Instabilities in Anisotropic Chiral Plasmas

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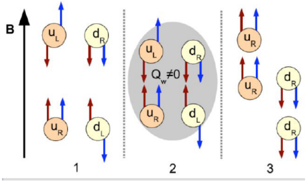
Motivation

- Why Chiral plasma?
  There is a theoretical proposal that P and CP violation can manifest itself in heavy-ion collision.



(a)

Figure: Chiral magnetic effect (CME): Blue-arrows denote direction of spin and red-arrows momentum. 1. B is strong & particles are in the lowest Landau level and initially number of left-handed and right-handed particles are same. 2. Finite topological charge $Q_w \neq 0 (= -1)$, will convert the left-handed particles to right-handed one by reversing the direction of momentum. 3. The right-handed up quarks will move upwards, the right-handed down quarks will move downwards. A charge difference of $Q = 2e$ will be created between two sides of a plane perpendicular to the magnetic field.(Fig. from Kharzeev, Mclerran & Warringa 08)

D. Kharzeev, Phys. Lett. B **633**, 260 (2006),
D. Kharzeev, L.D. Mclerran and H.J. Warringa, Nucl. Phys. A **803**, 227 (2008),
D. Kharzeev, A. Zhitnitsky, Nucl. Phys. A **797**, 67 (2007),

Motivation

- Three-particle correlator (P-even observable) measured at STAR collaboration indicate charge separation. However, more verifications are required.
- We shall focus on the kinetic theory which incorporate P-violating features and satisfy the anomaly equation:

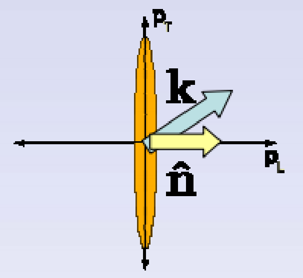
$$\partial_\mu j_5^\mu = C F^{\mu\nu} \tilde{F}_{\mu\nu} \tag{1}$$

- Such theory can have an instability arising due to imbalance in "chiral-chemical-potential"
- The number density: μT^2 & energy-density: $\mu^2 T^2$
- From anomaly Eq. no. density in the gauge field $\sim \alpha k A^2$ and comparing these two number-density: $k \sim \frac{\mu T^2}{\alpha A^2}$.
- Typical energy density in the gauge field $\epsilon_A \sim k^2 A^2 = \mu^2 T^2 \left(\frac{T^2}{\alpha^2 A^2}\right)$
- Thus for $\frac{T^2}{\alpha^2 A^2} < 1$, for the given value of k, the gauge-field can have lower energy than the particle energy $\mu^2 T^2$. This is an unstable situation.
- This instability is known in electroweak plasma (in context of primordial magnetic field)

e.g. M. Joyce & M. Shaposhnikov, PRL, 79, 1193, (1997)

Motivation

Weibel Instability:



(a)

Figure: Geometry for Weibel Instability.

- Momentum anisotropy can be present during an early stages of heavy-ion collision may cause Weibel instability (Abe & Niu 1980, Mrowczynski 1988 etc.) to grow.
- One considers the initial distribution function $n^0_{\mathbf{p}} = \frac{1}{[e^{(\tilde{p}-\mu_R)/T}+1]}$ where,

$\tilde{p} = p\sqrt{1+\xi(\mathbf{v}\cdot\hat{\mathbf{n}})^2}$.

E. S. Weibel, Phys. Rev. Lett. **2**, 83 (1959),
B. D. Fried, Phys. Fluids **2**, 337 (1959).

Chiral Kinetic Theory

$$\dot{n}_{\mathbf{p}} + \dot{\mathbf{x}} \cdot \frac{\partial n_{\mathbf{p}}}{\partial \mathbf{x}} + \dot{\mathbf{p}} \cdot \frac{\partial n_{\mathbf{p}}}{\partial \mathbf{p}} = 0,$$

$$\dot{\mathbf{x}} = \frac{1}{1 + e\mathbf{B} \cdot \mathbf{\Omega}_{\mathbf{p}}} \left(\tilde{\mathbf{v}} + e\tilde{\mathbf{E}} \times \mathbf{\Omega}_{\mathbf{p}} + e(\tilde{\mathbf{v}} \cdot \mathbf{\Omega}_{\mathbf{p}})\mathbf{B} \right),$$

$$\dot{\mathbf{p}} = \frac{1}{1 + e\mathbf{B} \cdot \mathbf{\Omega}_{\mathbf{p}}} \left[\left(e\tilde{\mathbf{E}} + e\tilde{\mathbf{v}} \times \mathbf{B} + e^2(\tilde{\mathbf{E}} \cdot \mathbf{B})\mathbf{\Omega}_{\mathbf{p}} \right) \right],$$

- where $\tilde{\mathbf{v}} = \frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{p}}$, $e\tilde{\mathbf{E}} = e\mathbf{E} - \frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{x}}$, $\epsilon_{\mathbf{p}} = p(1 - e\mathbf{B} \cdot \mathbf{\Omega}_{\mathbf{p}})$ and $\mathbf{\Omega}_{\mathbf{p}} = \pm \mathbf{p}/2p^3$. Here $\pm$ sign corresponds to right and lefted handed fermions respectively.
- If $\mathbf{\Omega}_{\mathbf{p}} = 0$, above equation reduces to Vlasov equation.
- From above equation it is easy to get,

$$\partial_t n + \nabla \cdot \mathbf{j} = \mathbf{e}^2 \int \frac{\mathbf{d}^3\mathbf{p}}{(2\pi)^3} \left(\mathbf{\Omega}_{\mathbf{p}} \cdot \frac{\partial \mathbf{n}_{\mathbf{p}}}{\partial \mathbf{p}} \right) \mathbf{E} \cdot \mathbf{B},$$

where,

$$n = \int \frac{d^3p}{(2\pi)^3} (1 + e\mathbf{B} \cdot \mathbf{\Omega}_{\mathbf{p}}) n_{\mathbf{p}},$$

D. T. Son and N. Yamamoto, Phys. Rev. D **87**, 085016 (2013) [arxiv:1210.815].,

Chiral kinetic theory

$$\mathbf{j} = -e\int \frac{d^3p}{(2\pi)^3}\left[\epsilon_{\mathbf{p}}\frac{\partial n_{\mathbf{p}}}{\partial p} + e\left(\boldsymbol{\Omega}_{\mathbf{p}}\cdot\frac{\partial \mathbf{n}_{\mathbf{p}}}{\partial \mathbf{p}}\right)\epsilon_{\mathbf{p}}\mathbf{B} + \epsilon_{\mathbf{p}}\boldsymbol{\Omega}_{\mathbf{p}}\times\frac{\partial n_{\mathbf{p}}}{\partial x}\right] + \mathbf{E}\times\boldsymbol{\sigma}.$$

$$\boldsymbol{\sigma} = \int \frac{d^3p}{(2\pi)^3}\boldsymbol{\Omega}_{\mathbf{p}} n_{\mathbf{p}}.$$

- Here onwards we use $v = \frac{\mathbf{p}}{p}$ (not to be confused with $\tilde{v}$).

D. T. Son and N. Yamamoto, Phys. Rev. D **87**, 085016 (2013) [arxiv:1210.815].,

$$\dot{n}_{\mathbf{p}} + \frac{1}{1+e\mathbf{B}\cdot\boldsymbol{\Omega}_{\mathbf{p}}}\left[\left(e\tilde{\mathbf{E}} + e\tilde{\mathbf{v}}\times\mathbf{B} + e^2(\tilde{\mathbf{E}}\cdot\mathbf{B})\boldsymbol{\Omega}_{\mathbf{p}}\right)\cdot\frac{\partial n_{\mathbf{p}}}{\partial \mathbf{p}} + \left(\tilde{\mathbf{v}} + e\tilde{\mathbf{E}}\times\boldsymbol{\Omega}_{\mathbf{p}} + e(\tilde{\mathbf{v}}\cdot\boldsymbol{\Omega}_{\mathbf{p}})\mathbf{B}\right)\cdot\frac{\partial n_{\mathbf{p}}}{\partial \mathbf{x}}\right] = 0,$$

,

Linear response analysis of anisotropic chiral plasma

- Linear response analysis:

$$j^i_{ind} = \Pi^{ij}(K)A_j(K),$$

$\Pi^{ij}(K)$ is the polarization tensor and for the present case:

$$\Pi^{ij}(K) = \Pi^{ij}_+(K) + \Pi^{im}_-(K) \tag{2}$$

- Parity even part: $\Pi^{ij}_+(K)$ & Parity odd: $\Pi^{ij}_-(K)$

$$\Pi^{ij}_+(K) = m_D^2 \int \frac{d\Omega}{4\pi} \frac{v^i(v^l + \xi(\mathbf{v}\cdot\hat{\mathbf{n}})\hat{n}^l)}{(1+\xi(\mathbf{v}\cdot\hat{\mathbf{n}})^2)^2} \left(\delta^{jl} + \frac{v^j k^l}{v.k + i\epsilon}\right),$$

This expression of Π^{ij}_+ matches with Romatschke & Strickland PRD68, 08. What is new is the following (*Weibel parameters enters parity odd physics*):

$$\Pi^{im}_-(K) = C_E \int \frac{d\Omega}{4\pi}\left[\frac{i\epsilon^{jlm}k^l v^j v^i(\omega + \xi(\mathbf{v}\cdot\hat{\mathbf{n}})(\mathbf{k}.\hat{\mathbf{n}}))}{(v.k+i\epsilon)(1+\xi(\mathbf{v}\cdot\hat{\mathbf{n}})^2)^{3/2}} + \left(\frac{v^j + \xi(\mathbf{v}\cdot\hat{\mathbf{n}})\hat{n}^j}{(1+\xi(\mathbf{v}\cdot\hat{\mathbf{n}})^2)^{3/2}}\right) i\epsilon^{iml}k^l v^j \right.$$
$$\left. - i\epsilon^{ijl}k^l v^j \left(\delta^{mn} + \frac{v^m k^n}{v.k+i\epsilon}\right)\left(\frac{v^n + \xi(\mathbf{v}\cdot\hat{\mathbf{n}})\hat{n}^n}{(1+\xi(\mathbf{v}\cdot\hat{\mathbf{n}})^2)^{3/2}}\right)\right]$$

where, $m_D^2 = \frac{\mu^2}{2\pi^2} + \frac{T^2}{6}$ and $C_E = \frac{\mu_5}{4\pi^2}$.

Linear response analysis of anisotropic chiral plasma

- Thus the results should depend upon μ_5, ξ & θ_n where, θ_n is the angle between wave-vector and anisotropy-direction.
- Weibel instability grows maximally for $\theta_n = 0$ whereas it damps for $\theta_n = \pi/2$.
- Chiral-plasma instability (CPI) can exist when $\xi = 0$.
- Ratio of maximum growth rates of both the instabilities: $\frac{\Gamma_{ch}}{\Gamma_w} \approx \frac{1}{4\pi^3}\left(\frac{\alpha}{\xi}\right)^{3/2}\left(\frac{\mu_5}{T}\right)^3$ where, α is the coupling constant.
- For $\xi > 1$ and $\mu_5 \leq T$ the Weibel modes can dominate over CPI.
- For certain values of θ_n the Weibel modes may not dominate. For $\xi \gg 1$, and setting $\omega = 0$ in dispersion relation, one can obtain $\theta_{nc} \sim \left(\frac{\pi m_D^2}{2k^2}\right)^{1/2} \xi^{-1/4}$

Results

- In small ξ limit ($\xi \ll 1$), it is possible to express analytical dispersion relation ($\omega = i\rho$):

$$\rho(k) = \left(\frac{4\alpha^3\mu_5^3}{\pi^4 m_D^2}\right) k_N^2 \left[1 - k_N + \frac{\xi}{12}(1+5\cos 2\theta_n) + \frac{\xi}{12}(1+3\cos 2\theta_n)\frac{\pi^2 m_D^2}{\mu_5^2\alpha^2 k_N}\right]. \quad (3)$$

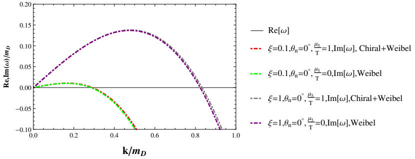
where, $k_N = \frac{\pi k}{\mu_5 \alpha}$.

- Here first term (unity) in the square bracket is due to pure chiral-mode. The factor $\frac{\xi}{12}(1+5\cos 2\theta_n)$ is due to coupling between the two instabilities. Last term is due to pure-Weibel instability.
- For $\theta_n = 0$ comparing the maximum growth-rate of both the instabiliites one finds: $\xi_c \approx 2^{2/3}\left(\frac{\alpha}{4\pi^2}\right)\left(\frac{\mu_5}{T}\right)^2$ which is a small number for $\mu_5 \leq 1$. For $\xi > \xi_c$ the Weibel modes will dominate.
- Similarly one can find critical value for $\theta_{nc} \sim \frac{1}{2}\cos^{-1}\left[\left(\frac{2}{27}\right)^{2/3}\frac{12\mu_5^2\alpha^2}{\xi\pi^2 m_D^2} - \frac{1}{3}\right]$

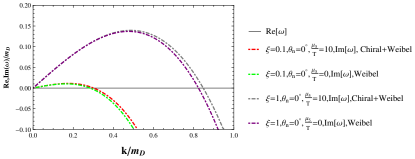
Results and conclusions

Results for large ξ in the case of the quasi-stationary limit ($|\omega| << k$) can obtained by numerically solving the dispersion relation:

- **Case-I: When propagation vector k is parallel to anisotropy vector**



(a)

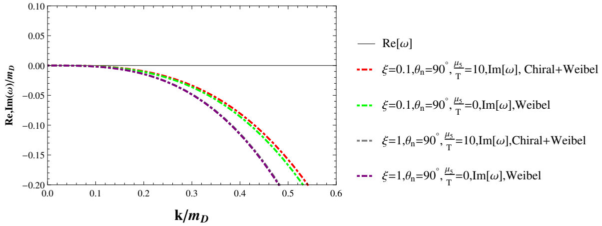


(b)

Figure: Shows plots of real and imaginary part of the transverse dispersion relation for the case when the angle θ_n between the propagation vector **k** of the perturbation and the anisotropy direction is zero. The modes are purely imaginary and the real part of frequency $\omega = 0$. Fig. (1a) shows comparison between pure Weibel modes (μ_5=0) with the cases when both the Weibel and chiral-imbalance instabilities are present when μ_5/T =1 and ξ =0.1,1 . Fig. (1b) depicts the similar comparison when μ_5/T =10. It shows that by increasing μ_5/T the chiral-imbalance instability become stronger.

Results and conclusions

- **Case-II: When propagation vector k is perpendicular to anisotropy vector**



(a)

Figure: Shows plots of the dispersion relation when $\theta_n = \pi/2$. The pure Weibel modes are known to give damping when $\theta_n = \pi/2$. For the instances when both the chiral-imbalance and Weibel instabilities are present (μ_5/T =10 and $\xi = 0.1,1$) the damping can become weaker.

For $\xi \gg 1$, one can estimate particular range of θ_n where the chiral modes could be dominant by setting $\omega = 0$, in pure-Weibel modes, one obtains $\theta_{nc} \sim \left(\frac{\pi m_D^2}{2k^2}\xi^{-1/4}\right)$.

THANK YOU

Linear response analysis of anisotropic chiral plasma

$$\Pi^{ij}(K) = \Pi^{ij}_{+}(K) + \Pi^{im}_{-}(K) \tag{4}$$

$$\Pi^{ij}_{+}(K) = m_D^2 \int \frac{d\Omega}{4\pi} \frac{v^i(v^l + \xi(\mathbf{v}\cdot\hat{\mathbf{n}})\hat{n}^l)}{(1+\xi(\mathbf{v}\cdot\hat{\mathbf{n}})^2)^2} \left(\delta^{jl} + \frac{v^j k^l}{v.k + i\epsilon}\right),$$

$$\Pi^{im}_{-}(K) = C_E \int \frac{d\Omega}{4\pi} \left[\frac{i\epsilon^{jlm} k^l v^j v^i(\omega + \xi(\mathbf{v}\cdot\hat{\mathbf{n}})(\mathbf{k}.\hat{\mathbf{n}}))}{(v.k+i\epsilon)(1+\xi(\mathbf{v}\cdot\hat{\mathbf{n}})^2)^{3/2}} + \left(\frac{v^j + \xi(\mathbf{v}\cdot\hat{\mathbf{n}})\hat{n}^j}{(1+\xi(\mathbf{v}\cdot\hat{\mathbf{n}})^2)^{3/2}}\right) i\epsilon^{iml} k^l v^j \right.$$
$$\left. - i\epsilon^{ijl} k^l v^j \left(\delta^{mn} + \frac{v^m k^n}{v.k + i\epsilon}\right)\left(\frac{v^n + \xi(\mathbf{v}\cdot\hat{\mathbf{n}})\hat{n}^n}{(1+\xi(\mathbf{v}\cdot\hat{\mathbf{n}})^2)^{3/2}}\right)\right]$$

where,

$$m_D^2 = -\frac{e^2}{2\pi^2}\int_0^\infty d\tilde{p}\tilde{p}^2 \left[\frac{\partial n^{0(0)}_{\tilde{\mathbf{p}}}(\tilde{p}-\mu_R)}{\partial \tilde{p}} + \frac{\partial n^{0(0)}_{\tilde{\mathbf{p}}}(\tilde{p}+\mu_R)}{\partial \tilde{p}} + \frac{\partial n^{0(0)}_{\tilde{\mathbf{p}}}(\tilde{p}-\mu_L)}{\partial \tilde{p}} + \frac{\partial n^{0(0)}_{\tilde{\mathbf{p}}}(\tilde{p}-\mu_L)}{\partial \tilde{p}}\right]$$

$$C_E = -\frac{e^2}{4\pi^2}\int_0^\infty d\tilde{p}\tilde{p} \left[\frac{\partial n^{0(0)}_{\tilde{\mathbf{p}}}(\tilde{p}-\mu_R)}{\partial \tilde{p}} - \frac{\partial n^{0(0)}_{\tilde{\mathbf{p}}}(\tilde{p}+\mu_R)}{\partial \tilde{p}} - \frac{\partial n^{0(0)}_{\tilde{\mathbf{p}}}(\tilde{p}-\mu_L)}{\partial \tilde{p}} + \frac{\partial n^{0(0)}_{\tilde{\mathbf{p}}}(\tilde{p}-\mu_L)}{\partial \tilde{p}}\right].$$

- After performing above integrations one can get $m_D^2 = \frac{\mu_5^2}{2\pi^2} + \frac{T^2}{6}$ and $C_E = \frac{\mu_5}{4\pi^2}$. It can be noticed that the terms with anisotropy parameter ξ are contributing in both parity-even and odd part of the self-energy or polarization tensor.

$$j^{\mu}_{ind} = \Pi^{\mu\nu}(K) A_{\nu}(K),$$

.

Linear response analysis of anisotropic chiral plasma

- Maxwell equation,

$$\partial_\nu F^{\nu\mu} = j^\mu_{ind} + j^\mu_{ext}.$$
$$j^\mu_{ind} = \Pi^{\mu\nu}(K) A_\nu(K),$$

- $\Pi^{\mu\nu}(K)$ is the retarded self energy in Fourier space. Here we denote the Fourier transform as $F(K) = \int d^4x e^{-i(\omega t - \mathbf{k}\cdot\mathbf{x})} F(x,t)$.
- Choosing temporal gauge $A_0 = 0$

$$[(k^2 - \omega^2)\delta^{ij} - k^i k^j + \Pi^{ij}(K)]E^j = i\omega j^i_{ext}(k).$$

- From this one can define,

$$[\Delta^{-1}(K)]^{ij} = (k^2 - \omega^2)\delta^{ij} - k^i k^j + \Pi^{ij}(K).$$

- The poles of $[\Delta(K)]^{ij}$ will give us the dispersion relation.

Finding the Poles of $[\Delta(K)]^{ij}$ or Dispersion relation

- We decompose first $\Pi^{ij}(K)$ in following six tensorial basis,

$$\Pi^{ij} = \alpha P_T^{ij} + \beta P_L^{ij} + \gamma P_n^{ij} + \delta P_{kn}^{ij} + \lambda P_A^{ij} + \chi P_{An}^{ij}.$$

- Where,

$$\begin{aligned} P_T^{ij} &= \delta^{ij} - k^i k^j / k^2 \\ P_L^{ij} &= k^i k^j / k^2 \\ P_n^{ij} &= \tilde{n}^i \tilde{n}^j / \tilde{n}^2 \\ P_{kn}^{ij} &= k^i \tilde{n}^j + k^j \tilde{n}^i \\ P_A^{ij} &= i\epsilon^{ijk} \hat{k}^k \\ P_{An}^{ij} &= i\epsilon^{ijk} \tilde{n}^k. \end{aligned}$$

- α,β, γ, δ λ and χ are some scalar functions of k and ω which can be determined by $\alpha = (P_T^{ij} - P_n^{ij})\Pi^{ij}$, $\beta = P_L^{ij}\Pi^{ij}$, $\gamma = (2P_n^{ij} - P_T^{ij})\Pi^{ij}$, $\delta = \frac{1}{2k^2\tilde{n}^2} P_{kn}^{ij}\Pi^{ij}$ $\lambda = -\frac{1}{2} P_A^{ij}\Pi^{ij}$ and $\chi = -\frac{1}{2\tilde{n}^2} P_{An}^{ij}\Pi^{ij}$.

Finding the Poles of $[\Delta(K)]^{ij}$ or Dispersion relation

- We shall first do the analysis in the small ξ limit (Very weak anisotropy),

$$
\begin{aligned}
\alpha &= \Pi_T + \xi\Big[\frac{z^2}{12}(3+5\cos 2\theta_n)m_D^2 - \frac{1}{6}(1+\cos 2\theta_n)m_D^2 + \frac{1}{4}\Pi_T\left((1+3\cos 2\theta_n) - z^2(3+5\cos 2\theta_n)\right)\Big];\\
z^{-2}\beta &= \Pi_L + \xi\Big[\frac{1}{6}(1+3\cos 2\theta_n)m_D^2 + \Pi_L\left(\cos 2\theta_n - \frac{z^2}{2}(1+3\cos 2\theta_n)\right)\Big];\\
\gamma &= \frac{\xi}{3}(3\Pi_T - m_D^2)(z^2-1)\sin^2\theta_n;\\
\delta &= \frac{\xi}{3k}(4z^2 m_D^2 + 3\Pi_T(1-4z^2))\cos\theta_n;\\
\lambda &= -\Pi_A/2 - \xi\frac{\mu k e^2}{8\pi^2}\Big[(1-z^2)\frac{\Pi_L}{m_D^2}((3\cos 2\theta_n - 1)\\
&- 2z^2(1+3\cos 2\theta_n)) + \frac{2z^2}{3}(1-3\cos 2\theta_n) - \frac{43}{15} + \frac{22}{10}(1+\cos 2\theta_n)\Big];\\
\chi &= \xi\,[f(\omega,k)]\,,
\end{aligned}
$$

- Here θ_n is the angle between wave vector $\mathbf{k}$ and anisotropy vector $\mathbf{n}$.
- Expressions for Π_T, Π_L are given as,

$$\Pi_T = m_D^2\frac{\omega^2}{2k^2}\left[1 + \frac{k^2-\omega^2}{2\omega k}\ln\frac{\omega+k}{\omega-k}\right],$$

$$\Pi_L = m_D^2\left[\frac{\omega}{2k}\ln\frac{\omega+k}{\omega-k} - 1\right],\ \Pi_A = \frac{\mu k e^2}{2\pi^2}\left[(1-z^2)\frac{\Pi_L}{m_D^2}\right]$$

Finding the Poles of $[\Delta(K)]^{ij}$ or Dispersion relation

- Similarly we can write $[\Delta^{-1}(k)]^{ij}$ as

$$[\Delta^{-1}(K)]^{ij} = C_T P_T^{ij} + C_L P_L^{ij} + C_n P_n^{ij} + C_{kn} P_{kn}^{ij} + C_A P_A^{ij} + C_{An} P_{An}^{ij}.$$

- Coefficients C's and α's have the following relationship.

$$\begin{aligned} C_T &= k^2 - \omega^2 + \alpha \\ C_L &= -\omega^2 + \beta \\ C_n &= \gamma \\ C_{kn} &= \delta \\ C_A &= \lambda \\ C_{An} &= \chi. \end{aligned}$$

- So once we know α,β, γ, δ λ and χ we can determine coefficient C's.
- But In order to get dispersion relation we have to find poles of $[\Delta(K)]^{ij}$ not of $[\Delta^{-1}(K)]^{ij}$.

Finding the Poles of $[\Delta(K)]^{ij}$ or Dispersion relation

- Now using the fact that inverse of a vector should exist in same space, one can decompose $[\Delta(K)]^{ij}$.

$$[\Delta(K)]^{ij} = aP_L^{ij} + bP_T^{ij} + cP_n^{ij} + dP_{kn}^{ij} + eP_A^{ij} + fP_{An}^{ij}$$

Now, using the relation,

$$[\Delta^{-1}(K)]^{ij}[\Delta(K)]^{jl} = \delta^{il}$$

- One can find the following dispersion relation,

$$2k\tilde{n}^2 C_A C_{An} C_{kn} + C_A^2 C_L + \tilde{n}^2 C_{An}^2 (C_n + C_T) - C_T(-k^2\tilde{n}^2 C_{kn}^2 + C_L(C_n + C_T)) = 0. \qquad (5)$$

- In the weak anisotropy limit, one can write the dispersion relation as,

$$C_A^2 C_L - C_T C_L (C_n + C_T)) = 0,$$

- Which give following two branches of Dispersion relation,

$$C_A^2 - C_T^2 - C_n C_T = 0.$$
$$C_L = 0.$$

- When $C_A = 0$, above equations reduces to exactly the same dispersion relation discussed in Ref. given below for an anisotropic plasma where there is no parity violating effect.
- Equation for transverse modes give the following solution,

$$(k^2 - \omega^2) = \frac{-(2\alpha + \gamma) \pm 2\lambda}{2}.$$

7

[7] P. Romatschke, M. Strickland, Phys. Rev. D **68** 036004 (2003)

Dispersion relation

- In the quasi stationary limit $|\omega| << k$ one can get the final form of dispersion relation as $\omega = i\rho(k)$, where $\rho(k)$ is given by.

$$\rho(k) = \left(\frac{4\alpha^3\mu_5^3}{\pi^4 m_D^2}\right) k_N^2 \left[1 - k_N + \frac{\xi}{12}(1 + 5\cos 2\theta_n) + \frac{\xi}{12}(1 + 3\cos 2\theta_n)\frac{\pi^2 m_D^2}{\mu_5^2\alpha^2 k_N}\right]. \qquad (6)$$

- Where $k_N = \frac{\pi k}{\mu_5 \alpha}$, and $\alpha = \frac{e^2}{4\pi}$ is the electromagnetic coupling.

- In the limit $\xi \to 0$ we will get,

$$\rho(k) = \left(\frac{4\alpha^3\mu_5^3}{\pi^4 m_D^2}\right) k_N^2 \left[1 - k_N\right]$$

- In the limit $\mu \to 0$ we will get,

$$\rho(k) = \left(\frac{4\alpha_e^3\mu_5^3}{\pi^4 m_D^2}\right) k_N^2 \left[-k_N + \frac{\xi}{12}\left(1 + 3\cos 2\theta_n\right)\frac{\pi^2 m_D^2}{\mu_5^2\alpha_e^2 k_N}\right].$$

P. Romatschke, M. Strickland, Phys. Rev. D **68** 036004 (2003),

Y. Akamatsu and N. Yamamoto, Phys. Rev. Lett. **111**, 052002 (2013).

Analysis of the instabilities

- Weibel instability grows maximally for $\theta_n = 0$.
- Weibel instability gets suppressed when $\cos 2\theta_n = -1/3$ i.e $\theta_n \approx 55^\circ$.
- The ratio of maximum growth rates for chiral and Weibel comes out to be $\frac{\Gamma_{ch}}{\Gamma_w} \approx \frac{1}{4\pi^3} \left(\frac{\alpha_e}{\xi}\right)^{3/2} \left(\frac{\mu_5}{T}\right)^3$.
- One can find for $\theta_n = 0$ the critical value $\xi_c \approx 2^{2/3} \left(\frac{\alpha_e}{4\pi^2}\right) \left(\frac{\mu_5}{T}\right)^2$ at which the maximum growth rates of the two instabilities become comparable.
- Two instabilities will have comparable growth at a critical angle $\theta_c = \frac{1}{2} \cos^{-1} \left[\left(\frac{2}{27}\right)^{2/3} \frac{12\mu_5^2\alpha^2}{\xi\pi^2 m_D^2} - \frac{1}{3} \right]$.

Results and conclusions

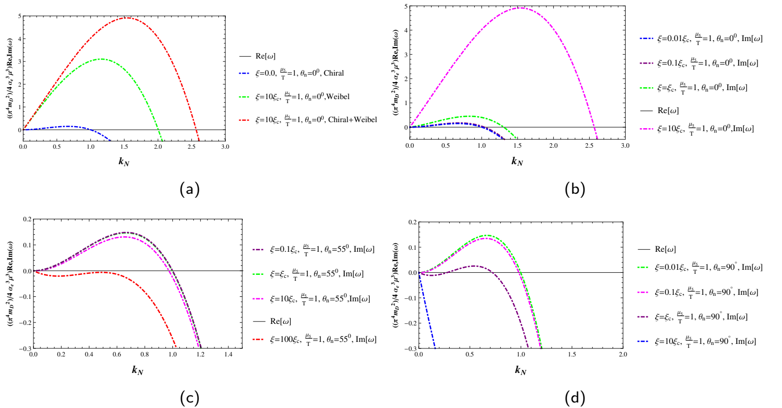
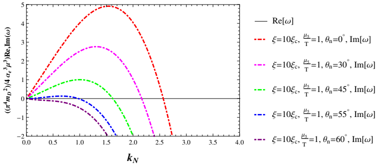
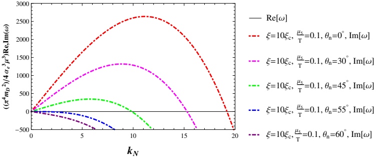


Figure: Shows plots of real and imaginary part of the dispersion relation. Here θ_n is the angle between the wave vector k and the anisotropy vector. Real part of dispersion relation is zero. Fig. (a) show plots for three cases: (i) Pure chiral (no anisotropy), (ii) Pure Weibel (chiral chemical potential=0) and (iii) When both chiral and Weibel instabilities at $\theta_n = 0$. Fig (b-d) represents the case when both instabilities are present but the anisotropy parameter varries at different values of θ_n for fixed $\mu_5/T = 1$. Here frequency is normalized in unit of $\omega / \left(\frac{4\alpha^3\mu_5^3}{\pi^4 m_D^2}\right)$ and wave-number k by $k_N = \frac{\pi}{\mu_5 \alpha} k$

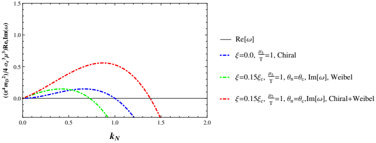
Results and conclusions



(a)



(b)



(c)

Figure: Shows plots of real and imaginary part of the dispersion relation. Here θ_n is the angle between the wave vector k and the anisotropy vector. Real part of dispersion relation is zero. Fig. (a-b) represent the case when both the instabilities are present for fixed $\xi = 10\xi_c$ and $\mu_5/T = 1,\ 0.1$ by varying θ_n respectively. Fig. (c) represents the case when for a particular value of $\theta_n \sim \theta_c$ two instabilities have equal growth at different ξ values. Here frequency is normalized in unit of $\omega / \left(\frac{4\alpha^3\mu_5^3}{\pi^4 m_D^2}\right)$ and wave-number k by $k_N = \frac{\pi}{\mu_5\alpha} k$.