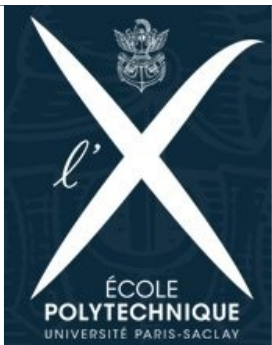


Small-x observable in pA collisions: Medium-induced coherent gluon radiation

Elena Petreska

Universidade de Santiago de Compostela and École Polytechnique

arXiv: 1603.01028, with Stéphane Munier and Stéphane Peigné



Initial Stages 2016
Lisbon, May 25



Disentangle hot medium from cold nuclear matter effects

Hadron suppression in AA and pA collisions with respect to pp

- *Hot medium (QGP) effects;*
- *Cold nuclear matter (CNM) effects:*

Parton shadowing in nuclear PDFs;

J. L. Albacete et al. (2013).

R. Vogt (2015)

E. G. Ferreira, F. Fleuret, J. P. Lansberg and A. Rakotozafindrabe (2013)

Gluon saturation at small x ;

H. Fujii and K. Watanabe, (2013).

B. Ducloue, T. Lappi and H. Mantysaari, (2015)

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This work
(1 \rightarrow 1) +

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Gluon saturation at small x ;

*T. Liou and A. H. Mueller,
(2014): (1 \rightarrow 2)*

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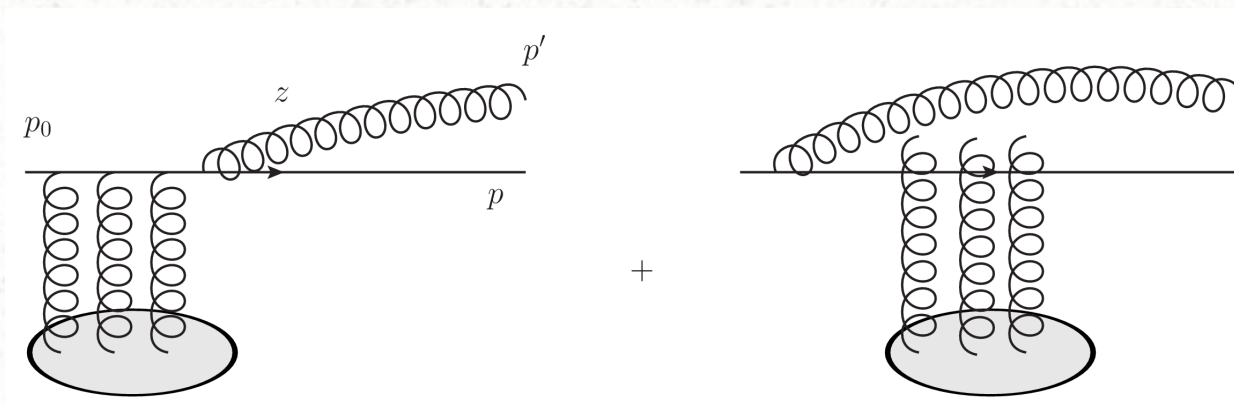
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Medium-induced coherent gluon radiation in this work

Associated with $q \rightarrow q$ and $g \rightarrow g$ hard forward scattering:

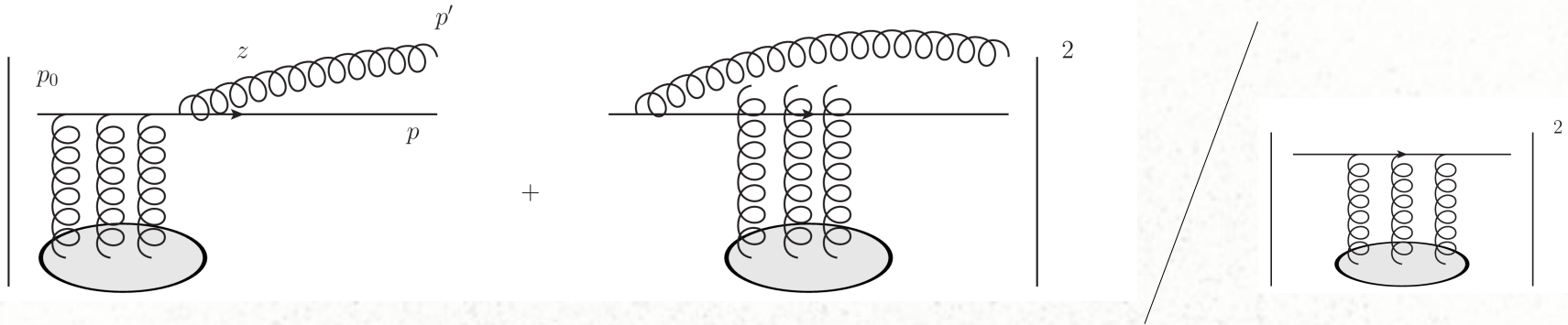
- in the saturation framework;
- MV classical model and Gaussian approximation for the nucleus;
- finite N_c ;
- to leading logs + magnitude of corrections.



$$a(p_0) + A \rightarrow a(p) + g(p') + X$$

- **Forward scattering (large p^+)**;
- **Hard scattering, $p_\perp \gg Q_s$** ;
- **Soft gluon radiation, $z \equiv p'^+/p_0^+ \ll 1$** .

Medium-induced spectrum



- proton

Radiation energy spectrum:

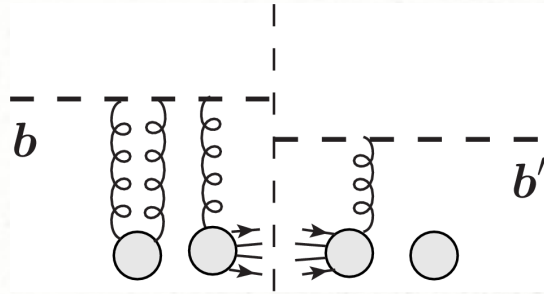
$$z \frac{dI_A}{dz} = p' + \frac{dI_A}{dp'} = \frac{\frac{d\sigma(a+A \rightarrow a+g+X)}{dy dy' d^2 p_{\perp}}}{\frac{d\sigma(a+A \rightarrow a+X)}{dy d^2 p_{\perp}}} = \frac{\frac{d\sigma(p+A \rightarrow a+g+X)}{dy dy' d^2 p_{\perp}}}{\frac{d\sigma(p+A \rightarrow a+X)}{dy d^2 p_{\perp}}}$$

Medium induced spectrum, nucleus target – proton target :

$$z \frac{dI}{dz} \Big|_{ind} \equiv z \frac{dI_A}{dz} - z \frac{dI_p}{dz}$$

$$Q_s^2(x) \sim A^{1/3} x^{-0.3}$$

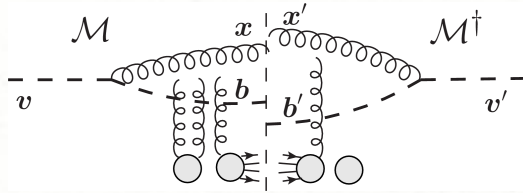
Single parton scattering cross section (denominator)



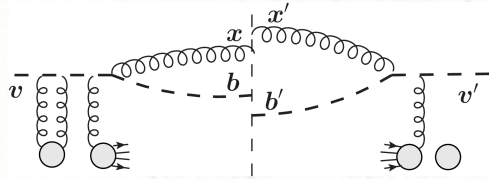
$$S^{(2)} = \frac{1}{d_R} \left\langle \text{tr} \left(U_R(v) U_R^\dagger(v') \right) \right\rangle$$

$$\frac{d\sigma(\text{pA} \rightarrow a + X)}{dy d^2p} = \frac{x_p f_{a/p}(x_p, \mu_F^2)}{(2\pi)^2} \int_b \int_{b'} e^{-ip \cdot (b-b')} S_a^{(2)}[b, b']$$

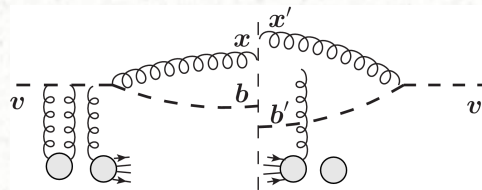
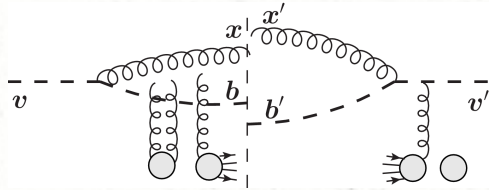
$a \rightarrow a + g$ forward production (numerator)



$$S^{(4)} = \frac{1}{d_R C_R} \left\langle \text{tr} \left(T_R^c U_R(b) U_R^\dagger(b') T_R^d \right) [V(x) V^\dagger(x')]^{cd} \right\rangle$$



$$S^{(2)} = \frac{1}{d_R} \left\langle \text{tr} \left(U_R(v) U_R^\dagger(v') \right) \right\rangle$$



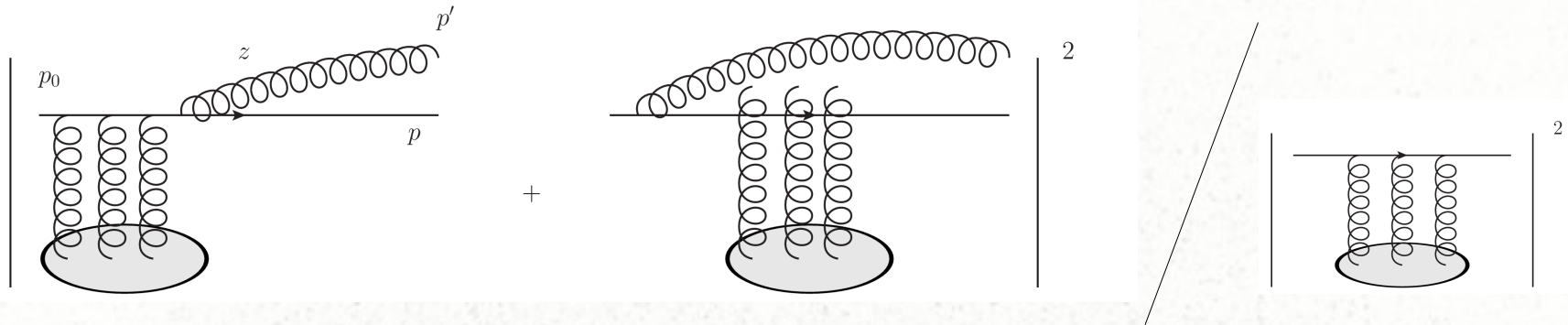
$$S^{(3)} = \frac{1}{d_R C_R} \left\langle \text{tr} \left(T_R^c U_R(b) T_R^d U_R^\dagger(v') \right) [V(x)]^{cd} \right\rangle$$

$$\frac{d\sigma(pA \rightarrow ag + X)}{dy dy' d^2p} = \frac{\alpha_s}{(2\pi)^4} x_p f_{a/p}(x_p, \mu_F^2) z (1-z) \Phi_a^g(z)$$

$$\times \int_x \int_b \int_{b'} e^{-ip \cdot (b-b')} \frac{(x-b) \cdot (x-b')}{(x-b)^2 (x-b')^2} \left\{ S_a^{(2)}[b, b'] + S_a^{(2)}[v, v'] - 2S_a^{(3)}[b, x, v'] \right\}$$

$$x' \rightarrow x' + b' \quad r = b' - b$$

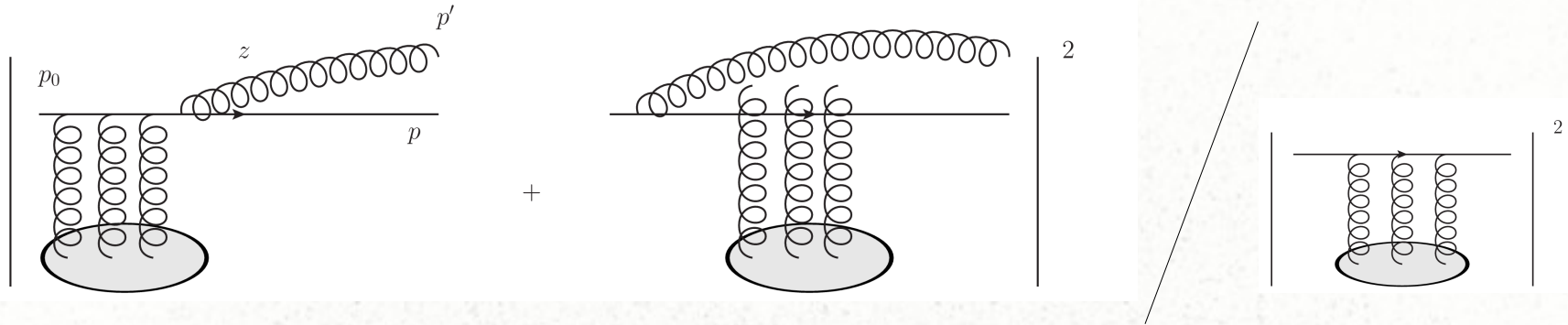
Medium-induced spectrum: General expression in coordinate space



$$z \frac{dI}{dz} = \frac{\frac{d\sigma(p+A \rightarrow a+g+X)}{dy dy' d^2 p_{\perp}}}{\frac{d\sigma(p+A \rightarrow a+X)}{dy d^2 p_{\perp}}}$$

$$z \frac{dI}{dz} = \frac{2C_R \alpha_s}{\pi^2} \frac{\int_r e^{ip \cdot r} \int_x \frac{x \cdot (x+r)}{x^2 (x+r)^2} \left\{ S_a(r) - S_a^{(3)}[0, x+r, zx+r] \right\}}{\int_r e^{ip \cdot r} S_a(r)} \Big|_{ind}$$

Medium-induced spectrum: General expression in coordinate space



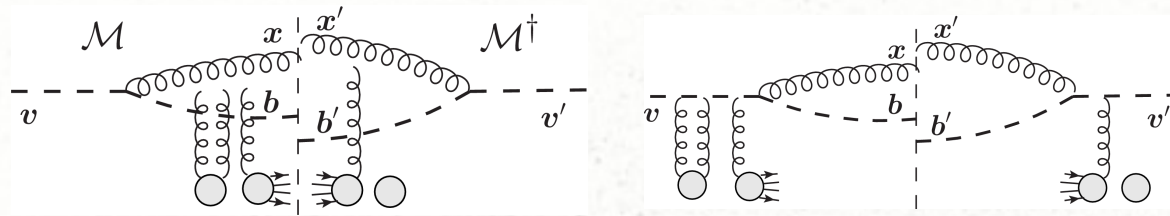
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Separation between purely initial/final and interference contribution:

$$z \frac{dI}{dz} \Big|_{ind} = z \frac{dI}{dz} \Big|_{IS} + z \frac{dI}{dz} \Big|_{FS} + z \frac{dI}{dz} \Big|_{INT}$$

Medium-induced spectrum: Purely initial/final radiation



$$r = b' - b$$

$$r \sim 1/p \ll 1/Q_s$$

$$z \frac{dI}{dz} \Big|_{IS} = z \frac{dI}{dz} \Big|_{FS} = \frac{C_R \alpha_s}{\pi^2} \frac{\int_r e^{ip \cdot r} [S_a(r) - 1] \int_x \frac{x \cdot (x+r)}{x^2 (x+r)^2}}{\int_r e^{ip \cdot r} S_a(r)} \Big|_{ind}$$

→ *Gaussian distribution of sources in the nucleus (MV model):*

$$S_a(x) \simeq \exp \left[-\frac{Q_a^2}{8} x^2 \log \left(\frac{1}{x^2 \mu^2} \right) \right] \approx 1 - \frac{Q_a^2}{8} x^2 \log \left(\frac{1}{x^2 \mu^2} \right)$$

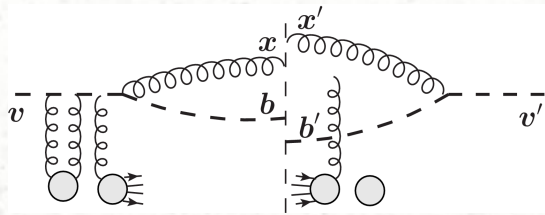
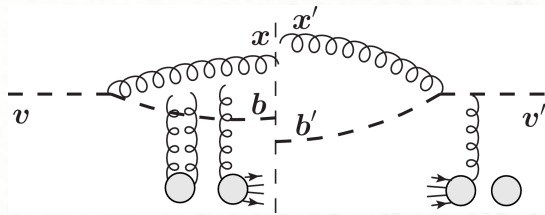
$$\int_r e^{ip \cdot r} [S_a(r) - 1] \int_x \frac{x \cdot (x + r)}{x^2 (x + r)^2} \simeq \frac{2\pi Q_a^2}{|p|^4} \left[\log \left(\frac{p^2}{\mu^2} \right) + \mathcal{O} \left(\frac{Q_a^2}{p^2} \log^2 \left(\frac{p^2}{\mu^2} \right) \right) \right]$$

$$\int_r e^{ip \cdot r} S_a(r) \simeq \frac{2\pi Q_a^2}{|p|^4} \left[1 + \mathcal{O} \left(\frac{Q_a^2}{p^2} \log \left(\frac{p^2}{\mu^2} \right) \right) \right]$$

$$z \frac{dI}{dz} \Big|_{IS} = z \frac{dI}{dz} \Big|_{FS} = 2 \frac{\alpha_s C_R}{\pi} \left\{ \log \left(\frac{p^2}{\mu^2} \right) + \mathcal{O} \left(\frac{Q_a^2}{p^2} \log^2 \left(\frac{p^2}{\mu^2} \right) \right) \right\} \Big|_{ind}$$

Medium-induced spectrum: Interference

The main contribution comes from the interference terms



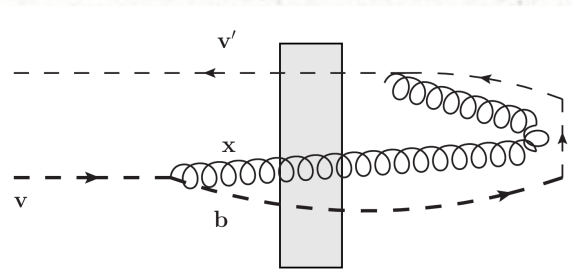
$$r \sim 1/p \ll 1/Q_s,$$

$$r \ll x \text{ but not } r \ll zx$$

$$z \frac{dI}{dz} \Big|_{INT} = \frac{2C_R \alpha_s}{\pi^2} \frac{\int_r e^{ip \cdot r} \int_x \frac{x \cdot (x+r)}{x^2 (x+r)^2} \left\{ 1 - S_a^{(3)}[0, x+r, zx+r] \right\}}{\int_r e^{ip \cdot r} S_a(r)} \Big|_{ind}$$

→ Gaussian distribution of sources in the nucleus (MV model):

$$S_a^{(3)}[b, x, v'] = [S_a(b-x)]^{\frac{C_A}{2C_R}} [S_a(x-v')]^{\frac{C_A}{2C_R}} [S_a(v'-b)]^{\frac{2C_R-C_A}{2C_R}}$$



A. Kovner and U. A. Wiedemann, (2001)

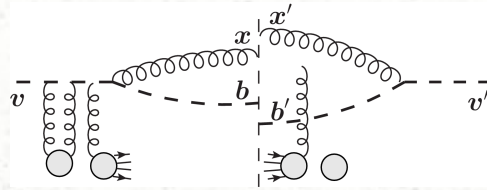
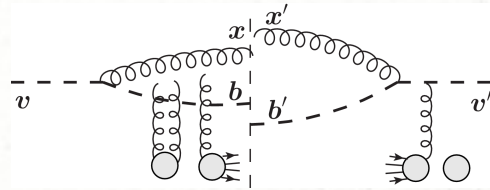
J. P. Blaizot, F. Gelis and R. Venugopalan (2004)

C. Marquet and H. Weigert (2010)

$$z \frac{dI}{dz} \Big|_{INT} = -(2C_R - C_A) \frac{\alpha_s}{\pi^2} \int \frac{d^2 x}{x^2} e^{-izp \cdot x} S_g(x)$$

Medium-induced spectrum: Interference

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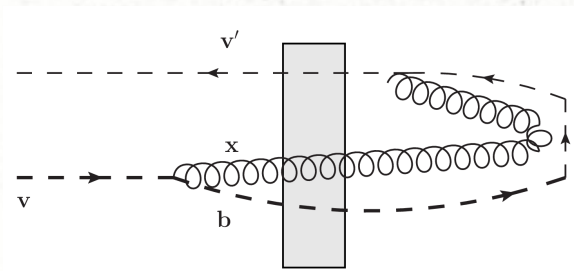
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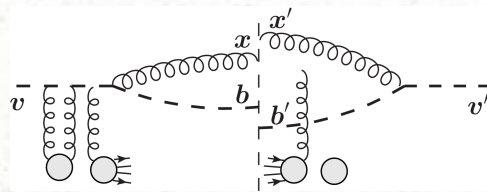
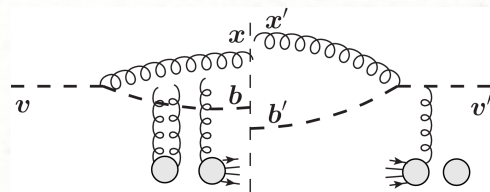
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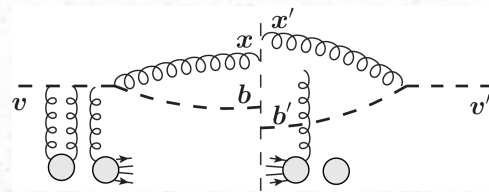
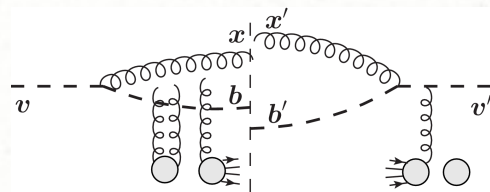
$$z \frac{dI}{dz} \Big|_{INT} = (2C_R - C_A) \frac{\alpha_s}{\pi^2} \int \frac{d^2x}{x^2} e^{-izp \cdot x} [S_{gp}(x) - S_g(x)]$$

$$g : N_c$$

$$q : -\frac{1}{N_c}$$

Medium-induced spectrum: Interference

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$$r \sim 1/p \ll 1/Q_s,$$

$$r \ll x \text{ but not } r \ll zx$$

$$z \frac{dI}{dz} \Big|_{INT} = \frac{2C_R \alpha_s}{\pi^2} \frac{\int_r e^{ip \cdot r} \int_x \frac{x \cdot (x+r)}{x^2 (x+r)^2} \left\{ 1 - S_a^{(3)}[0, x+r, zx+r] \right\}}{\int_r e^{ip \cdot r} S_a(r)} \Big|_{ind}$$

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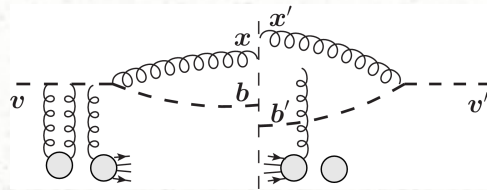
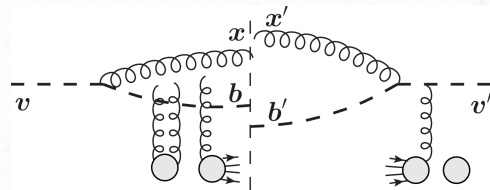
Opacity expansion:

F. Arleo, R. Kolevaton and S. Peigne, (2016)

N. Armesto, H. Ma, M. Martinez, Y. Mehtar-Tani and C. A. Salgado, (2012, 2013)

Medium-induced spectrum: Interference

The main contribution comes from the interference terms



$$r \sim 1/p \ll 1/Q_s,$$

$$r \ll x$$

Parametric limit:

$$z \frac{dI}{dz} \simeq (2C_R - C_A) \frac{\alpha_s}{\pi} \log \left(\frac{Q_s^2}{2z^2 p^2} \right)$$

Order of corrections:

$$\sim \alpha_s \mathcal{O} \left(\frac{Q_a^2}{p^2} \log^2 \left(\frac{p^2}{\mu^2} \right) \right)$$

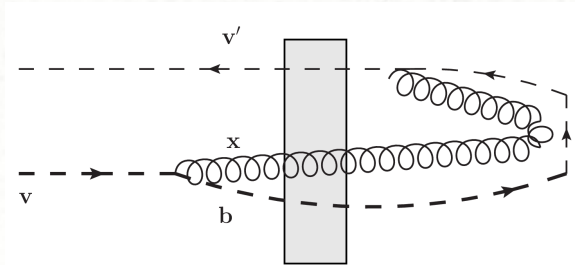
Saturation formalism vs opacity expansion

$$z \frac{dI}{dz} \Big|_{INT} = \frac{2C_R \alpha_s}{\pi^2} \frac{\int_r e^{ip \cdot r} \int_x \frac{x \cdot (x+r)}{x^2 (x+r)^2} \left\{ 1 - S_a^{(3)}[0, x+r, zx+r] \right\}}{\int_r e^{ip \cdot r} S_a(r)} \Big|_{ind}$$

$$S_a^{(3)}[b, x, v'] = [S_a(x+r)]^{\frac{C_A}{2C_R}} [S_a(x)]^{\frac{C_A}{2C_R}} [S_a(zx+r)]^{\frac{2C_R - C_A}{2C_R}}$$

$$S_a(x) = e^{-C_R \hat{\Gamma}(x)}$$

$$z \frac{dI}{dz} = \sum_{m \geq 2} \frac{2\alpha_s}{\pi^2} \frac{\int_r e^{ip \cdot r} \int \frac{d^2 x}{x^2} e^{-izp \cdot x} \frac{(-1)^m}{m!} \left\{ \frac{2C_R - C_A}{2} \hat{\Gamma}(r) + C_A \hat{\Gamma}(x) \right\}^m}{\int_r e^{ip \cdot r} \hat{\Gamma}(r)}$$



Dominant term: $m \hat{\Gamma}(x)^{m-1} \hat{\Gamma}(r)$

$$z \frac{dI}{dz} = (2C_R - C_A) \frac{\alpha_s}{\pi^2} \int \frac{d^2 x}{x^2} e^{-izp \cdot x} \sum_{n \geq 1} \frac{(-1)^{n+1}}{n!} \left[C_A \hat{\Gamma}(x) \right]^n$$

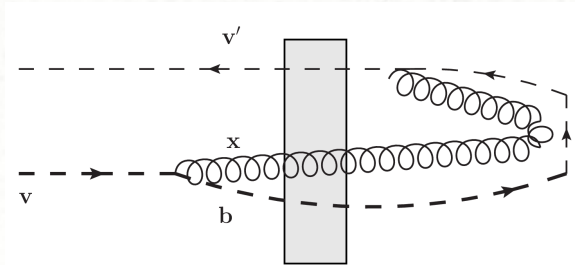
Saturation formalism vs opacity expansion

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$$S_a^{(3)}[b, x, v'] = [S_a(x+r)]^{\frac{C_A}{2C_R}} [S_a(x)]^{\frac{C_A}{2C_R}} [S_a(zx+r)]^{\frac{2C_R - C_A}{2C_R}}$$

$$S_a(x) = e^{-C_R \hat{\Gamma}(x)}$$

$$z \frac{dI}{dz} = \sum_{m \geq 2} \frac{2\alpha_s}{\pi^2} \frac{\int_r e^{ip \cdot r} \int \frac{d^2 x}{x^2} e^{-izp \cdot x} \frac{(-1)^m}{m!} \left\{ \frac{2C_R - C_A}{2} \hat{\Gamma}(r) + C_A \hat{\Gamma}(x) \right\}^m}{\int_r e^{ip \cdot r} \hat{\Gamma}(r)}$$



Dominant term: $m \hat{\Gamma}(x)^{m-1} \hat{\Gamma}(r)$

↓
Order m in opacity

(First order in opacity: $m=2$)

$$z \frac{dI}{dz} = (2C_R - C_A) \frac{\alpha_s}{\pi^2} \int \frac{d^2 x}{x^2} e^{-izp \cdot x} \sum_{n \geq 1} \frac{(-1)^{n+1}}{n!} \left[C_A \hat{\Gamma}(x) \right]^n$$

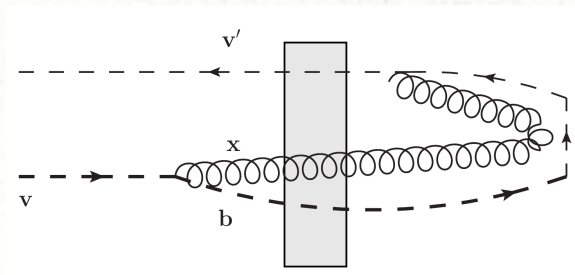
Saturation formalism vs opacity expansion

Saturation formalism:

- the calculation is performed in coordinate space;
- the scatterings are resummed (exponentiated) from the start;

Opacity expansion formalism:

- the calculation is performed in momentum space;
- a hard scattering is assumed from the start;
- calculation at first order in opacity, and then generalized to any order n by recurrency relations; the exponentiation appears at a later stage.



Dominant term: $m \hat{\Gamma}(x)^{m-1} \hat{\Gamma}(r)$

$$z \frac{dI}{dz} = (2C_R - C_A) \frac{\alpha_s}{\pi^2} \int \frac{d^2x}{x^2} e^{-izp \cdot x} \sum_{n \geq 1} \frac{(-1)^{n+1}}{n!} \left[C_A \hat{\Gamma}(x) \right]^n$$

Conclusions

We derive the induced coherent radiation spectrum for an energetic quark or gluon scattering off a nucleus;

- Saturation framework;*
- Hard, forward scattering;*
- Finite N_c ;*
- Gaussian approximation for the field of the nucleus.*

- General expression in coordinate space;*
- Leading asymptotics from the interference diagrams;*
- Order of magnitude of the corrections;*
- Explicit connection between the saturation and opacity expansion formalisms.*

Outlook

- *Implementing small- x evolution in the calculation;*
- *Phenomenology;*
- *Energy loss in gluon \rightarrow quark-antiquark scattering in the saturation framework and at finite N_c .*

The main contribution to the induced spectrum comes from the interference term

$$z \frac{dI}{dz} = \frac{\frac{d\sigma(p+A \rightarrow a+g+X)}{dy dy' d^2 p_{\perp}}}{\frac{d\sigma(p+A \rightarrow a+X)}{dy d^2 p_{\perp}}} = \frac{2C_R \alpha_s}{\pi^2} \frac{\int_r e^{ip \cdot r} \int_x \frac{x \cdot (x+r)}{x^2 (x+r)^2} \left\{ S_a(r) - S_a^{(3)}[0, x+r, zx+r] \right\}}{\int_r e^{ip \cdot r} S_a(r)} \Big|_{ind}$$

$$z \frac{dI}{dz} \Big|_{INT} = (2C_R - C_A) \frac{\alpha_s}{\pi^2} \int \frac{d^2 x}{x^2} e^{-izp \cdot x} [S_{gp}(x) - S_g(x)]$$

Parametric limits:

$$z \frac{dI}{dz} \simeq (2C_R - C_A) \frac{\alpha_s}{\pi} \times \begin{cases} \log \left(\frac{Q_s^2}{2z^2 p^2} \right) & \text{if } z \ll \frac{Q_s}{p}, \\ \frac{Q_s^2}{2z^2 p^2} & \text{if } z \gg \frac{Q_s}{p}, \end{cases}$$

Order of corrections:

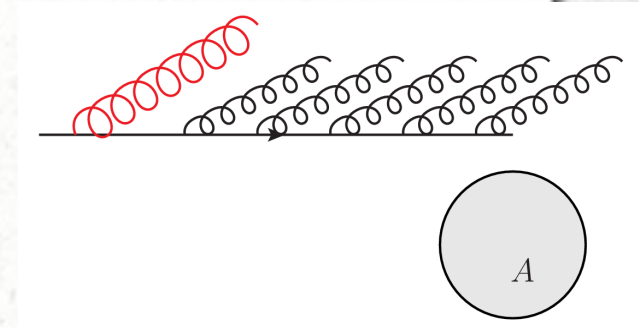
$$\sim \alpha_s \mathcal{O} \left(\frac{Q_a^2}{p^2} \log^2 \left(\frac{p^2}{\mu^2} \right) \right)$$

MV model → The saturation scale does not depend on energy.

Small-x evolution

- ***In the nucleus rest frame***

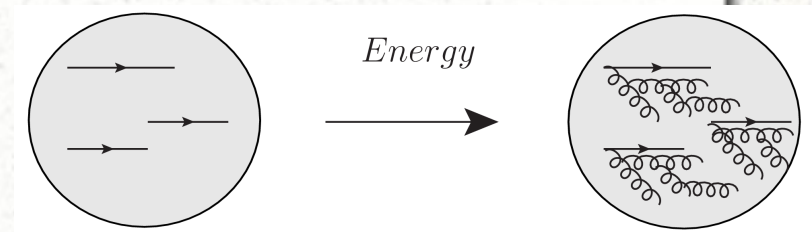
Softer gluons factorize in the nuclear parton density.



- ***Boosted nucleus***
Non-linear evolution equations
for the correlators:

→ ***JIMWLK***

→ ***Balitsky- Kovchegov (large N_c)***



J. Jalilian-Marian, A. Kovner, A. Leonidov, and H. Weigert, (1997, 1999)
E. Iancu, A. Leonidov, and L. D. McLerran, (2001)

I. Balitsky, (1996)

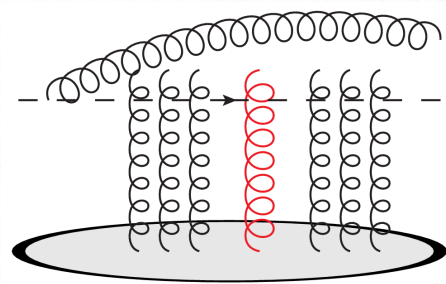
Y. V. Kovchegov, (1999)

- ***MV model + x-dependent saturation scale (finite N_c);***

E. Iancu and D. N. Triantafyllopoulos, (2012)

Medium induced radiation

- **General $1 \rightarrow 1$ hard forward scattering in the opacity expansion formalism.**



S. Peigne, F. Arleo and R. Koleyatov, (2016)

Rule for the color factor:

$$C_R + C_{R'} - C_t$$

- **$q \rightarrow q$ hard forward scattering in the opacity expansion formalism with a color singlet exchange in the t channel.**

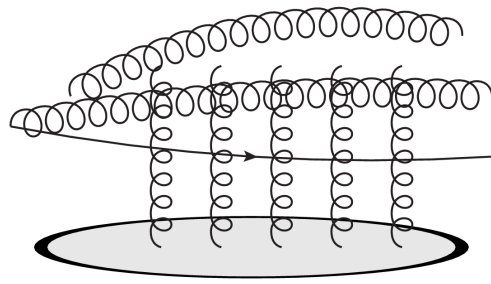
N. Armesto, H. Ma, M. Martinez, Y. Mehtar-Tani and C. A. Salgado, (2012, 2013)

- **$q \rightarrow qg$ and $g \rightarrow gg$ in the opacity expansion formalism.**

Rule for the color factor for $1 \rightarrow n$ scattering.

S. Peigne and R. Koleyatov, (2015)

- **$q \rightarrow qg$ and $g \rightarrow q \text{ anti-}q$ in the saturation formalism in the large- N_c limit.**

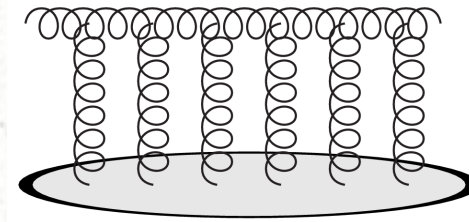
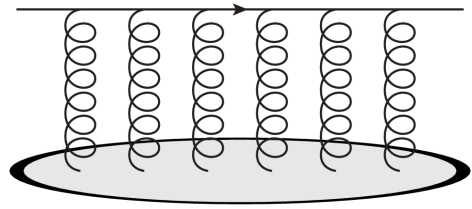


T. Liou and A. H. Mueller, (2014)

Medium-induced coherent gluon radiation in this work

Associated with $q \rightarrow q$ and $g \rightarrow g$ hard forward scattering:

- in the saturation framework;*
- MV classical model and Gaussian approximation for the nucleus;*
- finite N_c ;*
- to leading logs + magnitude of corrections.*



$$U_F = \mathcal{P} \exp \left[ig \int dx^+ A_c^-(x^+, x_\perp) T_F^c \right]$$

$$U_A = \mathcal{P} \exp \left[ig \int dx^+ A_c^-(x^+, x_\perp) T_A^c \right]$$

- Large ultra-relativistic nucleus; perturbative calculation.*

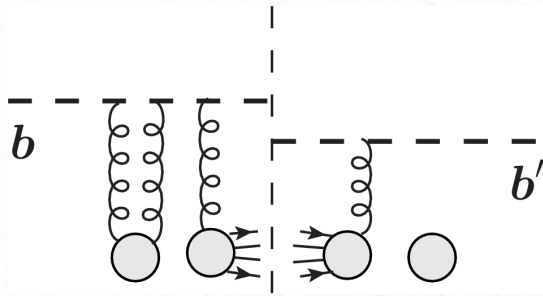
$$Q_s^2(x) \sim A^{1/3} x^{-0.3} \gg \Lambda_{QCD}^2 \quad \alpha_s(Q_s^2) \ll 1$$

- High occupation number of gluons; The small-x gluon field is classical.*

McLerran-Venugopalan model

L. D. McLerran and R. Venugopalan, (1994)

Parton elastic scattering cross section (denominator)



$$\frac{d\sigma(\text{pA} \rightarrow a + X)}{dy d^2p} \sim \int_b \int_{b'} e^{-ip \cdot (b-b')} S_a^{(2)}[b, b']$$

Assumptions:

→ *Infinite transverse nuclear size;*

$$p_{\perp} \gg Q_s; \quad r_{\perp} = b'_{\perp} - b_{\perp} \sim 1/p \ll 1/Q_s < 1/\Lambda_{QCD};$$

$$S_a^{(2)}[b, b'] \equiv S_a(b' - b)$$

→ *Gaussian distribution of sources in the nuclear wave function;*

$$S_a(x) \simeq \exp \left[-\frac{Q_a^2}{8} x^2 \log \left(\frac{1}{x^2 \mu^2} \right) \right]$$

$$S_a^{(3)}[b, x, v'] = [S_a(b - x)]^{\frac{C_A}{2C_R}} [S_a(x - v')]^{\frac{C_A}{2C_R}} [S_a(v' - b)]^{\frac{2C_R - C_A}{2C_R}} \quad 27$$