3-parton azimuthal angular correlations as a probe of gluon saturation

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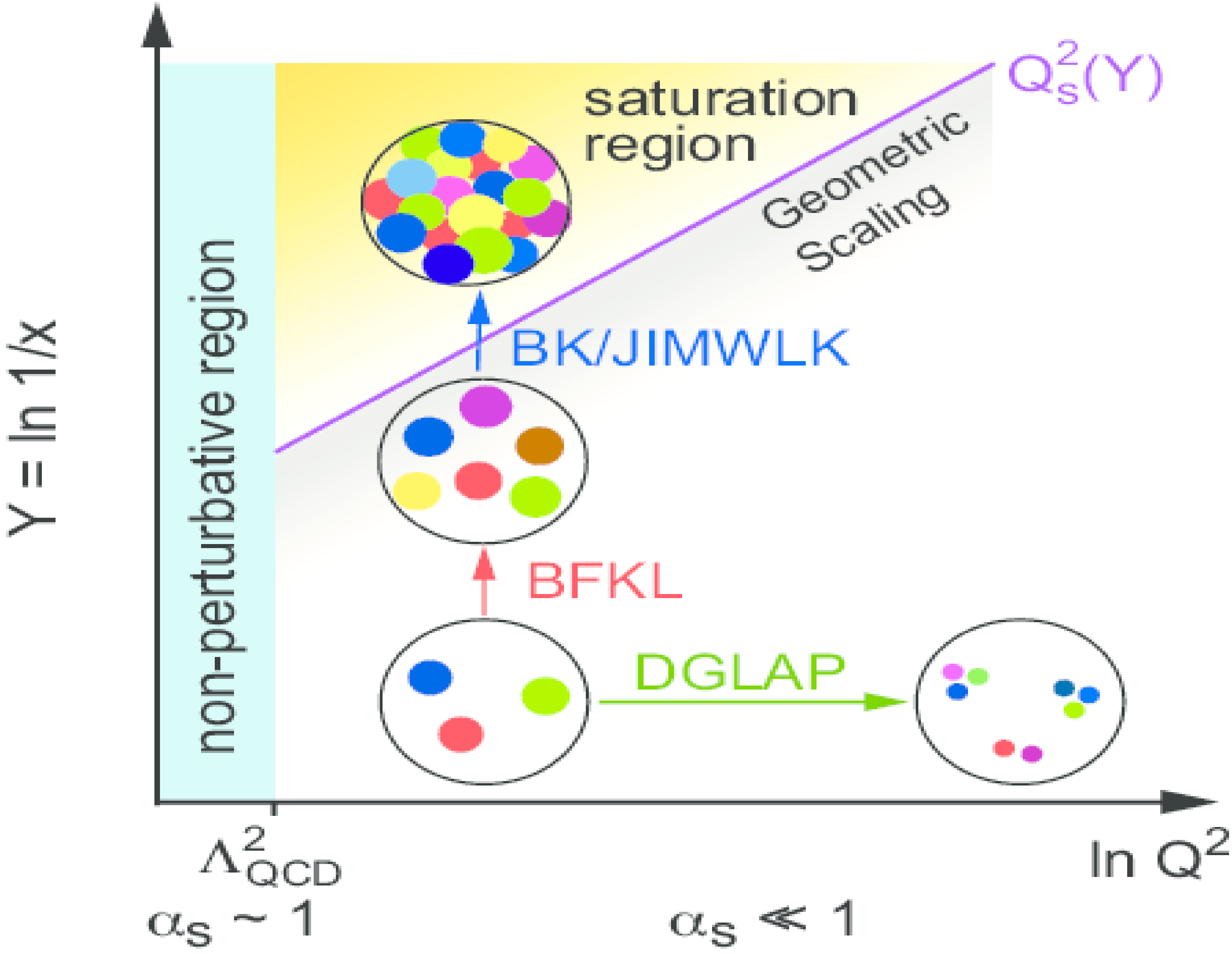
based on arXiv:1604.08526
with *A. Ayala, M. Hentschinski and M.E. Tejeda-Yeomans*

A proton at high energy: *saturation*



Signatures?

two main effects:

multiple scatterings
evolution with x (rapidity)

dense-dense (AA, pA, pp) collisions

initial conditions

dilute-dense (pA, forward pp) collisions

multiplicities
p_t *spectra*
angular correlations

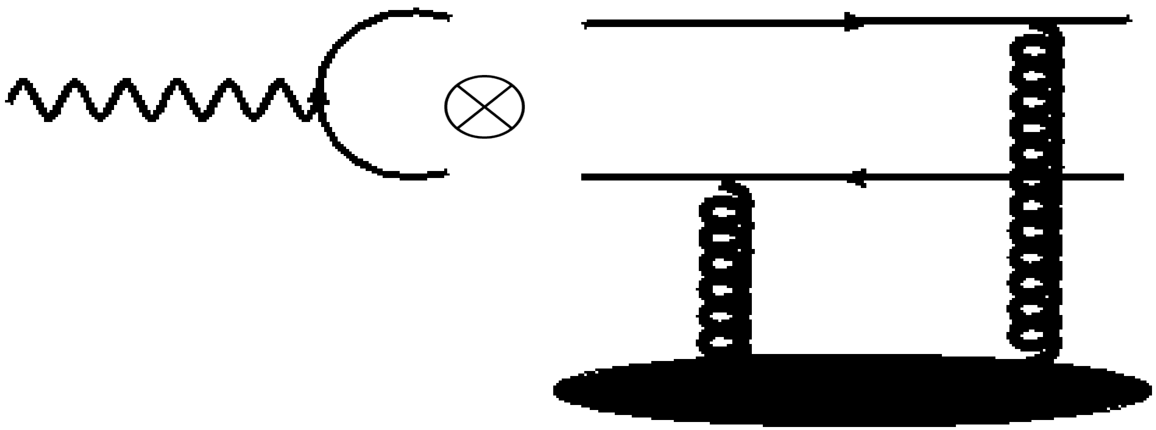
DIS

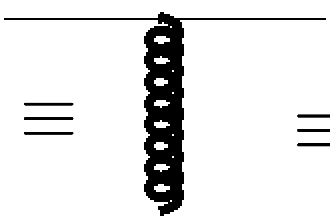
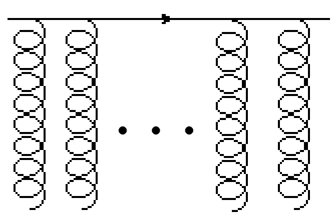
structure functions (diffraction)
NLO di-hadron correlations
3-hadron/jet correlations

DIS total cross section

$$\sigma^{\mathrm{total}}_{\mathrm{DIS}} = 2\int_0^1 dz \int d^2x_t\, d^2y_t \left|\Psi(k^{\pm}, k_t | z, x_t, y_t)\right|^2 T(x_t, y_t)$$

***dipole* cross section** $\quad T(x_t, y_t) \equiv \frac{1}{N_c}\mathrm{Tr}\left\langle 1 - V(x_t)V^{\dagger}(y_t)\right\rangle$



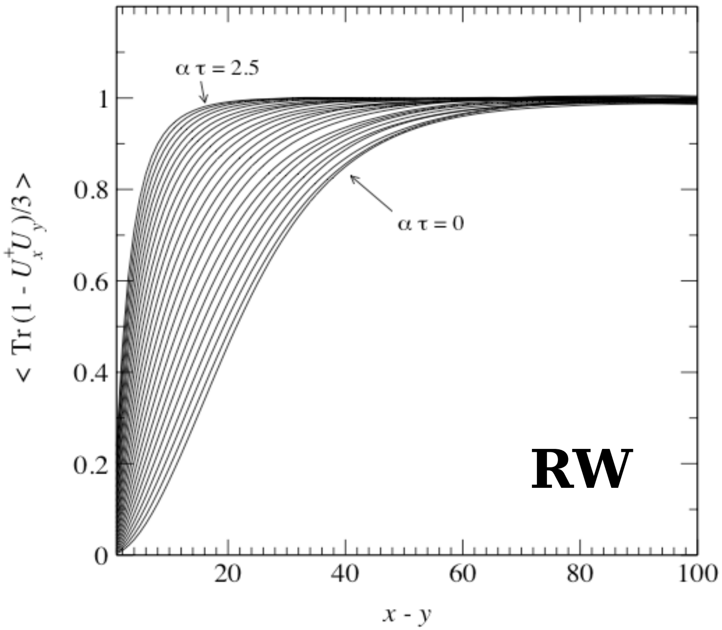
$V(x_t) \equiv$  $\equiv$  $\sim 1 + O(g\,A) + O(g^2\,A^2)$

Wilson line encodes multiple scatterings from the color field of the target

Dipoles at large N_c : BK eq.

$$\frac{d}{dy}T(x_t-y_t)=\frac{\bar{\alpha}_s}{2\pi}\int d^2z_t\,\frac{(x_t-y_t)^2}{(x_t-z_t)^2(y_t-z_t)^2}\times$$

$$\left[T(x_t-z_t)+T(z_t-y_t)-T(x_t-y_t)-T(x_t-z_t)T(z_t-y_t)\right]$$



$\tilde{T}(p_t)\to\log\left[\frac{Q_s^2}{p_t^2}\right]$ **saturation region**

$\tilde{T}(p_t)\to\frac{1}{p_t^2}\left[\frac{Q_s^2}{p_t^2}\right]^{\gamma}$ **extended scaling region**

$\tilde{T}(p_t)\to\frac{1}{p_t^2}\left[\frac{Q_s^2}{p_t^2}\right]$ **pQCD region**

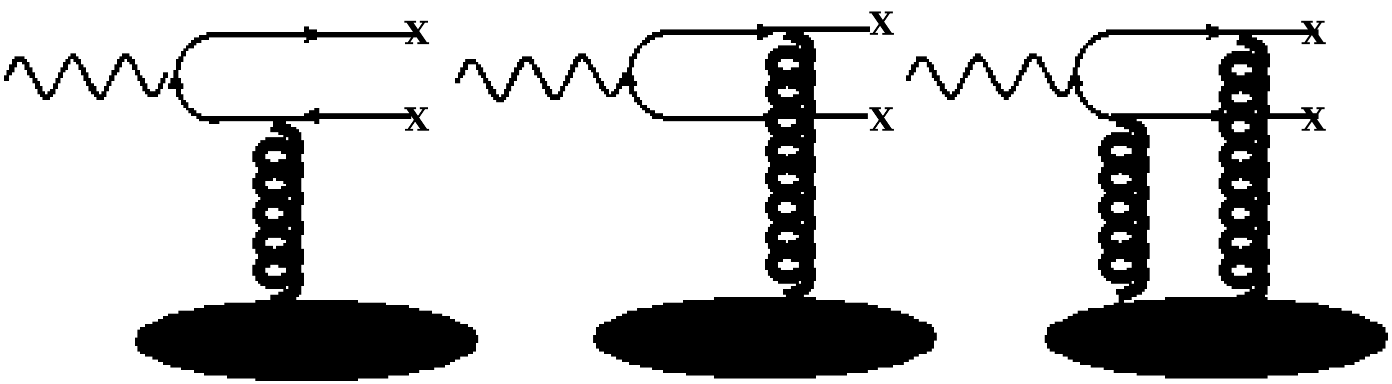
R*ummukainen-****W****eigert, NPA739 (2004) 183*

NLO: *Balitsky-Kovchegov-Weigert-Gardi-Chirilli (2007-2008)*

something with more discriminating power

di-hadron (azimuthal) angular correlations in DIS

LO: $\gamma^\star(\mathbf{k})\,\mathbf{p} \to \mathbf{q}(\mathbf{p})\,\bar{\mathbf{q}}(\mathbf{q})\,\mathbf{X}$



new d.o.f

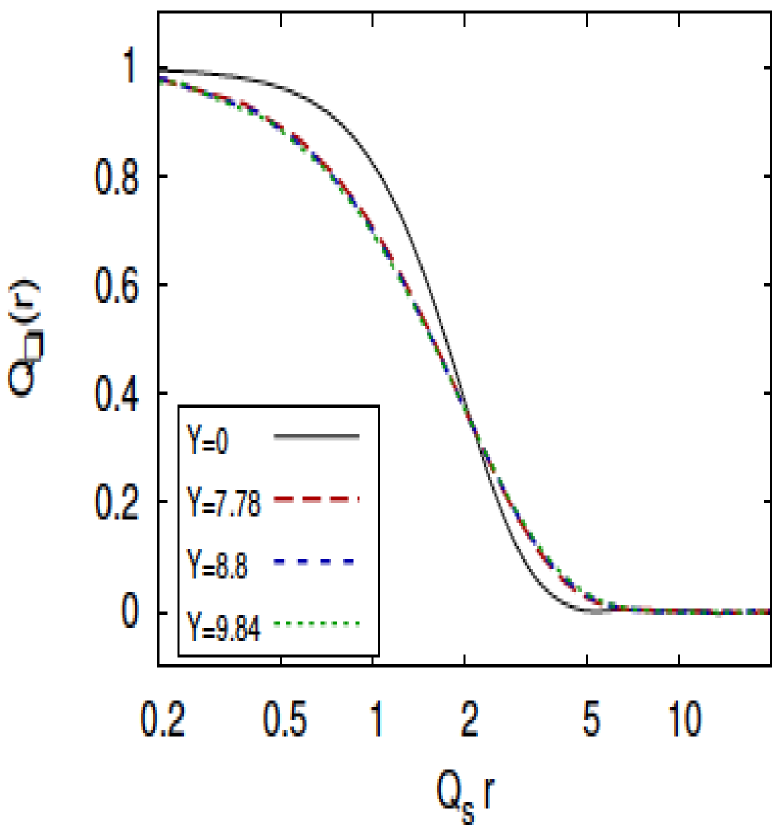
quadrupoles

quark propagator in the background color field

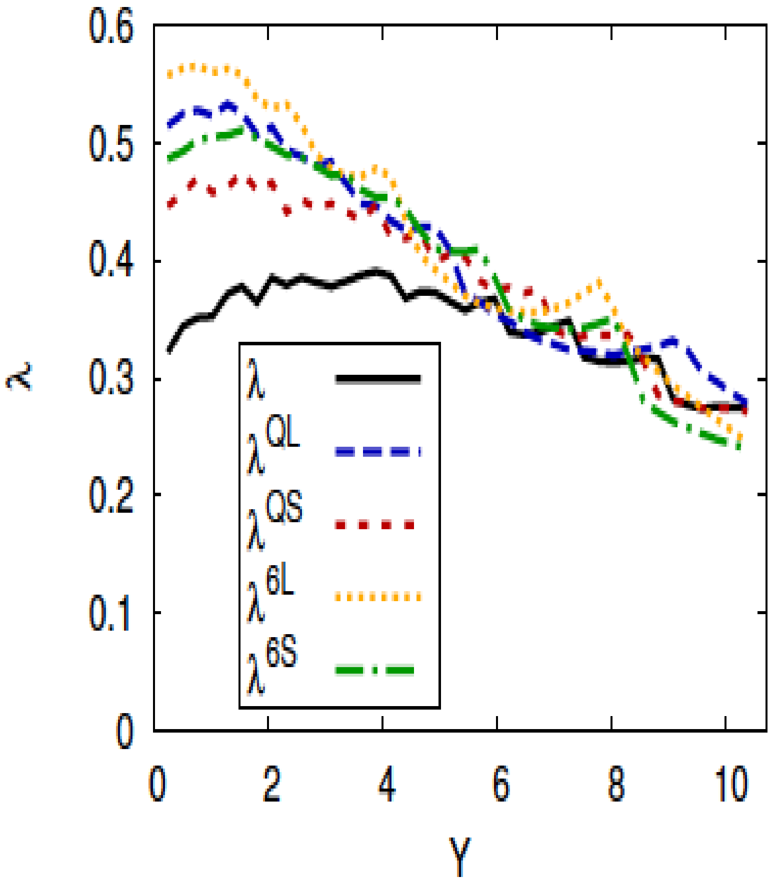
$$\mathrm{S}_F(q,p) \equiv (2\pi)^4\delta^4(p-q)\,S_F^0(p) + S_F^0(q)\,\tau_f(q,p)\,S_F^0(p)$$

$$\tau_f(q,p) \;\; \equiv \;\; (2\pi)\delta(p^- - q^-)\,\gamma^- \int d^2x_t\, e^{i(q_t-p_t)\cdot x_t}$$
$$\left\{\theta(p^-)[V(x_t)-1] - \theta(-p^-)[V^\dagger(x_t)-1]\right\}$$

Quadrupole evolution: JIMWLK



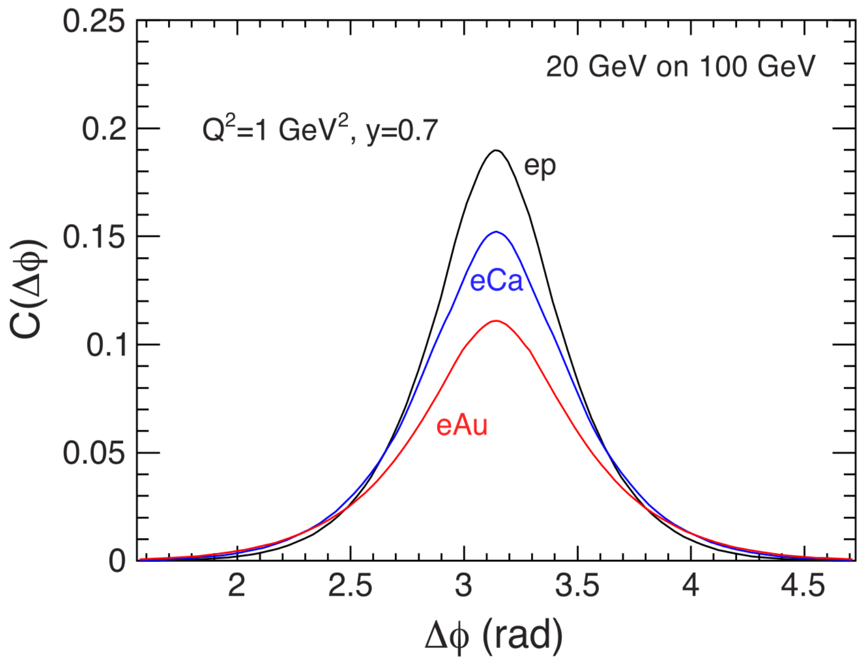
Geometric scaling also present in quadrupoles

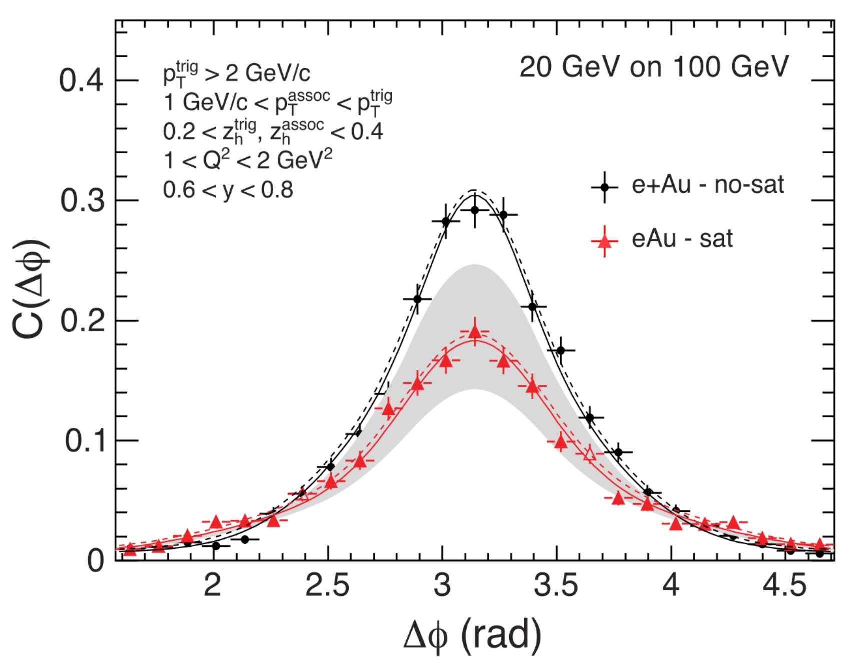


Energy dependence of saturation scale

Dumitru-Jalilian-Marian-Lappi-Schenke-Venugopalan:PLB706 (2011) 219

Di-hadron azimuthal correlations in DIS





Electron Ion Collider...., A. Accardi et al., arXiv:1212.1701

Zheng-Aschenauer-Lee-Xiao, PRD89 (2014)7, 074037

Precision CGC: <u>NLO</u> *corrections*

DIS total cross section:

photon impact factor
evolution equations

pA collisions:

Single inclusive particle production

<u>**NLO di-jet production in DIS**</u>

LO 3-jet production

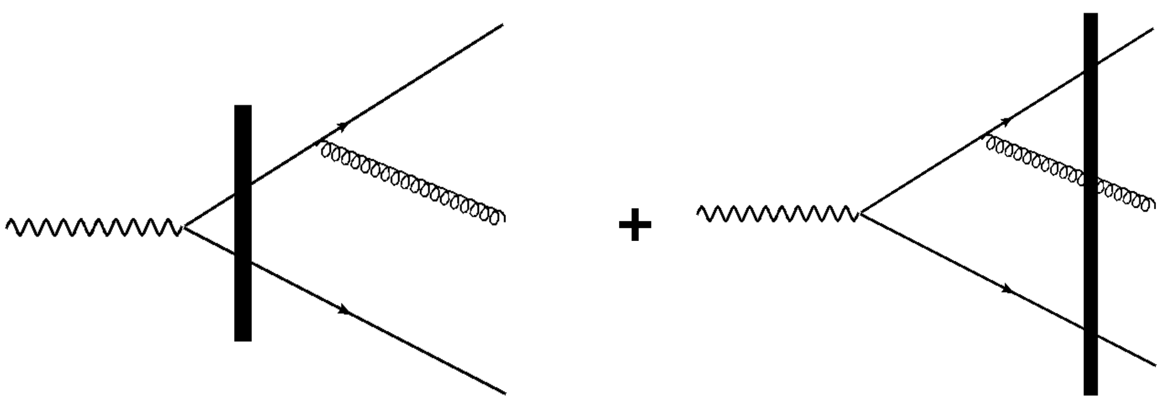
<u>*two away side hadrons: additional knob*</u>

Azimuthal correlations in DIS

*di-jet production in DIS: **NLO***

real contributions: $$\gamma^{\star}\mathbf{T} \rightarrow \mathbf{q}\,\bar{\mathbf{q}}\,\mathbf{g}\,\mathbf{X}$$

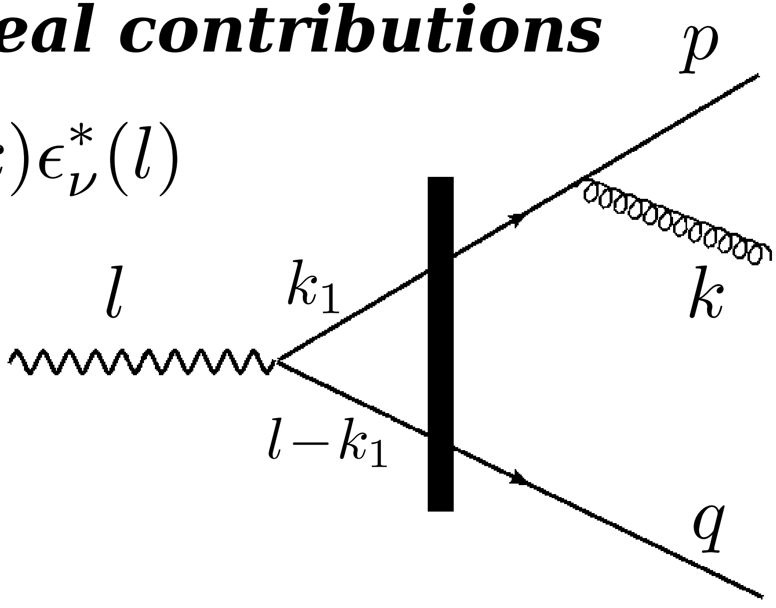
integrate out one of the produced partons



work in progress: Ayala, Hentschinski , Jalilian-Marian, Tejeda-Yeomans

LO <u>3-parton production</u>: real contributions

$$\mathcal{A} \equiv -eg\,\bar{u}(p)\,[A]^{\mu\nu}\,v(q)\,\epsilon_\mu\,(k)\epsilon^*_\nu(l)$$



$$A_1^{\mu\nu} = \gamma^\mu\,t^a\,S_F^0(p+k)\,\tau_F(p+k,k_1)\,S_F^0(k_1)\,\gamma^\nu\,S_F^0(l-k_1)\,\tau_F(l-k_1,q)\,\frac{d^4k_1}{(2\pi)^4}$$

$$= \frac{i}{2\,l^-}\frac{\delta(l^- - p^- - q^- - k^-)}{(p+k)^2}\int d^2x_t\,d^2y_t\,e^{-i(p_t+k_t)\cdot x_t}e^{-iq_t\cdot y_t}$$
$$\gamma^\mu\,t^a\,i(\not{p}+\not{k})\gamma^-\,i\not{k}_1\,\gamma^\nu\,i(\not{l}-\not{k}_1)\,\gamma^-\,K_0\,[L\,(x_t-y_t)]$$
$$V(x_t)\,V^\dagger(y_t)$$

with

$$L^2 = \frac{q^-(p^-+k^-)}{l^-\,l^-}Q^2 \qquad k_1^- = p^- - k^- \qquad k_1^+ = \frac{k_{1t}^2 - i\epsilon}{2(p^-+k^-)} \qquad k_{1t} = -i\,\partial_{x_t-y_t}$$

spinor helicity methods

Review:
L. Dixon, hep-ph/9601359

massless quarks: helicity eigenstates

$$u_\pm(k) \equiv \frac{1}{2}(1\pm\gamma_5)\,u(k) \qquad \overline{u_\pm(k)} \equiv \overline{u(k)}\,\frac{1}{2}(1\mp\gamma_5)$$

$$v_\mp(k) \equiv \frac{1}{2}(1\pm\gamma_5)\,v(k) \qquad \overline{v_\mp(k)} \equiv \overline{v(k)}\,\frac{1}{2}(1\mp\gamma_5)$$

$$u_+(k) = v_-(k) = \frac{1}{\sqrt{2}}\begin{bmatrix}\sqrt{k^+}\\ \sqrt{k^-}e^{i\phi_k}\\ \sqrt{k^+}\\ \sqrt{k^-}e^{i\phi_k}\end{bmatrix} \qquad u_-(k) = v_+(k) = \frac{1}{\sqrt{2}}\begin{bmatrix}\sqrt{k^-}e^{-i\phi_k}\\ -\sqrt{k^+}\\ -\sqrt{k^-}e^{-i\phi_k}\\ \sqrt{k^+}\end{bmatrix}$$

with $e^{\pm i\phi_k} \equiv \dfrac{k_x \pm i k_y}{\sqrt{k^+ k^-}}$

notation:

$$|i^\pm> \equiv |k_i^\pm> \equiv u_\pm(k_i) = v_\mp(k_i)$$

$$<i^\pm| \equiv <k_i^\pm| \equiv \overline{u_\pm(k_i)} = \overline{v_\mp(k_i)}$$

basic spinor products:

$$<i\,j> \equiv <i^-|j^+> = \overline{u_-(k_i)}\,u_+(k_j)$$

$$[i\,j] \equiv <i^+|j^-> = \overline{u_+(k_i)}\,u_-(k_j)$$

with

$$<ij> \equiv \sqrt{|s_{ij}|}\,e^{i\phi_{ij}}$$

$$[ij] \equiv \sqrt{|s_{ij}|}\,e^{-i(\phi_{ij}+\pi)}$$

where

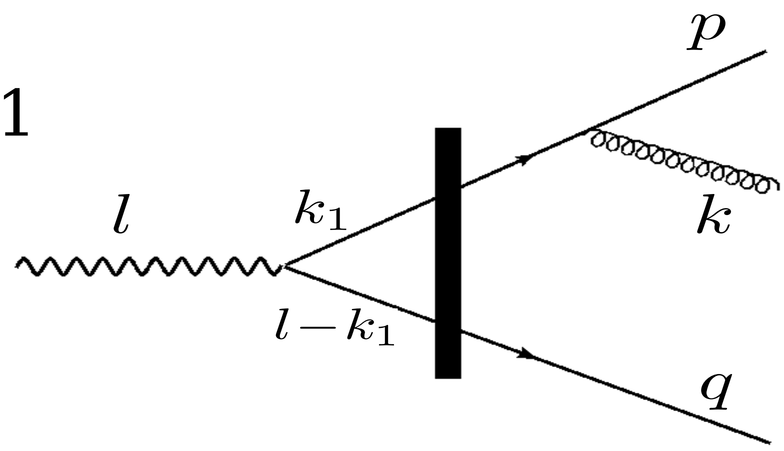
$$\cos\phi_{ij} = \frac{k_i^x k_j^+ - k_j^x k_i^+}{\sqrt{|s_{ij}|k_i^+ k_j^+}}$$

$$\sin\phi_{ij} = \frac{k_i^y k_j^+ - k_j^y k_i^+}{\sqrt{|s_{ij}|k_i^+ k_j^+}}$$

and

$$s_{ij} = (k_i + k_j)^2 = 2k_i\cdot k_j$$

Diagram A1



longitudinal photons

gluon, quark helicity

$g, h = \pm$

$$A^L_{1,hg} = \sqrt{2Q^2}\, e^{ix_t(k_t+p_t)+iq_t y_t}\, K_0\left[\sqrt{Q^2 x_{12}^2 z_2\,(z_1+z_3)}\right]\cdot a^L_{1,hg}$$

with $x_{12}^2 \equiv (x_t - y_t)^2$

$$a^L_{1,++} = \frac{z_1 z_2 \sqrt{z_1 z_2}\,(z_1+z_3)}{z_3 e^{-i\theta_p}|p_t| - z_1 e^{-i\theta_k}|k_t|}$$

$$a^L_{1,-+} = \frac{\sqrt{z_1 z_2}\, z_2\,(z_1+z_3)^2}{z_3 e^{-i\theta_p}|p_t| - z_1 e^{-i\theta_k}|k_t|}$$

$$a^L_{1,--} = (a^L_{1,++})^\star$$

$$a^L_{1,+-} = (a^L_{1,+--+})^\star$$

add up all the pieces, Fourier transform, square the amplitude,....
triple differential cross section

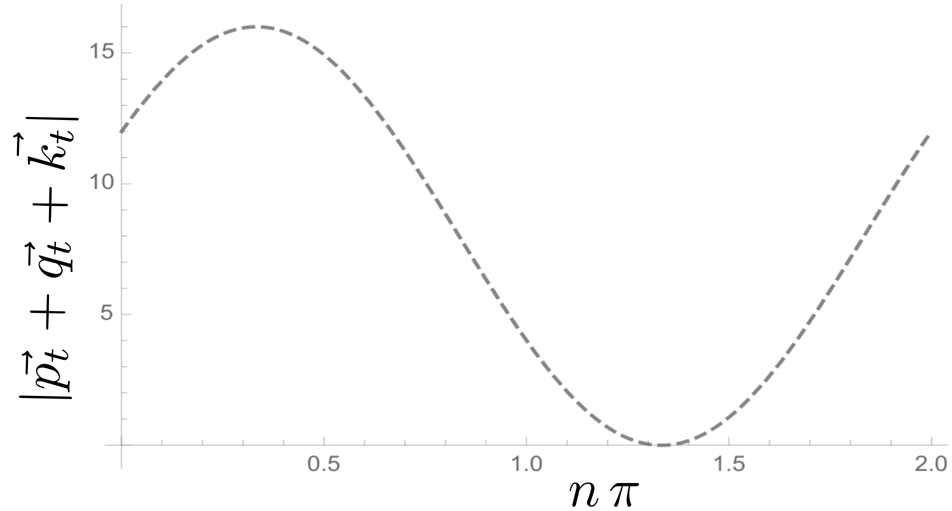
3-parton kinematics

<u>linear regime</u>: <u>*use ugd's*</u>

$$z_1 = z_2 = 0.2, z_3 = 0.6$$

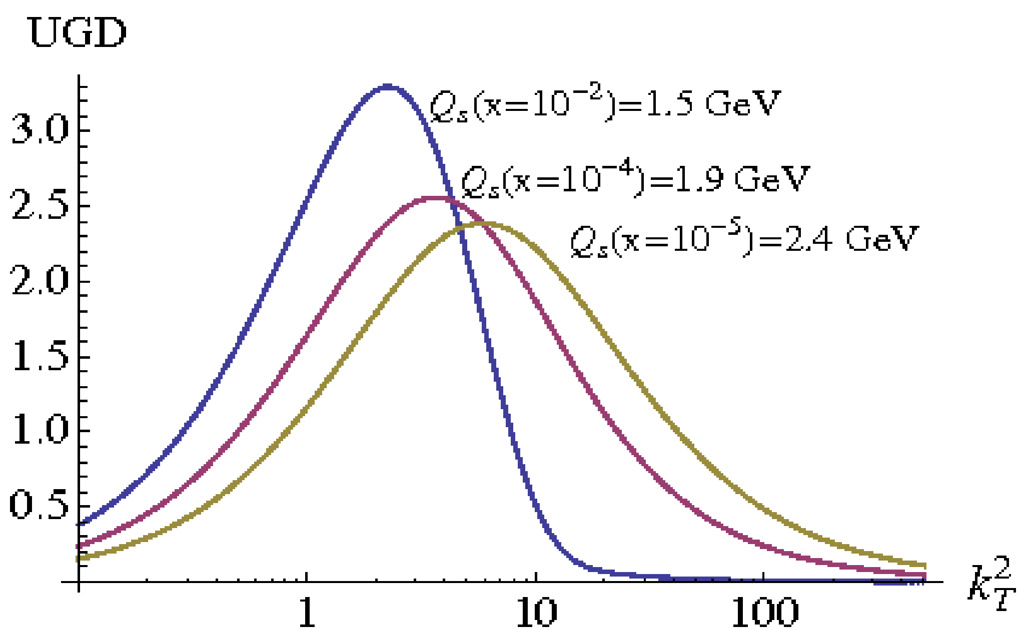
$$p_t = q_t = k_t = 4\,GeV$$

$$Q^2 = 16\,GeV^2$$



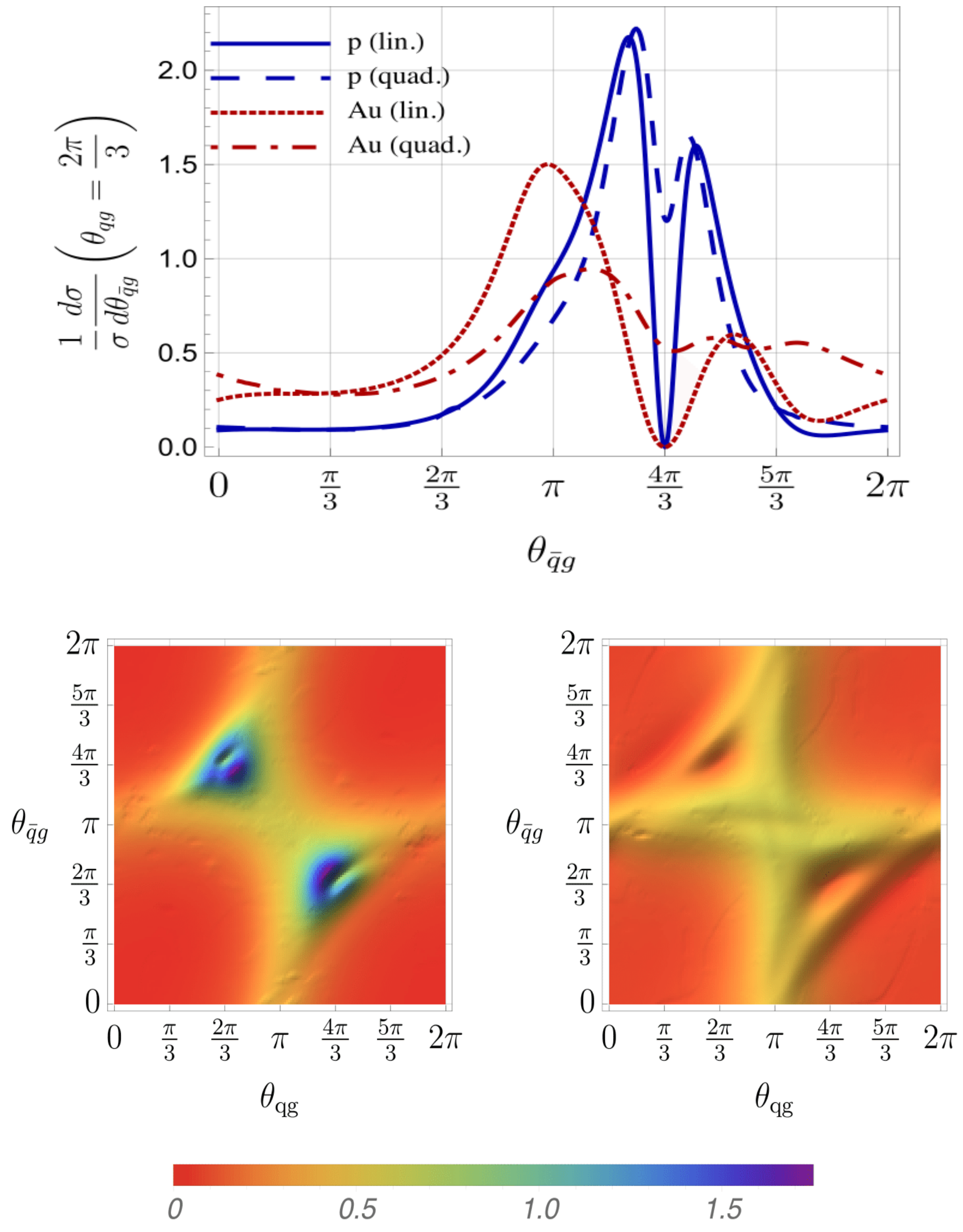
$$\Delta\phi_{12} = \frac{2\pi}{3} \qquad \text{vary} \quad \Delta\phi_{13}$$

$$k_t^2\,\tilde{T}(k_t)$$



multiple scattering: broadening of the peak
x-evolution: reduction of magnitude

3-parton azimuthal angular correlations



Possible extensions to other processes

real photons: $Q^2 \to 0$

ultra-peripheral nucleus-nucleus collisions

crossing symmetry:

$$\gamma^{(\star)} \longrightarrow q\,\bar{q}\,g \quad \Longleftrightarrow \quad \left\{ \begin{array}{lcl} q & \longrightarrow & q\,g\,\gamma^{(\star)} \\ \bar{q} & \longrightarrow & \bar{q}\,g\,\gamma^{(\star)} \\ g & \longrightarrow & q\,\bar{q}\,\gamma^{(\star)} \end{array} \right\}$$

proton-nucleus collisions (collinear factorization in proton?)

$$pA \longrightarrow h_1\,h_2\,\gamma^{(\star)}\,X$$

SUMMARY

CGC is a systematic approach to high energy collisions

Leading Log CGC works (too) well

- ***it has been used to fit a wealth of data; ep, eA, pp, pA, AA***
- ***need to eliminate/minimize late time/hadronization effects***

Precision (NLO) studies are needed

- ***signs of trouble at $p_t > Q_s$?***

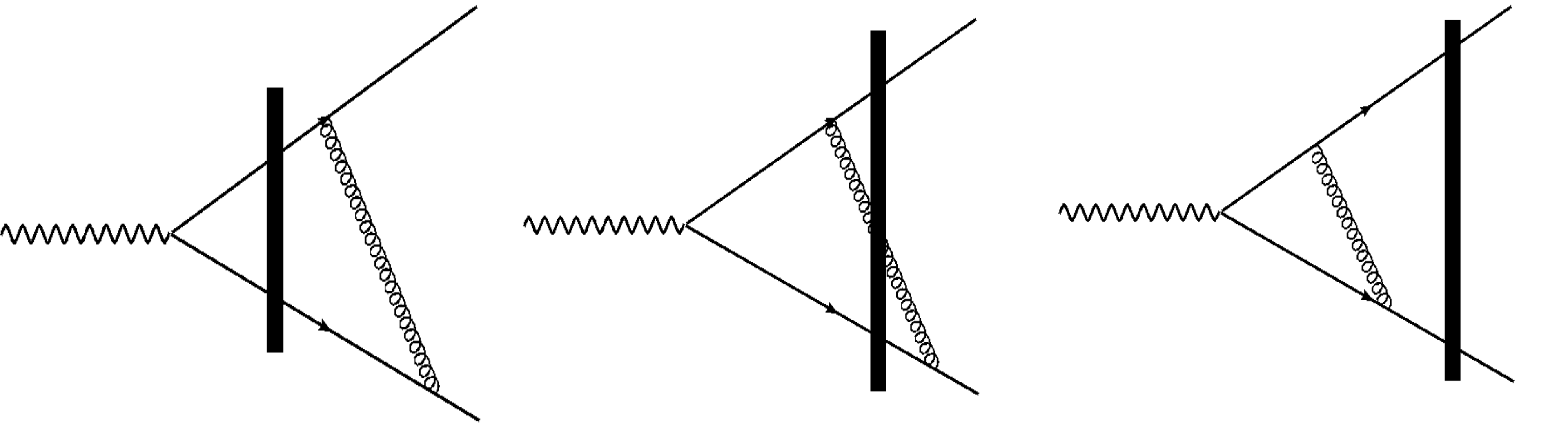
Azimuthal angular correlations offer a unique probe of CGC

- ***3-hadron/jet correlations should be even more discriminatory***

an EIC is needed for precision CGC studies

di-jet azimuthal correlations in DIS

virtual contributions: $\gamma^{\star}\mathbf{T} \to \mathbf{q}\,\bar{\mathbf{q}}\,\mathbf{X}$



+ "self-energy" diagrams

di-jet azimuthal correlations in DIS

NLO: $\gamma^{\star}\mathbf{p} \to \mathbf{h}\,\mathbf{h}\,\mathbf{X}$

integrate out one of the produced partons - there are divergences:

rapidity divergences: JIMWLK evolution of n-point functions

collinear divergences: DGLAP evolution of fragmentation functions

infrared divergences cancel

the finite pieces are written as a factorized cross section

related work by:

Boussarie, Grabovsky, Szymanowski, Wallon, JHEP1409, 026 (2014)
Balitsky, Chirilli, PRD83 (2011) 031502, PRD88 (2013) 111501
Beuf, PRD85, (2012) 034039

structure of Wilson lines: cross section

$$
\begin{aligned}
\mathrm{tr}\left[W_1 W_1^*\right] &= \frac{\left(N_c^2-1\right) S_Q(x_t,x'_t,y'_t,y_t)}{2N_c} \\
\mathrm{tr}\left[W_1 W_2^*\right] &= \frac{1}{4}\left(S_D(z'_t,x'_t) S_Q(x_t,z'_t,y'_t,y_t) - \frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right) \\
\mathrm{tr}\left[W_1 W_3^*\right] &= \frac{1}{2}\left(S_D(x_t,y) S_D(y'_t,x'_t) - \frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right) \\
\mathrm{tr}\left[W_1 W_4^*\right] &= \frac{1}{4}\left(S_D(z'_t,x'_t) S_Q(x_t,z'_t,y'_t,y_t) - \frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right) \\
\mathrm{tr}\left[W_2 W_1^*\right] &= \frac{1}{4}\left(S_D(x_t,z) S_Q(z_t,x'_t,y'_t,y_t) - \frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right) \\
\mathrm{tr}\left[W_2 W_2^*\right] &= \frac{1}{8}\left(S_Q(x_t,x'_t,z'_t,z_t) S_Q(z,z'_t,y'_t,y_t) - \frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right) \\
\mathrm{tr}\left[W_2 W_3^*\right] &= \frac{1}{4}\left(S_D(z,y_t) S_Q(x_t,x'_t,y'_t,z) - \frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right) \\
\mathrm{tr}\left[W_2 W_4^*\right] &= \frac{1}{8}\left(S_Q(x_t,x'_t,z'_t,z) S_Q(z_t,z'_t,y'_t,y_t) - \frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right) \\
\mathrm{tr}\left[W_3 W_1^*\right] &= \frac{1}{2}\left(S_D(x_t,y_t) S_D(y'_t,x'_t) - \frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right) \\
\mathrm{tr}\left[W_3 W_2^*\right] &= \frac{1}{4}\left(S_D(y'_t,z'_t) S_Q(x_t,x'_t,z'_t,y_t) - \frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right) \\
\mathrm{tr}\left[W_3 W_3^*\right] &= \frac{\left(N_c^2-1\right) S_Q(x_t,x'_t,y'_t,y_t)}{2N_c} \\
\mathrm{tr}\left[W_3 W_4^*\right] &= \frac{1}{4}\left(S_D(y'_t,z'_t) S_Q(x_t,x'_t,z'_t,y_t) - \frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right) \\
\mathrm{tr}\left[W_4 W_1^*\right] &= \frac{1}{4}\left(S_D(x_t,z_t) S_Q(z,x'_t,y'_t,y_t) - \frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right) \\
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\mathrm{tr}\left[W_4 W_3^*\right] &= \frac{1}{4}\left(S_D(z,y_t) S_Q(x_t,x'_t,y'_t,z) - \frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right) \\
\mathrm{tr}\left[W_4 W_4^*\right] &= \frac{1}{8}\left(S_Q(x_t,x'_t,z'_t,z_t) S_Q(z_t,z'_t,y'_t,y_t) - \frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right)
\end{aligned}
$$

$$\gamma^\star \mathbf{p} \to \mathbf{q}\,\bar{\mathbf{q}}\,\mathbf{g}\,\mathbf{X}$$

we are developing a Mathematica package to put all this together