Fluid dynamic propagation of initial baryon number perturbations

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Initial Stages 2016, Lisbon
mainly based on


Total baryon number $B - \bar{B}$ is conserved.

For $^{208}\text{Pb} - ^{208}\text{Pb}$ collisions $B - \bar{B} = 416$

Determines total integrated baryon number

Small compared to total number of baryons $B + \bar{B}$ and other produced particles at RHIC or LHC energies.

$\Rightarrow$ Standard assumption:

$$\mu_B = 0$$

What about local and event-by-event fluctuations?

Dynamics should be governed by universal fluid dynamics.
Evolution of baryon number in fluid dynamics

- Small perturbation in static medium with $u^\mu = (1, 0, 0, 0)$

$$\frac{\partial}{\partial t} \delta n(t, \vec{x}) = D \vec{\nabla}^2 \delta n(t, \vec{x})$$

- Baryon number diffusion constant

$$D = \kappa \left[ \frac{nT}{\epsilon + p} \right]^2 \left( \frac{\partial(\mu/T)}{\partial n} \right)_\epsilon$$

- Heat capacity $\kappa$ appears here because

baryon diffusion in Landau frame $\widehat{=}$ heat conduction in Eckart frame

- Is $D$ finite for $n \to 0$?
Heat conductivity

- Heat conductivity of QCD rather poorly understood theoretically so far.
- From perturbation theory [Danielewicz & Gyulassy, PRD 31, 53 (1985)]

\[ \kappa \sim \frac{T^4}{\mu^2 \alpha_s^2 \ln \alpha_s} \quad (\mu \ll T) \]

- From AdS/CFT [Son & Starinets, JHEP 0603 (2006)]

\[ \kappa = 8\pi^2 \frac{T}{\mu^2} \eta = 2\pi \frac{sT}{\mu^2} \quad (\mu \ll T) \]

- Baryon diffusion constant \( D \) finite for \( \mu \to 0 \)!
Evolution of baryon number density from conservation law

$$u^\mu \partial_\mu n + n\nabla_\mu u^\mu + \nabla_\mu \nu^\mu = 0$$

Diffusion current $\nu^\alpha$ determined by heat conductivity $\kappa$

$$\nu^\alpha = -\kappa \left[ \frac{nT}{\epsilon + p} \right]^2 \Delta^{\alpha\beta} \partial_\beta \left( \frac{\mu}{T} \right)$$
Bjorken expansion

Consider Bjorken type expansion

\[
\partial_\tau \epsilon + (\epsilon + p) \frac{1}{\tau} - \left( \frac{4}{3} \eta + \zeta \right) \frac{1}{\tau^2} = 0
\]
\[
\partial_\tau n + n \frac{1}{\tau} = 0
\]

Heat conductivity \( \kappa \) does not enter by symmetry argument

Compare ideal gas to lattice QCD equation of state

[Borsanyi et al., JHEP 08 (2012) 053]
Consider situation with $\langle n(x) \rangle = \langle \mu(x) \rangle = 0$

Local event-by-event fluctuation $\delta n \neq 0$

Concentrate now on Bjorken flow profile for $u^\mu$

Consider perturbation $\delta n$

$$\partial_\tau \delta n + \frac{1}{\tau} \delta n - D(\tau) \left( \partial_x^2 + \partial_y^2 + \frac{1}{\tau^2} \partial_\eta^2 \right) \delta n = 0$$

Structures in transverse and rapidity directions are “flattened out” by heat conductive dissipation
Solution by Bessel-Fourier expansion

- Expand perturbations like

\[ \delta n(\tau, r, \phi, \eta) = \int_0^\infty dk \ k \ \sum_{m=-\infty}^{\infty} \int \frac{dq}{2\pi} \delta n(\tau, k, m, q) e^{i(m\phi + q\eta)} J_m(kr) \]

- Leads to ODE

\[ \partial_\tau \delta n + \frac{1}{\tau} \delta n + D(\tau) \left( k^2 + \frac{q^2}{\tau^2} \right) \delta n = 0. \]

- For \( q = 0 \) and different \( k \approx 1/\text{fm} \), AdS/CFT value \( \kappa = 8\pi^2 \frac{T}{\mu^2} \eta \)

![Graphs for different values of \( \eta/s \)]
Evolution of perturbations

- For $k = 0$ and different $q = 1, 3, 5$, AdS/CFT value $\kappa = 8\pi^2 \frac{T}{\mu^2} \eta$

- At $\tau = 10\text{fm/c}$

- Only long-range fluctuations survive diffusive damping.
Initial baryon number fluctuation

- Initial baryon number density fluctuations must be known to learn about diffusive transport properties...
- What can be said from first principles?
- How are baryon number fluctuations generated by QCD processes?
- What is the dynamics at very early times?
- Maybe answers at this conference... ?
Glauber type model

- Fluctuations due to nucleon positions: used so far for energy density

\[ \epsilon(\tau_0, \mathbf{x}, \eta) = \sum_{i=1}^{N_{\text{part}}} \hat{\epsilon}_w(\mathbf{x} - \mathbf{x}_i) \]

- Can be generalized to baryon number fluctuations

\[ n(\tau_0, \mathbf{x}, \eta) = \sum_{i=1}^{N_{\text{part}}} \hat{n}_w(\mathbf{x} - \mathbf{x}_i) \]

- Would generate baryon number fluctuations on nucleon scale
- More general origin of fluctuations is initial state physics and early-time, non-equilibrium dynamics
Baryon number fluctuations at freeze-out

- On freeze-out surface

\[
\frac{dN_i}{d^3p d^3x} = f_i(p^\mu; T, u^\mu, \mu, \pi^{\mu\nu}, \pi_{\text{bulk}}, \nu^\mu)
\]

- Close-to-equilibrium expansion

\[
f_i = f_{i, \text{eq}} + \delta f_i
\]

- Equilibrium distribution functions

\[
f_{i, \text{eq}} = \frac{1}{e^{\frac{-p_\nu u^\nu - \mu_i}{T}} \pm 1}
\]

Baryons and anti-baryons have opposite baryon chemical potential

- Non-equilibrium correction

\[
\delta f_i = p_\mu p_\nu \pi^{\mu\nu} \tilde{g}_i(p_\mu u^\mu, T, \mu_i) + p_\mu p_\nu \Delta^{\mu\nu} \pi_{\text{bulk}} \tilde{h}_i(p_\mu u^\mu, T, \mu_i) \\
+ p_\mu \nu^\mu \tilde{k}_i(p_\mu u^\mu, T, \mu_i)
\]
Fluctuations at freeze-out

- Background-perturbation splitting can also be used at freeze-out
- Interesting observable is net baryon number

\[ n(\phi, \eta) = (B - \bar{B})(\phi, \eta) \]

- Correlation functions and distributions contain information about baryon number fluctuations
- Two-particle correlation function of baryons minus anti-baryons

\[ C_{\text{Baryon}}(\phi_1 - \phi_2, \eta_1 - \eta_2) = \langle n(\phi_1, \eta_1) n(\phi_2, \eta_2) \rangle_c \]
Baryon number correlation function

- In Fourier representation

\[ C_{\text{Baryon}}(\Delta \phi, \Delta \eta) = \sum_{m=-\infty}^{\infty} \int \frac{dq}{2\pi} \tilde{C}_{\text{Baryon}}(m, q) e^{im\Delta \phi + iq\Delta \eta} \]

heat conductivity leads to exponential suppression

\[ \tilde{C}_{\text{Baryon}}(m, q) = e^{-m^2I_1 - q^2I_2} \tilde{C}_{\text{Baryon}}(m, q) \bigg|_{\kappa=0} \]

- \( I_1 \) and \( I_2 \) can be approximated as

\[ I_1 \approx \int_{\tau_0}^{\tau_f} d\tau \frac{2}{R^2} \kappa \left[ \frac{nT}{\epsilon + p} \right]^2 \left( \frac{\partial(\mu/T)}{\partial n} \right)_{\epsilon} \]
\[ I_2 \approx \int_{\tau_0}^{\tau_f} d\tau \frac{2}{\tau^2} \kappa \left[ \frac{nT}{\epsilon + p} \right]^2 \left( \frac{\partial(\mu/T)}{\partial n} \right)_{\epsilon} \]

- \( I_2 \gg I_1 \) would lead to long-range correlations in rapidity direction ("baryon number ridge")
Conclusions

- Baryon number diffusion constant $\sim$ heat conductivity is well-defined transport property of the quark-gluon plasma for $\mu \rightarrow 0$
- Baryon number fluctuations could allow to constrain it
- Early time baryon diffusion should lead to long-range rapidity correlations in net baryon number
- More knowledge about initial state welcome
- Seems to be interesting topic for further experimental and theoretical studies