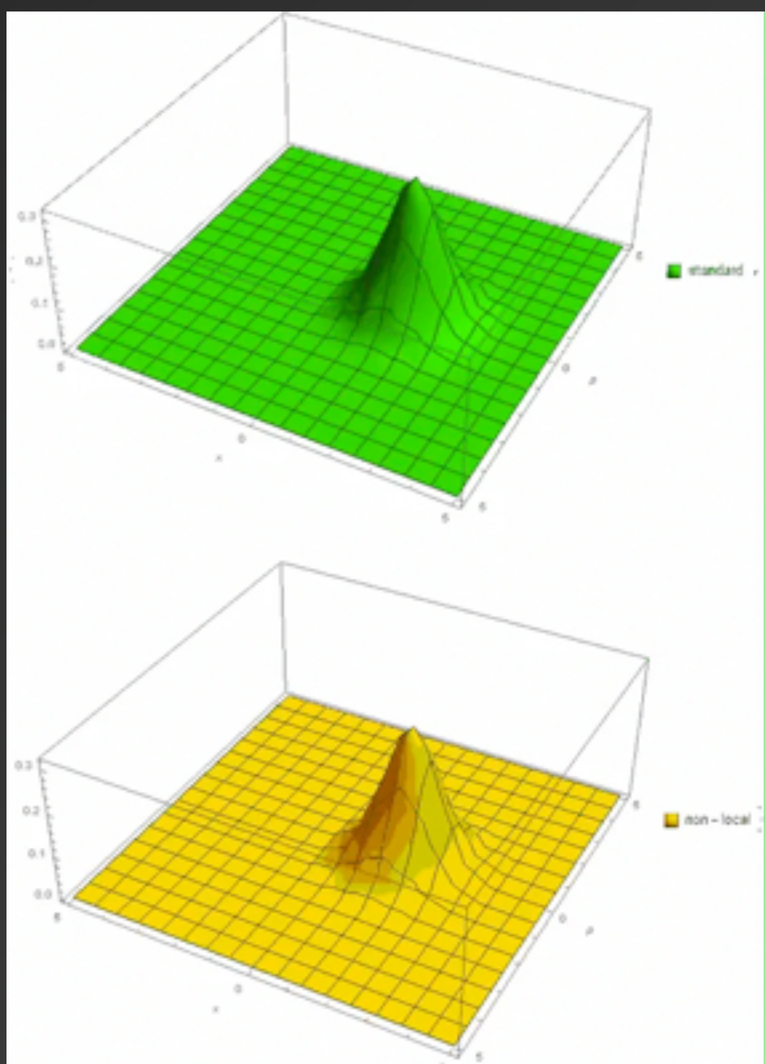
$$\mathrm{Var}(x) = \frac{1}{2}\left(1-\epsilon a_2\left[(6\alpha^2-1)\sin^2(t)\right]\right)+\mathcal{O}(\epsilon^2),$$

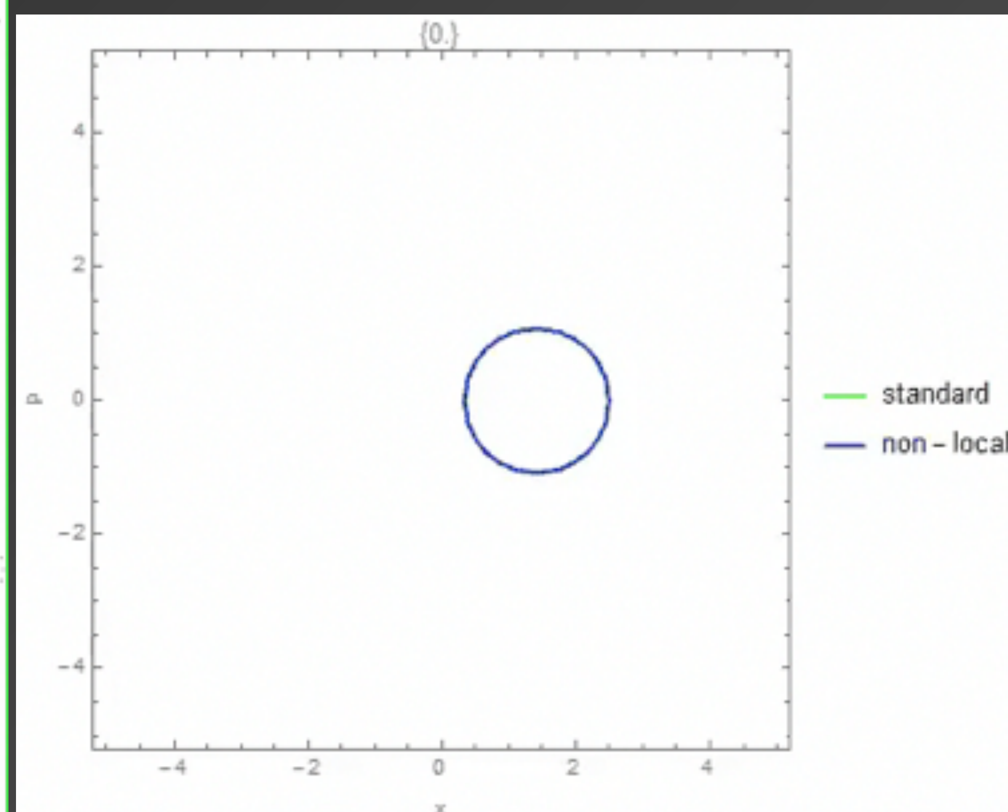
$$\mathrm{Var}(p) = \frac{1}{2}\left(1+\epsilon a_2\left[(6\alpha^2-1)\sin^2(t))\right]\right)+\mathcal{O}(\epsilon^2).$$

Let's consider Wigner quasi probability distribution for a coherent state of our quantum harmonic oscillator,

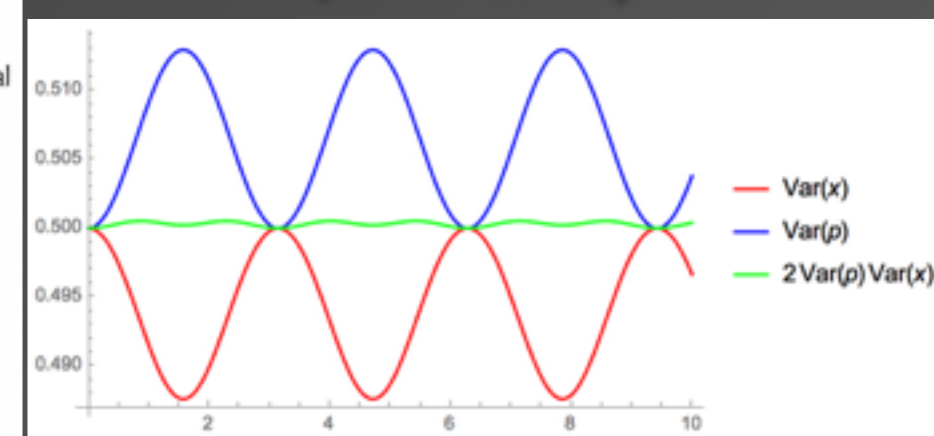
$$P(x,p;t)\frac{1}{\pi}\int_{-\infty}^{\infty}\mathrm{dy}\;\phi(x+y,t)^*\phi(x-y,t)\;e^{2ipy}$$

and confront its evolution for a coherent state (easier to experimental realise than the ground state) in the case of ***S*** and ***S***+ε***S***^{2}





The Coherent state Wigner function shows a periodic almost perfect squeezing.
Very difficult to produce spontaneously…



Current best bounds on the non-locality scale by comparing nonlocal relativistic EFTs to the 8 TeV LHC data $l_{nl}\leq 10^{-19}$m

Forecast with experiment in preparation (in absence of periodic squeezing) imply $l_{nl}\leq 10^{-29}$m !

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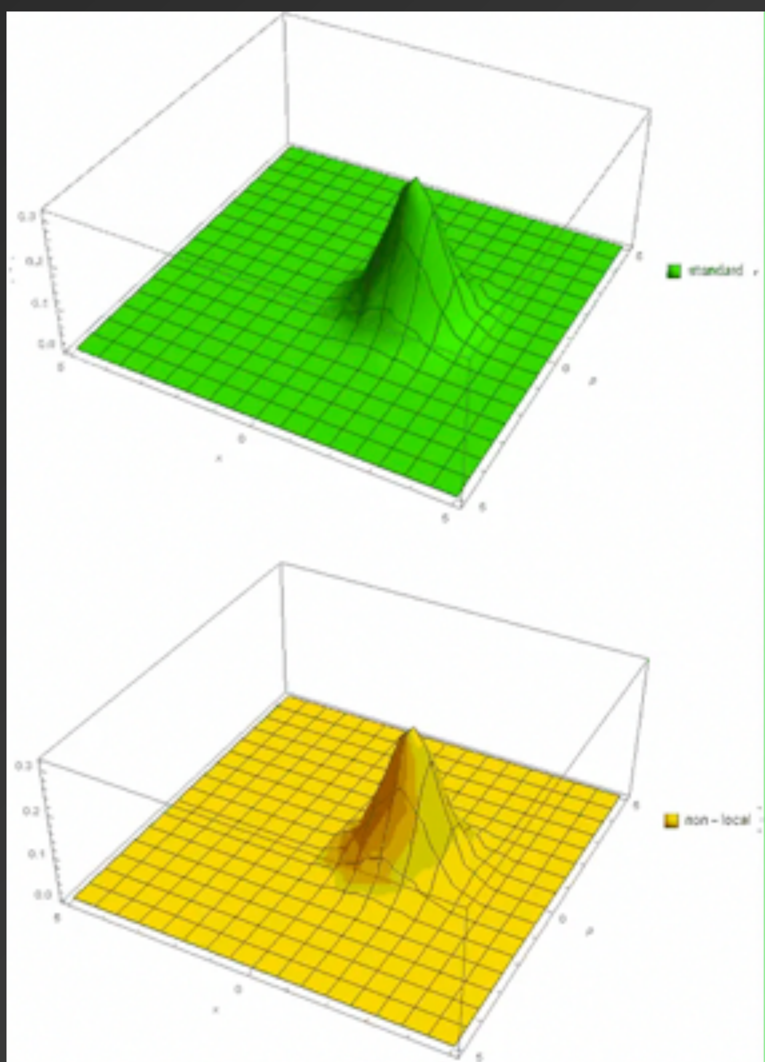

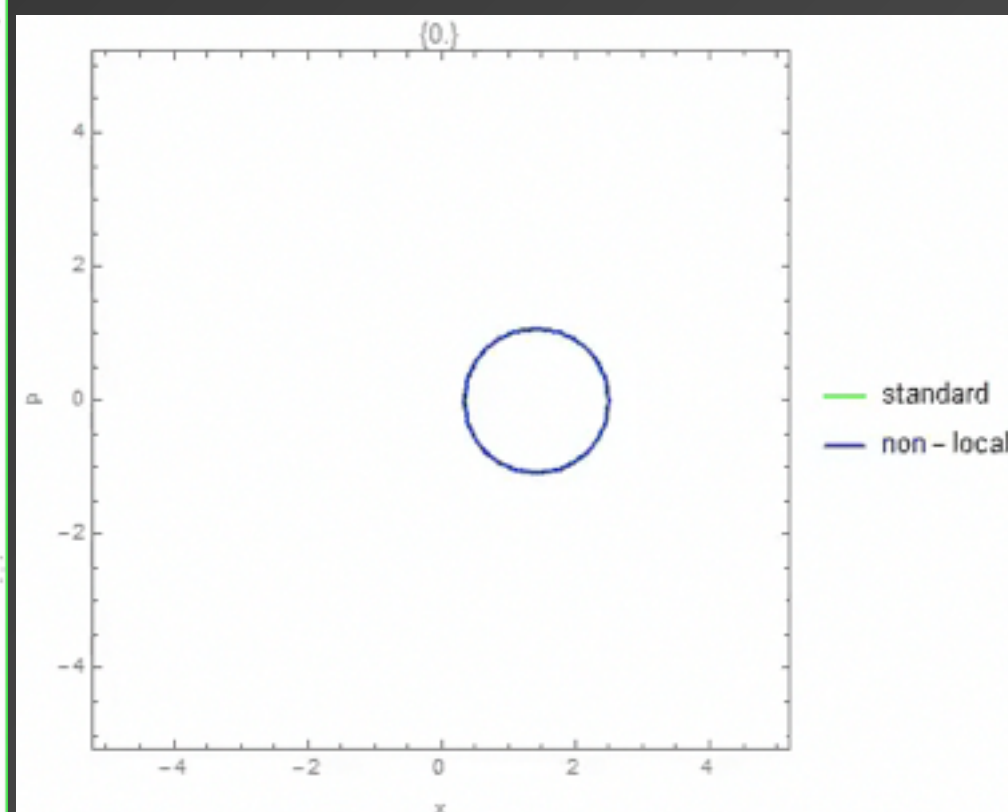

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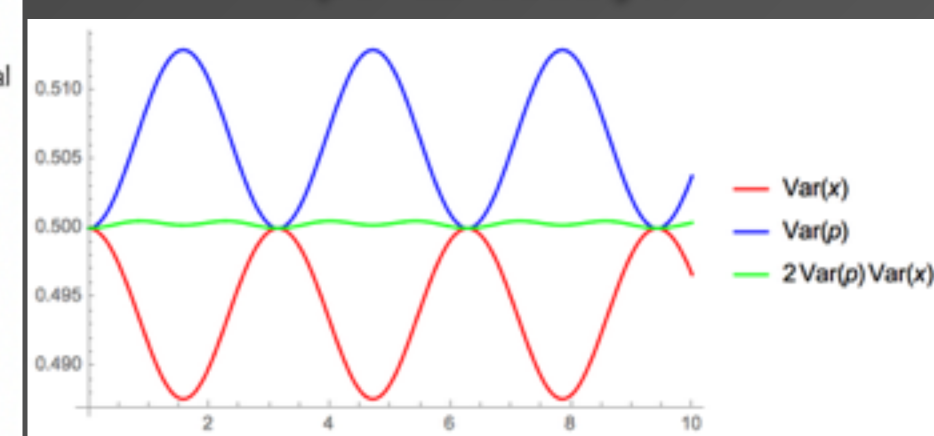

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The emergence of Emergent Gravity is still in the making...